Alexis Hudes
Winter 2025
Bayesian Statistical Modeling and Computation

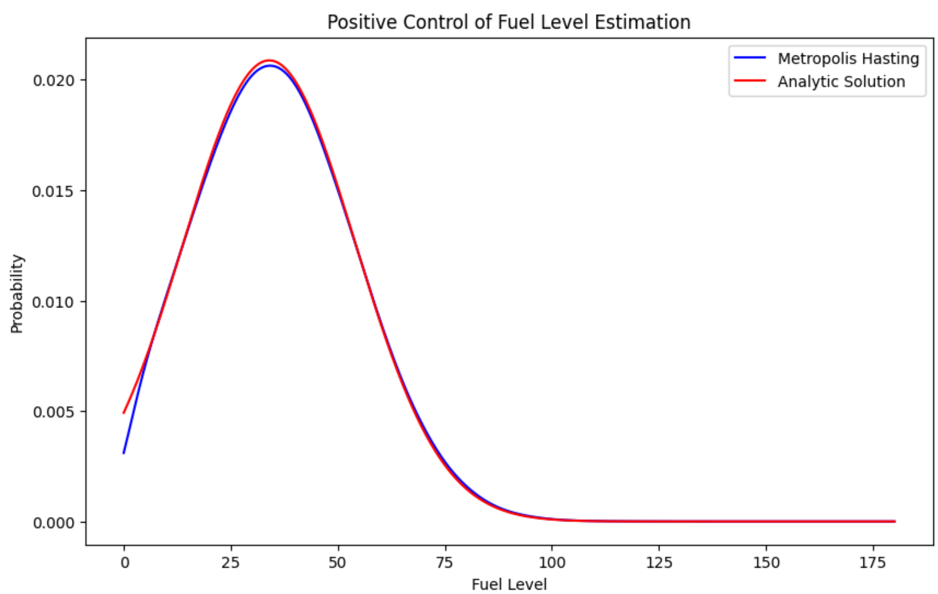
Problem Set #4: Markov Chain Monte Carlo

To implement a Metropolis Hasting algorithm for the fuel level in problem set 3, I used the same prior and likelihood function, but a different sampling strategy:

1. The initial fuel level was selected randomly from a uniform distribution from 0 to 182.
2. The next sample was selected by sampling from a normal distribution with a mean of the current sample and a standard deviation of 90 (selected based on trial and error to get an acceptance rate of 24.7%, which is close to, but above 23.4%).
3. Then an acceptance threshold was chosen from a uniform distribution from 0 to 1.
4. If the ratio of the posterior at the new sample divided by the posterior of the current sample was at or above the threshold, then the new sample was adopted. Otherwise, the current sample remained.
5. Steps 2-4 were repeated for 3 million samples, which was the lowest number tested to reach convergence. To test convergence, I calculated the confidence interval and was using a threshold of 0.05 based on the example given in Workflow_Example.R.

The required number of runs to reach convergence was actually the same for MCMC and Bayes Monte Carlo, which I found surprising, since I know that MCMC is supposed to be more efficient. This may be a sign that the model was not properly tuned.

To compare the accuracy of the implemented method to a positive control, I used a kernel density estimator to get a pdf from the MCMC samples. I then plotted the pdf against the analytical solution, which would be a normal distribution (μ = 34 l, σ = 20 l) that is cut off at 0 l and 180 l and normalized. As you can see, the result deviates the most at 0 L.



Reproducibility:
The analysis is reproducible. The code is thoroughly commented, has a starting seed of 42 for the random number generation, and is available on GitHub at:
https://github.com/alexis-hudes/BayesianHW4.git

Appendix: Code
Note: this is Python code, not R

```
# Bayesian HW Set 4: Markov Chain Monte Carlo
# February 2025

##  author: Alexis Hudes
##  copyright by the author
##  distributed under the GNU general public license
##  https://www.gnu.org/licenses/gpl.html
##  no warranty (see license details at the link above)

#Packages
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm, gaussian_kde, t
from scipy.integrate import cumtrapz
from scipy.interpolate import interp1d
from scipy.stats import truncnorm
import random

#set seed
np.random.seed(42)

# Metropolis Hasting Algorithm for fuel level
# Number of samples (determined by trial and error to converge)
n = 3*10**6

# uniform prior from 0 to 182
def prior(x):
   return 1 / 183 if 0 <= x <= 182 else 0

# likelihood function is a normal distribution with mean x and std 20, measured at 34
liters
def likelihood(x):
   return norm.pdf(x, loc=34, scale=20)
```

```python
# Metropolis-Hastings algorithm
def met(n, init):
   samples = []
   current_sample = init
   accept = []

   for i in range(n):
       # Propose a new sample
       proposed_sample = np.random.normal(current_sample, 90)

       # Calculate the acceptance ratio
       current_likelihood = likelihood(current_sample) * prior(current_sample)
       proposed_likelihood = likelihood(proposed_sample) * prior(proposed_sample)

       acceptance_ratio = proposed_likelihood / current_likelihood

       # Accept or reject
       if np.random.rand() < acceptance_ratio:
           current_sample = proposed_sample
           accept.append(1)
       else:
           accept.append(0)

       #creating a 'burn in' period of 1% of the samples
       if i > n*0.01:
           samples.append(current_sample)

   return np.array(samples), accept

# run with n samples and a random initial sample
samples_MH, acceptance = met(n, np.random.uniform(low=0, high=182))
#Plotting results
#acceptance rate
aceptance_rate=sum(acceptance)/len(acceptance)
print("The acceptance rate is:", aceptance_rate)

# Plot the histogram of the samples
plt.hist(samples_MH, bins=50, density=True, alpha=0.4, edgecolor='black')
```

```
# Title and labels
plt.title('Metropolis-Hastings Sampling')
plt.xlabel('Fuel Level')
plt.ylabel('Probability')

# Show plot and legend
plt.show()
plt.close()

# Convergence testing

# define the confidence interval convergence check, based on the example from
workflow.r
def check_convergence(chain, threshold):
   # Compute the confidence interval of the mean
   ci = 2 * t.ppf(0.975, len(chain)) * np.std(chain) / np.sqrt(len(chain))
   print("The confidence interval is:", ci)

   # Check if the confidence interval is less than the threshold
   if ci <= threshold:
       return True
   else:
       return False

# Check convergence
converged = check_convergence(samples_MH, threshold=0.05)

if converged:
   print("The chain has converged")
else:
   print("The chain has not converged")

#Positive Control

# KDE estimation from the samples
kde = gaussian_kde(samples_MH, bw_method=0.2)

# Create a range of x values to evaluate the pdf
x = np.linspace(0, 180, 1000)

# Evaluate the KDE on these values
kde_pdf = kde.evaluate(x)
```

```python
# Analytical truncated normal distribution
a, b = (0 - 34) / 20, (180 - 34) / 20
trunc_normal_pdf = truncnorm.pdf(x, a, b, loc=34, scale=20)

# Plot the KDE and truncated normal pdfs
plt.figure(figsize=(10, 6))
plt.plot(x, kde_pdf, label='Metropolis Hasting', color='blue')
plt.plot(x, trunc_normal_pdf, label='Analytic Solution', color='red')
plt.title("Positive Control of Fuel Level Estimation")
plt.xlabel("Fuel Level")
plt.ylabel("Probability")
plt.legend()
plt.show()
plt.close()
```