1a. $f(x) = \frac{1}{2} x^T Q x + b^T x + c, \quad Q \in \mathbb{R}^{n \times n}, \; b, x \in \mathbb{R}^n, \; c \in \mathbb{R}.$ Let $k, p \in \{1, \dots, n\}$.

$$= \frac{1}{2} (x_1 \cdots x_n) \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ \vdots & \ddots & \vdots \\ q_{n1} & \cdots & q_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + (b_1 \cdots b_n) \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + c$$

$$= \frac{1}{2} (x_1 \cdots x_n) \begin{pmatrix} \sum_{i=1}^n q_{1i} x_i \\ \vdots \\ \sum_{i=1}^n q_{ni} x_i \end{pmatrix} + \sum_{i=1}^n b_i x_i + c$$

$$= \frac{1}{2} \sum_{j=1}^n \sum_{i=1}^n x_j q_{ji} x_i + \sum_{i=1}^n b_i x_i + c.$$

$$\frac{\delta f}{\delta x_k} = \frac{1}{2} \left(\sum_{\substack{j=1 \\ j \neq k}}^n x_j q_{jk} + \sum_{\substack{i=1 \\ i \neq k}}^n x_i q_{ki} + 2 q_{kk} x_k \right) + b_k$$

$$= \frac{1}{2} \left(\sum_{j=1}^n x_j q_{jk} + \sum_{i=1}^n x_i q_{ki} \right) + b_k .$$ Therefore,

$$\nabla f(x) = \frac{1}{2} \left\{ \begin{pmatrix} q_{11} & q_{21} & \cdots & q_{n1} \\ q_{12} & q_{22} & \cdots & q_{n2} \\ \vdots & & & \\ q_{1n} & q_{2n} & \cdots & q_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} q_{11} & q_{12} & \cdots & q_{1n} \\ q_{21} & q_{22} & \cdots & q_{2n} \\ \vdots & & & \\ q_{n1} & q_{n2} & \cdots & q_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \right\} + b$$

$$= \frac{1}{2} \left\{ Q^T x + Qx \right\} + b$$

$$= \frac{1}{2} (Q^T + Q) x + b.$$

Furthermore, since

$$\frac{\delta}{\delta x_p} \left(\frac{\delta f}{\delta x_k} \right) = \frac{\delta}{\delta x_p} \left(\frac{1}{2} \left(\sum_{j=1}^n x_j q_{jk} + \sum_{i=1}^n x_i q_{ki} \right) + b_k \right)$$

$$= \frac{1}{2} (q_{pk} + q_{kp}) \quad \forall k, p,$$ then

$$\nabla^2 f(x) = \frac{1}{2} \begin{pmatrix} q_{11} & \cdots & q_{n1} \\ q_{12} & \cdots & q_{n2} \\ \vdots & & \vdots \\ q_{1n} & \cdots & q_{nn} \end{pmatrix} + \begin{pmatrix} q_{11} & \cdots & q_{1n} \\ q_{21} & & q_{2n} \\ \vdots & & \vdots \\ q_{n1} & \cdots & q_{nn} \end{pmatrix}$$

$$= \frac{1}{2} (Q^T + Q).$$