Technological Innovation and Bursting Bubbles[1]

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Bubble and technological innovation

- Many asset price booms seem to be related to technological innovation (general purpose technologies, GPTs) (Quinn and Turner, 2020)
- Examples:
  - 1720s French Mississippi bubble and British South Sea bubble: Atlantic trade, insurance
  - 1840s British railway mania: steam engine, railway network
  - 1890s British bicycle mania: pneumatic tire
  - 1920s U.S. stock price boom: electricity, consumer durables, automobile, etc.
  - 1990s U.S. dot-com bubble: Internet
  - Now: AI?

This paper

- Macro-finance model of innovation and stock bubble
  - Stock price $(Q) >$ fundamental value ($V :=$ PDV of dividends)
- Features:
  - Skilled agents choose to work in knowledge-intensive sector or establish new firms
  - Monopolistic competition: firm stocks pay dividends
  - Strength of knowledge spillover determines dividend growth rate
  - Agents expect spillover to eventually weaken
    (regime switching with absorbing state)

Main results

1. Agents rationally expect boom to eventually end, but bubble ($Q > V$) emerges as unique equilibrium outcome
   - Bubble necessity (Hirano and Toda, 2025a)
2. Long- and short-run effects of stock bubbles
   - Positive feedback between innovation and stock price
   - Despite inevitable collapse, bubble permanently increases output (because technology prevails) ▸ Bezos
   - Effect on wage inequality temporary
3. Implications for macro-financial modeling
   - Balanced growth is knife-edge (Uzawa, 1961; Schlicht, 2006)
   - Unbalanced growth and bubbles

Related literature

- Rational bubble: Samuelson (1958), Bewley (1980), Tirole (1985), Scheinkman and Weiss (1986), Kocherlakota (1992), Santos and Woodford (1997)
- Rational bubble attached to real assets: Hirano and Toda (2024, 2025a,b)
- Stochastic bubble: Blanchard (1979), Weil (1987)
- Technological innovation and asset boom: Olivier (2000), Pástor and Veronesi (2009)

Model

- Two period OLG model, $t = 0, 1, \dots$
- Young endowed with $e_t > 0$ units of good, old none
- Initial old endowed with asset with dividend $D_t > 0$
- $\{(e_t, D_t)\}_{t=0}^{\infty}$ follows some stochastic process
- Epstein-Zin utility with unit EIS

$$U(c_t^y, c_{t+1}^o) = (1-\beta)\log c_t^y + \beta \log \mathrm{E}_t[(c_{t+1}^o)^{1-\gamma}]^{\frac{1}{1-\gamma}}$$

Equilibrium

Definition

Stochastic process $\left\{(Q_t, c_t^y, c_t^o, n_t)\right\}_{t=0}^{\infty}$ is *rational expectations equilibrium* if

1. (Utility maximization) initial old consume $c_0^o = Q_0 + D_0$; for each $t \geq 0$, (c_t^y, n_t, c_{t+1}^o) maximizes utility subject to budget

$$\begin{aligned} &\text{Young:} && c_t^y + Q_t n_t = e_t, \\ &\text{Old:} && c_{t+1}^o = (Q_{t+1} + D_{t+1}) n_t, \end{aligned}$$

2. (Commodity market clearing) for each t, we have $c_t^y + c_t^o = e_t + D_t$,
3. (Asset market clearing) for each t, we have $n_t = 1$.

Unique equilibrium

- Due to unit EIS, optimal consumption of young is

$$c_t^y = (1-\beta)e_t$$

- Young budget constraint and $n_t = 1$ forces

$$Q_t = Q_t n_t = e_t - c_t^y = \beta e_t$$

Proposition

There exists unique rational expectations equilibrium. Asset price is $Q_t = \beta e_t$ and consumption is $(c_t^y, c_t^o) = ((1-\beta)e_t, \beta e_t + D_t)$.

Stochastic discount factor

- Let

$$m_{t\to t+1}=\frac{\partial U/\partial c^o_{t+1}}{\partial U/\partial c^y_t}=\frac{\beta}{1-\beta}\frac{c^y_t(c^o_{t+1})^{-\gamma}}{\mathrm{E}_t[(c^o_{t+1})^{1-\gamma}]}$$

be stochastic discount factor (SDF) between time t and $t+1$

- Using equilibrium conditions,

$$\begin{aligned}m_{t\to t+1}&=\frac{\beta}{1-\beta}\frac{(1-\beta)e_t(\beta e_{t+1}+D_{t+1})^{-\gamma}}{\mathrm{E}_t[(\beta e_{t+1}+D_{t+1})^{1-\gamma}]}\\&=\frac{Q_t(Q_{t+1}+D_{t+1})^{-\gamma}}{\mathrm{E}_t[(Q_{t+1}+D_{t+1})^{1-\gamma}]}\end{aligned}$$

- Depends only on asset price, dividend, and risk aversion

No-arbitrage condition

- Let $m_{t\to t+1}$ be SDF between t and $t+1$
- No-arbitrage condition is

$$Q_t = \mathrm{E}_t[m_{t\to t+1}(Q_{t+1} + D_{t+1})]$$

- Iteration yields

$$Q_0 = \mathrm{E}_0 \sum_{s=1}^{t} m_{0\to s} D_s + \mathrm{E}_0[m_{0\to t} Q_t],$$

where

$$m_{t\to t+s} := m_{t\to t+1} \times \cdots \times m_{t+s-1\to t+s}$$

is SDF between t and $t+s$

Fundamental value and bubble

- Letting $t \to \infty$ (+ dominated convergence theorem), get

$$Q_0 = \underbrace{\mathrm{E}_0 \sum_{s=1}^{\infty} m_{0\to s} D_s}_{=: V_0} + \underbrace{\lim_{t\to\infty} \mathrm{E}_0[m_{0\to t} Q_t]}_{=: B_0},$$

  where
  - V_0: fundamental value,
  - B_0: bubble
- By definition, no bubble if and only if

$$\lim_{t\to\infty} \mathrm{E}_0[m_{0\to t} Q_t] = 0$$

Emergence of stochastic bubbles

- We now put more structure to derive stochastic bubbles

Assumption

There are two states denoted by u, b. Letting $z_t \in \{u, b\}$ denote state at time t, transition probabilities given by

$$\Pr[z_{t+1} = u \mid z_t = u] = \pi \in (0, 1),$$
$$\Pr[z_{t+1} = b \mid z_t = b] = 1.$$

- State u persists with probability π
- State b absorbing

State b exhibits balanced growth

Assumption

For any τ, conditional on $z_\tau = b$, sequence $\{(e_t, D_t)\}_{t=\tau}^{\infty}$ is deterministic and $e_{t+1}/e_t = D_{t+1}/D_t$ for all $t \geq \tau$.

Proposition

Once state b is reached, no bubble: $Q_t = V_t$.

- Intuition: $Q_t = \beta e_t$ grows with endowment
- In state b, uncertainty resolved and gross risk-free rate

$$R_{t+1} = \frac{\beta e_{t+1} + D_{t+1}}{\beta e_t} = \frac{e_{t+1}}{e_t}\left(1 + \frac{1}{\beta}\underbrace{\frac{D_{t+1}}{e_{t+1}}}_{\text{constant}}\right)$$

exceeds endowment growth, so discounting rules out bubbles

Condition for bubbles in state u

Assumption

Conditional on time $t-1$ information, endowment e_t and dividend D_t depend only on state $z_t \in \{u, b\}$.

Theorem

For $z \in \{u, b\}$, let (e_t^z, D_t^z) be value of (e_t, D_t) conditional on $z_0 = \cdots = z_{t-1} = u$ and $z_t = z$ and let $c_t^z := \beta e_t^z + D_t^z$. If $z_0 = u$, then there is a bubble at $t = 0$ if and only if

Vanishing dividends: $$\sum_{t=1}^{\infty} D_t^u / e_t^u < \infty,$$

Large crash: $$\sum_{t=1}^{\infty} (c_t^b / c_t^u)^{1-\gamma} < \infty.$$

Intuition and implications

1. Noting $Q_t = \beta e_t$, $\sum D_t^u / e_t^u < \infty$ implies $Q_t^u / D_t^u \to \infty$. Hence bubble can be understood as temporary deviation from balanced growth and explosive dynamics in P/D ratio
2. Equilibrium is unique. Hence (under these conditions) asset price bubble is necessity, not possibility
3. Conditions for stochastic bubbles stronger than deterministic case (Montrucchio, 2004, Proposition 7); if $\gamma < 1$, need crash to be larger the longer the bubble lasts

Model with innovation and intangible capital

- Extend toy model to production, innovation, intangible capital (Grossman and Helpman, 1991)
  - R&D
  - Monopolistic competition
- Mass $L > 0$ unskilled agents work in consumption good sector
- Mass $H > 0$ skilled agents either
  - Work in knowledge-intensive intermediate good firms, or
  - Engage in R&D and establish new firms

Consumption good sector

- Representative firm produces output (consumption good)

$$
\begin{aligned}
Y_t &= F(A_{Xt}X_t, A_{Lt}L_t) \\
&= \left(\alpha(A_{Xt}X_t)^{1-\rho} + (1-\alpha)(A_{Lt}L_t)^{1-\rho}\right)^{\frac{1}{1-\rho}}
\end{aligned}
$$

  where
  - X_t: knowledge-intensive good, L_t: unskilled labor
  - A_{Xt}, A_{Lt}: factor-augmenting productivities
  - $1/\rho < 1$: elasticity of substitution
- Maximizes profit $Y_t - P_tX_t - w_{Lt}L_t$, where
  - P_t: price of knowledge-intensive good
  - w_{Lt} unskilled wage
- Zero profit

Knowledge-intensive good sector

- Representative firm produces knowledge-intensive good

$$X_t = n_t^{1-1/\theta} \left(\int_0^{n_t} [x_t(j)]^\theta \, \mathrm{d}j \right)^{1/\theta},$$

where

  - n_t: "knowledge" (accumulates over time)
  - $x_t(j)$: knowledge-intensive intermediate good produced by firm j
  - $\theta \in (0, 1)$: elasticity parameter

- Maximizes profit

$$P_t X_t - \int_0^{n_t} p_t(j) x_t(j) \, \mathrm{d}j$$

- Zero profit

Knowledge-intensive intermediate good sector

- Intermediate goods differentiated by $j \in [0, n_t]$
- Skilled labor produces intermediate good $1 : 1$
- Firm j maximizes profit

$$d_t(j) = (p_t(j) - w_{Ht})x_t(j)$$

  by setting $p_t(j)$ (monopolistic competition), taking wage w_{Ht} and demand $x_t(j)$ as given
- Profit $d_t(j)$ paid as dividend to firm j stock

R&D sector

- New intermediate good varieties created through R&D
- 1 unit of skilled labor $\to an_t$ new varieties (firms)
- Founder sells stocks (claim to monoply profits) at IPO
- Hence indifference condition

$$w_{Ht} = Q_t a n_t,$$

where $Q_t = q_t(j)$ stock price

Equilibrium

Proposition

There exists unique equilibrium. Letting $g(x) = (F_X/F_L)(x,1)$*, fraction of skilled who work* ϕ_t *solves*

$$\frac{1}{aH} = \phi_t - 1 + \beta + \beta\left[\theta\frac{A_{Xt}H}{A_{Lt}L}g\left(\frac{A_{Xt}H}{A_{Lt}L}\phi_t\right)\right]^{-1}.$$

Equilibrium prices are

Knowledge-intensive good price:	$P_t = p_t(j) = F_X A_{Xt}$,
Skilled wage:	$w_{Ht} = \theta F_X A_{Xt}$,
Unskilled wage:	$w_{Lt} = F_L A_{Lt}$,
Stock price:	$Q_t = \frac{w_{Ht}}{an_t} = \frac{\theta}{an_t}F_X A_{Xt}$,

where F_X, F_L *are evaluated at* $(A_{Xt}H\phi_t, A_{Lt}L)$.

Knowledge spillover

Assumption

Let there be two states indexed by $z \in \{u, b\}$. There exist constants $A_X, A_L > 0$ and $\xi_u, \xi_b, \lambda_u, \lambda_b \geq 0$ such that

$$(A_{Xt}, A_{Lt}) = (A_X n_t^{\xi_{z_t}}, A_L n_t^{\lambda_{z_t}}),$$

where

$$\xi_u > \lambda_u > \lambda_b = \xi_b.$$

- Spillover as in Frankel (1962) and Romer (1986)
- State b (absorbing) is balanced growth ($\xi_b = \lambda_b$)
- Spillover stronger in state u and in knowledge-intensive good sector

Equilibrium

- Equilibrium conditions

$$u: \quad \frac{1}{aH} = \phi_t - 1 + \beta + \frac{\beta(1-\alpha)}{\theta\alpha}\left(\frac{A_X H}{A_L L}\right)^{\rho-1} n_t^{(\xi_{z_t}-\lambda_{z_t})(\rho-1)} \phi_t^{\rho},$$

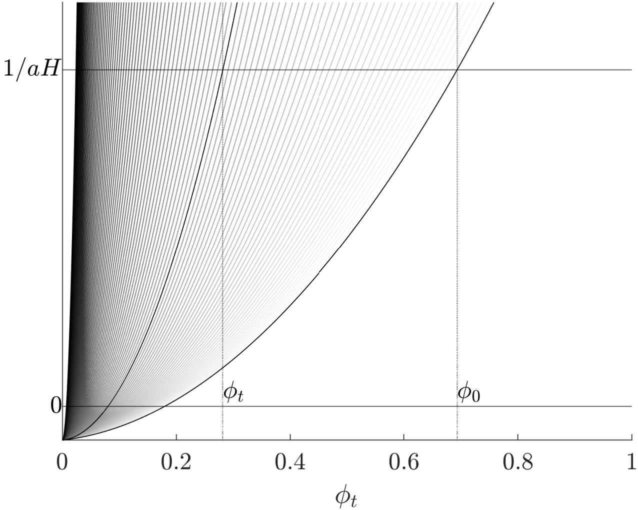
$$b: \quad \frac{1}{aH} = \phi_b - 1 + \beta + \frac{\beta(1-\alpha)}{\theta\alpha}\left(\frac{A_X H}{A_L L}\right)^{\rho-1} \phi_b^{\rho}.$$

Proposition

Under maintained assumptions, following statements are true.

1. *Conditional on staying in state u, $\{\phi_t\}$ monotonically converges to zero and knowledge n_t asymptotically grows at rate $G_u := 1 + aH$.*
2. *In state b, $\{\phi_t\}$ is constant at ϕ_b and knowledge n_t grows at rate $G_b := 1 + a(1-\phi_b)H < G_u$.*

Dynamics of ϕ_t



Inevitable emergence of stock price bubbles

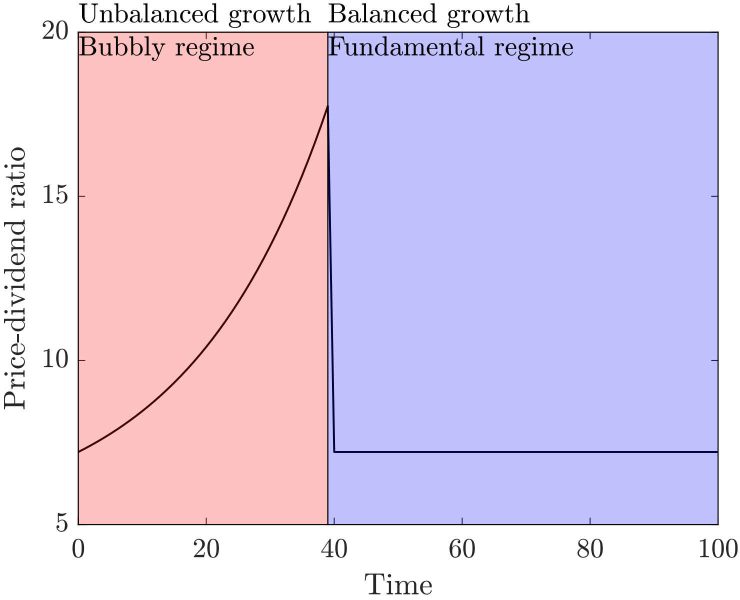
- Equilibrium dynamics reduces to toy model

Theorem

Suppose relative risk aversion is $\gamma < 1$. Let Q_t be stock price in unique equilibrium and V_t fundamental value. Then

1. *In state $z_t = u$, stock price exhibits a bubble: $Q_t > V_t$ and price-dividend ratio Q_t/D_t grows exponentially.*
2. *In state $z_t = b$, stock price reflects fundamentals: $Q_t = V_t$ and price-dividend ratio Q_t/D_t is constant.*

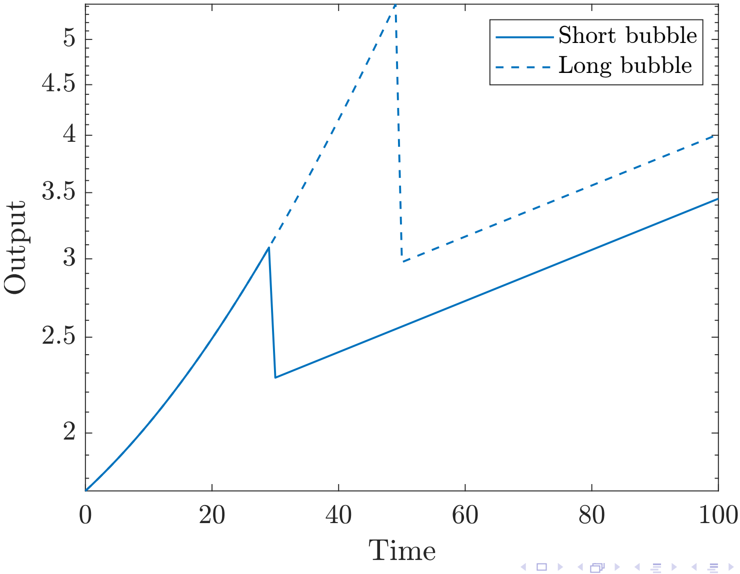
Numerical example



Implications of stock bubbles

- Indifference condition $w_{Ht} = an_t Q_t$, so $Q_t \uparrow$ implies high skilled wage and more innovation
- Unskilled labor relatively abundant, so wage inequality $\uparrow$ during bubble
- In long run, $\phi_t = \phi_b$ constant, so wage inequality unaffected but GDP permanently higher with longer bubble

Dynamics of output



Balanced growth is knife-edge

- In macro, there is strong presupposition in balanced growth
- But balanced growth is knife-edge (Uzawa (1961) steady state growth theorem)

Proposition

Assume only Epstein-Zin utility and neoclassical production function F. Then price-dividend ratio Q_t/D_t is constant over time if and only if either relative productivity A_{Xt}/A_{Lt} is constant or production function F is Cobb-Douglas.
In particular, in our setting, parameters need to satisfy

$$\xi_u = \lambda_u \quad \text{or} \quad \rho = 1.$$

Conclusion

- Any balanced growth model is knife-edge theory
- Once we adopt unbalanced growth (here due to uneven technological spillover), asset price bubble becomes necessity
- Tight connection between technological innovation and stock bubble
- Innovation-driven stock bubble has many benefits (e.g., higher long-run output because more innovation) despite inevitable collapse

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"The [bubbles] that are industrial are not nearly as bad, it can even be good, because when the dust settles and you see who are the winners, societies benefits from those inventions." return