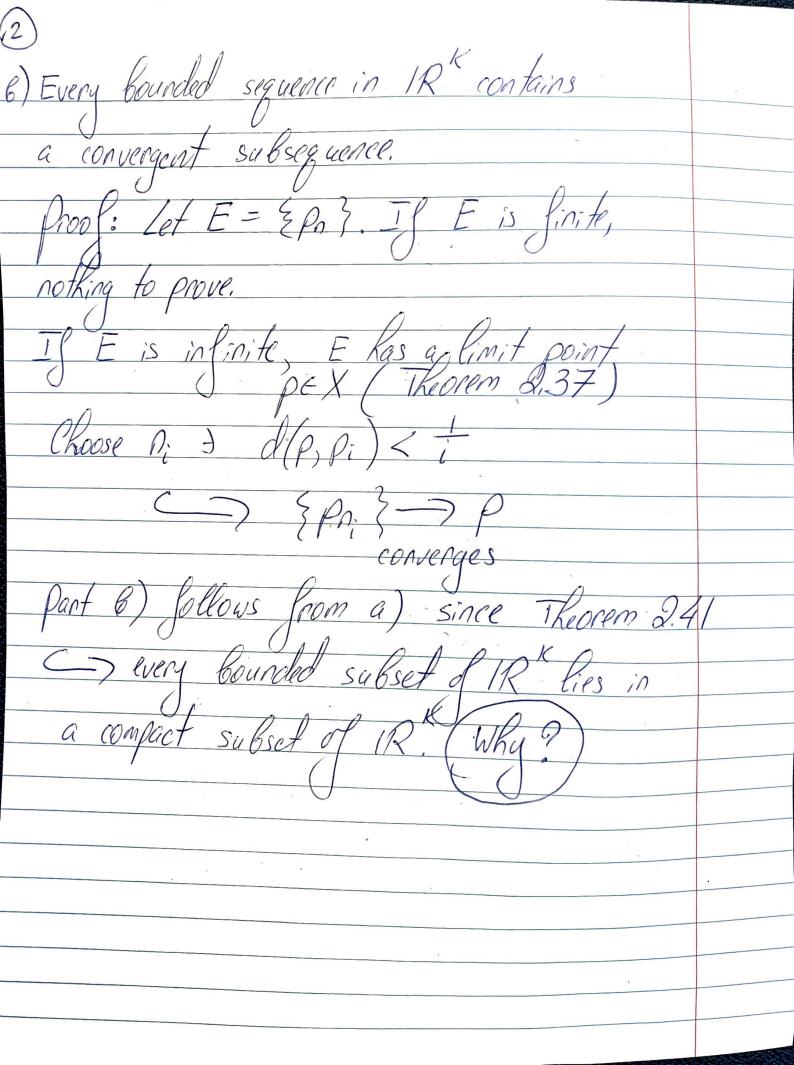
Math 265H, Detober 12, 2022 Subsequences; lonsider Epat and consider Sequence n, 2 n2 < 111 2 nx < 111 Then Epn; 3, is called a subsequence The spring converges, the limit is called a subsequential limit of ξp_n ?

Example: $p_n = (-1)$ does not converge But 3 PZK 3 Converges just fine! metric space X, then some subsequence converges to a point in X.



Theorem: The subsequential limits of a sequence spit in a metric space X form a closed subset of X. Proof: E = subsequential limits of EPs?

g = limbt point of Eth We must show that q E Choose n, + pn, + g (what if it does not exist? Let $\delta = A(\mathbf{p}, \mathbf{p}_0)$. Assume that ni, nz, ne, kave been Since q is a limit point point of E, J x & E + d(x,q) < 2 8. Since x e E*, 3 n, > n; , 3 d(x, pn;) < 2 8. Thus a (8, Pn) = 2.2.5 ~ Pn. ~ > 9 L

Cauchy sequences:

A sequences 3 pn 3 in a metric space is said to be a Cauchy sequence if 7 N + d(pn, pm) < E. if n > N of metair space $S = \{d(p, g): p \in E, g \in E\}$ sup(S) = diam(E) Observation: 3pn 3 C X sequence; EN = 3 PN, PNH, III) Then Epn? is laucky ill lim diam En = 0 Theorem: a) E = X1 metric space Then diam E = diam (E) b) I(K_n) sequence of compact sets ∂ $K_n \supset K_{n+1}$ and f $\lim_{n\to\infty} diam K_n = 0$, then 1) Kn consists of exactly one point. Proof: Since E C E diam (E) = diam (E)

Fix e>0, and choose PEE, gEE. Jp', GEE → d(P,P') < E, d(E,E') < E, so d(p,q) = d(p,p') + d(p',g') + d(g',g)<2E+ d(p',g') = 2E+diam(E) So, diam(\bar{E}) = $2\epsilon + diam(\bar{E})$, and we are done.

