Undersampled Reconstruction Techniques to Speed up MRI

Claudia Prieto, PhD
Division of Imaging Sciences and Biomedical Engineering
King's College London

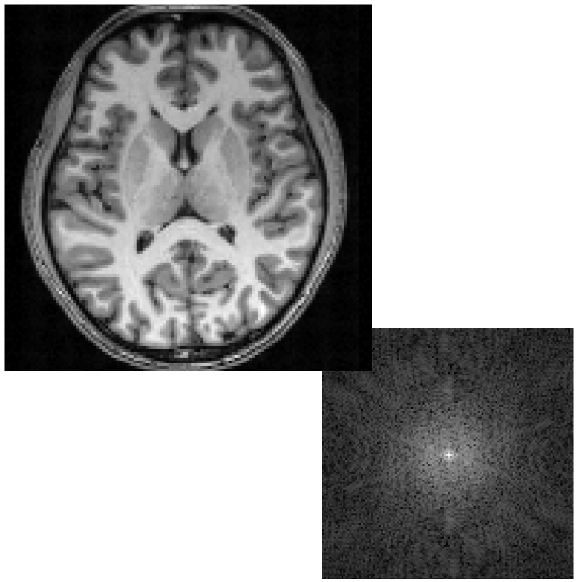
Speeding up the acquisition in MRI

In MRI we need to undersample the data to speed up the acquisition

Why to speed up the MRI acquisition?

- To achieve simultaneous high spatial and temporal resolution
- To reduce motion artefacts
- To provide more comfort to the patients
- To decrease operating costs

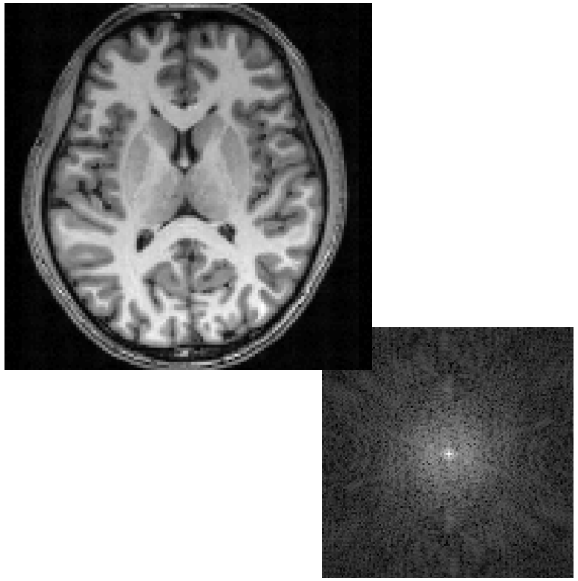
Undersampled reconstruction in MRI



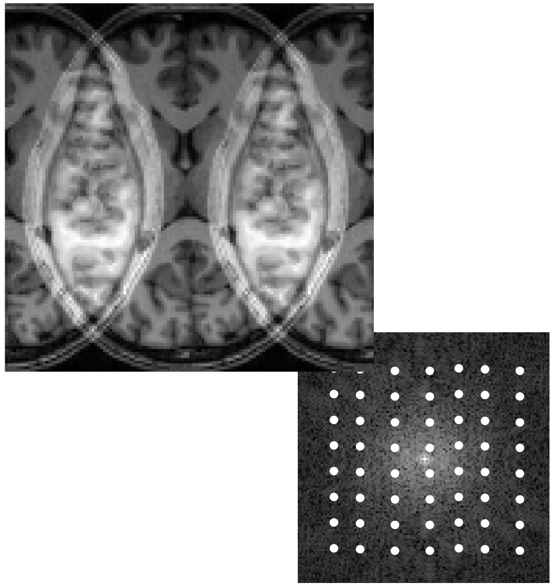
Fully Sampled

Undersampled

Undersampled reconstruction in MRI

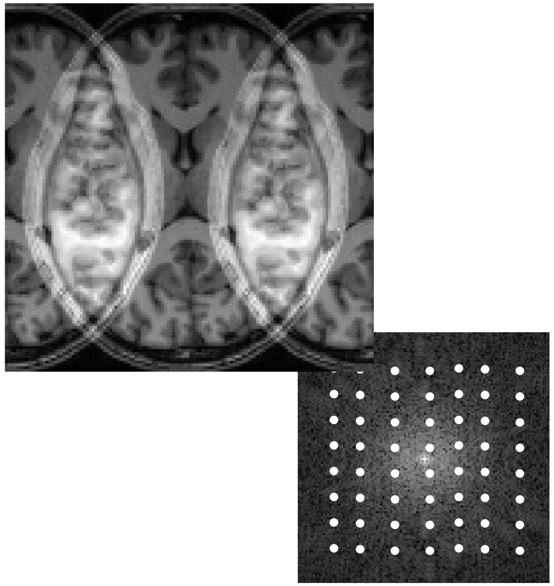


Fully Sampled



Undersampled

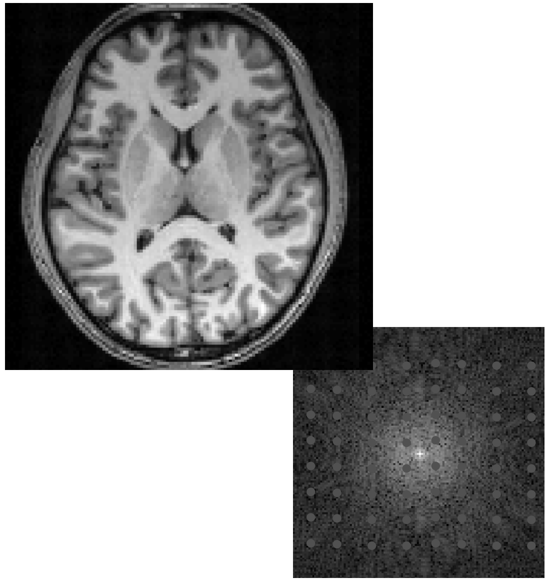
Undersampled reconstruction in MRI



Undersampled data
(fast but aliased)

Constraints
and
assumptions

Reconstruction
Techniques



Unaliased
reconstruction

Contents

Forward model: Signal equation

- Under **ideal conditions** the **MRI signal equation** for a **single receiver** coil is given by:

$$s(t) = \int_V m(\mathbf{r}) e^{-i2\pi \mathbf{k}(t)\cdot\mathbf{r}} \, d\mathbf{r}$$

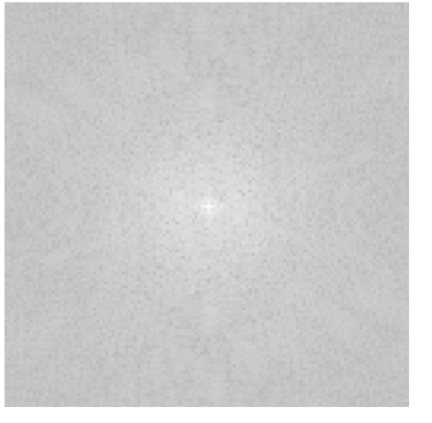
m: transverse magn. of the object
r: spatial position
k: k-space trajectory

During MRI acquisition M noisy samples of this signal are collected

$$b_i = s(t_i) + \varepsilon_i, \quad i = 1, \ldots, M$$

b_i: i^{th} sample in k-space
ε_i: additive complex white Gaussian noise
M: number of measurements (k-space samples)

k-space



Forward model: Signal equation

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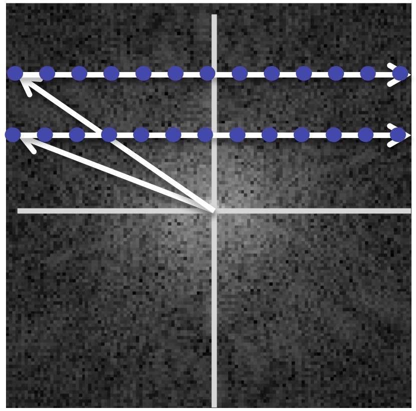
- During MRI acquisition **M noisy samples** of this signal are collected in **k-space**

k-space



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b_i: i^{th} sample in k-space
ε_i: additive complex white Gaussian noise
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Discrete formulation of MRI reconstruction

- Using one-to-one correspondence between **signal equation** and **Fourier transform** :

$$\mathbf{b} = \mathbf{UFx} + \varepsilon$$

$$\boxed{\mathbf{b} = \mathbf{Ex} + \varepsilon}$$

b: vector of sampled data
U: k-space sampling and **F**: FT
x: vector of unknown parameters $\mathbf{x} = (x_1,\ldots,x_N)$
ε: additive Gaussian noise
E: Encoding matrix

Where we use a **discrete approximation of the true object**

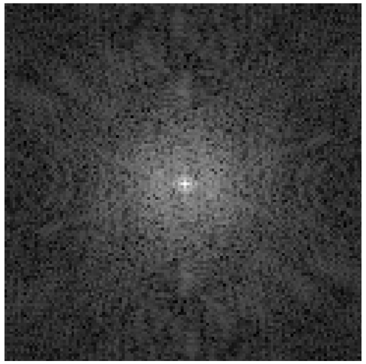
- MRI reconstruction involves the **inversion of the encoding matrix**, which contains the relevant information of the imaging process

Fully sampled MRI reconstruction

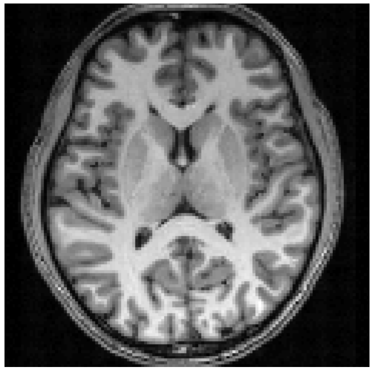
- For fully sampled acquisitions **E** correspond to the **Fourier transform** and the inversion can be evaluated by the **inverse discrete Fourier transform**

$$\mathbf{x} = \mathbf{F}^{-1}\mathbf{b}$$

$$\mathrm{FT}^{-1}\Bigg($$



$$\Bigg) =$$

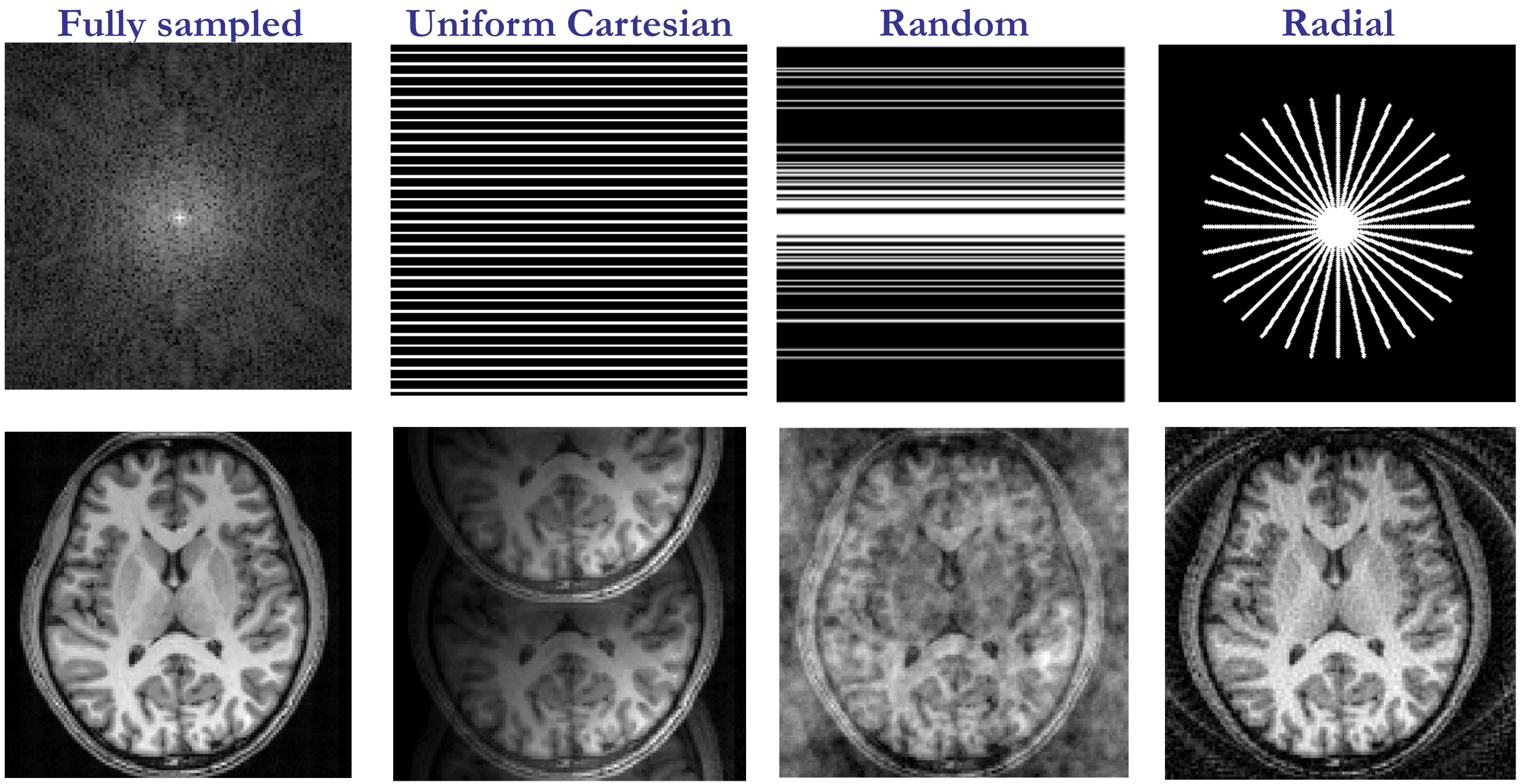


- In more general approaches **E is not well conditioned** and **direct inversion is not possible**

Contents

What happened if we undersample the data?

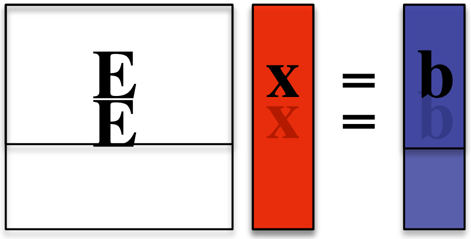
- **Undersampling** in k-space means **aliasing in image domain**



Aliasing appearance depends on how the samples are taken

Undersampled MRI reconstruction

- When undersampling, the linear system becomes underdetermined:



$$M < N$$

- There is an **infinite number of solutions** that satisfy: $\mathbf{b} = \mathbf{E}\mathbf{x}$
- How do we find **x** in this case?

  The least square solution is given by:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\| \mathbf{b} - \mathbf{E}\mathbf{x} \right\|_2^2$$

Undersampled MRI reconstruction

- This leads to the **closed-form solution**:

$$\hat{\mathbf{x}} = (\mathbf{E}^H\mathbf{E})^{-1}\mathbf{E}^H\mathbf{b} \qquad \hat{\mathbf{x}} = \mathbf{E}^{+}\mathbf{b}$$

  where $\mathbf{E}^{+}$ is the Moore-Penrose pseudo inverse of $\mathbf{E}$

- In many cases explicitly computing $\mathbf{E}^{+}$ is expensive, so **iterative optimization methods** are employed

- **E is typically ill-conditioned**: the least square solution is very sensitive to any data perturbation (**noisy measurements**!!)

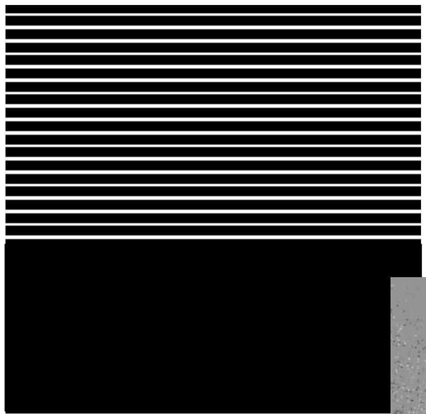
- We need to add **stronger constraints** to solve the undersampled reconstruction

Assumptions and prior information

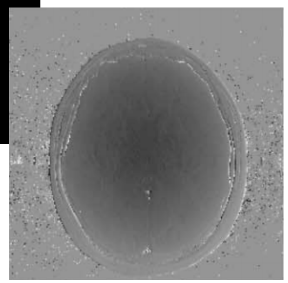
- **Assumptions** or **prior-information** can be incorporated into the reconstruction process to **isolate** or **stabilize the solution**

- Assumptions and prior information in MRI reconstruction:

  - **Phase of the object is smooth**:



Partial k-space

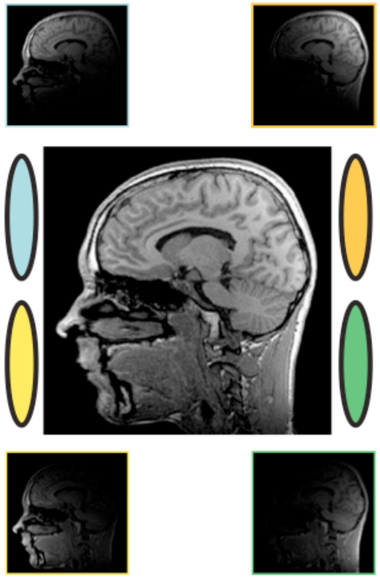


[1]Homodyne reconstruction

[1]Noll et. al, IEEE TMI, 10:154-163, 1991.

Assumptions and prior information

- Assumptions and prior information in MRI reconstruction:
  - **Spatial Encoding:**



[2]SENSE, [3]GRAPPA, [4]SMASH

[2]Pruessmann et. al, MRM, 42:952-962, 1999.

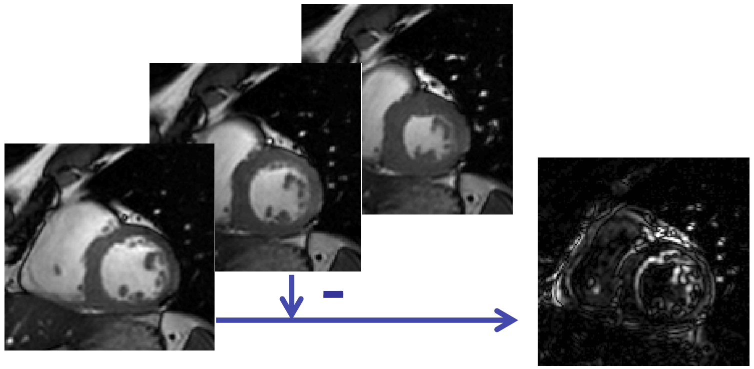
[3]Griswold et. al, MRM, 47:1202-1210, 2002.

[4]Sodickson et. al, MRM, 38:591-603, 1997.

Assumptions and prior information

- Assumptions and prior information in MRI reconstruction:

  - **Temporal correlation:**



[5]Unfold, [6]k-t BLAST, [7]HYPR, etc.

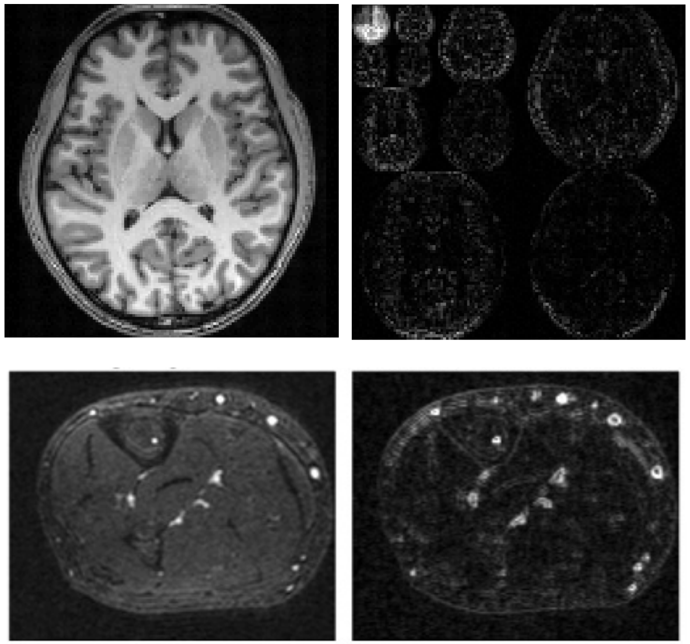
[5]Madore et. al, MRM, 42:813-28, 1999.

[6]Tsao et. al, MRM, 50:1031-42, 2003.

[7]Mistreta et. al,, MRM, 55:30-40, 2006.

Assumptions and prior information

- Assumptions and prior information in MRI reconstruction:
  - **Sparse representation**:



[8]Compressed Sensing

[8]Lustig et. al, MRM, 58:1182-95, 2007.

Constrained and regularization methods

- These assumptions can be incorporated in the reconstruction via a cost function using a **constrained formulation**, that is:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}}\ A(\mathbf{x}) \qquad s.t. \quad \mathbf{b} = \mathbf{E}\mathbf{x}$$

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}}\ A(\mathbf{x}) \qquad s.t. \quad \left\| \mathbf{b} - \mathbf{E}\mathbf{x} \right\|_2^2 \leq \varepsilon$$

- Among all feasible solutions we want to choose the one with the **smallest cost function A for a given assumption**

- The term $\mathbf{b} = \mathbf{E}\mathbf{x}$ or $\left\| \mathbf{b} - \mathbf{E}\mathbf{x} \right\|_2^2 \leq \varepsilon$ is called **data consistency**

Constrained and regularization methods

- Alternatively, we can use the **regularization** approach:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \underbrace{\left\|\mathbf{b} - \mathbf{E}\mathbf{x}\right\|_2^2}_{\text{How well the solution predicts the noisy measurements}} + \underbrace{\lambda A(\mathbf{x})}_{\text{measures the regularity of the solution}}$$

- The regularization parameter $\lambda > 0$ controls the trade-off between data consistency and faithfulness to the optimization criterion $A(\mathbf{x})$

Constrained and regularization methods

Which cost function we could use?

- **Assumption**: extra spatial encoding is provided by **parallel imaging** (acquisition of multiple images in parallel using different coils):

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\| \mathbf{b} - \mathbf{UFSx} \right\|_2^2$$

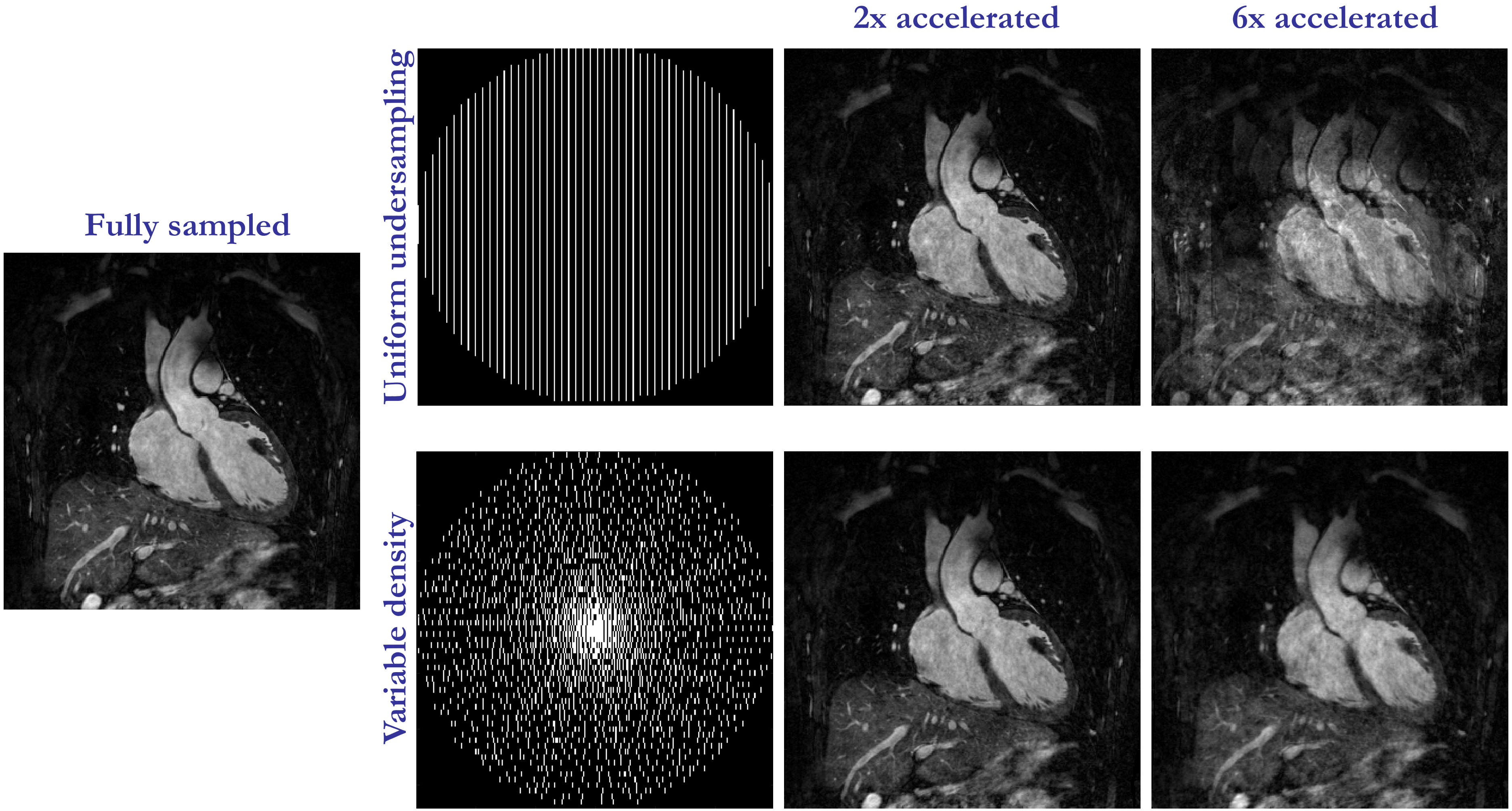
b: sampled data **from all coils**
E: Encoding matrix where **U**: k-space sampling, **F**: FT and **S: coil sensitivities**)
x: unknown parameters

$$\boxed{\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\| \mathbf{b} - \mathbf{Ex} \right\|_2^2}$$

- In general, **direct inversion is not feasible** due to matrix dimensionality. Common solutions: **decoupling encoding matrix**, **iterative optimization**

Constrained and regularization methods

SENSE Reconstruction



Constrained and regularization methods

Which cost function we could use?

- **Assumption**: the reconstructed image is somehow **similar to a reference image** x_0 that is known, e.g. from prior information :

$$\boxed{\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\| \mathbf{b} - \mathbf{E}\mathbf{x} \right\|_2^2 + \lambda \left\| L(\mathbf{x} - \mathbf{x}_0) \right\|_2^2}$$

Which closed-form solution for is given by:

$$\hat{\mathbf{x}} = \left(\lambda L^T L + \mathbf{E}^H \mathbf{E} \right)^{-1} \left(\lambda L^T L \mathbf{x}_0 + \mathbf{E}^H \mathbf{b} \right)$$

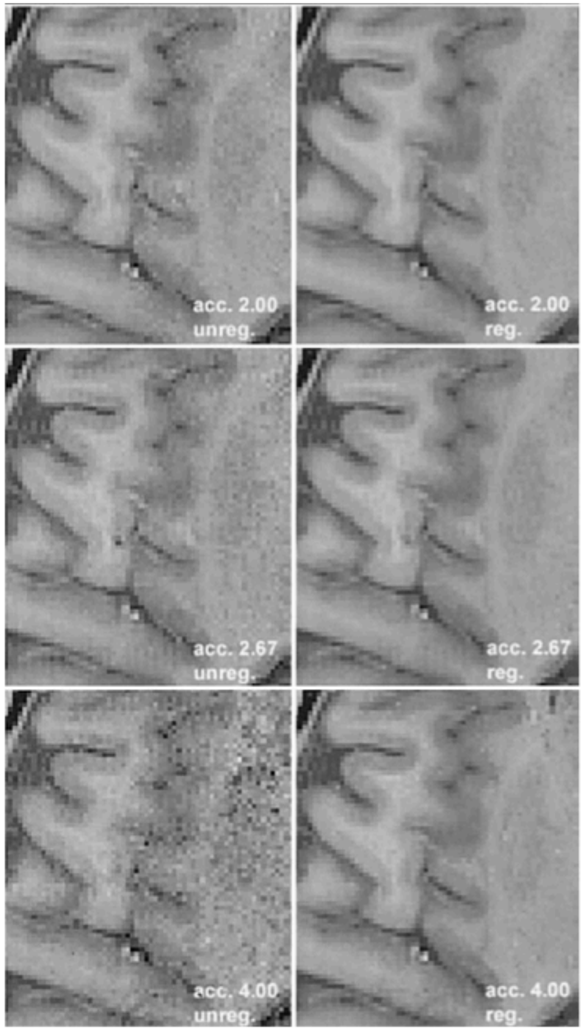
This is known as Tikhonov regularization

Constrained and regularization methods

Regularized SENSE Reconstruction

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\| \mathbf{b} - \mathbf{E}\mathbf{x} \right\|_2^2 + \lambda \left\| \mathbf{x} - \mathbf{x}_0 \right\|_2^2$$

- Reference scan is used as prior information in Thikonov regularization



Constrained and regularization methods

Which cost function we could use?

- **Assumption**: the reconstructed image has **minimum Total Variation**, that means:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\| \mathbf{b} - \mathbf{E}\mathbf{x} \right\|_2^2 + \lambda TV(\mathbf{x})$$

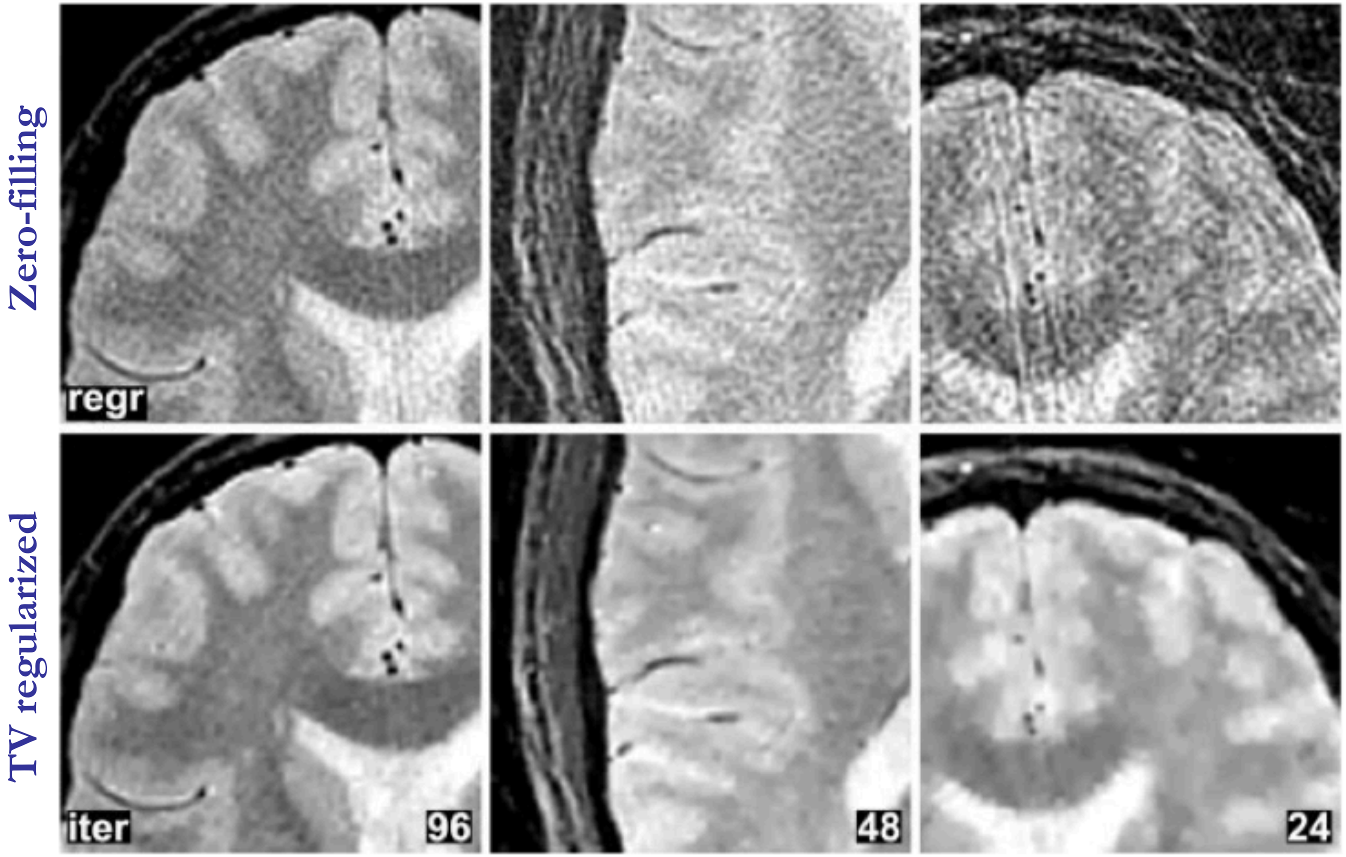
Where TV may be approximated by:

$$TV(\mathbf{x}) = \sum_{i,j} \sqrt{\left| \nabla \mathbf{x} \right|^2 + \varepsilon}$$

Assume piecewise constant or smoothly varying data

Constrained and regularization methods

Iterative image reconstruction using TV constraint



2.6x accelerated 5.33x accelerated 10.6x accelerated

Constrained and regularization methods

Which cost function we could use?

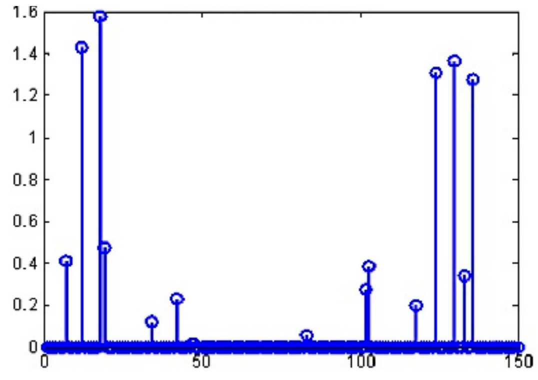
- **Assumption**: the reconstructed image has a **sparse representation** in some domain ψ.

- A signal is called sparse if most of its elements are zero, i.e. if its l_0-"norm" is small:

$$\|\alpha\|_0 = \sum_k |\alpha_k|^0$$

- Regularized reconstruction:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{b} - \mathbf{E}\mathbf{x}\|_2^2 + \lambda \|\Psi\mathbf{x}\|_0$$



Constrained and regularization methods

Which cost function we could use?

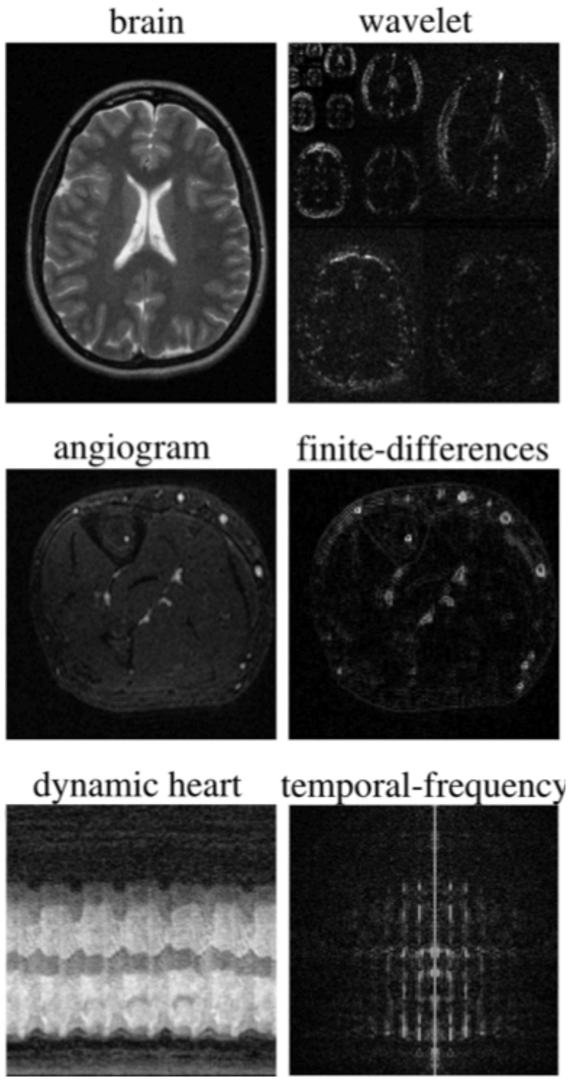
- **Assumption**: the reconstructed image has a **compressible representation** in ψ.
- A signal is called compressible if most of its components are concentrated around zero:

$$\|\alpha\|_1 = \sum_k |\alpha_k| \approx \|\tilde{\alpha}\|_1$$

$\tilde{\alpha}$ retains only the large components of α

- Regularized reconstruction:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{b} - \mathbf{E}\mathbf{x}\|_2^2 + \lambda \|\Psi\mathbf{x}\|_1$$

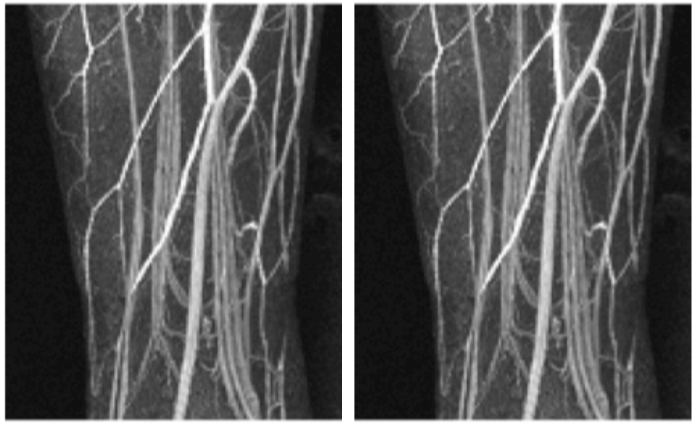


Lustig et al, MRM 2007

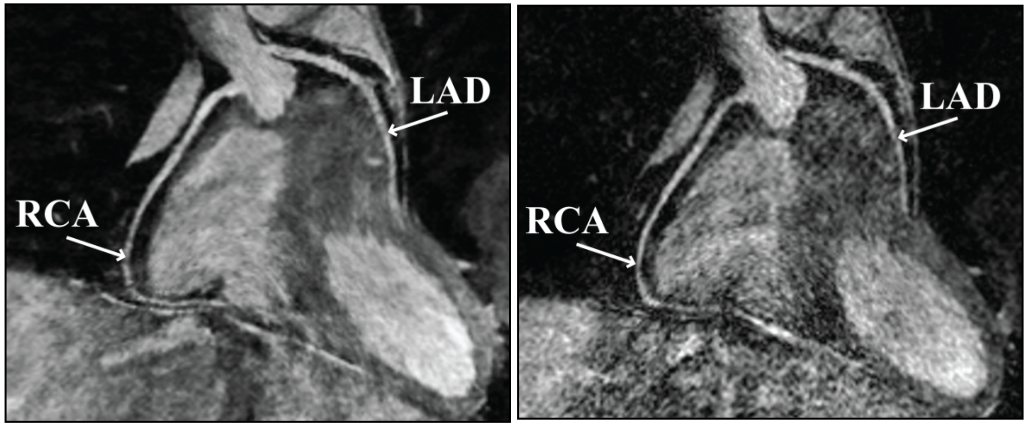
Constrained and regularization methods

Compressed Sensing examples

Fully sampled 10x CS

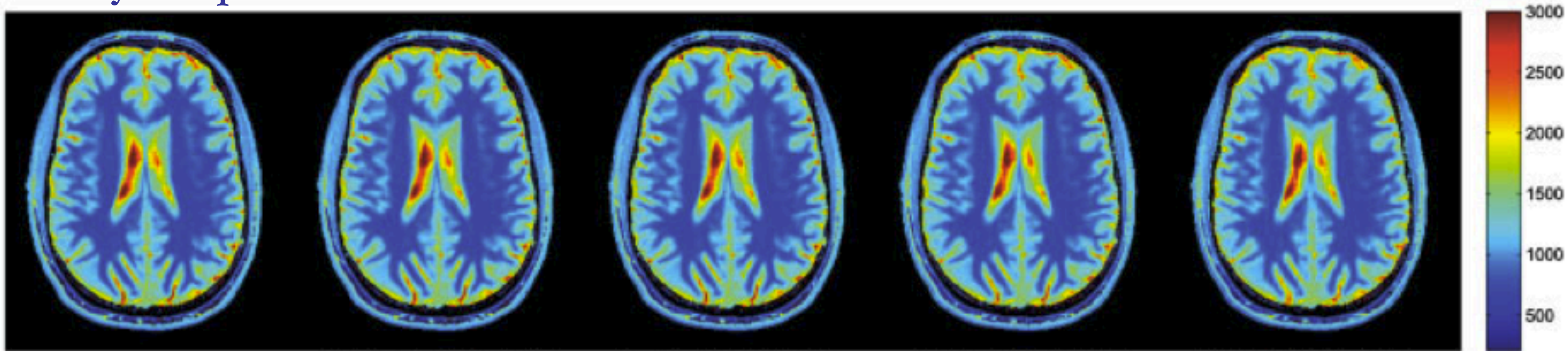


6x CS-SENSE 6x SENSE



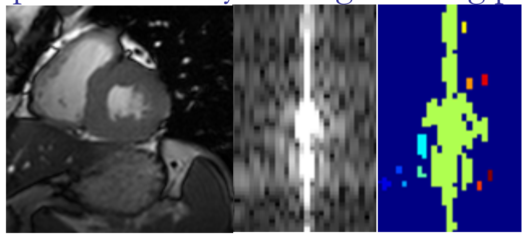
Fully sampled 2x CS 4x CS 6x CS 8x CS

T1 mapping



Constrained and regularization methods

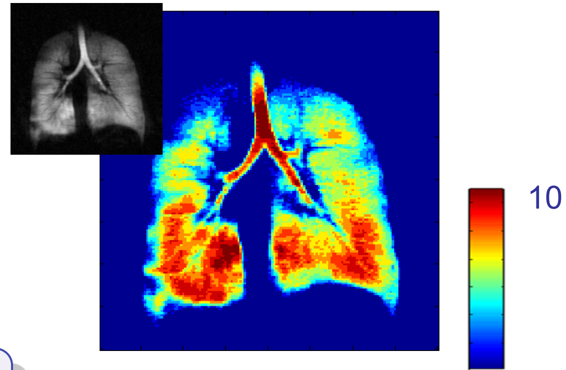
Groups: connectivity of neighbouring pixels



Which cost function we could use?

- **Assumption**: the reconstructed image has a **structured compressible representation**

Groups: **intensity based clustering**



- Components of the sparse domain are in **groups**

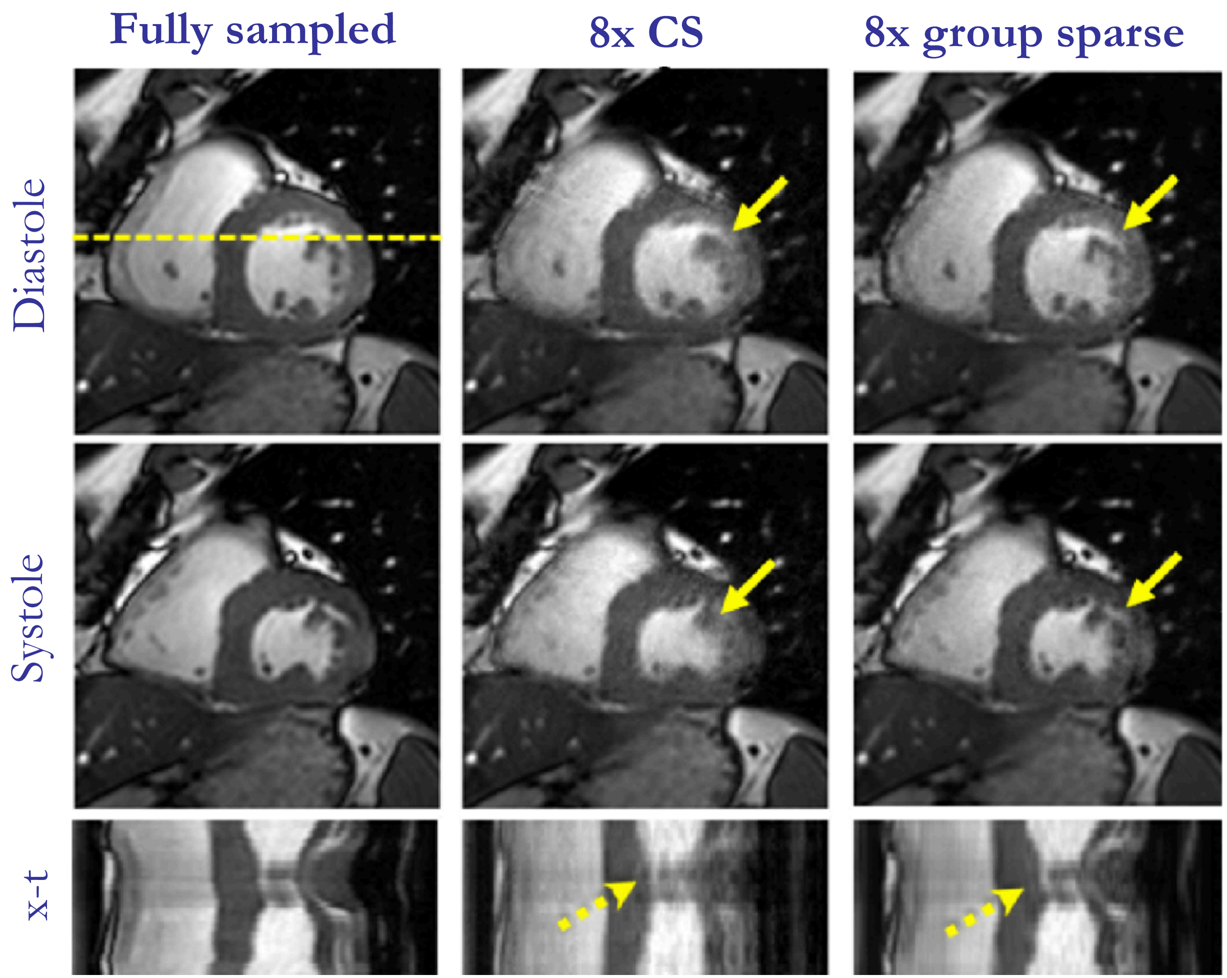
- Group sparse formulation:

$$Min_{\mathbf{x}} \left\| \mathbf{x}^g \right\|_{1,2} = \left\| \mathbf{x}_1^g \right\|_2 + \left\| \mathbf{x}_2^g \right\|_2 + \ldots \left\| \mathbf{x}_k^g \right\|_2$$
$$\text{s.t. } \left\| \mathbf{Ex} - \mathbf{b} \right\|_2 \le \varepsilon$$

x : unknown parameters
$\mathbf{x}^{\mathbf{g}}_j$: elements in j[th] group
$\mathbf{E}_u$: Encoding matrix
b : acquired data

Constrained and regularization methods

Group sparsity for cardiac CINE imaging



Constrained and regularization methods

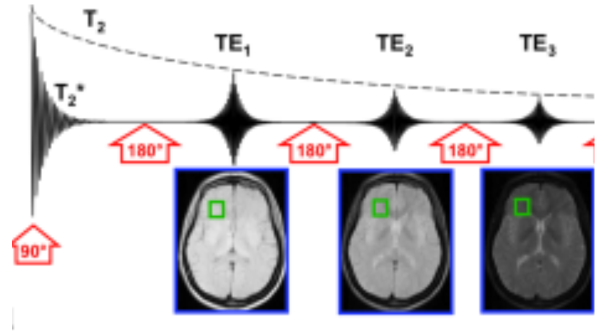
Which cost function we could use?

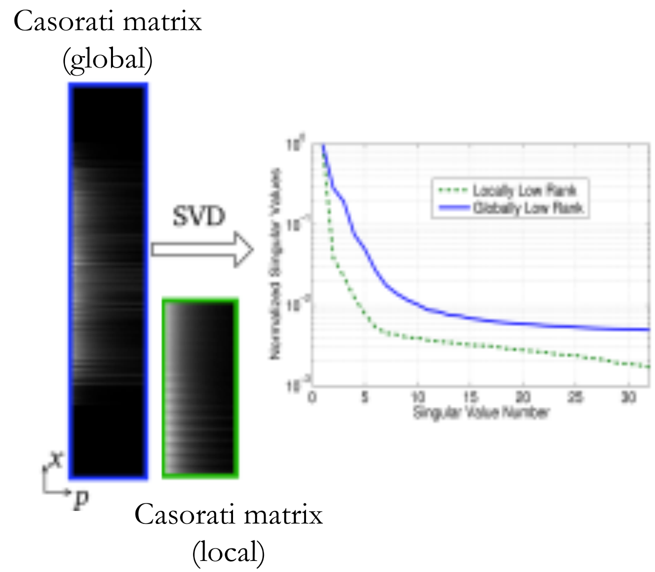
- **Assumption**: the Casorati matrix (**C**) of the reconstructed image(s) is **low-rank**
- **C** can be represented by a few dominant singular values (and vectors), achieved e.g. by minimizing the nuclear norm:

$$\|\mathbf{C}\|_* = \sum_i \sigma_i$$

- Regularized reconstruction:

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \|\mathbf{b} - \mathbf{E}\mathbf{x}\|_2^2 + \lambda \|\mathbf{C}\mathbf{x}\|_*$$

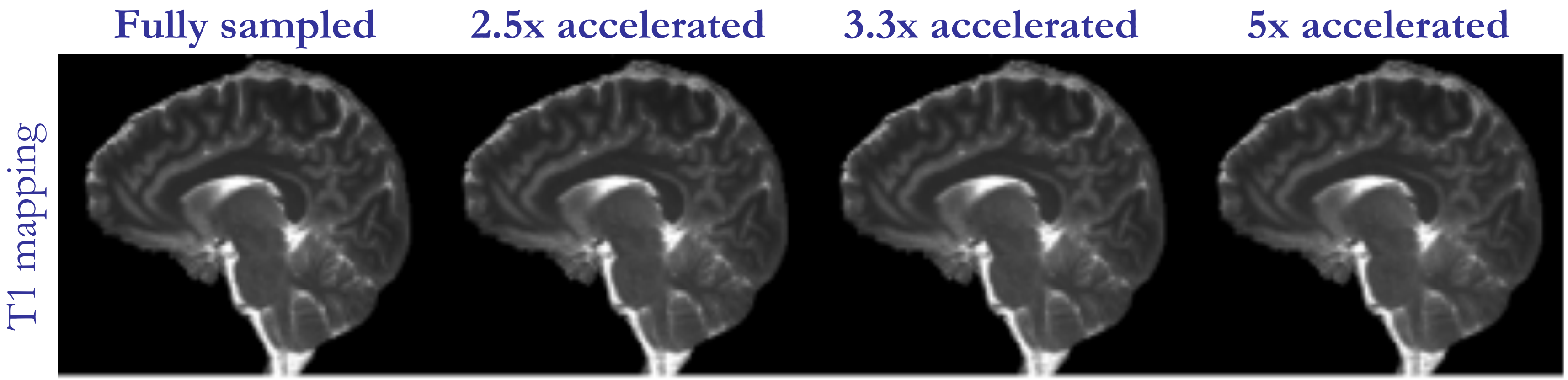


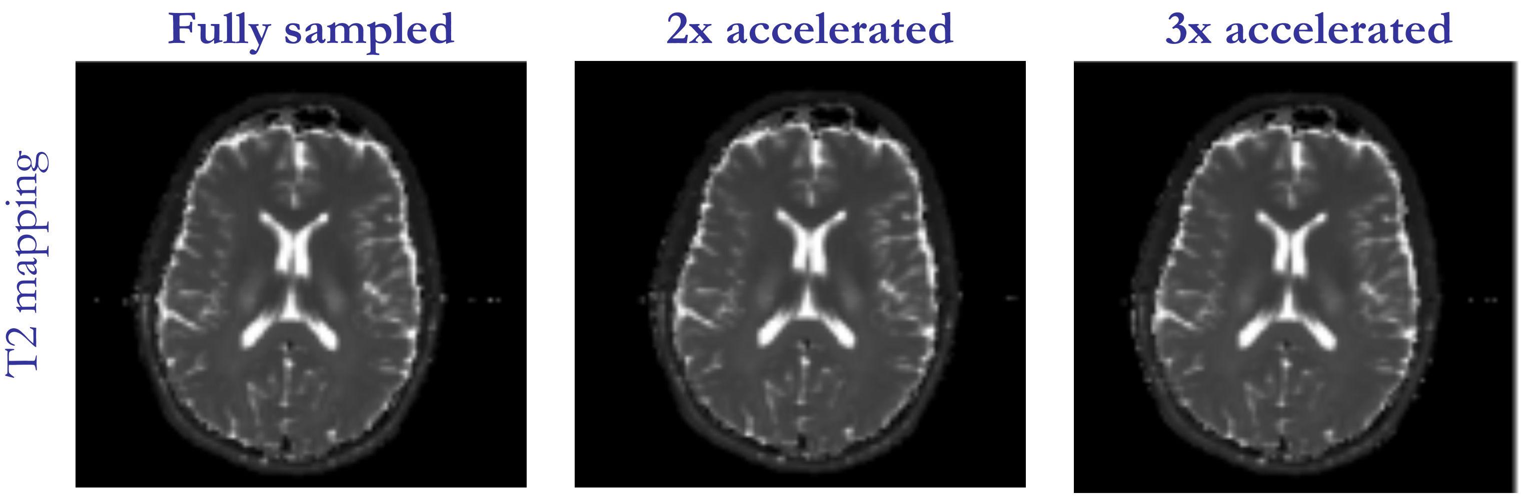


Zhang T et al, MRM 2015

Constrained and regularization methods

Parameter mapping (T1 and T2 map) with locally low rank constraint

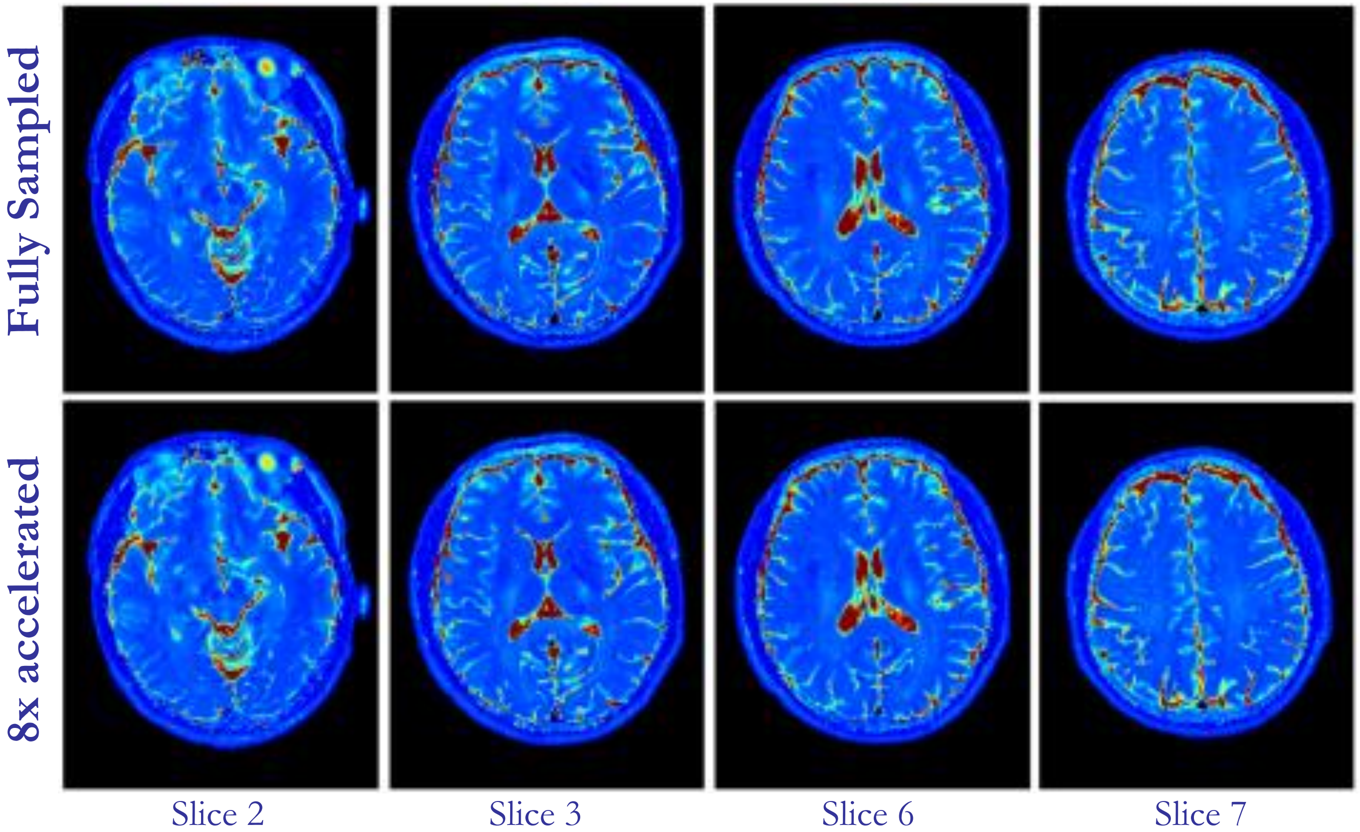




Constrained and regularization methods

Parameter T2 mapping with low rank and sparsity constraints

$$\hat{\mathbf{x}} = \underset{\mathbf{x}}{\operatorname{argmin}} \left\| \mathbf{b} - \mathbf{E}\mathbf{x} \right\|_2^2 + \lambda_1 \left\| \Psi \mathbf{x} \right\|_1 + \lambda_2 \left\| \mathbf{C}\mathbf{x} \right\|_*$$

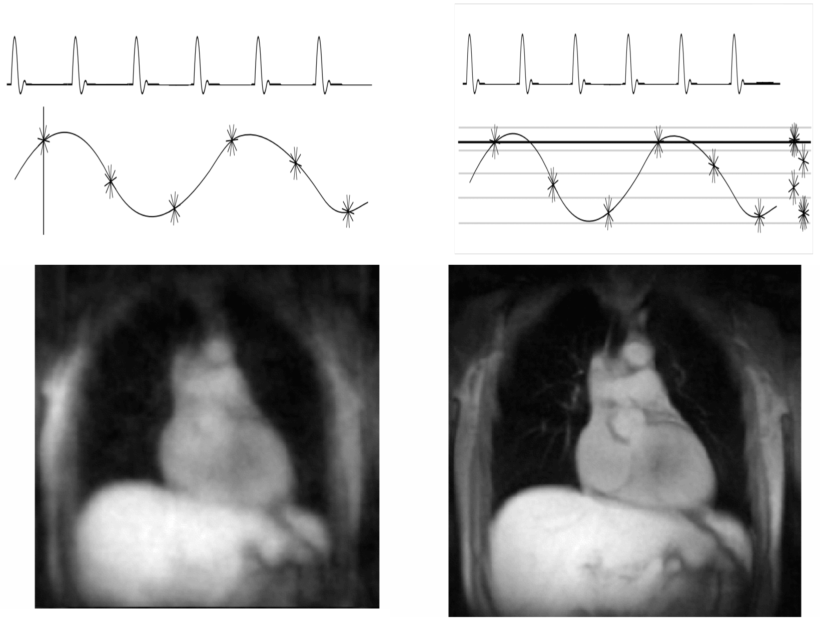


Zhao B et al, "Accelerated MR Parameter Mapping with Low-Rank and Sparsity Constraints", MRM 2015; 74: 489-98

Contents

Motion Corrected Compressed Sensing

$$\hat{\mathbf{x}} = \operatorname{argmin}_{\mathbf{x}} \left\| \mathbf{b} - \sum_t \mathbf{U}_t \mathbf{F} \mathbf{T}_t \mathbf{S} \mathbf{x} \right\|_2 + \lambda \left\| \Psi \mathbf{x} \right\|_1$$



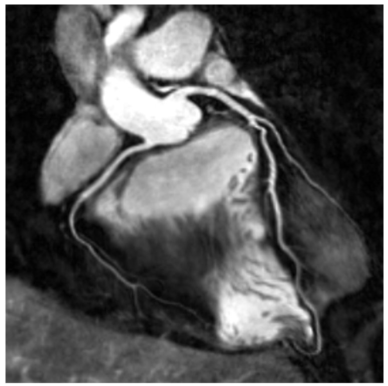
Motion estimation beat-to-beat and/or bin-to-bin from image navigators or data itself

Registration to a ref. position

$$\xrightarrow{\mathbf{T}_t}$$

Non-rigid motion model

Registration/model



MC-CS

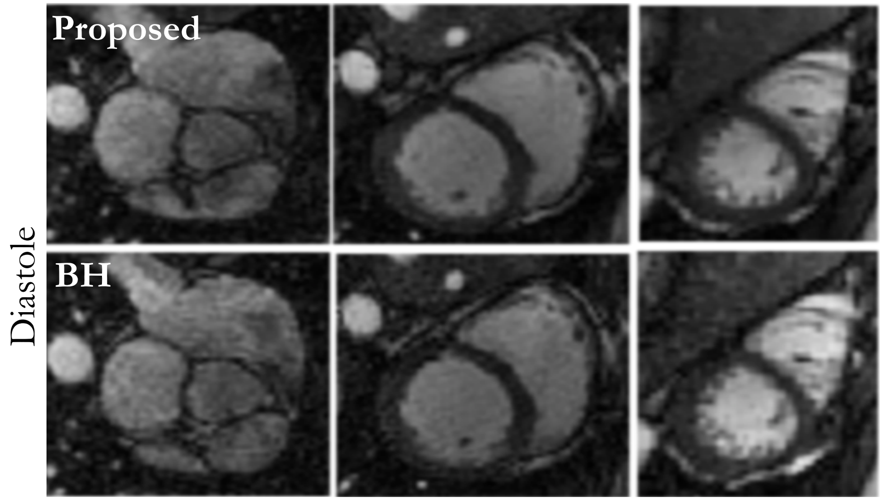
Usman M, Prieto C et al, "Motion corrected compressed sensing for free-breathing dynamic cardiac MRI", MRM 2013; 70: 504-16

M2D and 3D Free-breathing Cine MRI

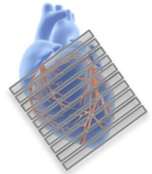
- M2D Cine: requires several breath-holds

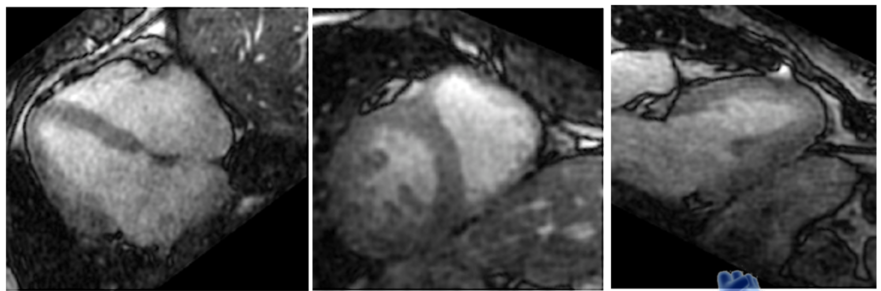
Our approach:

- **2 min M2D free breathing** acquisition
- **4 min** free breathing **3D** isotropic whole heart
- **Non-rigid respiratory motion** estimated from **acquired data**

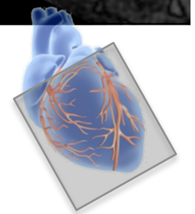


BH ~ 10min, MC-SS ~ 2 min for 12 slices





3D MC-SS ~ 4 min isotropic whole-heart



M2D Single Breath-hold CINE MRI

- M2D Cine: requires several BH

Our approach:

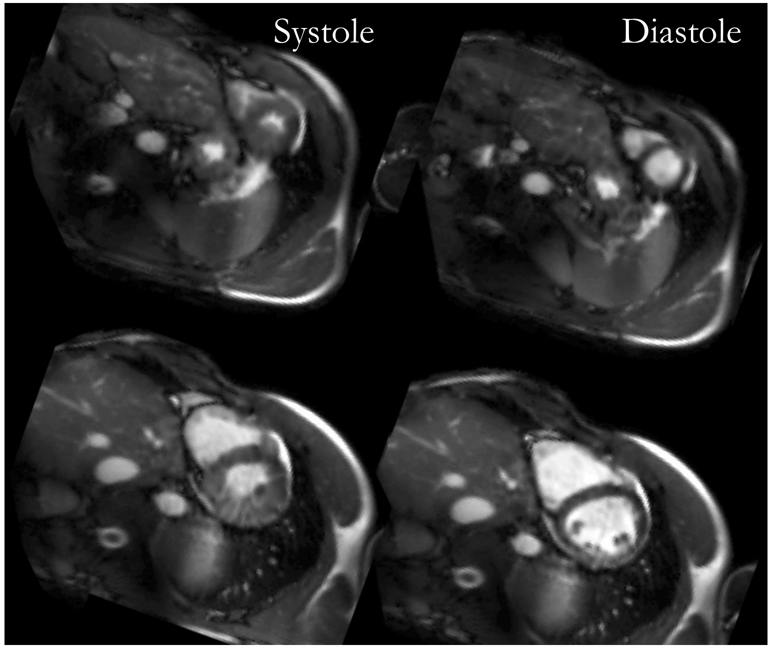
- Non-rigid **cardiac motion** estimation from acquired data
- **Alternate** between **MC-CS** and **motion estimation**

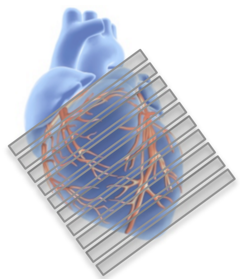
$$\min_\theta H\left(\mathbf{T}_\theta;\mathbf{m}\right)+C(\mathbf{T}_\theta)$$

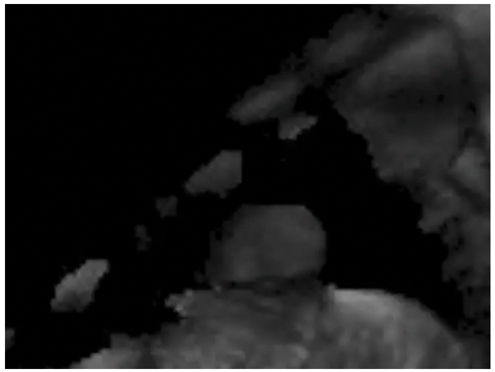
$$\min_\mathbf{m}\left\|\Psi^{-1}\mathbf{T}_\theta\mathbf{m}\right\|_1+\left\|\mathbf{b}-\mathbf{E}\mathbf{m}\right\|_2$$

- **All slices in single BH ~11s**

K-t WiSE 10x accelerated





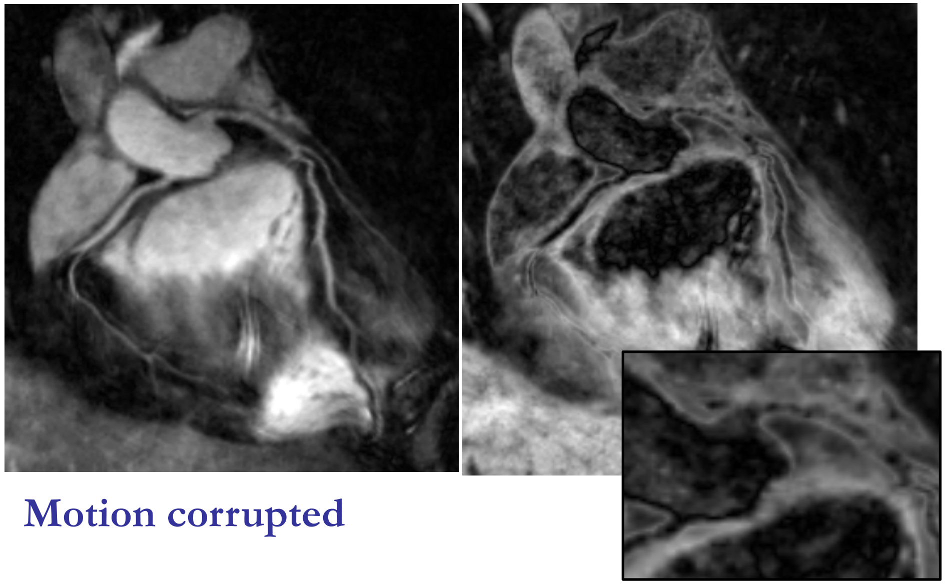


3D Coronary lumen and wall imaging

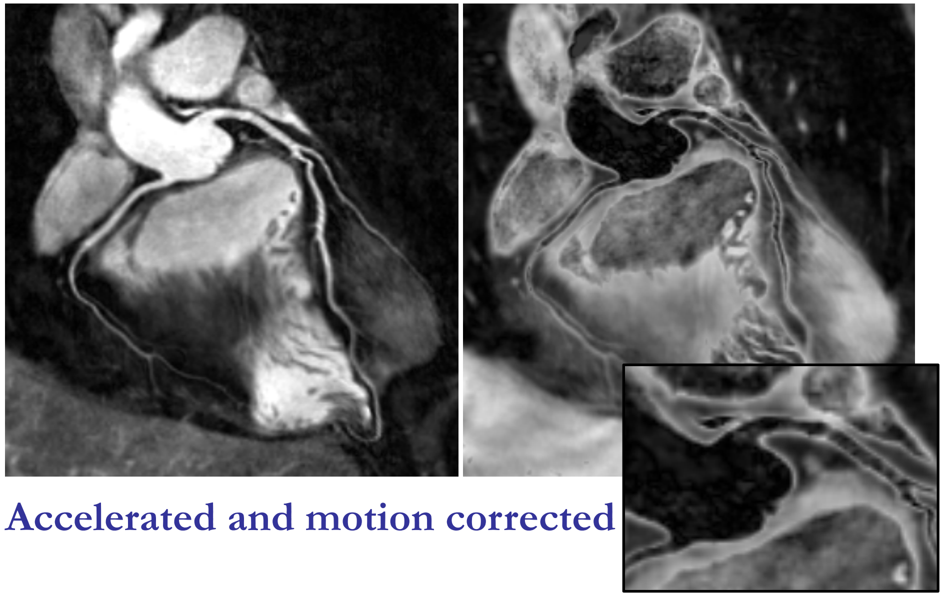
- Independent acquisitions that requires long scan times

Our approach:

- Simultaneous acquisition of co-registered CMRA and vessel wall imaging
- Image navigator acquire at every heart beat
- Non-rigid respiratory motion estimated from data itself



Motion corrupted



Accelerated and motion corrected

Cruz G, Prieto C et al, “Highly efficient nonrigid motion-corrected 3D whole-heart coronary vessel wall imaging ” MRM 2016 doi: 10.1002/mrm.

Summary

Reconstruction Methods for Undersampled MR Data

- Undersampling introduce **aliasing in the image domain**
- Undersampling recon: **linear system becomes underdetermined**, and typically ill-conditioned
- Assumptions or prior-information can be incorporated into the reconstruction process to **constraint or regularize the solution**:
  - Prior image, minimum TV, sparsity/compressibility, low rank
- Motion correction can be incorporated directly in undersampled reconstruction to further accelerate the scan
- Since all MRI reconstructions involve constraints, the question is **which constraints** are appropriate for a given application

KING'S College LONDON

Thanks!!

Claudia Prieto, PhD
claudia.prieto@kcl.ac.uk
https://kclpure.kcl.ac.uk/portal/claudia.prieto.html