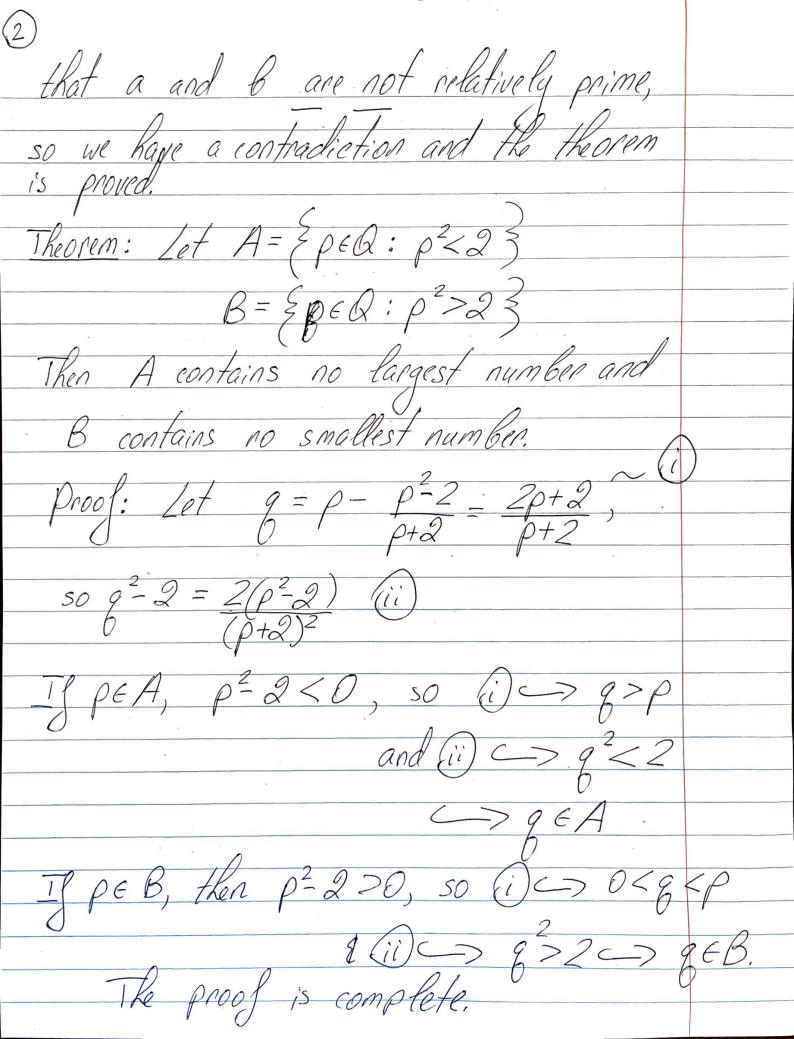
Math 265H August 31, 2022 Q = field of rational numbers 5 a: 6≠0 a, 6 € 2/3 integers

One gateway to analysis is the observation that the eguation x. -2 = 0 has no retional solutions Proof: Suppose that $X = \frac{a}{6}$, w/ Then $(a)^2 - 2$, so $a^2 = 26$. This implies that a is even. Since a' is even, a is even also (why?) which means that a is divisible by 4. This means that 6 is even, which implies that 6 is even, We conclude



A set, XEA p = empty set A is contained in B ACB & BCACT A=B Orden: An orden on a set S is a relation, denoted by < > i) x < y, x = y, or y < x one of these is true ii) x,y, 2 ∈ S, if x < y & y < Z, then X < Z. Upper bound: S $E \subset S$ ordered

If $\exists \beta \in S$ $\ni x \leq \beta$ for every $x \in E$ there exists above $\beta = upper$ bound. Lower bounds are defined the same way,

Least upper bound: S ordered set, ECS, E Counded Suppose that IXES W/ following properties: i) & is an upper bound of E ii) If I(x) then I is not an upper bound of E. The $\chi = Reast upper bound of E.$ notation The greatest lower bound (infinum) way w/ respect to the lower bound,

Example: $\frac{5}{5}$ n=1,2, $\frac{3}{5}$ Proof: $0 = \inf(E)$ Proof: $0 < \pi$ for any $n = 1, 2, ..., \infty$ so 0 is a lower bound. Let 1>0. Phoose a positive integer 1 I his is accomplished By taking $n > \frac{1}{x}$ which is always possible since integers are not bounded above. We conclude that 0 = inf(E), as

Another example: A = \ \ \rho \in \mathcal{Q} : \rho^2 > 23 We already saw that A does not kave the blast greatest lower bound in Q. think this through!!! Definition: An ordered set 3 is said to have the least-upper-bound property of: i) If ECS, E + p, E bounded above, then sup(E) exists in S = Theorem: Sordered w/ least upper bound property, BCS, B + Ø, B bound be four.

Let L = set of all lower bounds of B.

Then d = sup L exists in S, and In panticular, in B exists in S.

B bounded be for > 2 + 6 Every XEB is an upper bound for 2 -> 2 is bounded above C > d = sup / exists in S. If Y < d, then Y is not an upper bound for 2, so 8 & B. It follows that $\alpha \leq x$ for every $x \in B$, il dEL. If $d < \beta$, $\beta \notin Z$ since d is an upper bound for Z. Therefore, del, but Bdl if B> By definition, $d = \inf B$, as desired.