2. Let $f(x) = \boxed{\frac{1}{2}\|\sin(Ax) - b\|_2^2 + \frac{\beta}{2}\|Lx\|_2^2}$ $\quad \beta \in \mathbb{R}_{>0},\ L \in \mathbb{R}^{n \times n},\ A \in \mathbb{R}^{m \times n},\ b \in \mathbb{R}^m,\ x \in \mathbb{R}^n.$

$$= \frac{1}{2}\left\{\langle \sin(Ax) - b,\ \sin(Ax) - b\rangle\right\} + \frac{\beta}{2}\langle Lx, Lx\rangle$$

$$= \frac{1}{2}\left\{\langle \sin(Ax), \sin(Ax)\rangle - 2\langle \sin(Ax), b\rangle + \langle b, b\rangle\right\} + \frac{\beta}{2}\langle Lx, Lx\rangle$$

$$= \frac{1}{2}\left\{\sum_{j=1}^{m} \sin^2\left(\sum_{i=1}^{n} a_{ji}x_i\right) - 2\sum_{j=1}^{m} b_j \sin\left(\sum_{i=1}^{n} a_{ji}x_i\right) + \sum_{j=1}^{m} b_j^2\right\} + \frac{\beta}{2}\sum_{j=1}^{n}\left(\sum_{i=1}^{n} l_{ji}x_i\right)^2.$$

Let $k \in \{1, \dots, n\}$. Then

$$\frac{\delta f(x)}{\delta x_k} = \frac{1}{2}\left\{\sum_{j=1}^{m} 2\sin\left(\sum_{i=1}^{n} a_{ji}x_i\right)\cos\left(\sum_{i=1}^{n} a_{ji}x_i\right)a_{jk} - 2\sum_{j=1}^{m} b_j \cos\left(\sum_{i=1}^{n} a_{ji}x_i\right)a_{jk}\right\} + \frac{\beta}{2}\sum_{j=1}^{n}\left\{2\left(\sum_{i=1}^{n} l_{ji}x_i\right)l_{jk}\right\}$$

$*$ $$= \sum_{j=1}^{m}\left\{a_{jk}\cos\left(\sum_{i=1}^{n} a_{ji}x_i\right)\left(\sin\left(\sum_{i=1}^{n} a_{ji}x_i\right) - b_j\right)\right\} + \beta\sum_{j=1}^{n} l_{jk}\sum_{i=1}^{n} l_{ji}x_i.$$

$$\frac{\delta f(x)}{\delta x} = \begin{pmatrix} a_{11} & \cdots & a_{m1} \\ a_{12} & & \vdots \\ \vdots & & \vdots \\ a_{1n} & a_{2n} \cdots & a_{mn} \end{pmatrix}\begin{pmatrix} \cos\left(\sum_{i=1}^{n} a_{1i}x_i\right) & 0 & \cdots & 0 \\ 0 & \cos\left(\sum_{i=1}^{n} a_{2i}x_i\right) & & \vdots \\ & & \ddots & \\ 0 & \cdots & & \cos\left(\sum_{i=1}^{n} a_{mi}x_i\right) \end{pmatrix}\begin{pmatrix} \sin\left(\sum_{i=1}^{n} a_{1i}x_i\right) - b_1 \\ \vdots \\ \sin\left(\sum_{i=1}^{n} a_{mi}x_i\right) - b_m \end{pmatrix} + \beta\begin{pmatrix} l_{11} & \cdots & l_{n1} \\ l_{12} & & l_{n2} \\ \vdots & & \vdots \\ l_{1n} & l_{2n} \cdots & l_{nn} \end{pmatrix}\begin{pmatrix} \sum_{i=1}^{n} l_{1i}x_i \\ \vdots \\ \sum_{i=1}^{n} l_{ni}x_i \end{pmatrix}$$

$$= \boxed{\overset{n \times m}{A^T}\ \overset{m \times m}{\mathrm{diag}(\cos(Ax))}\ \overset{m \times 1}{(\sin(Ax) - b)} + \beta L^T L x.}$$ // n-vector

Let $p \in \{1, \dots, n\}$. Then

$$\frac{\delta}{\delta x_p}\left(\frac{\delta f}{\delta x_k}\right) = \frac{\delta}{\delta x_p}(*) = \sum_{j=1}^{m}\left\{a_{jk}\left[-\sin\left(\sum_{i=1}^{n} a_{ji}x_i\right)a_{jp}\left(\sin\left(\sum_{i=1}^{n} a_{ji}x_i\right) - b_j\right) + \cos^2\left(\sum_{i=1}^{n} a_{ji}x_i\right)a_{jp}\right]\right\} + \beta\sum_{j=1}^{n} l_{jk}l_{jp}$$

$$= \sum_{j=1}^{m}\left\{a_{jk}a_{jp}\left[1 - 2\sin^2\left(\sum_{i=1}^{n} a_{ji}x_i\right) + b_j\sin\left(\sum_{i=1}^{n} a_{ji}x_i\right)\right]\right\} + \beta L^T L$$

$$= \boxed{A^T A\left\{1 - 2\sin(Ax)(\sin(Ax))^T + b^T\sin(Ax)\right\} + \beta L^T L.}$$