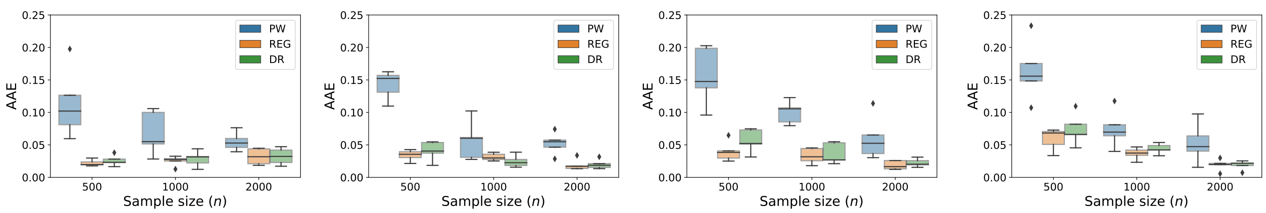
Additional Materials for "Estimators for Bounding the Effect of Policies: A Graphical Approach"

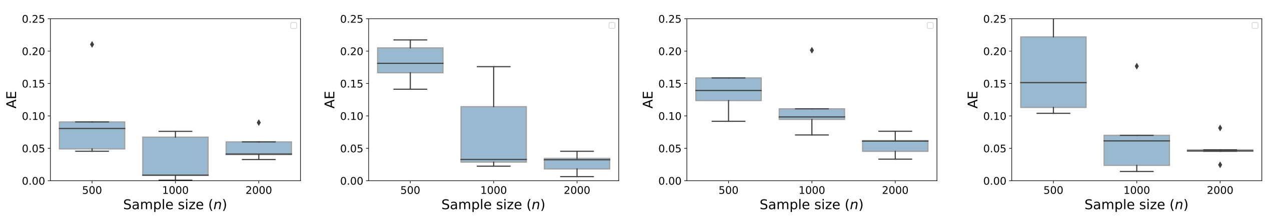
This document makes additional error evaluations for estimators for the effect of policies and atomic interventions and provides graphs to illustrate the notion of partial instrumental sets.

We consider the simulation set-up presented in Sec. 5 with performance evaluated for different dimensionalities of the covariate vector $\boldsymbol{W}$, without misspecification of any of the nuisance functions. For the effect of atomic interventions, we follow Sec. 5.2 for the design of the experiment, while for the effect of policies, we follow Sec. 5.1 for the design of the experiment. See also Appendix C.1 for further data generation details. Performance results are given in Figures 1 and 2 for the effect of atomic interventions and policies, respectively. In each panel, box plots summarize the minimum, the maximum, the sample median, and the first and third quartiles of performance across 10 different dataset seeds. In Figures 1 and 2, we observe overall a mild increase in average error with the dimensionality of $\boldsymbol{W}$ (especially for lower sample sizes), but also find that errors are relatively robust to increasing dimensionality for larger sample sizes. In Figure 1, the increase in error is most pronounced for probability-weighted estimators ("PW") where the nuisance function appears in the denominator, increasing the variance whenever the probability mass function is small.



(a) $\mathrm{Dim}(\boldsymbol{W}) = 5$. (b) $\mathrm{Dim}(\boldsymbol{W}) = 50$. (c) $\mathrm{Dim}(\boldsymbol{W}) = 100$. (d) $\mathrm{Dim}(\boldsymbol{W}) = 200$.

Figure 1: Error comparisons for estimators bounding the effect of atomic interventions (Def. 9) for varying dimensionality of covariate vector $\boldsymbol{W}$.



(a) $\mathrm{Dim}(\boldsymbol{W}) = 5$. (b) $\mathrm{Dim}(\boldsymbol{W}) = 50$. (c) $\mathrm{Dim}(\boldsymbol{W}) = 100$. (d) $\mathrm{Dim}(\boldsymbol{W}) = 200$.

Figure 2: Error evaluations for the estimator for bounding the effect of policies (Def. 6) for varying dimensionality of covariate vector $\boldsymbol{W}$.

(a) $\mathcal{G}$ (b) $\mathcal{G}_{\boldsymbol{\pi}}$

Figure 3: Causal diagrams used to illustrate the definition of partial instrumental sets. In this example, $\boldsymbol{\pi} := (\pi_{X_1}(C), \pi_{X_2}(C))$. A set $\boldsymbol{Z} \subseteq \boldsymbol{V}$ is said to be a partial instrumental set for a policy $\boldsymbol{\pi}$ in $\mathcal{G}$ if $(Y \perp\!\!\!\perp_d \boldsymbol{Z})_{\mathcal{G}_{\boldsymbol{\pi}}}$. To be informative, this set $\boldsymbol{Z}$ should also satisfy Y is not d-separated from $\boldsymbol{Z}$ in $\mathcal{G}$. In this figure, for $\boldsymbol{Z} = \{A\}$, we can establish that Y is not d-separated from A in $\mathcal{G}$ as $\{A \to X \to Y\}$ is an open path in $\mathcal{G}$, and we can establish that $(Y \perp\!\!\!\perp_d A)_{\mathcal{G}_{\boldsymbol{\pi}}}$ as A and Y are disconnected in $\mathcal{G}_{\boldsymbol{\pi}}$.