Technological Innovation and Bursting Bubbles[1]

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Bubble and technological innovation

- Many asset price booms seem to be related to technological innovation (general purpose technologies, GPTs) (Quinn and Turner, 2020)
- Examples:
  - 1720s French Mississippi bubble and British South Sea bubble: Atlantic trade, insurance
  - 1840s British railway mania: steam engine, railway network
  - 1890s British bicycle mania: pneumatic tire
  - 1920s U.S. stock price boom: electricity, consumer durables, automobile, etc.
  - 1990s U.S. dot-com bubble: Internet
  - Now: AI?

This paper

- Macro-finance model of innovation and stock bubble
  - Stock price (Q) $>$ fundamental value ($V :=$ PDV of dividends)
- Features:
  - Skilled agents choose to work in knowledge-intensive sector or establish new firms
  - Monopolistic competition: firm stocks pay dividends
  - Strength of knowledge spillover determines dividend growth rate
  - Agents expect spillover to eventually weaken (regime switching with absorbing state)

Main results

1. Agents rationally expect boom to eventually end, but bubble ($Q > V$) emerges as unique equilibrium outcome
   - Bubble necessity (Hirano and Toda, 2025a)
2. Long- and short-run effects of stock bubbles
   - Positive feedback between innovation and stock price
   - Despite inevitable collapse, bubble permanently increases output (because technology prevails)
   - Effect on wage inequality temporary
3. Implications for macro-financial modeling
   - Balanced growth is knife-edge (Uzawa, 1961; Schlicht, 2006)
   - Unbalanced growth and bubbles

Related literature

- Rational bubble: Samuelson (1958), Bewley (1980), Tirole (1985), Scheinkman and Weiss (1986), Kocherlakota (1992), Santos and Woodford (1997)
- Rational bubble attached to real assets: Hirano and Toda (2024, 2025a,b)
- Stochastic bubble: Blanchard (1979), Weil (1987)
- Technological innovation and asset boom: Olivier (2000), Pástor and Veronesi (2009)

Model

- Two period OLG model, $t = 0, 1, \dots$
- Young endowed with $e_t > 0$ units of good, old none
- Initial old endowed with asset with dividend $D_t > 0$
- $\{(e_t, D_t)\}_{t=0}^{\infty}$ follows some stochastic process
- Epstein-Zin utility with unit EIS

$$U(c_t^y, c_{t+1}^o) = (1-\beta)\log c_t^y + \beta \log \mathrm{E}_t[(c_{t+1}^o)^{1-\gamma}]^{\frac{1}{1-\gamma}}$$

Equilibrium

Definition

Stochastic process $\left\{(Q_t, c_t^y, c_t^o, n_t)\right\}_{t=0}^{\infty}$ is *rational expectations equilibrium* if

1. (Utility maximization) initial old consume $c_0^o = Q_0 + D_0$; for each $t \geq 0$, (c_t^y, n_t, c_{t+1}^o) maximizes utility subject to budget

$$\begin{aligned} &\text{Young:} && c_t^y + Q_t n_t = e_t, \\ &\text{Old:} && c_{t+1}^o = (Q_{t+1} + D_{t+1}) n_t, \end{aligned}$$

2. (Commodity market clearing) for each t, we have $c_t^y + c_t^o = e_t + D_t$,
3. (Asset market clearing) for each t, we have $n_t = 1$.

Unique equilibrium

- Due to unit EIS, optimal consumption of young is

$$c_t^y = (1-\beta)e_t$$

- Young budget constraint and $n_t = 1$ forces

$$Q_t = Q_t n_t = e_t - c_t^y = \beta e_t$$

Proposition

There exists unique rational expectations equilibrium. Asset price is $Q_t = \beta e_t$ and consumption is $(c_t^y, c_t^o) = ((1-\beta)e_t, \beta e_t + D_t)$.

Stochastic discount factor

- Let
$$m_{t\to t+1}=\frac{\partial U/\partial c^o_{t+1}}{\partial U/\partial c^y_t}=\frac{\beta}{1-\beta}\frac{c^y_t(c^o_{t+1})^{-\gamma}}{\mathrm{E}_t[(c^o_{t+1})^{1-\gamma}]}$$
be stochastic discount factor (SDF) between time t and $t+1$
- Using equilibrium conditions,
$$\begin{aligned}m_{t\to t+1}&=\frac{\beta}{1-\beta}\frac{(1-\beta)e_t(\beta e_{t+1}+D_{t+1})^{-\gamma}}{\mathrm{E}_t[(\beta e_{t+1}+D_{t+1})^{1-\gamma}]}\\&=\frac{Q_t(Q_{t+1}+D_{t+1})^{-\gamma}}{\mathrm{E}_t[(Q_{t+1}+D_{t+1})^{1-\gamma}]}\end{aligned}$$
- Depends only on asset price, dividend, and risk aversion

No-arbitrage condition

- Let $m_{t\to t+1}$ be SDF between t and $t+1$
- No-arbitrage condition is

$$Q_t = \mathrm{E}_t[m_{t\to t+1}(Q_{t+1} + D_{t+1})]$$

- Iteration yields

$$Q_0 = \mathrm{E}_0 \sum_{s=1}^{t} m_{0\to s} D_s + \mathrm{E}_0[m_{0\to t} Q_t],$$

where $m_{t\to t+s} = m_{t\to t+1} \times \cdots \times m_{t+s-1\to t+s}$ is SDF between t and $t+s$

Fundamental value and bubble

- Letting $t \to \infty$, get

$$Q_0 = \underbrace{\mathrm{E}_0 \sum_{s=1}^{\infty} m_{0\to s} D_s}_{=: V_0} + \underbrace{\lim_{t\to\infty} \mathrm{E}_0[m_{0\to t} Q_t]}_{=: B_0},$$

  where
  - V_0: fundamental value,
  - B_0: bubble
- By definition, no bubble if and only if

$$\lim_{t\to\infty} \mathrm{E}_0[m_{0\to t} Q_t] = 0$$

Emergence of stochastic bubbles

- We now put more structure to derive stochastic bubbles

Assumption

There are two states denoted by u, b. Letting $z_t \in \{u, b\}$ denote state at time t, transition probabilities given by

$$\Pr[z_{t+1} = u \mid z_t = u] = \pi \in (0, 1),$$
$$\Pr[z_{t+1} = b \mid z_t = b] = 1.$$

- State u persists with probability π
- State b absorbing

State b exhibits balanced growth

Assumption

For any τ, conditional on $z_\tau = b$, sequence $\{(e_t, D_t)\}_{t=\tau}^\infty$ is deterministic and $e_{t+1}/e_t = D_{t+1}/D_t$ for all $t \geq \tau$.

Proposition

Once state b is reached, no bubble: $Q_t = V_t$.

- Intuition: $Q_t = \beta e_t$ grows with endowment
- In state b, uncertainty resolved and gross risk-free rate

$$R_{t+1} = \frac{\beta e_{t+1} + D_{t+1}}{\beta e_t} = \frac{e_{t+1}}{e_t}\left(1 + \frac{1}{\beta}\underbrace{\frac{D_{t+1}}{e_{t+1}}}_{\text{constant}}\right)$$

exceeds endowment growth, so discounting rules out bubbles

Condition for bubbles in state u

Assumption

Conditional on time $t-1$ information, endowment e_t and dividend D_t depend only on state $z_t \in \{u, b\}$.

Theorem

For $z \in \{u, b\}$, let (e_t^z, D_t^z) be value of (e_t, D_t) conditional on $z_0 = \cdots = z_{t-1} = u$ and $z_t = z$ and let $c_t^z := \beta e_t^z + D_t^z$. If $z_0 = u$, then there is a bubble at $t = 0$ if and only if

Vanishing dividends: $$\sum_{t=1}^{\infty} D_t^u / e_t^u < \infty,$$

Large crash: $$\sum_{t=1}^{\infty} (c_t^b / c_t^u)^{1-\gamma} < \infty.$$

Intuition and implications

1. Noting $Q_t = \beta e_t$, $\sum D_t^u / e_t^u < \infty$ implies $Q_t^u / D_t^u \to \infty$. Hence bubble can be understood as temporary deviation from balanced growth and explosive dynamics in P/D ratio
2. Equilibrium is unique. Hence (under these conditions) asset price bubble is necessity, not possibility
3. Conditions for stochastic bubbles stronger than deterministic case (Montrucchio, 2004, Proposition 7); if $\gamma < 1$, need crash to be larger the longer the bubble lasts

Model with innovation and intangible capital

- Extend toy model to production, innovation, intangible capital (Grossman and Helpman, 1991)
  - R&D
  - Monopolistic competition
- Mass $L > 0$ unskilled agents work in consumption good sector
- Mass $H > 0$ skilled agents either
  - Work in knowledge-intensive intermediate good firms, or
  - Engage in R&D and establish new firms

Consumption good sector

- Representative firm produces output (consumption good)

$$
\begin{aligned}
Y_t &= F(A_{Xt}X_t, A_{Lt}L_t) \\
&= \left(\alpha(A_{Xt}X_t)^{1-\rho} + (1-\alpha)(A_{Lt}L_t)^{1-\rho}\right)^{\frac{1}{1-\rho}}
\end{aligned}
$$

  where
    - X_t: knowledge-intensive good, L_t: unskilled labor
    - A_{Xt}, A_{Lt}: factor-augmenting productivities
    - $1/\rho < 1$: elasticity of substitution
- Maximizes profit $Y_t - P_tX_t - w_{Lt}L_t$, where
    - P_t: price of knowledge-intensive good
    - w_{Lt} unskilled wage
- Zero profit

Knowledge-intensive good sector

- Representative firm produces knowledge-intensive good

$$X_t = n_t^{1-1/\theta} \left(\int_0^{n_t} [x_t(j)]^\theta \, \mathrm{d}j \right)^{1/\theta},$$

where

  - n_t: "knowledge" (accumulates over time)
  - $x_t(j)$: knowledge-intensive intermediate good produced by firm j
  - $\theta \in (0, 1)$: elasticity parameter

- Maximizes profit

$$P_t X_t - \int_0^{n_t} p_t(j) x_t(j) \, \mathrm{d}j$$

- Zero profit

Knowledge-intensive intermediate good sector

- Intermediate goods differentiated by $j \in [0, n_t]$
- Skilled labor produces intermediate good $1:1$
- Firm j maximizes profit

$$d_t(j) = (p_t(j) - w_{Ht})x_t(j)$$

  by setting $p_t(j)$ (monopolistic competition), taking wage w_{Ht} and demand $x_t(j)$ as given
- Profit $d_t(j)$ paid as dividend to firm j stock

R&D sector

- New intermediate good varieties created through R&D
- 1 unit of skilled labor $\to an_t$ new varieties (firms)
- Founder sells stocks (claim to monoply profits) at IPO
- Hence indifference condition

$$w_{Ht} = Q_t a n_t,$$

where $Q_t = q_t(j)$ stock price

Equilibrium

Proposition

There exists unique equilibrium. Letting $g(x) = (F_X/F_L)(x,1)$, fraction of skilled who work ϕ_t solves

$$\frac{1}{aH} = \phi_t - 1 + \beta + \beta \left[\theta \frac{A_{Xt}H}{A_{Lt}L} g\left(\frac{A_{Xt}H}{A_{Lt}L}\phi_t \right) \right]^{-1}.$$

Equilibrium prices are

Knowledge-intensive good price:	$P_t = p_t(j) = F_X A_{Xt}$,
Skilled wage:	$w_{Ht} = \theta F_X A_{Xt}$,
Unskilled wage:	$w_{Lt} = F_L A_{Lt}$,
Stock price:	$Q_t = \dfrac{w_{Ht}}{a n_t} = \dfrac{\theta}{a n_t} F_X A_{Xt}$,

where F_X, F_L are evaluated at $(A_{Xt}H\phi_t, A_{Lt}L)$.

Knowledge spillover

Assumption

Let there be two states indexed by $z \in \{u, b\}$*. There exist constants* $A_X, A_L > 0$ *and* $\xi_u, \xi_b, \lambda_u, \lambda_b \geq 0$ *such that*

$$(A_{Xt}, A_{Lt}) = (A_X n_t^{\xi_{z_t}}, A_L n_t^{\lambda_{z_t}}),$$

as in Frankel (1962) and Romer (1986), where

$$\xi_u > \lambda_u > \lambda_b = \xi_b.$$

- State b (absorbing) is balanced growth ($\xi_b = \lambda_b$)
- Spillover stronger in state u and in knowledge-intensive good sector

Equilibrium

- Equilibrium conditions

$$u: \quad \frac{1}{aH} = \phi_t - 1 + \beta + \frac{\beta(1-\alpha)}{\theta\alpha}\left(\frac{A_X H}{A_L L}\right)^{\rho-1} n_t^{(\xi_{z_t}-\lambda_{z_t})(\rho-1)}\phi_t^{\rho},$$

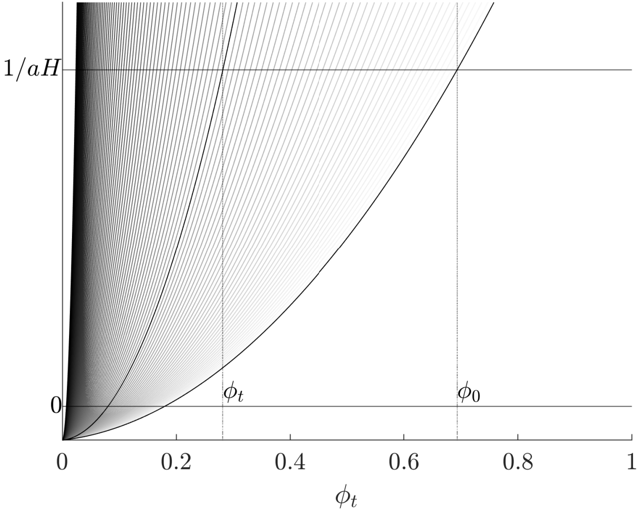
$$b: \quad \frac{1}{aH} = \phi_b - 1 + \beta + \frac{\beta(1-\alpha)}{\theta\alpha}\left(\frac{A_X H}{A_L L}\right)^{\rho-1} \phi_b^{\rho}.$$

Proposition

Under maintained assumptions, following statements are true.

1. *Conditional on staying in state u, $\{\phi_t\}$ monotonically converges to zero and knowledge n_t asymptotically grows at rate $G_u := 1 + aH$.*
2. *In state b, $\{\phi_t\}$ is constant at ϕ_b and knowledge n_t grows at rate $G_b := 1 + a(1-\phi_b)H < G_u$.*

Dynamics of ϕ_t



Inevitable emergence of stock price bubbles

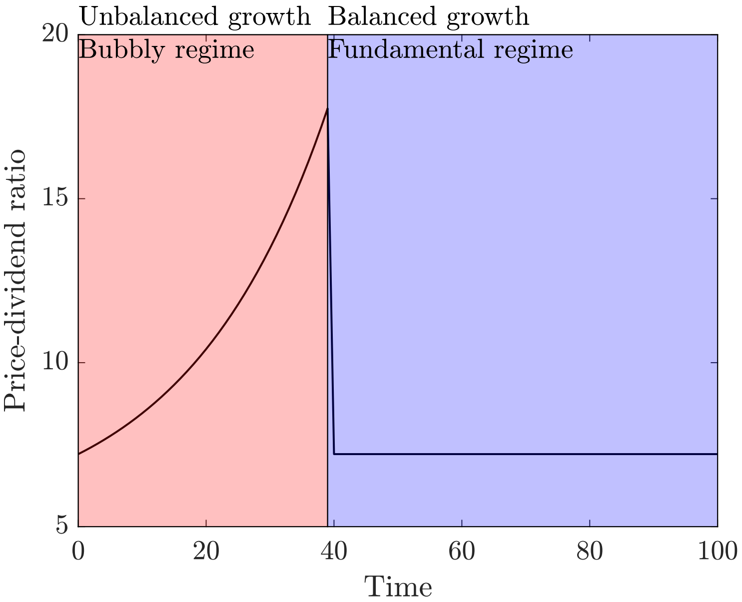
- Equilibrium dynamics reduces to toy model

Theorem

Suppose relative risk aversion is $\gamma < 1$. Let Q_t be stock price in unique equilibrium and V_t fundamental value. Then

1. *In state $z_t = u$, stock price exhibits a bubble: $Q_t > V_t$ and price-dividend ratio Q_t/D_t grows exponentially.*
2. *In state $z_t = b$, stock price reflects fundamentals: $Q_t = V_t$ and price-dividend ratio Q_t/D_t is constant.*

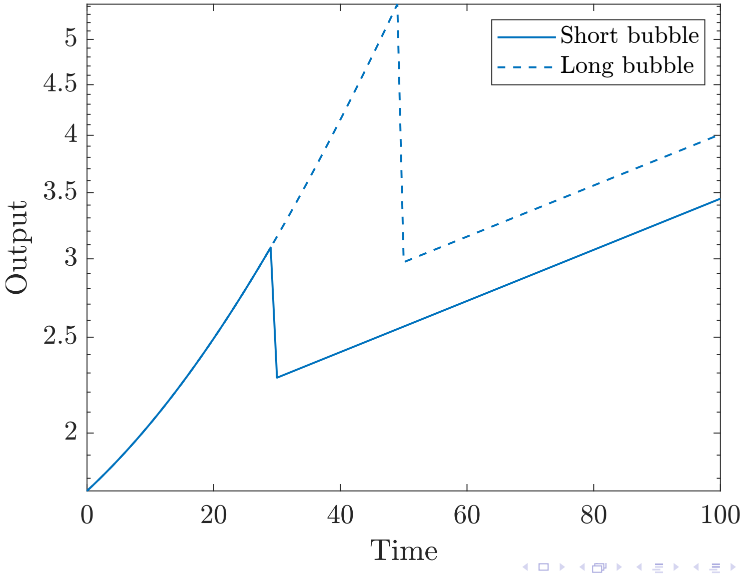
Numerical example



Implications of stock bubbles

- Indifference condition $w_{Ht} = an_t Q_t$, so $Q_t \uparrow$ implies high skilled wage and more innovation
- Unskilled labor relatively abundant, so wage inequality $\uparrow$ during bubble
- In long run, $\phi_t = \phi_b$ constant, so wage inequality unaffected but GDP permanently higher with longer bubble

Dynamics of output



Balanced growth is knife-edge

- In macro, there is strong presupposition in balanced growth
- But balanced growth is knife-edge (Uzawa (1961) steady state growth theorem)

Proposition

Assume only Epstein-Zin utility and neoclassical production function F. Then price-dividend ratio Q_t/D_t is constant over time if and only if either relative productivity A_{Xt}/A_{Lt} is constant or production function F is Cobb-Douglas.
In particular, in our setting, parameters need to satisfy

$$\xi_u = \lambda_u \quad \text{or} \quad \rho = 1.$$

Conclusion

- Any balanced growth model is knife-edge theory
- Once we adopt unbalanced growth (here due to uneven technological spillover), asset price bubble becomes necessity
- Tight connection between technological innovation and stock bubble
- Innovation-driven stock bubble has many benefits (e.g., higher long-run output because more innovation) despite inevitable collapse

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