Housing Bubbles with Phase Transitions[1]

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[1]Link to paper: https://arxiv.org/abs/2303.11365

Housing price and rent

- Connection between housing price and rent is not tight
- Examples:
  - Trend in housing price and rent indexes ▸ Image
  - Cross-section of housing price and rent ▸ Image
  - Upward trend in price-rent ratio in many countries during past three decades (Amaral et al., 2024)
- In popular press, often referred to as "housing bubble"
- Understanding why and how housing bubbles emerge is of interest because housing booms and bust often associated with macroeconomic problems (Jordà et al., 2015)

Rational asset price bubbles

- Bubble: asset price (P) $>$ fundamental value (V)
  - $V =$ present value of dividends (D)
- Fundamental difficulty in generating asset price bubbles in real assets
  - Santos and Woodford, 1997: bubble impossible if dividends nonnegligible relative to endowments
  - See Hirano and Toda (2024a, JME) for illustration
- Theory of rational asset price bubbles attached to dividend-paying assets (including housing) largely underdeveloped
  - See Wilson (1981, JET), Hirano and Toda (2024b, JPE)

Questions

1. How can housing prices be disconnected from fundamentals in rational general equilibrium model?
2. How is disconnection related to economic conditions (e.g., income, credit) and expectation formation?
3. What are efficiency properties of equilibria with housing?

This paper

- Theoretically study equilibrium housing price in plain-vanilla OLG model with housing
- Main results
    1. Two-stage phase transition: as income share of home buyers $\uparrow$, equilibrium regime transitions

  fundamental equilibrium ($P = V$) only

  $\rightarrow$coexistence of fundamental & bubbly eq. (bubble possibility)

  $\rightarrow$bubbly equilibrium ($P > V$) only (bubble necessity)

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  2. Expectation- or credit-driven housing bubbles: if home buyers expect high future income or access to credit, housing bubbles emerge
  3. Welfare analysis: in bubble possibility regime, fundamental equilibria inefficient (overturn McCallum (1987))

Related literature

- Housing: Piazzesi and Schneider (2016)
- Monetary/bubble theory: Samuelson, 1958, Bewley, 1980, Tirole, 1985
- Housing as pure bubble: Kocherlakota (2009, 2013), Arce and López-Salido (2011), Chen and Wen (2017), etc.
- Bubble Necessity: Hirano and Toda (2024b)

Model

- Time: $t = 0, 1, \ldots$
- Two period overlapping generations (OLG) model (young & old) with two goods (consumption & housing service)
- Utility $U(y_t, z_{t+1}, h_t)$, where y_t: young consumption, z_{t+1}: old consumption, h_t: housing service
- Endowment of consumption good: $a_t > 0$ for young, $b_t > 0$ for old
- Housing stock in unit supply, initially owned by old; housing stock produces housing service every period

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- Housing stock in unit supply, initially owned by old; housing stock produces housing service every period
  - Think of consumption good as apple, housing service as banana, and housing stock as banana tree

Markets

- Perfect commodity, housing, and rental markets
- Budget constraints:

$$\begin{aligned} &\text{Young:} \quad && y_t + P_t x_t + r_t h_t \leq a_t, \\ &\text{Old:} \quad && z_{t+1} \leq b_{t+1} + (P_{t+1} + r_{t+1}) x_t, \end{aligned}$$

  where x_t: housing stock, P_t: housing price, r_t: housing rent
- Gross risk-free rate $R_t = (P_{t+1} + r_{t+1})/P_t$
- Can combine budget constraint as

$$y_t + \frac{z_{t+1}}{R_t} + r_t h_t \leq a_t + \frac{b_{t+1}}{R_t}$$

Equilibrium

- As usual, equilibrium defined by
  - optimization
  - market clearing

Definition

Rational expectations equilibrium consists of prices $\{(P_t, r_t)\}_{t=0}^{\infty}$ and allocations $\{(x_t, y_t, z_t, h_t)\}_{t=0}^{\infty}$ such that for each t,

1. (Individual optimization) Young maximize utility $U(y_t, z_{t+1}, h_t)$ subject to budget constraints,
2. (Commodity market clearing) $y_t + z_t = a_t + b_t$,
3. (Rental market clearing) $h_t = 1$,
4. (Housing market clearing) $x_t = 1$

Equilibrium characterization

- Let $S_t = P_t + r_t$ be *housing expenditure*
- Market clearing implies $x_t = h_t = 1$ and hence $y_t = a_t - S_t$, $z_{t+1} = b_{t+1} + S_{t+1}$
- First-order conditions imply $1/R_t = U_z/U_y$ and $r_t = U_h/U_y$
- Combining these with $R_t = S_{t+1}/P_t$ yields

$$S_{t+1}U_z = S_t U_y - U_h$$

- Hence equilibrium fully characterized by sequence of housing expenditure $\{S_t\}_{t=0}^{\infty}$ satisfying this difference equation
  - Can show existence of equilibrium

Definition of housing bubble

- With Arrow-Debreu (date-0) price $q_t > 0$, no-arbitrage implies

$$q_t P_t = q_{t+1}(P_{t+1} + r_{t+1}), \quad \text{so}$$
$$P_0 = \sum_{t=1}^{T} q_t r_t + q_T P_T \quad \text{by iteration}$$

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- Letting $T \to \infty$, get

$$P_0 = \underbrace{\sum_{t=1}^{\infty} q_t r_t}_{=: V_0 = \text{fundamental value}} + \underbrace{\lim_{T\to\infty} q_T P_T}_{\text{bubble component}}$$

- If $\lim_{T\to\infty} q_T P_T = 0$, transversality condition holds and no bubble; if > 0, bubble

Assumptions

Assumption (Endowments)

There exist $G > 1$, $a, b > 0$, and $T > 0$ such that the endowments are $(a_t, b_t) = (aG^t, bG^t)$ for $t \geq T$

- Constant income ratio and growth in long run
- Justification of $G > 1$ ▸ Image

Assumptions

Assumption (Utility)

The utility function takes form

$$U(y, z, h) = u(c(y, z)) + v(h),$$

where

1. *composite consumption $c(y, z)$ is homogeneous of degree 1, quasi-concave (and differentiable, Inada condition)*
2. *utility of composite consumption is $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ for some $\gamma \in (0, 1)$,*
3. *utility of housing service satisfies $v' > 0$.*

Justification of $\gamma < 1$

- Interpretations of γ:
  - reciprocal of elasticity of substitution between consumption and housing service
  - elasticity of rent with respect to income
- (Empirical)
  - Ogaki and Reinhart (1998) find $\gamma = 1/1.24 = 0.81$ from ES between durable & non-durable consumption
  - Piazzesi et al. (2007) find $\gamma = 1/1.27 = 0.79$ from cointegration of price & quantity of housing service
  - Howard and Liebersohn (2021) find $\gamma = 0.79$ from cross-sectional regression
- (Theoretical) $\gamma > 1$ is pathological
  - price/rent $\to 0$ as $t \to \infty$
  - interest rate $\to \infty$ as $t \to \infty$

Two useful lemmas

Lemma (Backward induction)

For any equilibrium $\mathcal{S}_T = \{S_t\}_{t=T}^{\infty}$ *starting at* $t = T$*, there exists a unique equilibrium* $\mathcal{S}_0 = \{S_t\}_{t=0}^{\infty}$ *starting at* $t = 0$ *that agrees with* $\mathcal{S}_T$ *for* $t \geq T$*.*

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For any equilibrium $\mathcal{S}_T = \{S_t\}_{t=T}^{\infty}$ starting at $t = T$, there exists a unique equilibrium $\mathcal{S}_0 = \{S_t\}_{t=0}^{\infty}$ starting at $t = 0$ that agrees with $\mathcal{S}_T$ for $t \geq T$.

- Suffices to study equilibrium behavior near steady state
- Hence without loss of generality assume $(a_t, b_t) = (aG^t, bG^t)$ for all t

Two useful lemmas

Lemma (Bounds on rents)

In any equilibrium, $0 < \limsup_{t\to\infty} G^{-\gamma t} r_t < \infty$.

Two useful lemmas

Lemma (Bounds on rents)

In any equilibrium, $0 < \limsup_{t \to \infty} G^{-\gamma t} r_t < \infty$*.*

- Hence $\exists 0 < \underline{r} \leq \bar{r} < \infty$ such that
  - $r_t \leq \bar{r} G^{\gamma t}$ for all t,
  - $r_t \geq \underline{r} G^{\gamma t}$ infinitely often
- Intuition:
  - endowment grows at rate G
  - hence rent (MRS between housing and consumption) grows at rate G^γ
- Roughly speaking, rent grows at rate $G^\gamma < G$ because $\gamma < 1$

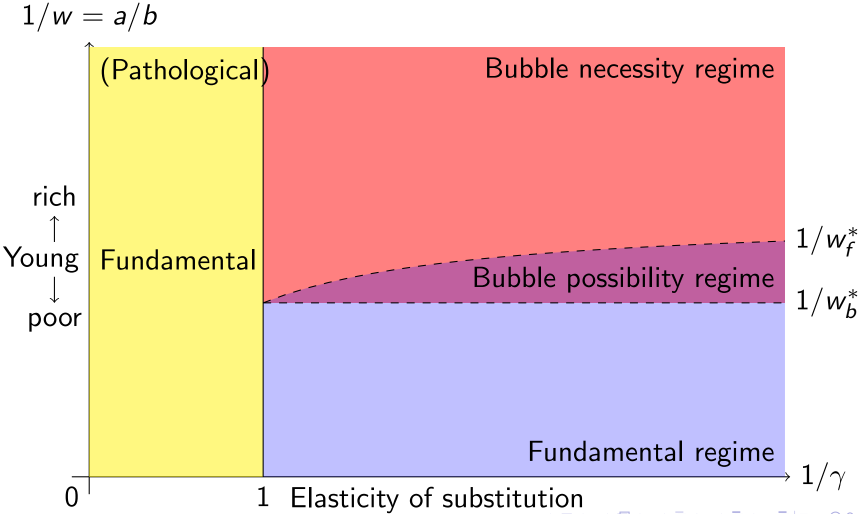
(Non)existence of fundamental equilibria

- Let $w := b/a$ be old to young income ratio
- By previous lemma, rent grows at rate $G^\gamma < G$
- Hence if housing price reflects fundamental value, S_t grows at rate G^γ and $S_t \ll a_t$
- Economy becomes "house-less" in long run and interest rate becomes

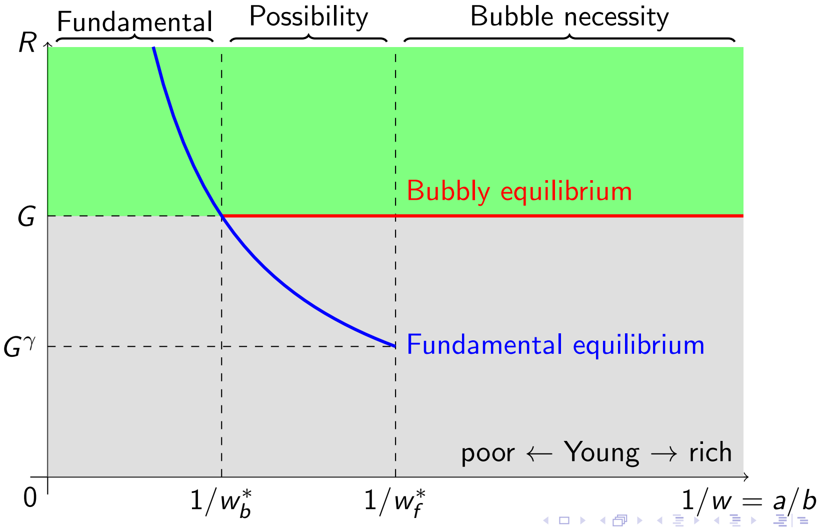
$$R_t = \frac{c_y}{c_z}(y_t, z_{t+1}) \sim \frac{c_y}{c_z}(aG^t, bG^{t+1}) = \frac{c_y}{c_z}(1, Gw),$$

- If w sufficiently low, $R_t < G^\gamma$, implying infinite fundamental value, which is impossible in equilibrium

Phase transition of equilibrium housing price regimes



Housing price regimes and equilibrium interest rate



(Non)existence of fundamental equilibria

Theorem

Let $m = v'(1)$ and $w = b/a$.

1. *There exists unique $w_f^* > 0$ satisfying $(c_y/c_z)(1, Gw_f^*) = G^\gamma$.*
2. *If $w > w_f^*$, there exists fundamental long run equilibrium such that*

$$(y_t, z_t) \sim (aG^t, awG^t), \qquad P_t \sim ma^\gamma \frac{G^\gamma c_z}{c_y - G^\gamma c_z} \frac{c^\gamma}{c_y} G^{\gamma t},$$

$$r_t \sim ma^\gamma \frac{c^\gamma}{c_y} G^{\gamma t}, \qquad R_t \sim \frac{c_y}{c_z} > G^\gamma,$$

   where c, c_y, c_z are evaluated at $(y, z) = (1, Gw)$.

3. *If $w < w_f^*$, there exist no fundamental equilibria, and all equilibria bubbly with $\liminf_{t\to\infty} G^{-t} P_t > 0$.*

Discussion of (non)existence

- Existence part ($w > w_f^*$, so young poor enough) just says we can construct fundamental equilibrium with intuitive order of magnitude, so no big deal
- But nonexistence part ($w < w_f^*$, so young rich enough) much stronger: no fundamental equilibria can exist at all, regardless of long run behavior such as
  - convergent,
  - cyclic,
  - chaotic
- Nonexistence part based on Bubble Necessity Theorem of Hirano and Toda (2024b, JPE)

Existence of bubbly equilibrium

- Since economy grows at rate G, if bubbly equilibrium exists, housing expenditure S_t must grow at rate G
- Then $R_t = S_{t+1}/P_t \to G$
- Define detrended variable $s_t = S_t/(aG^t)$
- Then equilibrium condition is nonlinear difference equation

$$Gs_{t+1}c_z = s_t c_y - ma^{\gamma-1}G^{(\gamma-1)t}c^\gamma,$$

where functions evaluated at $(y_t, z_{t+1}) = (1 - s_t, G(w + s_{t+1}))$

Existence of bubbly equilibrium

Theorem

Let $m = v'(1)$ and $w = b/a$.

1. *There exists unique $w_b^* > w_f^*$ satisfying $\frac{c_y}{c_z}(1, Gw_b^*) = G$. Let $s^* = \frac{w_b^* - w}{w_b^* + 1}$.*
2. *For generic $G > 1$ and $w < w_b^*$, there exists bubbly long run equilibrium such that*

$$(y_t, z_t) \sim (a(1-s^*)G^t, a(w+s^*)G^t), \qquad P_t \sim as^*G^t,$$
$$r_t \sim ma^\gamma \frac{c^\gamma}{c_y} G^{\gamma t}, \qquad R_t \sim G,$$

   where c, c_y are evaluated at $(y, z) = (1 - s^*, G(w + s^*))$.

3. *In bubbly equilibrium, there is housing bubble and the price-rent ratio P_t/r_t diverges to ∞.*

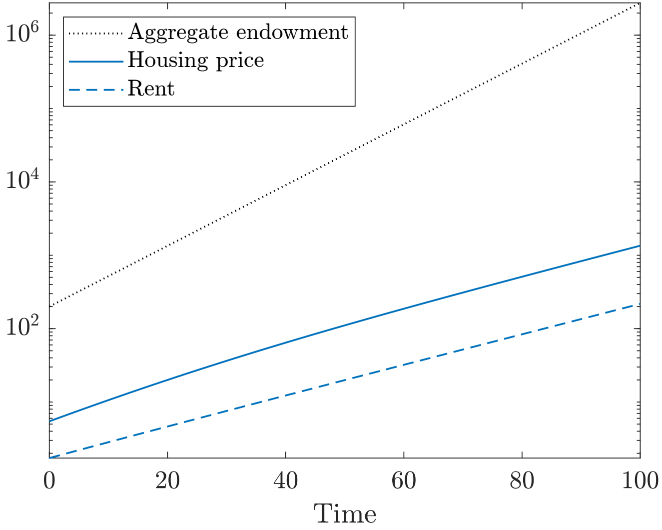
Numerical example

- Suppose utility is CES, so

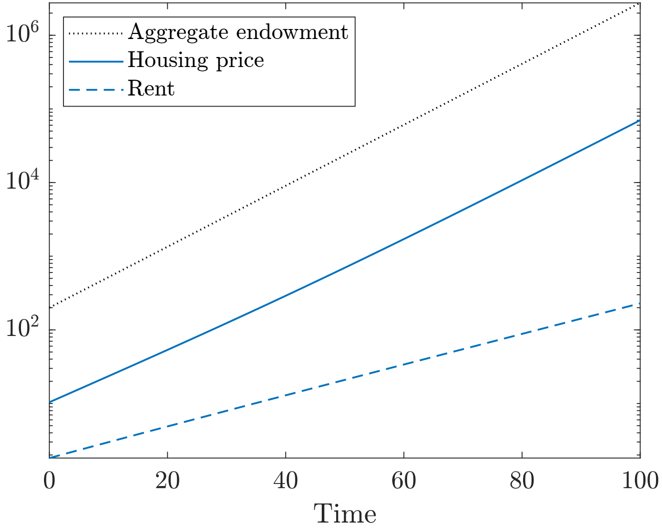
$$c(y,z)=\begin{cases}((1-\beta)y^{1-\sigma}+\beta z^{1-\sigma})^{\frac{1}{1-\sigma}} & \text{if } 0<\sigma\neq 1,\\ y^{1-\beta}z^{\beta} & \text{if } \sigma=1\end{cases}$$

- Set $\beta=1/2$, $\sigma=1$, $\gamma=1/2$, $m=0.1$, and $G=1.1$
- Then $w_b^*=1$; consider $(a,b)=(95,105)$ (fundamental) or $(a,b)=(105,95)$ (bubbly)

Fundamental equilibrium dynamics



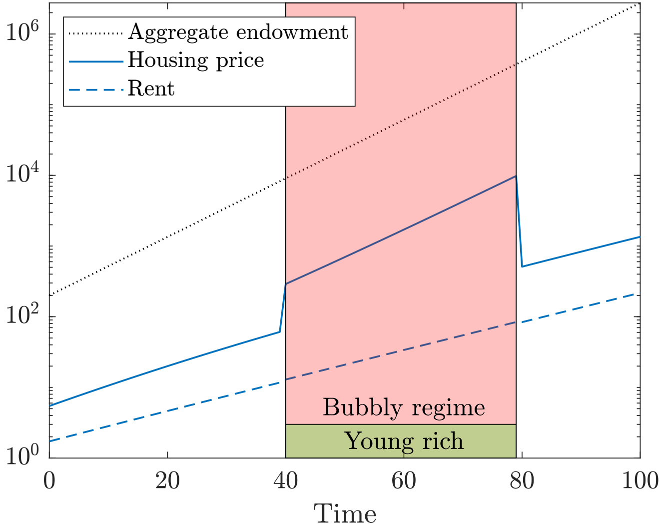
Bubbly equilibrium dynamics



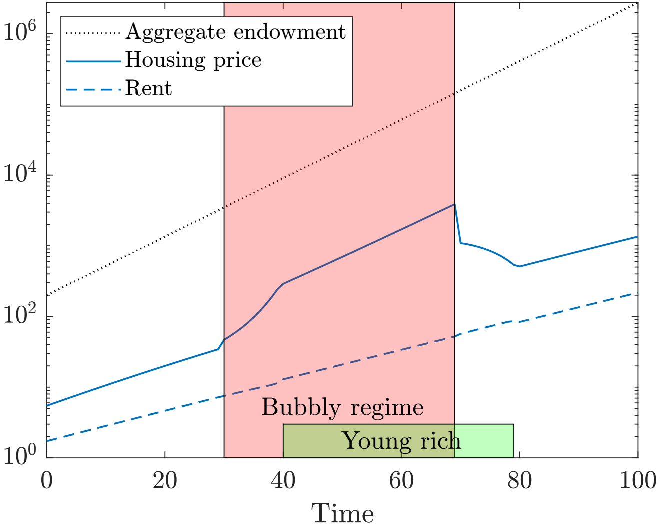
Expectation-driven housing bubbles

- Suppose income distribution between young and old changes between $(95, 105)$ (fundamental) and $(105, 95)$ (bubbly)
- Consider both unexpected change and expected change for 10 periods
- Even if income does not change, access to credit has same effect (see paper)

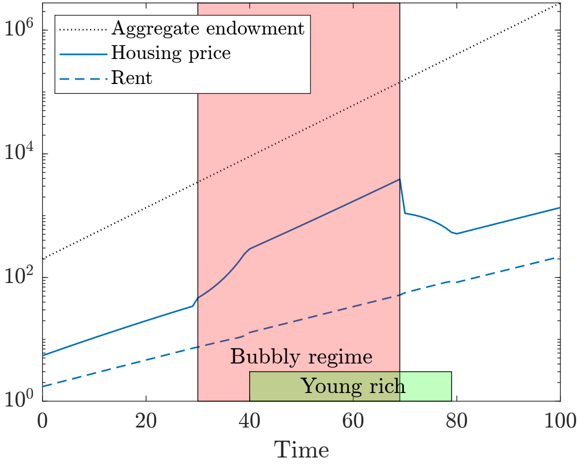
Unexpected income change



Expected income change



Expected income change



- Irving Fisher was right to proclaim "prices have reached what looks like a permanently high plateau"

Welfare analysis

- Housing (and land) is durable non-reproducible asset
- McCallum, 1987 showed land restores dynamic efficiency in (particular) OLG model
- This widely believed result is not true in general

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1. *If $w \geq w_b^*$, any equilibrium is efficient.*
2. *If $w < w_b^*$, any bubbly equilibrium is efficient.*
3. *If $w < w_b^*$, any fundamental equilibrium is inefficient.*

Welfare analysis

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2. *If $w < w_b^*$, any bubbly equilibrium is efficient.*
3. *If $w < w_b^*$, any fundamental equilibrium is inefficient.*

- McCallum (1987) implicitly assumed steady state growth, which need not hold
- Hence policymakers have role in guiding equilibrium selection

Concluding remarks

- Theory of housing bubbles remains largely underdeveloped due to the fundamental difficulty of attaching bubbles to dividend-paying assets
- Presented bare-bones model of housing bubbles with phase transitions
- Welcome generalizations and quantitative & empirical analysis
- Some testable implications:

Testable implications

1. Income (or available funds) of home buyers $\uparrow \implies$ bubble more likely
   - Gyourko et al., 2013 document correlation between income growth and housing appreciation
   - Barlevy and Fisher, 2021 document correlation between availability of interest-only mortgage and housing appreciation

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   - Gyourko et al., 2013 document correlation between income growth and housing appreciation
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2. If bubble, both price-rent ratio and price-income ratio increase
   - Amaral et al. (2024) show upward trend in price-rent ratio

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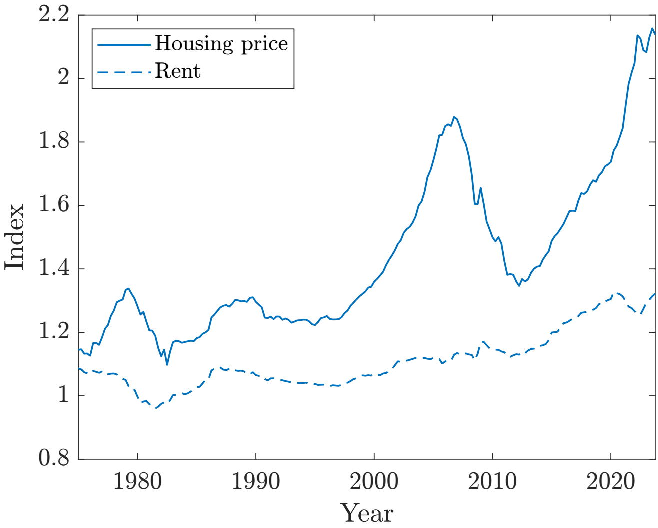
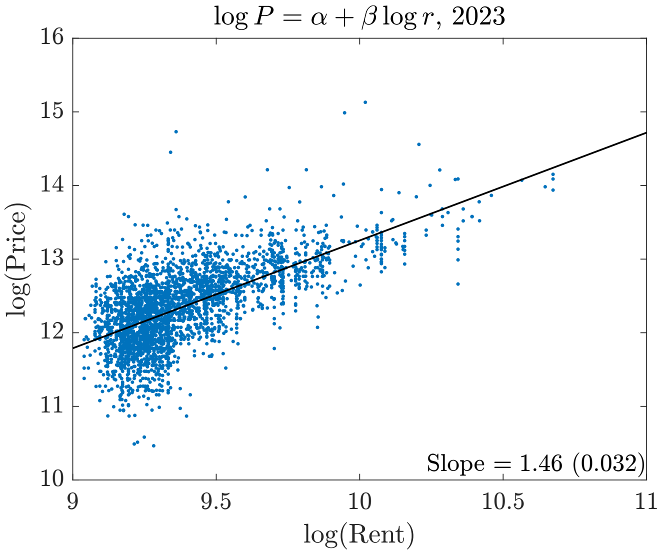
Housing price and rent index



Return

Housing price and rent in U.S. counties



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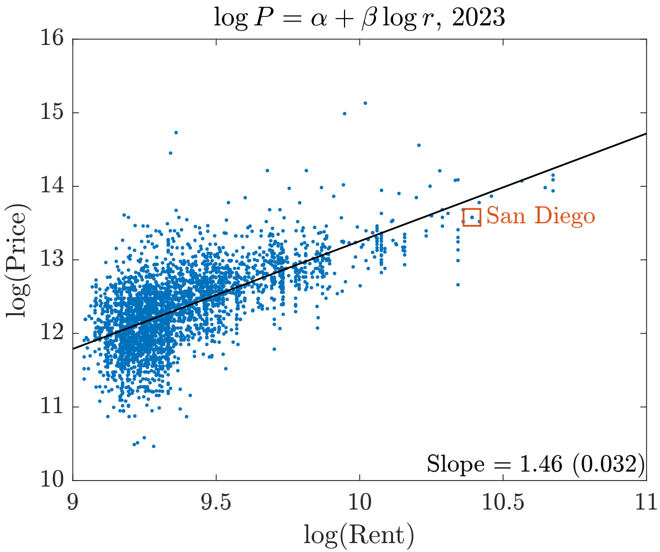

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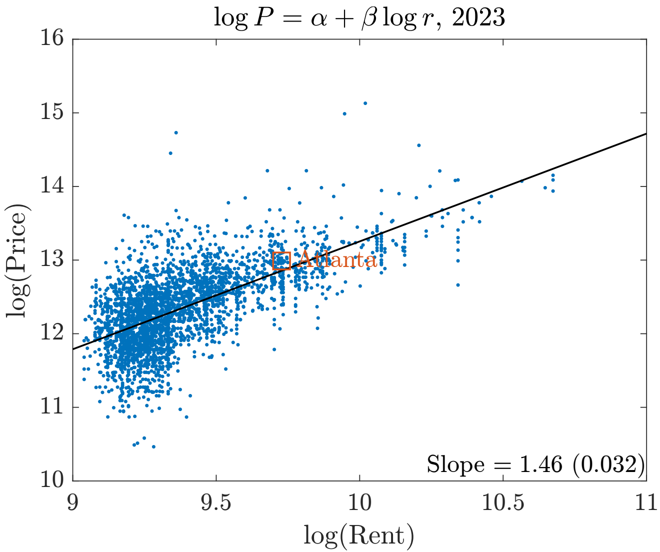

Return

Housing units and GDP