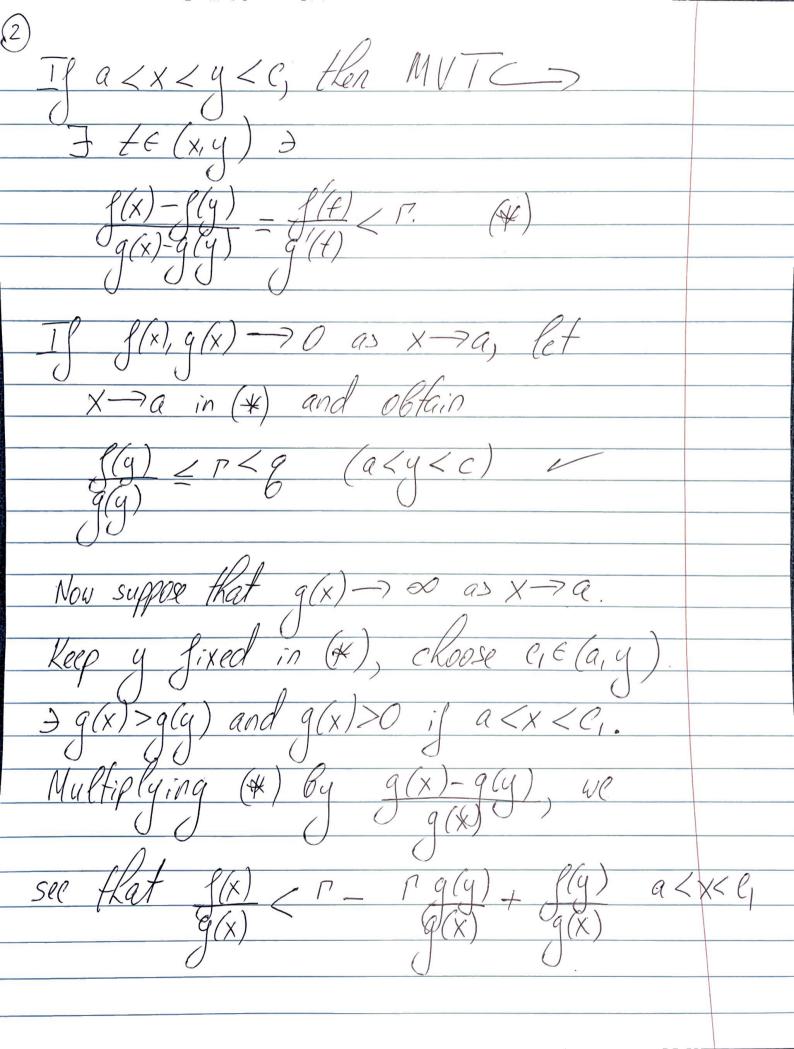
Math 265H, Fall 2022, November 21 Theorem: $\begin{cases} q \text{ are real and differentiable in (a,6)}, \\ and g'(x) \neq 0 \end{cases}$ for all $x \in (a,6)$, where $-\infty \leq a \leq b \leq \infty$, Suppose If $f(x) \rightarrow 0$, $g(x) \rightarrow 0$ as $x \rightarrow a$,

or if g(x), $f(x) \rightarrow \infty$ as $x \rightarrow a$,

then $f(x) \rightarrow A$ as $x \rightarrow a$. Proof: Let $-\infty \leq A < \infty$ Choose $g \ni A < g \nmid p \ni A < p < g$ Then $\exists c \in (a, b) \ni a < x < c \text{ implies } f \land a f$



Now let x-a and use the assumption $g(x) \rightarrow \infty$ as $x \rightarrow a$ to see that \exists $c_2 \in (a, c_1) \rightarrow f(x) < g \quad (a < x < c_2).$ Puffing everytking together, for any g > $A < Q, f < C_2 \rightarrow f(x) < G if a < x < C_2$ In the same way, $f - \infty < A \leq \infty$, and $\rho < A$, we can find $c_3 \rightarrow$ P<(x) (a<x<c3) and the

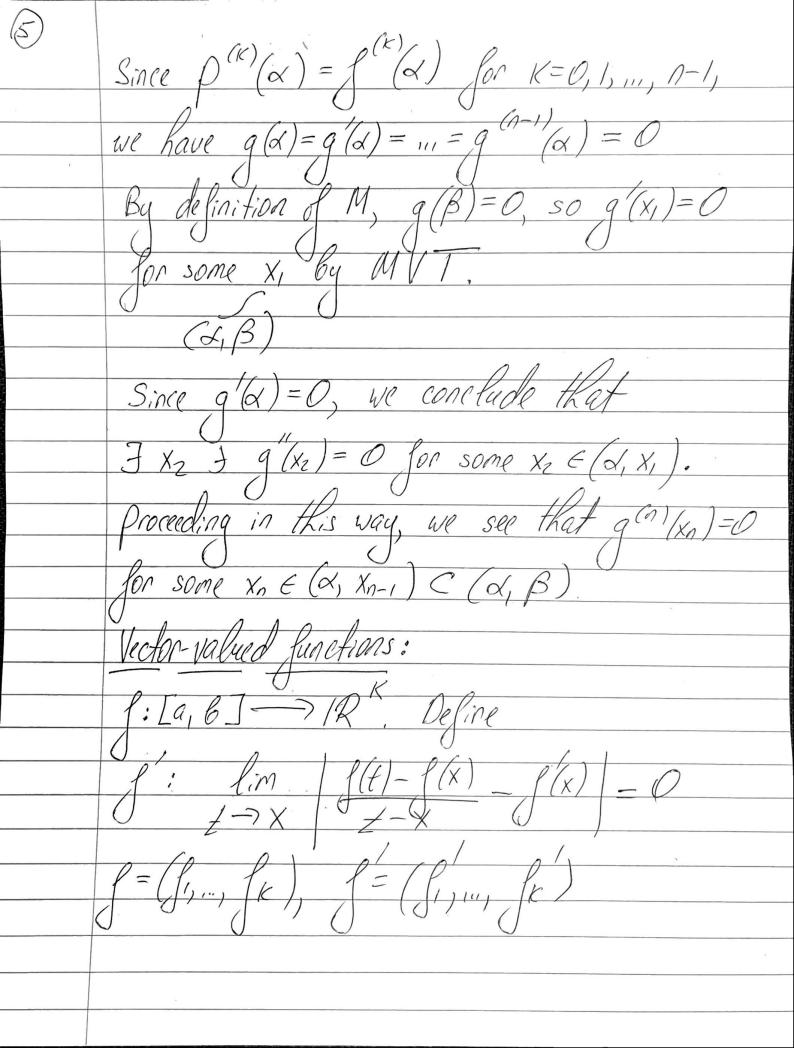
(a) (a) proof is complete.

Theorem: Suppose f is a real-valued function on [a,6], no positive integer, for every fe(a,6).

Let &, & be distinct points on [a,6], and $\rho(t) = \sum_{K=0}^{n-1} \int_{K!}^{(K)} (\alpha) (t-\alpha)^{K}$ Then I x between & and B > $f(\beta) = \rho(\beta) + f(x)(x)(\beta - \alpha).$ Proof: When n=1, this is just MVT, so

for higher on we must somehow reduce to
that case. Let M be given by $\int (\beta) = \rho(\beta) + M(\beta - \lambda)^2 \text{ and } \ell \text{et}$ $g(t) = f(t) - \rho(t) - M(t - \lambda) \quad (a \le t \le b)$ Claim: n; M = f(n)(x), for some xe(xB) Differentiating q & plugging in the definition

of P, we see that q(t) - f(t) - n. M. and we are left to show that g'(x) = 0for some $X \in AB$



Things are difficult in kigher dimensions: Define $f(x) = e^{iX} = \cos X + i \sin X$ $f(l_{1}) - f(0) = 1 - 1 = 0$, but $f(x) = ie^{ix}, so |f(x)| = 1$,
so MVI fails in this case. More disasters: On (0,1), define $f(x) = x \text{ and } g(x) = x + x^2 e^{\frac{i}{x^2}}$ Also, $g(x) = 1 + \sum_{i=1}^{\infty} 2x - \frac{2i}{X} e^{\frac{i}{X^2}}$ (0<x<1) $\frac{S_0}{S_0} = \frac{(q(x))^2}{2x - \frac{2i}{x}} = \frac{1}{2}$ It follows that $|g(x)| = \frac{1}{|g(x)|} = \frac{x}{2-x}$

We conclude that $\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$, and L'Hospital fails! But what is true? Theorem: $f: [a, 6] \rightarrow IR^{K}$ differentiable

Then $\exists x \in (a, 6) \Rightarrow$ $|(6)-(a)| \leq (6-a)|(x)|$ Proof: Let 2 = (16)- ((a), and $Q(t) = 2. f(t) \quad a \in t \in b$ Continuous differentiable on (a,b)(B-a) 2. P(x) for som(xE(a,6)) Also, $\varrho(b) - \varrho(a) = 2 \cdot \varrho(b) - 2 \cdot \varrho(a) = |2|$ 121 = (6-a) 12. 1 (x) 1 / (6-a) 12 13 (x) 1 by Gucky-Schwerz