CLARIFICATION OF THE PROOF OF THEOREM WE PROVED IN CLASS ON JANUARY 31

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In class we proved that $(A \cup B) \backslash B \subseteq A$. Let's go over the proof to make sure that the logic is clear. Suppose that $X \in (A \cup B) \backslash B$. Let P be the statement that $x \in A$, and Q the statement that $x \in B$. Then the expression

$$(A \cup B) \backslash B$$

is encoded by

$$P \vee Q \wedge \neg Q.$$

The point that I did not make sufficiently clear in class is the following. The truth table for P IS NOT the same as the truth table for $P \vee Q \wedge \neg Q$, but this IS NOT THE POINT. The point is that P is TRUE whenever $P \vee Q \wedge \neg Q$ is TRUE, which is all we need because we only need to conclude that $x \in A$, which is encoded by P.

Indeed, $P \vee Q \wedge \neg Q$ is TRUE only when P is TRUE and Q is FALSE. Under these conditions P is TRUE and this all we need to know because out goal is to show that if $x \in (A \cup B) \backslash B$, then $x \in A$. This is precisely what we just did.

Let's take a step back and try to understand this from the point of view of sets. The expression P and the expression $P \vee Q \wedge \neg Q$ CANNOT have the same truth table because when P and Q are both true, x is contained in the intersection of A and B. In particular, this means that $x \in B$, which in incompatible with $P \vee Q \wedge \neg Q$ where the possibility that $x \in B$ is excluded by the condition $\neg Q$.