

MATH 238: FINAL PROJECT

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This is the final project for Math 238, Fall 2016. You may talk to each other, but the final drafts must be put together individually. You may use all possible electronic resources and you may even ask other mathematicians for hints (but not solutions).

Problem #1: Write a complete proof of Theorem 1.1 in my paper with Hart and Solymosi.

Problem #2: Prove that there exists $C_k > 0$, independent of p , such that if $\#E > C_k p^{\frac{d+1}{2}}$, then

$$\{(\|x^1 - x^2\|, \|x^2 - x^3\|, \dots, \|x^k - x^{k+1}\|) : x^j \in E; 1 \leq j \leq k+1\} = \mathbb{Z}_p^k.$$

Problem #3: Let $E \subset \mathbb{Z}_p^2$, p prime, congruent to 3 modulo 4. Prove that there exists $\alpha < 2$ and a constant C independent of p such that if $|E| \geq Cp^\alpha$, then

$$\mathbb{Z}_p^3 \subset \{(\|x - u\|, \|x - v\|, \|x - w\|) : x, u, v, w \in E\}.$$