Exam #3

#1 $\lim_{t\to\infty} 2t-\sqrt{4t^2+5} = \lim_{t\to\infty} \frac{(2t-\sqrt{4t^2+5})(2t+\sqrt{4t^2+5})}{2t+\sqrt{4t^2+5}}$

$= \lim_{t\to\infty} \frac{4t^2-(4t^2+5)}{2t+\sqrt{4t^2+5}} = \lim_{t\to\infty} \frac{-5}{2t+\sqrt{4t^2+5}} = 0$

b/c of $\frac{\text{constant}}{\infty}$ type.

#2 $f(x) = x^3-3x^2+6x+9$ is continuous & differentiable.

$\left.\begin{array}{l} f(0)=9>0 \\ f(-1)=-1<0 \end{array}\right)$ By IVT, there exists a root of f on $(-1,0)$.

Let a be a root of f, then $f(a)=0$.

For any $b(\neq a)$, by MVT,

$\frac{f(b)-f(a)}{b-a} = f'(c)$ for some c

$f(b)-0 = (b-a)f'(c) = (b-a)(3c^2-6c+6)$.

$\uparrow$

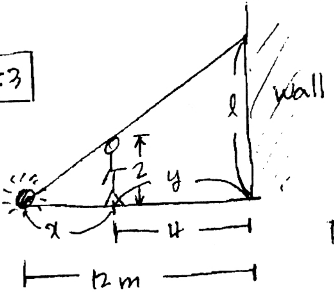
$f'(x) = 3x^2-6x+6$.

$f(b) = (b-a)\left(3(c^2-2c+1-1)+6\right)$

$= (b-a)\{3(c-1)^2+3\} \neq 0$ b/c $b-a\neq 0$, $3(c-1)^2+3>0$.

Therefore, $f(x)$ has exactly one real root.

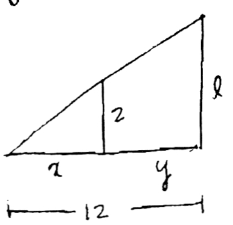
#3



$\frac{dx}{dt} = 1.6\ m/s$

Find $\frac{dl}{dt}$ when $y=4$?

By similar triangles,

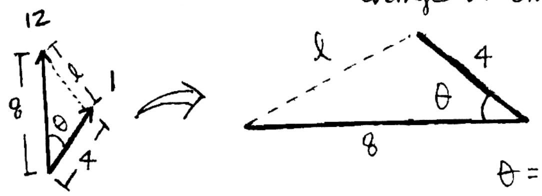


$\frac{2}{l} = \frac{x}{12} \Rightarrow l = \frac{24}{x}$

$\frac{dl}{dt} = \frac{dl}{dx}\cdot\frac{dx}{dt} = -24\frac{1}{x^2}\cdot(1.6)$

$= -24\cdot\frac{1}{8^2}\cdot\frac{16}{10} = -\frac{3}{5}\ m/s$

#4 Find $\frac{dl}{dt}$ when the tips of the hands change at one o'clock.



$\theta = \frac{360^\circ}{12} = 30^\circ = \frac{\pi}{6}$.

By the law of cosines,

$l^2 = 4^2+8^2-2\cdot4\cdot8\cos\theta$

$2l\frac{dl}{dt} = -64(-\sin\theta)\frac{d\theta}{dt}$ — (*)

Let θ_1 be an angle related to the hour hand

& " θ_2 " the minute ".

$\frac{d\theta}{dt} = \frac{d\theta_1}{dt}-\frac{d\theta_2}{dt} = \frac{2\pi}{12}-\frac{2\pi}{1} = \frac{\pi}{6}-2\pi = -\frac{11}{6}\pi$

when $\theta=\frac{\pi}{6}$, $l^2 = 16+64-2\cdot4\cdot8\cos\frac{\pi}{6}$

$= 80-32\sqrt{3} \Rightarrow l = \sqrt{80-32\sqrt{3}}$

From (*),

$\frac{dl}{dt} = \frac{64\cdot\sin\frac{\pi}{6}\cdot(-\frac{11}{6}\pi)}{2\sqrt{80-32\sqrt{3}}} = \frac{\cancel{64}^{\cancel{32}\,16}(\frac{1}{\cancel{2}})(-\frac{11}{6}\pi)}{\cancel{2}\sqrt{80-32\sqrt{3}}}$

$= \frac{-\frac{88}{3}\pi}{\sqrt{80-32\sqrt{3}}}$ "

#5

(a) ① Any constant function works for it

Ex. $f(x)=1, 2,$ or 5 and so on.

② Ex. $f(x) = -(x-5)^4$

(b) $y = \frac{1}{x}$ Domain: $x\neq 0$.

$f'(x) = -\frac{1}{x^2} < 0$ for all x in the domain.