MATH471: HOMEWORK #1

ALEX IOSEVICH

Please do the following problems and turn them in on Monday, September 26:

Problem #1: Use the Borel-Cantelli lemma to prove that for any $p > 2, c > 0$, the set
$$\left\{ x \in [0,1] : \left| x - \frac{a}{q} \right| \leq \frac{c}{q^p} \text{ for infinitely many } (a, q) \in \mathbb{N} \times \mathbb{N} \right\}$$
has Lebesgue measure 0.

Problem #2: Let E be a subset of $[0, 1]$ consisting of real numbers that DO NOT have the digit 5 in their base 10 expansions. Compute the Lebesgue measure of E and prove your assertion.

Food for thought: Do problems #1 and #2 make you wish for a measure of "smallness" more refined than measure 0?

Problem #3: Prove that
$$\lim_{R \to \infty} \int_0^1 \sin(Rx^2)dx = 0.$$
Can you make a stronger statement and prove it?

Do problems 31 and 33 on page 49.

Do problems 34, 37, 38, 40, and 46 on page 53.