MATH 238: HOMEWORK #3 DUE MONDAY, 10/03/16

ALEX IOSEVICH

Problem #1: Let $A, B \subset \mathbb{R}^2$. Suppose that $\#A = \#B = n$. Define

$$\Delta(A,B) = \{|x-y| : x \in A, y \in B\}.$$

Prove that $\#\Delta(A,B) \geq Cn^{\frac{6}{7}}$. What happens if the sizes of A and B are very different?

Problem #2: Do Exercise 6.2 on page 69.

Problem #3: Do Exercise 6.4 on page 69.

Problem #4: Let $\mathbb{Z}_p$ denote the integers modulo a prime number p. Let $\mathbb{Z}_p^2$ denote the two-dimensional vector space over $\mathbb{Z}_p$. A line in $\mathbb{Z}_p^2$ is the set $L_{x,v} = \{x + tv : t = 0, 1, \ldots, p-1\}$ where x is a fixed elements of $\mathbb{Z}_p^2$ and v is a fixed non-zero element of $\mathbb{Z}_p^2$. Suppose that v is not a multiple of w. Then

$$\#(L_{x,v} \cap L_{y,w}) = 1.$$

Problem #5: Count the total number of distinct lines in $\mathbb{Z}_p^2$, p prime.