1

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

$$\int_a^b g(x)\,dx = G(b) - G(a), \text{ where } G'(x) = g(x)$$

$$\int_{-1}^{5} x^2\,dx = \left.\frac{x^3}{3}\right|_{-1}^{5} = \frac{5^3}{3} - \frac{(-1)^3}{3}$$

$$\int_0^{\frac{\pi}{4}} \sec\theta \tan\theta\,d\theta = \left.\sec\theta\right|_0^{\frac{\pi}{4}} = \sec\left(\frac{\pi}{4}\right) - \sec(0)$$

$$= \sqrt{2} - 1$$

Fundamental Theorem of Calculus and friends

②

$$\int x 2^x \, dx \qquad 2^x = e^{x\ln(2)}$$

$$= \int \underline{x} e^{x\ln(2)} \, dx \sim \underline{\text{Integration by parts}}$$

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$$f(x) = x \qquad g'(x) = e^{x\ln(2)}$$

$$f'(x) = 1 \qquad g(x) = \frac{e^{x\ln(2)}}{\ln(2)}$$

$$\frac{x e^{x\ln(2)}}{\ln(2)} - \int \frac{e^{x\ln(2)}}{\ln(2)} \, dx$$

$$= \frac{x e^{x\ln(2)}}{\ln(2)} - \frac{e^{x\ln(2)}}{(\ln(2))^2} + C$$

③

$$\int \frac{x^3}{x-1}\,dx$$

Partial fractions

$$\begin{array}{r} x^2 + x + 1 \\ x-1 \overline{) \; x^3 \qquad\quad} \\ x^3 - x^2 \\ \hline 0 + x^2 \\ x^2 - x \\ \hline x \\ x - 1 \\ \hline \boxed{1} \end{array}$$

remainder

$$\frac{x^3}{x-1} = x^2 + x + 1 + \frac{1}{x-1}$$

$$\int \left(x^2 + x + 1 + \frac{1}{x-1} \right) dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + x + \ln(|x-1|) + C$$

$$\int \frac{x^2-3x+7}{(x^2-4x+6)^2}\,dx$$

too elaborate for the exam

$$\int \frac{y}{(y+4)(2y-1)}\,dy$$

partial fractions

$$\frac{y}{(y+4)(2y-1)} = \frac{A}{y+4} + \frac{B}{2y-1}$$

$$y = A(2y-1) + B(y+4)$$

$y=-4$: $\quad -4 = A(2(-4)-1)$

$$-4 = A(-9) \qquad A = \frac{4}{9}$$

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$y = A(2y-1) + B(y+4)$

$y=-4: \quad A = \frac{4}{9}$

$y=\frac{1}{2}: \quad \frac{1}{2} = 0 + B\left(\frac{1}{2}+4\right)$

$\frac{1}{2} = B\left(\frac{9}{2}\right) \qquad B = \frac{1}{9}$

$\int \frac{y}{(y+4)(2y-1)}\, dy =$

$\frac{4}{9}\int \frac{dy}{y+4} + \frac{1}{9}\int \frac{dy}{2y-1}$

$\frac{4}{9}\ln(|y+4|) + \frac{1}{9}\cdot\frac{1}{2}\ln(|2y-1|)$

Clarification: $\int \frac{dy}{2y-1} \qquad u = 2y-1$

$du = 2dy$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln(|u|) = \frac{1}{2}\ln(|2y-1|)$

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$$\int \frac{x}{x^2+8x+25} = \int \underbrace{\frac{x}{(x+4)^2+9}}\, dx$$

Complete the square:

$(x+4)^2 + 9$

trigonometric substitution

$x+4 = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$$\int \frac{(3\tan\theta - 4)\, 3\sec^2\theta\, d\theta}{9\tan^2\theta + 9} =$$

$$\int \frac{(3\tan\theta - 4)\, 3\cancel{\sec^2\theta}\, d\theta}{9\underbrace{\cancel{(\tan^2\theta + 1)}}_{\sec^2\theta}} = \frac{1}{3}\int (3\tan\theta - 4)\, d\theta$$

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$$\frac{1}{3}\int (3\tan\theta - 4)\,d\theta =$$

$$\int \tan\theta\,d\theta \quad - \quad \frac{4}{3}\int d\theta$$

$$\shortparallel$$

$$-\frac{4}{3}\theta$$

$$\int \tan\theta\,d\theta = \int \frac{\sin\theta}{\cos\theta}\,d\theta \qquad u = \cos\theta$$

$$du = -\sin\theta\,d\theta$$

$$-\int \frac{du}{u} = -\ln(|u|) = \ln\left(\frac{1}{|u|}\right) = \ln(|\sec\theta|) + C$$

In total, we have

$$\ln(|\sec\theta|) - \frac{4}{3}\theta + C$$

$$x + 4 = 3\tan\theta$$

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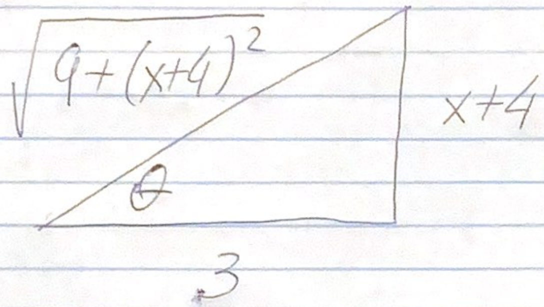
$$\ln(|\sec\theta|) - \frac{4}{3}\theta + C$$

$$x+4 = 3\tan\theta$$



$$\sec\theta = \frac{\sqrt{9+(x+4)^2}}{3}$$

$$\ln\left(\frac{\sqrt{9+(x+4)^2}}{3}\right) - \frac{4}{3}\tan^{-1}\left(\frac{x+4}{3}\right) + C$$

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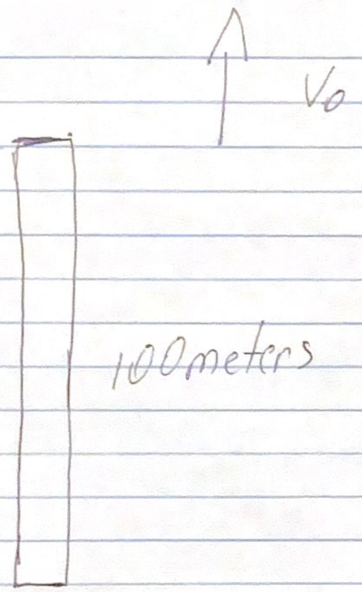

How should we choose v_0 so that the object hits the ground in 10 seconds?

$$g = -9.8 \, m/s^2$$

$$v(t) = -9.8t + v_0$$

$$s(t) = -\frac{9.8t^2}{2} + v_0 t + 100$$

$$0 = s(10) = -\frac{9.8}{2}10^2 + 10v_0 + 100$$

solve for $\underline{v_0}$.