Math 265 H, Fall 2022, Oct 24 Theorem: $\sum a_n$ converges iff for every $e>0 \ni N \ni \sum_{k=n}^{\infty} a_k < e$. Proof: This is just the lauchy Carterion. Theorem: If D'an converges, then $\lim_{n\to\infty} a_n = 0$ Proof: Take m=n in the previous theorem. Theorem: A series of non-negative terms

coverges if partial sums \{\sigma_s\}\} are bounded.

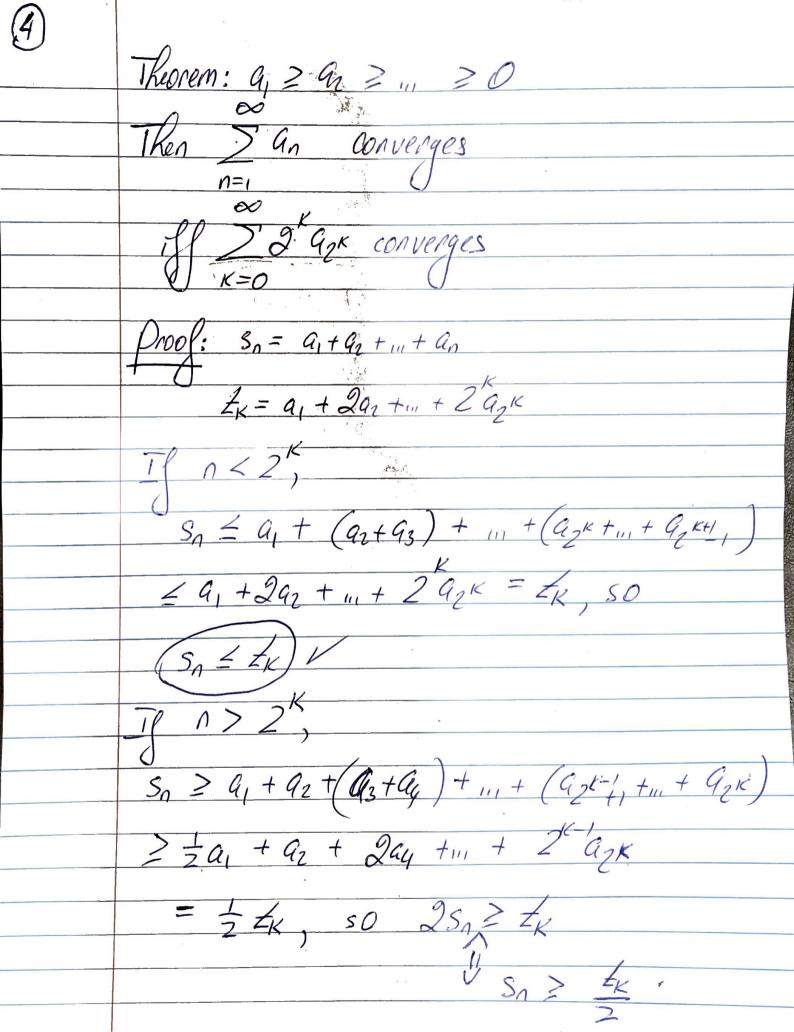
Proof: Theorem 3.14 implies this instantly

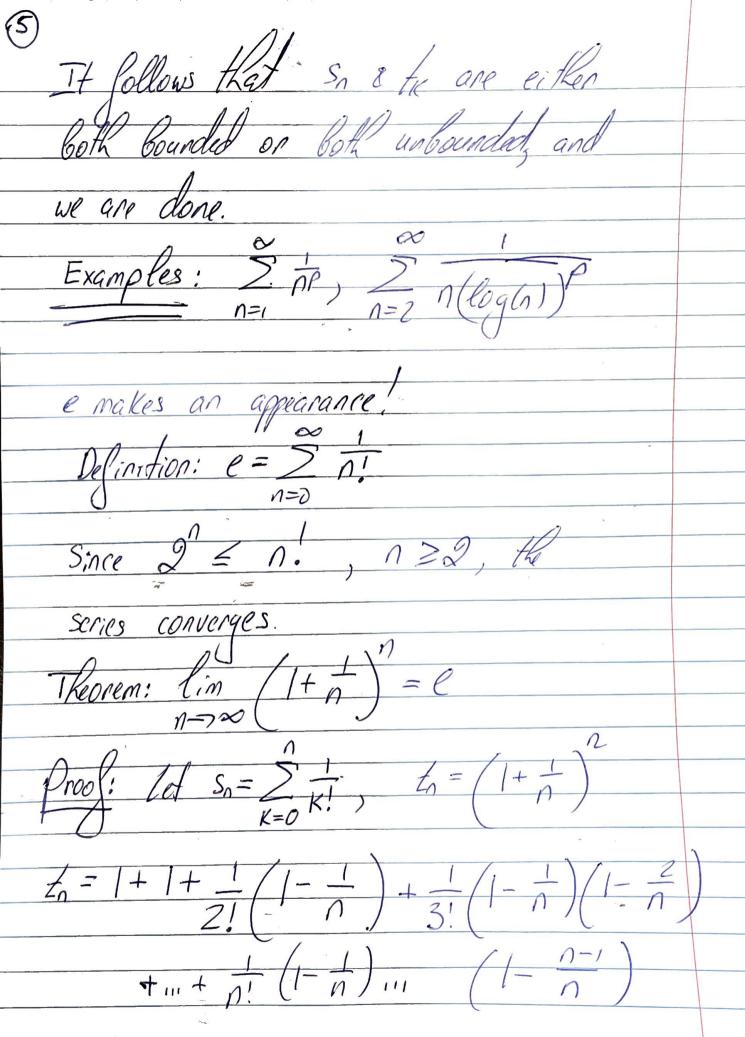
Theorem: a) If $|a_n| \leq e_n$ for $n \geq N_0$ and if $2e_n$ converges, Hen $2e_n$ econverges.

Proof: By Cauchy, given $e > 0 \neq N \geq N_0 \Rightarrow$ $m \geq N \geq N \geq N_0$ C_K ≤ € This implies that |M| = |M| = |M| = |M| |M|and we are done! il an 2 dn 20 for n2 No, and
il 2 dn diverges, then 2 an diverges.

Proof: Follows directly from a).

Theorem: If 0=x<1, 1 ×≥1, the series diverges $Droof: S_n = \sum_{k=1}^{N} x^k =$ 1+ X+ X + ... + X $XS_{n} = X + X + \dots + X + X$ $S_n(I-x) = I-x$ $S_n = I-x$ $3_n \rightarrow \frac{1}{1-x}$ if |x| < 1If x=1, we get sn=n -> diverges





(1) Toked,
$$(1+\frac{1}{n}) = 1 \cdot (\frac{1}{n}) + n \cdot 1 \cdot (\frac{1}{n}) + \frac{1}{n^{2}} + \frac{1}{n^{2}} + \frac{1}{n^{3}} + \frac{1}{n^{3}} + \frac{1}{n^{2}} + \frac{1}{n^{$$

It follows that $t_n = s_n$, so $l_m \neq l_n = e$ (Theorem \$19) $\frac{1}{2n} \ge \frac{1+1+\frac{1}{2n}(1-\frac{1}{n})}{2n} + \frac{1}{2n} = \frac{1}{2n} = \frac{1}{2n} + \frac{1}{2n} = \frac{1}{2n} = \frac{1}{2n} + \frac{1}{2n} = \frac{1}{2n}$ $\frac{1}{m!}\left(\frac{1}{n}\right) \cdots \left(\frac{1-m-1}{n}\right)$ lot n-sex (fixed m) $\lim_{n \to \infty} \frac{1}{2!} + \frac{1}{m!} + \frac{1}{m!}$ so Sm = lin In, Kence lim Sm = lim to, so. e = Pim to 9 we are done!