

# Technological Innovation and Bursting Bubbles<sup>1</sup>

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## Bubble and technological innovation

- Many asset price booms seem to be related to technological innovation (general purpose technologies, GPTs) (Quinn and Turner, [2020](#))

## Bubble and technological innovation

- Many asset price booms seem to be related to technological innovation (general purpose technologies, GPTs) (Quinn and Turner, 2020)
- Examples:
  - 1720s French Mississippi bubble and British South Sea bubble: Atlantic trade, insurance
  - 1840s British railway mania: steam engine, railway network
  - 1890s British bicycle mania: pneumatic tire
  - 1920s U.S. stock price boom: electricity, consumer durables, automobile, etc.
  - 1990s U.S. dot-com bubble: Internet
  - Now: AI?

## Rational asset price bubbles

- Bubble: asset price ( $Q$ )  $>$  fundamental value ( $V$ )
  - $V$  = present value of dividends ( $D$ )
- Fundamental difficulty in generating asset price bubbles in real assets
  - Santos and Woodford ([1997](#)): bubble impossible if dividends nonnegligible relative to endowments
  - See Hirano and Toda ([2024](#), JME) for illustration
- Theory of rational asset price bubbles attached to dividend-paying assets (including housing) largely underdeveloped
  - See Wilson ([1981](#), JET), Hirano and Toda ([2025a](#), JPE), Hirano and Toda ([2025b](#), PNAS)

## This paper

- Macro-finance model of innovation and stock bubble
- Features:
  - Skilled agents choose to work in knowledge-intensive sector or establish new firms
  - Monopolistic competition: firm stocks pay dividends
  - Strength of knowledge spillover determines dividend growth rate
  - Agents expect spillover to eventually weaken (regime switching with absorbing state)

## Main results

1. Agents rationally expect boom to eventually end, but bubble ( $Q > V$ ) emerges as unique equilibrium outcome
  - **Bubble necessity** (Hirano and Toda, 2025a)
2. Long- and short-run effects of stock bubbles
  - Positive feedback between innovation and stock price
  - Despite inevitable collapse, bubble permanently increases output (because technology prevails)
  - Effect on wage inequality temporary
3. Implications for macro-financial modeling
  - Balanced growth is knife-edge (Uzawa, 1961; Schlicht, 2006)
  - Unbalanced growth and bubbles

## Related literature

- **Rational bubble:** Samuelson (1958), Bewley (1980), Tirole (1985), Scheinkman and Weiss (1986), Kocherlakota (1992), Santos and Woodford (1997)
- **Rational bubble attached to real assets:** Hirano and Toda (2024, 2025a,b)
- **Stochastic bubble:** Blanchard (1979), Weil (1987)
- **Technological innovation and asset boom:** Olivier (2000), Pástor and Veronesi (2009)

## Model: agents and preferences

- Time:  $t = 0, 1, \dots$
- Two period overlapping generations (OLG) model
- Aggregate uncertainty
- Epstein-Zin utility with unit EIS

$$U(c_t^y, c_{t+1}^o) = (1 - \beta) \log c_t^y + \beta \log E_t[(c_{t+1}^o)^{1-\gamma}]^{\frac{1}{1-\gamma}}$$



## Model: endowments and dividends

- Young endowed with  $e_t > 0$  units of good, old none
- Initial old endowed with unit supply of long-lived asset that pays dividend  $D_t > 0$
- $\{(e_t, D_t)\}_{t=0}^{\infty}$  follows some stochastic process
- Budget constraints

$$\text{Young:} \quad c_t^y + Q_t n_t = e_t,$$

$$\text{Old:} \quad c_{t+1}^o = (Q_{t+1} + D_{t+1})n_t,$$

where  $Q_t$ : asset price,  $n_t$ : asset holdings

# Equilibrium

## Definition

Stochastic process  $\{(Q_t, c_t^y, c_t^o, n_t)\}_{t=0}^{\infty}$  is *rational expectations equilibrium* if

1. (Utility maximization) initial old consume  $c_0^o = Q_0 + D_0$ ; for each  $t \geq 0$ ,  $(c_t^y, n_t, c_{t+1}^o)$  maximizes utility subject to budget constraints,
2. (Commodity market clearing) for each  $t$ , we have  $c_t^y + c_t^o = e_t + D_t$ ,
3. (Asset market clearing) for each  $t$ , we have  $n_t = 1$ .

## Unique equilibrium

- Due to unit EIS, optimal consumption of young is

$$c_t^y = (1 - \beta)e_t$$

- Young budget constraint and  $n_t = 1$  forces

$$Q_t = Q_t n_t = e_t - c_t^y = \beta e_t$$

### Proposition

*There exists unique rational expectations equilibrium. Asset price is  $Q_t = \beta e_t$  and consumption is  $(c_t^y, c_t^o) = ((1 - \beta)e_t, \beta e_t + D_t)$ .*

## Stochastic discount factor

- Let

$$m_{t \rightarrow t+1} = \frac{\partial U / \partial c_{t+1}^o}{\partial U / \partial c_t^y} = \frac{\beta}{1 - \beta} \frac{c_t^y (c_{t+1}^o)^{-\gamma}}{\mathbb{E}_t[(c_{t+1}^o)^{1-\gamma}]}$$

be stochastic discount factor (SDF) between time  $t$  and  $t + 1$

- Using equilibrium conditions,

$$\begin{aligned} m_{t \rightarrow t+1} &= \frac{\beta}{1 - \beta} \frac{(1 - \beta) e_t (\beta e_{t+1} + D_{t+1})^{-\gamma}}{\mathbb{E}_t[(\beta e_{t+1} + D_{t+1})^{1-\gamma}]} \\ &= \frac{Q_t (Q_{t+1} + D_{t+1})^{-\gamma}}{\mathbb{E}_t[(Q_{t+1} + D_{t+1})^{1-\gamma}]} \end{aligned}$$

- Useful later

## No-arbitrage condition

- Let  $m_{t \rightarrow t+1}$  be SDF between  $t$  and  $t + 1$
- Let  $m_{t \rightarrow t+s} = m_{t \rightarrow t+1} \times \cdots \times m_{t+s-1 \rightarrow t+s}$  be SDF between  $t$  and  $t + s$
- No-arbitrage condition is

$$Q_t = E_t[m_{t \rightarrow t+1}(Q_{t+1} + D_{t+1})]$$

- Iteration yields

$$Q_0 = E_0 \sum_{s=1}^t m_{0 \rightarrow s} D_s + E_0[m_{0 \rightarrow t} Q_t]$$

## Fundamental value and bubble

- Letting  $t \rightarrow \infty$ , get

$$Q_0 = E_0 \underbrace{\sum_{s=1}^{\infty} m_{0 \rightarrow s} D_s}_{=: V_0} + \underbrace{\lim_{t \rightarrow \infty} E_0[m_{0 \rightarrow t} Q_t]}_{=: B_0},$$

where

- $V_0$ : fundamental value,
  - $B_0$ : bubble
- By definition, no bubble if and only if

$$\lim_{t \rightarrow \infty} E_0[m_{0 \rightarrow t} Q_t] = 0$$

## Emergence of stochastic bubbles

- We now put more structure to derive stochastic bubbles

### Assumption

*There are two states denoted by  $u, b$ . Letting  $z_t \in \{u, b\}$  denote state at time  $t$ , transition probabilities given by*

$$\Pr[z_{t+1} = u \mid z_t = u] = \pi \in (0, 1),$$

$$\Pr[z_{t+1} = b \mid z_t = b] = 1.$$

- State  $u$  persists with probability  $\pi$
- State  $b$  absorbing

## State $b$ exhibits balanced growth

### Assumption

*For any  $\tau$ , conditional on  $z_\tau = b$ , sequence  $\{(e_t, D_t)\}_{t=\tau}^\infty$  is deterministic and  $e_{t+1}/e_t = D_{t+1}/D_t$  for all  $t \geq \tau$ .*



## State $b$ exhibits balanced growth

### Assumption

*For any  $\tau$ , conditional on  $z_\tau = b$ , sequence  $\{(e_t, D_t)\}_{t=\tau}^\infty$  is deterministic and  $e_{t+1}/e_t = D_{t+1}/D_t$  for all  $t \geq \tau$ .*

### Proposition

*Once state  $b$  is reached, no bubble:  $Q_t = V_t$ .*

## State $b$ exhibits balanced growth

### Assumption

For any  $\tau$ , conditional on  $z_\tau = b$ , sequence  $\{(e_t, D_t)\}_{t=\tau}^\infty$  is deterministic and  $e_{t+1}/e_t = D_{t+1}/D_t$  for all  $t \geq \tau$ .

### Proposition

Once state  $b$  is reached, no bubble:  $Q_t = V_t$ .

- Intuition:  $Q_t = \beta e_t$  grows with endowment
- In state  $b$ , uncertainty resolved and gross risk-free rate

$$R_{t+1} = \frac{\beta e_{t+1} + D_{t+1}}{\beta e_t} = \frac{e_{t+1}}{e_t} \left( 1 + \frac{1}{\beta} \underbrace{\frac{D_{t+1}}{e_{t+1}}}_{\text{constant}} \right)$$

exceeds endowment growth, so discounting rules out bubbles

## Condition for bubbles in state $u$

### Assumption

*Conditional on time  $t - 1$  information, endowment  $e_t$  and dividend  $D_t$  depend only on state  $z_t \in \{u, b\}$ .*

### Theorem

*For  $z \in \{u, b\}$ , let  $(e_t^z, D_t^z)$  be value of  $(e_t, D_t)$  conditional on  $z_0 = \dots = z_{t-1} = u$  and  $z_t = z$  and let  $c_t^z := \beta e_t^z + D_t^z$ . If  $z_0 = u$ , then there is a bubble at  $t = 0$  if and only if*

$$\sum_{t=1}^{\infty} D_t^u / e_t^u < \infty,$$

$$\sum_{t=1}^{\infty} (c_t^b / c_t^u)^{1-\gamma} < \infty.$$

## Intuition and implications

1. Noting  $Q_t = \beta e_t$ ,  $\sum D_t^u / e_t^u < \infty$  implies  $Q_t^u / D_t^u \rightarrow \infty$ .  
Hence bubble can be understood as temporary deviation from balanced growth and explosive dynamics in P/D ratio
2. Equilibrium is unique. Hence (under these conditions) asset price bubble is **necessity**, not possibility
3. Conditions for stochastic bubbles stronger than deterministic case (Montrucchio, [2004](#), Proposition 7); if  $\gamma < 1$ , need crash to be larger the longer the bubble lasts

# Model with innovation and intangible capital

- Endowment economy highly stylized
- We extend toy model to production, innovation, intangible capital (Grossman and Helpman, [1991](#))
  - R&D
  - Monopolistic competition

## Agents and preferences

- Basically same as toy model
  - Two-period OLG model
  - Epstein-Zin preferences
- Mass  $L > 0$  unskilled agents work in consumption good sector
- Mass  $H > 0$  skilled agents either
  - Work in knowledge-intensive intermediate good firms, or
  - Engage in R&D and establish new firms

## Consumption good sector

- Representative firm produces output (consumption good)

$$Y_t = F(A_{X_t}X_t, A_{L_t}L_t),$$

where

- $F$ : neoclassical production function (e.g., CES)
  - $X_t$ : knowledge-intensive good,  $L_t$ : unskilled labor
  - $A_{X_t}, A_{L_t}$ : factor-augmenting productivities
- Maximizes profit

$$Y_t - P_tX_t - w_{L_t}L_t,$$

where

- $P_t$ : price of knowledge-intensive good
  - $w_{L_t}$  unskilled wage
- Zero profit

## Knowledge-intensive good sector

- Representative firm produces knowledge-intensive good

$$X_t = n_t^{1-1/\theta} \left( \int_0^{n_t} [x_t(j)]^\theta dj \right)^{1/\theta},$$

where

- $n_t$ : “knowledge” (accumulates over time)
  - $x_t(j)$ : knowledge-intensive intermediate good produced by firm  $j$
  - $\theta \in (0, 1)$ : elasticity parameter
- Maximizes profit

$$P_t X_t - \int_0^{n_t} p_t(j) x_t(j) dj$$

- Zero profit



## Knowledge-intensive intermediate good sector

- Intermediate goods differentiated by  $j \in [0, n_t]$
- Skilled labor produces intermediate good 1 : 1
- Firm  $j$  maximizes profit

$$d_t(j) = (p_t(j) - w_{Ht})x_t(j)$$

by setting  $p_t(j)$  (monopolistic competition), taking wage  $w_{Ht}$  and demand  $x_t(j)$  as given

- Profit  $d_t(j)$  paid as dividend to firm  $j$  stock

## R&D sector

- New intermediate good varieties created through R&D
- 1 unit of skilled labor  $\rightarrow an_t$  new varieties (firms)
- Founder sells stocks (claim to monopoly profits) at IPO
- Hence indifference condition

$$w_{Ht} = Q_t an_t,$$

where  $Q_t = q_t(j)$  stock price

# Equilibrium

- First-order conditions of consumption good

$$P_t = F_X(A_{Xt}X_t, A_{Lt}L)A_{Xt},$$
$$w_{Lt} = F_L(A_{Xt}X_t, A_{Lt}L)A_{Lt}$$

- FOC of intermediate good  $j$

$$p_t(j) = P_t(X_t/n_t)^{1-\theta} x_t(j)^{\theta-1}$$

- Hence demand for intermediate good  $j$

$$x_t(j) = (X_t/n_t)(p_t(j)/P_t)^{-\frac{1}{1-\theta}}$$

# Equilibrium

- Monopolistic competition implies

$$p_t(j) = w_{Ht}/\theta$$

- Because  $p_t(j)$  common across  $j$ , so is  $x_t(j) = x_t$
- Dividend

$$d_t(j) = (p_t(j) - w_{Ht})x_t(j) = \frac{1 - \theta}{\theta} w_{Ht} x_t$$

also common across  $j$

- Hence may focus on symmetric equilibrium  $Q_t = q_t(j)$

## Optimal consumption and saving

- Skilled agents indifferent between working and R&D:

$$an_t Q_t = w_{Ht}$$

- Budget constraints of type  $i \in \{H, L\}$

$$\text{Young:} \quad c_{it}^y + Q_t n_{it} = w_{it},$$

$$\text{Old:} \quad c_{i,t+1}^o = (Q_{t+1} + D_{t+1})n_{it}.$$

- Optimal consumption  $c_{it}^y = (1 - \beta)w_{it}$ , so stock demand

$$n_{Ht} = \beta w_{Ht} / Q_t = \beta a n_t,$$

$$n_{Lt} = \beta w_{Lt} / Q_t = (w_{Lt} / w_{Ht}) \beta a n_t.$$

## Market clearing

- Let  $\phi_t \in (0, 1]$  be fraction of skilled agents working
- Market clearing for skilled labor:

$$X_t = n_t x_t = \phi_t H$$

- Fraction  $1 - \phi_t$  engage in R&D, so

$$n_{t+1} = (1 + a(1 - \phi_t)H)n_t$$

- Stock market clearing:

$$\underbrace{n_{t+1}}_{\text{supply}} = \underbrace{Hn_{Ht} + Ln_{Lt}}_{\text{demand}}$$

# Equilibrium

## Proposition

*There exists unique equilibrium. Letting  $g(x) = (F_X/F_L)(x, 1)$ ,  $\phi_t$  solves*

$$\frac{1}{aH} = \phi_t - 1 + \beta + \beta \left[ \theta \frac{A_{Xt}H}{A_{Lt}L} g \left( \frac{A_{Xt}H}{A_{Lt}L} \phi_t \right) \right]^{-1}.$$

*Equilibrium prices are*

*Knowledge-intensive good price:*  $P_t = p_t(j) = F_X A_{Xt},$

*Skilled wage:*  $w_{Ht} = \theta F_X A_{Xt},$

*Unskilled wage:*  $w_{Lt} = F_L A_{Lt},$

*Stock price:*  $Q_t = \frac{w_{Ht}}{an_t} = \frac{\theta}{an_t} F_X A_{Xt},$

*where  $F_X, F_L$  are evaluated at  $(A_{Xt}H\phi_t, A_{Lt}L)$ .*

## Production function and productivities

- Specialize production function and productivities  $(A_{X_t}, A_{L_t})$
- Production function is CES:

$$F(X, L) = (\alpha X^{1-\rho} + (1-\alpha)L^{1-\rho})^{\frac{1}{1-\rho}},$$

where  $\alpha \in (0, 1)$  and  $1/\rho$  is elasticity of substitution

- As before, two states  $z \in \{u, b\}$



# Knowledge spillover

## Assumption

There exist constants  $A_X, A_L > 0$  and  $\xi_u, \xi_b, \lambda_u, \lambda_b \geq 0$  such that

$$(A_{Xt}, A_{Lt}) = (A_X n_t^{\xi_{z_t}}, A_L n_t^{\lambda_{z_t}}).$$

Furthermore,

$$\begin{aligned}\psi &:= (\xi_u - \lambda_u)(\rho - 1) > 0, \\ \lambda_u &> \lambda_b = \xi_b.\end{aligned}$$

- State  $b$  is balanced growth ( $\xi_b = \lambda_b$ )
- Suffices to assume  $1/\rho < 1$  (complement) and  $\xi_u > \lambda_u > \lambda_b$  (spillover stronger in state  $u$  and in knowledge-intensive good sector)

# Equilibrium

- Equilibrium conditions

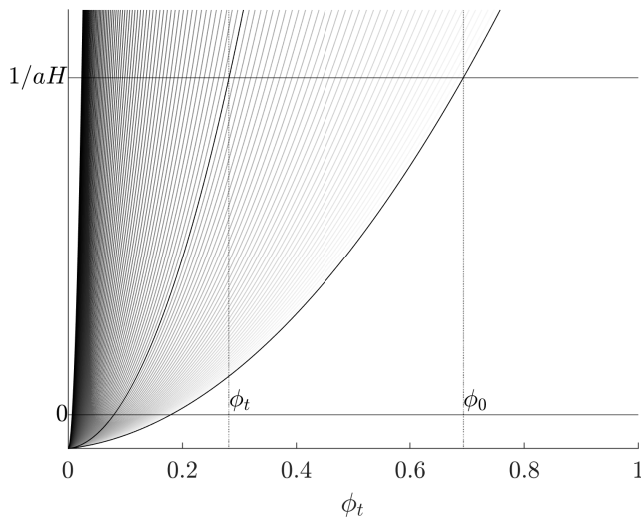
$$u : \quad \frac{1}{aH} = \phi_t - 1 + \beta + \frac{\beta(1-\alpha)}{\theta\alpha} \left( \frac{A_X H}{A_L L} \right)^{\rho-1} n_t^{(\xi_{z_t} - \lambda_{z_t})(\rho-1)} \phi_t^\rho,$$

$$b : \quad \frac{1}{aH} = \phi_b - 1 + \beta + \frac{\beta(1-\alpha)}{\theta\alpha} \left( \frac{A_X H}{A_L L} \right)^{\rho-1} \phi_b^\rho.$$

## Proposition

*Under maintained assumptions, following statements are true.*

- Conditional on staying in state  $u$ ,  $\{\phi_t\}$  monotonically converges to zero and knowledge  $n_t$  asymptotically grows at rate  $G_u := 1 + aH$ .*
- In state  $b$ ,  $\{\phi_t\}$  is constant at  $\phi_b$  and knowledge  $n_t$  grows at rate  $G_b := 1 + a(1 - \phi_b)H < G_u$ .*

Dynamics of  $\phi_t$ 

# Inevitable emergence of stock price bubbles

- Equilibrium dynamics reduces to toy model

## Theorem

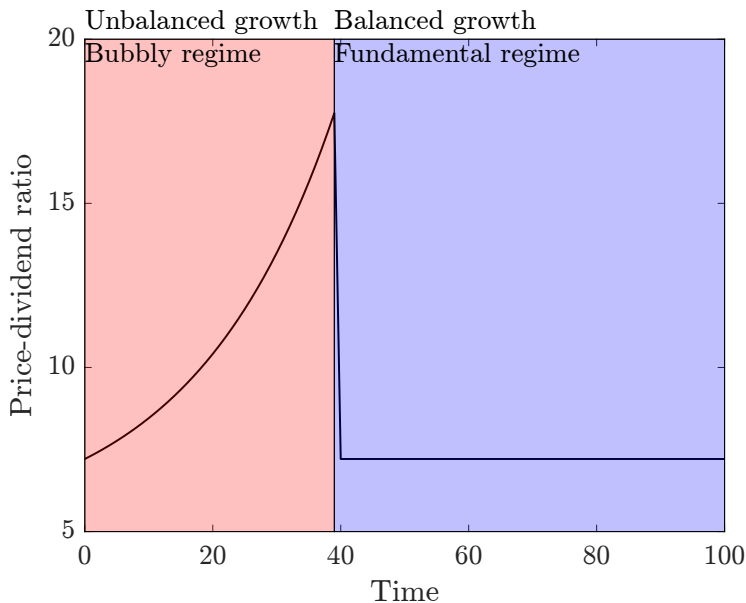
*Suppose production function CES and relative risk aversion is  $\gamma < 1$ . Let  $Q_t$  be stock price in unique equilibrium and  $V_t$  fundamental value. Then*

- 1. In state  $z_t = u$ , stock price exhibits a bubble:  $Q_t > V_t$  and price-dividend ratio  $Q_t/D_t$  grows exponentially.*
- 2. In state  $z_t = b$ , stock price reflects fundamentals:  $Q_t = V_t$  and price-dividend ratio  $Q_t/D_t$  is constant.*

## Intuition and implications

1. Temporary unbalanced technological growth driven by regime switching and some conditions on elasticities necessarily generate stock price bubble
2. Dynamics of price-dividend ratio markedly different:
  - In state  $u$ , exponential growth
  - In state  $b$ , constant
3. Stock price bubble can be understood as temporary deviation from balanced growth; agents willing to buy overpriced stocks despite expecting collapse

# Numerical example



## Implication for skilled wage

- What is effect of stock price bubble?
- Stock price  $Q_t$  pushed above fundamental value  $V_t$
- Indifference condition  $w_{Ht} = an_t Q_t$ , so  $Q_t \uparrow \implies w_{Ht} \uparrow$

### Observation

*The stock market bubble tends to increase the skilled wage.*

## Implication for innovation

- Skilled wage  $w_{Ht} = \theta F_X A_{Xt}$ , where  $F$  evaluated at  $(A_{Xt} H \phi_t, A_{Lt} L)$
- Noting  $F$  concave and  $(A_{Xt}, A_{Lt})$  predetermined,  $w_{Ht} \uparrow \implies \phi_t \downarrow$
- Hence fraction of skilled agents in R&D,  $1 - \phi_t \uparrow$

### Observation

*The stock market bubble tends to promote innovation.*



## Implication for wage inequality

- Unskilled wage  $w_{Lt} = F_L A_{Lt}$ , where  $F$  evaluated at  $(A_{Xt} H \phi_t, A_{Lt} L)$
- Hence  $\phi_t \downarrow \implies w_{Lt} \downarrow$  (fixing  $n_t$ )

### Observation

*The stock market bubble tends to increase the wage gap between skilled and unskilled agents.*

## Implication for short-run output

- Assume state switches from  $u$  to  $b$  (bubble bursts) at  $t$
- After burst, output is

$$Y_t = F(A_X H \phi_b, A_L L) n_t^{\lambda_b} \sim n_t^{\lambda_b}$$

- Before burst, output is

$$Y_{t-1} = F(A_{X,t-1} H \phi_t, A_{L,t-1} L) \sim n_{t-1}^{\lambda_u}$$

- Hence output growth has order of magnitude  $n_t^{\lambda_b - \lambda_u}$

### Observation

*The longer the stock market bubble lasts (with higher  $n_t$ ), the more severe the economic contraction when it bursts.*

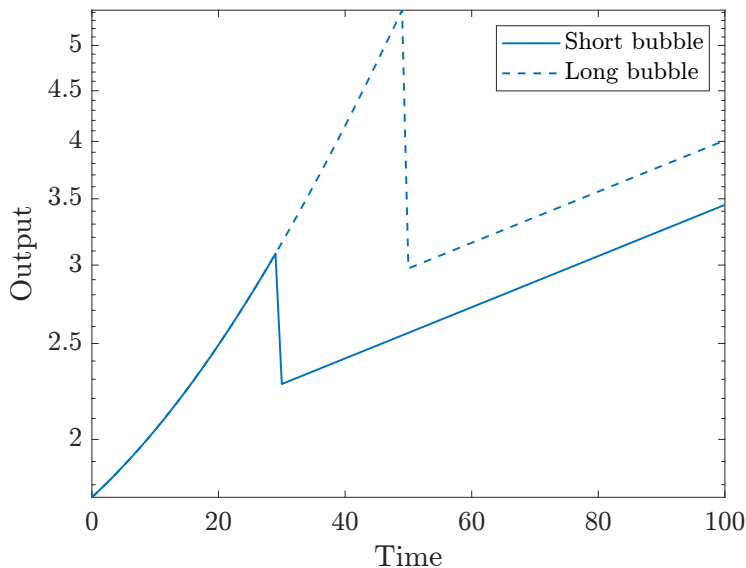
## Implication for long-run output

- In long run, state is  $b$  and output  $Y_t \sim n_t^{\lambda_b}$
- $n_t$  larger (more innovation) the longer bubble lasts

### Observation

*The stock market bubble tends to increase the output in the long run.*

## Dynamics of output



## Implication for long-run wages

- In long run,  $\phi_t = \phi_b$  constant
- Hence relative wage

$$\frac{w_{Ht}}{w_{Lt}} = \theta \frac{F_X}{F_L} \frac{A_X}{A_L}$$

constant

### Observation

*The stock market bubble tends to increase wages in the long run but does not affect the wage gap between skilled and unskilled agents.*

## Balanced growth is knife-edge

- In macro, there is strong presupposition in balanced growth
- But balanced growth is knife-edge (Uzawa (1961) steady state growth theorem)

### Proposition

*Assume only Epstein-Zin utility and neoclassical production function  $F$ . Then price-dividend ratio  $Q_t/D_t$  is constant over time if and only if either relative productivity  $A_{Xt}/A_{Lt}$  is constant or production function  $F$  is Cobb-Douglas.*






*In particular, in our setting, parameters need to satisfy*

$$\psi = (\xi_u - \lambda_u)(\rho - 1) = 0.$$

## Conclusion

- Any balanced growth model is knife-edge theory
- Once we adopt unbalanced growth (here due to uneven technological spillover), asset price bubble becomes necessity
- Tight connection between technological innovation and stock bubble
- Innovation-driven stock bubble has many benefits (e.g., higher long-run output because more innovation) despite inevitable collapse

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