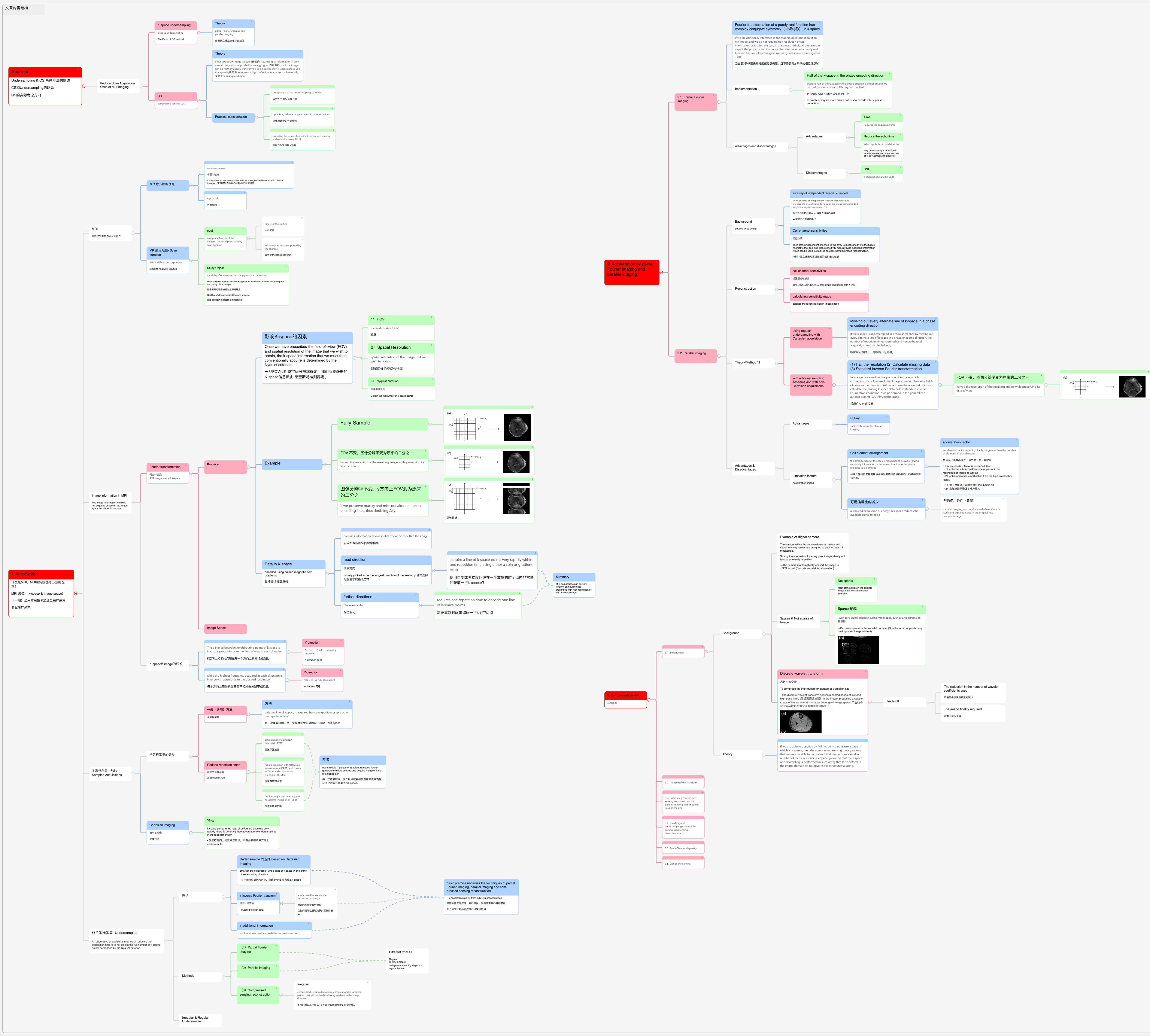
文章内容结构
Abstract
Undersampling & CS 两种方法的概述
CS和Undersampling的联系
CS的实际考虑方向
Reduce Scan Acquisition times of MR imaging
K-space undersampling
k-space undersampling
The Basic of CS method
Theory
partial Fourier imaging and parallel imaging
局部傅立叶成像和平行成像
CS
compressed sensing (CS)
Theory
if our target MR image is sparse稀疏的, having signal information in only a small proportion of pixels (like an angiogram血管造影), or if the image can be mathematically transformed to be sparse then it is possible to use that sparsity稀疏性 to recover a high definition image from substantially 实质上 less acquired data
Practical consideration
designing k-space undersampling schemes
设计K 空间欠采样方案
optimizing adjustable parameters in reconstructions
优化重建中的可调参数
exploiting the power of combined compressed sensing and parallel imaging (CS-PI
利用 CS-PI 的强大功能
1. Introduction
什么是MRI，MRI和传统医疗方法的区别?
MRI 成像 （k-space & Image space）
（一般）全采样采集 &加速全采样采集
非全采样采集
MRI
在医疗中的优点以及局限性
在医疗方面的优点
non-invasiveness
非侵入性的
it is feasible to use quantitative MRI as a longitudinal biomarker in trials of therapy。定量MRI作为纵向生物标记是可行的
repeatable
可重复的
MRI的局限性- Scan duration
'MRI is difficult and expensive
remains relatively unused
cost
cost per volunteer of the imaging (dictated principally by scan duration
nature of the staffing
人员配备
infrastructure costs supported by the charges
收费支持的基础设施成本
Study Object
the ability of study subjects to comply with scan procedure
since subjects have to lie still throughout an acquisition in order not to degrade the quality of the images
图像采集过程中被摄对象保持静止
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Image information in MRI
The image information in MRI is not acquired directly in the image space but rather in k-space
Fourier transformation
傅立叶变换
关联 Image space & k-space
K-space
影响K-space的因素
Once we have prescribed the field-of- view (FOV) and spatial resolution of the image that we wish to obtain, the k-space information that we must then conventionally acquire is determined by the Nyquist criterion
一旦FOV和期望空间分辨率确定，我们所要获得的K-space信息就由 奈奎斯特准则界定。
1：FOV
the field-of- view (FOV)
视野
2：Spatial Resolution
spatial resolution of the image that we wish to obtain
期望图像的空间分辨率
3：Nyquist criterion
奈奎斯特准则
Collect the full number of k-space points
Example
Fully Sample
FOV 不变，图像分辨率变为原来的二分之一
halved the resolution of the resulting image while preserving its field of view
图像分辨率不变，y方向上FOV变为原来的二分之一
if we preserve max ky and miss out alternate phase encoding lines, thus doubling Δky
(a)
(b)
(c)
max k_y
Δk_y
导致翻折
Data in K-space
encoded using pulsed magnetic field gradients
脉冲磁场梯度编码
contains information about spatial frequencies within the image
包含图像内的空间频率信息
read direction
读取方向
usually picked to be the longest direction of the anatomy 通常选择为解刨学的最长方向
acquire a line of k-space points very rapidly within one repetition time using either a spin or gradient echo
使用自旋或者梯度回波在一个重复的时间点内非常快的获取一行k-space点
further directions
Phase encoded
相位编码
requires one repetition time to encode one line of k-space points
需要重复时间来编码一行k个空间点
Summary
MRI acquisitions can be very lengthy, particular those prescribed with high resolution or with wide coverage.
Image Space
K-space和image的联系
The distance between neighbouring points of k-space is inversely proportional to the field of view in each direction
K空间上相邻的点和在每一个方向上的视场成反比
Y-direction
Δk_{y} ∝ 1/(field of view in y direction)
X-direction 同理
while the highest frequency acquired in each direction is inversely proportional to the desired resolution
每个方向上获得的最高频率和所需分辨率成反比
Y-direction
max k_{y} ∝ 1/(y-resolution)
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全采样采集 - Fully Sampled Acquisitions
全采样采集的分类
一般（通用）方法
全采样采集
方法
only one line of k-space is acquired from one gradient or spin echo per repetition time1
每一次重复时间，从一个梯度或者自旋回波中获取一行K-space
Reduce repetition times
加速全采样采集
加速Nyquist-rule
echo planar imaging (EPI) (Mansfield 1977)
回波平面成像
rapid acquisition with relaxation enhancement (RARE, also known as fast or turbo spin echo) (Hennig et al 1986
快速自旋转回波
fast low angle shot imaging and its variants (Haase et al 1986)
快速低角度拍摄
方法
use multiple rf pulses or gradient refocussings to generate multiple echoes and acquire multiple lines of k-space per
每一次重复时间，多个脉冲或者梯度重新聚焦从而生成多个回波并获取多行k-space。
Cartesian imaging
笛卡尔成像
成像方法
特点
k-space points in the read direction are acquired very quickly, there is generally little advantage to undersampling in the read dimension
- 在读取方向上的获取速度快，没有必要在读取方向上undersample
非全采样采集- Undersampled
An alternative or additional method of reducing the acquisition time is to not collect the full number of k-space points demanded by the Nyquist criterion
理论
Under-sample 的选择 based on Cartesian Imaging
omit忽略 the collection of whole lines of k-space in one of the phase encoding directions
- 在一条相位编码方向上，忽略k空间的整条线的k-space
+ inverse Fourier transform
傅立叶逆变换
（Applied to such data)
artefacts will be seen in the reconstructed image
重建的图像中看到伪影
伪影的确切性质取决于欠采样的模式
+ additional information
additional information to stabilise the reconstruction
basic premise underlies the techniques of partial Fourier imaging, parallel imaging and com- pressed sensing reconstruction
—>Acceptable quality from sub-Nyquist acquisition
是部分傅立叶成像，并行成像，压缩感重建的基础前提
部分傅立叶和并行成像已经实现应用
Methods
（1）Partial Fourier imaging
（2）Parallel imaging
（3）Compressed sensing reconstruction
Different from CS
Regular
规则欠采样模式
omit phase encoding steps in a regular fashion
Irregular
compressed sensing demands an irregular under-sampling pattern that will not lead to aliasing artefacts in the image domain.
不规则的欠采样模式—>不会导致图像域中的混叠伪像。
Irregular & Regular Undersample
2. Acceleration by partial Fourier imaging and parallel imaging
2.1 . Partial Fourier imaging
Fourier transformation of a purely real function has complex conjugate symmetry（共轭对称） in k-space
If we are principally interested in the magnitude information of an MR image, and we do not require high resolution phase information, as is often the case in diagnostic radiology, then we can exploit the property that the Fourier transformation of a purely real function has complex conjugate symmetry in k-space (Feinberg et al 1986)
当主要对MR图像的幅度信息感兴趣，且不需要高分辨率的相位信息时
Implementation
Half of the k-space in the phase encoding direction
acquire half of the k-space in the phase encoding direction and we can reduce the number of TRs required twofold
相位编码方向上获取K-space 的一半
In practice, acquire more than a half —>To provide robust phase correction.
Advantages and disadvantages
Advantages
Time
Reduces the acquisition time
Reduce the echo time
When apply this in read direction
may permit a slight reduction in repetition time per phase encode 减少每个相位编码的重复时间
Disadvantages
SNR
a corresponding fall in SNR
2.2. Parallel imaging
Background
phased array design
an array of independent receiver channels
using an array of independent receiver channels could increase the overall signal to noise of the image compared to a single homogeneous volume coil.
单个均匀体积线圈—>一组独立接收器通道
=>增加图片整体信噪比
Coil channel sensitivities
相控阵设计
each of the independent channels in the array is most sensitive to the tissue nearest to that coil, and these sensitivity maps provide additional information which can be used to stabilise an undersampled image reconstruction。
阵列中独立通道对靠近线圈的组织最为敏感
Reconstruction
coil channel sensitivities
线圈通道敏感度
单独获取低分辨率扫描 从而获取线圈通道敏感度的相关信息。
calculating sensitivity maps
stabilise the reconstruction in image space
Theory/Method *2
using regular undersampling with Cartesian acquisition
Missing out every alternate line of k-space in a phase encoding direction
If the k-space is undersampled in a regular manner by missing out every alternate line of k-space in a phase encoding direction, the number of repetition times required (and hence the total acquisition time) can be halved。
相位编码方向上，每相隔一行遗漏。
with arbitrary sampling schemes and with non-Cartesian acquisitions
(1) Half the resolution (2) Calculate missing data (3) Standard inverse Fourier transformation
fully acquire a small central portion of k-space, which corresponds to a low resolution image covering the same field- of- view as the main acquisition, and use the acquired points to calculate the missing k-space data before standard inverse Fourier transformation, as is performed in the generalized autocalibrating (GRAPPA) techniques .
采用广义自动校准
FOV 不变，图像分辨率变为原来的二分之一
halved the resolution of the resulting image while preserving its field of view
(b)
Advantages & Disadvantages
Advantages
Robust
sufficiently robust for clinical imaging
Limitation factors
Acceleration limited
Coil element arrangement
the arrangement of the coil elements has to provide varying sensitivity information in the same direction as the phase encodes to be omitted
线圈元件的布置需要提供会被省略的相位编码方向上的敏感度变化信息。
acceleration factor
acceleration factor cannot typically be greater than the number of elements in that direction
加速因子通常不能大于该方向上的元素数量。
If this acceleration factor is exceeded, then （1）coherent artefact will become apparent in the reconstructed image as well as （2）enhanced noise amplification from the high acceleration factor.
（1）相干伪像会在重构图像中变得非常明显；
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可用信噪比的减少
a reduced acquisition of energy in k-space reduces the available signal to noise
PI的使用条件（背景）
parallel imaging can only be used where there is sufficient signal to noise in the original fully sampled image.
3. Compressed sensing
压缩感测
3.1 . Introduction
Background
Example of digital camera
The sensors within the camera detect an image and signal intensity values are assigned to each of, say, 15 megapixels.
Storing the information for every pixel independently will lead to extremely large files
->The camera mathematically convert the image to JPEG format (Discrete wavelet transformation)
Sparse & Not sparse of Image
Not sparse
Most of the pixels in the original image have non-zero signal intensity.
Sparse 稀疏
(With zero signal intensity.)Some MR images, such as angiograms. 血管造影
->Becomes sparse in the wavelet domain. (Small number of pixels carry the important image content)
(b)
Discrete wavelet transform
离散小波变换
To compress the information for storage at a smaller size.
- The discrete wavelet transform applies a nested series of low and high pass filters (低通高通滤波器）to the image, producing a wavelet space of the same matrix size as the original image space. 产生的小波空间与原始图像空间有相同的矩阵大小。
(a)
(b)
Trade-off
The reduction in the number of wavelet coefficients used
所使用小波系数数量的减少
The image fidelity required
所需图像保真度
Theory
If we are able to describe an MR image in a transform space in which it is sparse, then the compressed sensing theory argues that we may be able to reconstruct that image from a smaller number of measurements in k-space, provided that the k-space undersampling is performed in such a way that the artefacts in the image domain do not give rise to structured aliasing.
3.2. The sparsifying transform
3.3. Combining compressed sensing reconstruction with parallel imaging and/or partial Fourier imaging
3.4. The design of undersampling schemes for compressed sensing reconstruction
3.5. Spatio Temporal sparsity
3.6. Dictionary learning

Abstract

Undersampling & CS 两种方法的概述
CS和Undersampling的联系
CS的实际考虑方向

Reduce Scan Acquisition times of MR imaging

K-space undersampling

k-space undersampling

The Basic of CS method

- Theory

  partial Fourier imaging and parallel imaging

  局部傅立叶成像和平行成像

CS

compressed sensing (CS)

- Theory

  if our target MR image is sparse稀疏的, having signal information in only a small proportion of pixels (like an angiogram血管造影), or if the image can be mathematically transformed to be sparse then it is possible to use that sparsity稀疏性 to recover a high definition image from substantially实质上 less acquired data

Practical consideration

- designing k-space undersampling schemes

  设计K 空间欠采样方案

- optimizing adjustable parameters in reconstructions

  优化重建中的可调参数

- exploiting the power of combined compressed sensing and parallel imaging (CS-PI

  利用 CS-PI 的强大功能

1. Introduction

什么是MRI，MRI和传统医疗方法的区别?
MRI 成像 （k-space & Image space）
（一般）全采样采集 &加速全采样采集
非全采样采集

MRI

在医疗中的优点以及局限性

在医疗方面的优点

- non-invasiveness

  非侵入性的

  it is feasible to use quantitative MRI as a longitudinal biomarker in trials of therapy。定量MRI作为纵向生物标记是可行的

- repeatable
  可重复的

MRI的局限性- Scan duration

'MRI is difficult and expensive

remains relatively unused

cost

cost per volunteer of the imaging (dictated principally by scan duration

- nature of the staffing
  人员配备

- infrastructure costs supported by the charges
  收费支持的基础设施成本

- Study Object
  the ability of study subjects to comply with scan procedure
  since subjects have to lie still throughout an acquisition in order not to degrade the quality of the images
  图像采集过程中被摄对象保持静止
  Hold breath for abdominal/thoracic imaging
  胸腹部影像成像需要被试者屏住呼吸

Image information in MRI

The image information in MRI is not acquired directly in the image space but rather in k-space

Fourier transformation

傅立叶变换
关联 Image space & k-space

K-space

影响K-space的因素

Once we have prescribed the field-of- view (FOV) and spatial resolution of the image that we wish to obtain, the k-space information that we must then conventionally acquire is determined by the Nyquist criterion
一旦FOV和期望空间分辨率确定，我们所要获得的K-space信息就由 奈奎斯特准则界定。

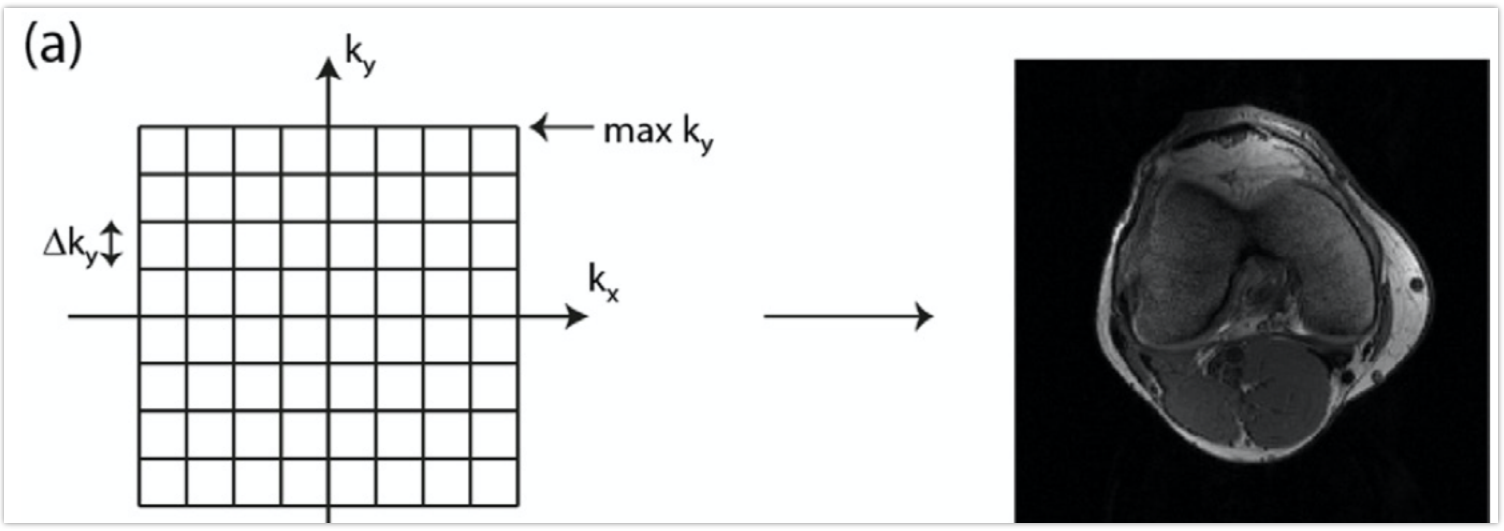
- 1： FOV
  the field-of- view (FOV)
  视野

- 2：Spatial Resolution
  spatial resolution of the image that we wish to obtain
  期望图像的空间分辨率

- 3： Nyquist criterion
  奈奎斯特准则
  Collect the full number of k-space points

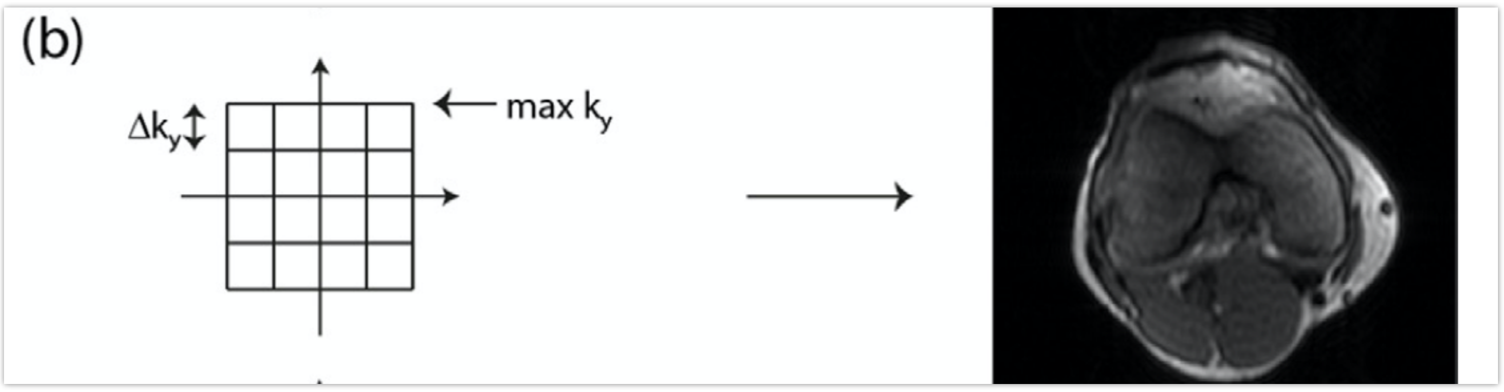
Example

Fully Sample

- 

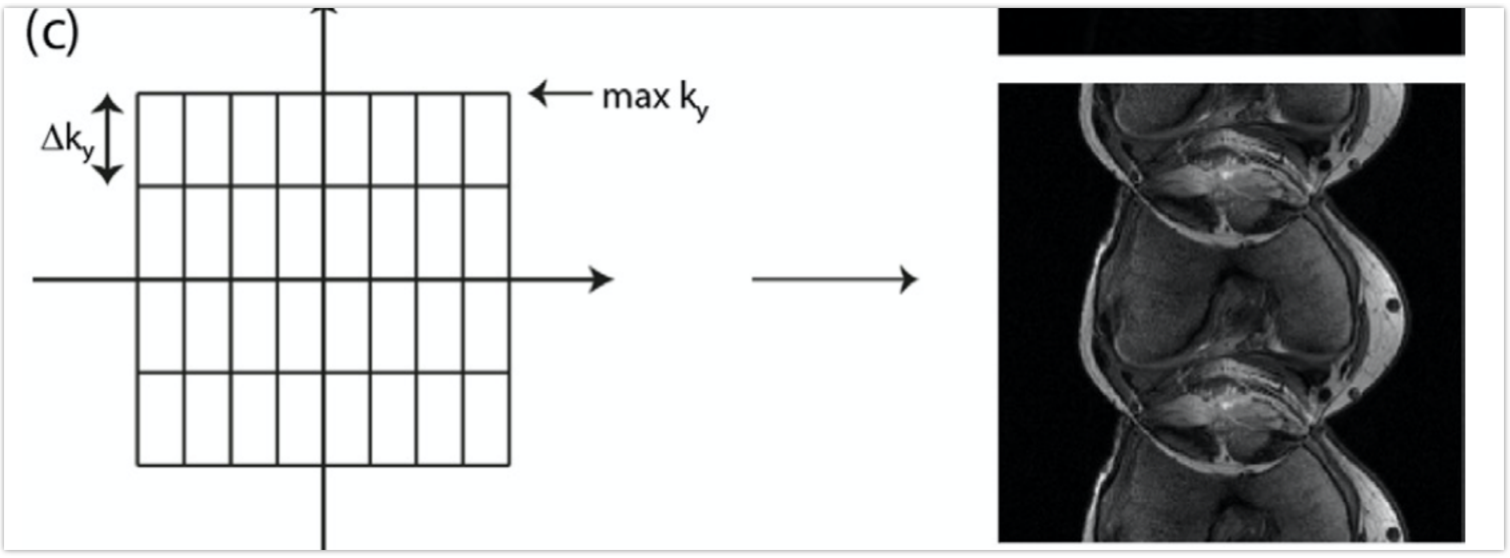
FOV 不变，图像分辨率变为原来的二分之一

> halved the resolution of the resulting image while preserving its field of view

- 

图像分辨率不变，y方向上FOV变为原来的二分之一

> if we preserve max ky and miss out alternate phase encoding lines, thus doubling Δky

- 

  导致翻折

Data in K-space

encoded using pulsed magnetic field gradients
脉冲磁场梯度编码

- > contains information about spatial frequencies within the image

  包含图像内的空间频率信息

read direction

> 读取方向

usually picked to be the longest direction of the anatomy 通常选择为解刨学的最长方向

- > acquire a line of k-space points very rapidly within one repetition time using either a spin or gradient echo

  使用自旋或者梯度回波在一个重复的时间点内非常快的获取一行k-space点

- Summary

  MRI acquisitions can be very lengthy, particular those prescribed with high resolution or with wide coverage.

- further directions

  > Phase encoded

  相位编码

  - > requires one repetition time to encode one line of k-space points

    需要重复时间来编码一行k个空间点

- Image Space

- K-space和image的联系

  - > The distance between neighbouring points of k-space is inversely proportional to the field of view in each direction

    K空间上相邻的点和在每一个方向上的视场成反比

    - Y-direction

      > Δk_{y} α 1/(field of view in y direction)

      X-direction 同理

  - > while the highest frequency acquired in each direction is inversely proportional to the desired resolution

    每个方向上获得的最高频率和所需分辨率成反比

    - Y-direction

      > max k_{y} α 1/(y-resolution)

      x-direction 同理

- 全采样采集 - Fully Sampled Acquisitions

  - 全采样采集的分类

    - 一般（通用）方法

      全采样采集

      - 方法

        > only one line of k-space is acquired from one gradient or spin echo per repetition time1

        每一次重复时间，从一个梯度或者自旋回波中获取一行K-space

    - Reduce repetition times

      加速全采样采集
      加速Nyquist-rule

      - 方法

        use multiple rf pulses or gradient refocussings to generate multiple echoes and acquire multiple lines of k-space per
        每一次重复时间，多个脉冲或者梯度重新聚焦从而生成多个回波并获取多行k-space。

      - > echo planar imaging (EPI) (Mansfield 1977)

        回波平面成像

- rapid acquisition with relaxation enhancement (RARE, also known as fast or turbo spin echo) (Hennig et al 1986 4

  快速自旋转回波

- fast low angle shot imaging and its variants (Haase et al 1986) 4

  快速低角度拍摄

Cartesian imaging 6

笛卡尔成像

成像方法

- 特点

  k-space points in the read direction are acquired very quickly, there is generally little advantage to undersampling in the read dimension
  - 在读取方向上的获取速度快，没有必要在读取方向上undersample

非全采样采集- Undersampled

An alternative or additional method of reducing the acquisition time is to not collect the full number of k-space points demanded by the Nyquist criterion

- basic premise underlies the techniques of partial Fourier imaging, parallel imaging and com- pressed sensing reconstruction

  —>Acceptable quality from sub-Nyquist acquisition
  是部分傅立叶成像，并行成像，压缩感重建的基础前提
  部分傅立叶和并行成像已经实现应用

理论

- Under-sample 的选择 based on Cartesian Imaging

  omit忽略 the collection of whole lines of k-space in one of the phase encoding directions
  - 在一条相位编码方向上，忽略k空间的整条线的k-space

+ inverse Fourier transform 6

傅立叶逆变换

（Applied to such data)

- artefacts will be seen in the reconstructed image 6

  重建的图像中看到伪影
  伪影的确切性质取决于欠采样的模式

- + additional information 6

  additional information to stabilise the reconstruction

Methods

- Different from CS

  Regular
  规则欠采样模式
  omit phase encoding steps in a regular fashion

- （1）Partial Fourier imaging 6

- (2) Parallel imaging

- (3) Compressed sensing reconstruction

  - Irregular

    > compressed sensing demands an irregular under-sampling pattern that will not lead to aliasing artefacts in the image domain.

    不规则的欠采样模式—>不会导致图像域中的混叠伪像。

- Irregular & Regular Undersample

2. Acceleration by partial Fourier imaging and parallel imaging

2.1 . Partial Fourier imaging

- Fourier transformation of a purely real function has complex conjugate symmetry（共轭对称） in k-space

  > If we are principally interested in the magnitude information of an MR image, and we do not require high resolution phase information, as is often the case in diagnostic radiology, then we can exploit the property that the Fourier transformation of a purely real function has complex conjugate symmetry in k-space (Feinberg et al 1986)

  当主要对MR图像的幅度信息感兴趣，且不需要高分辨率的相位信息时

- Implementation

  - Half of the k-space in the phase encoding direction

    > acquire half of the k-space in the phase encoding direction and we can reduce the number of TRs required twofold

    相位编码方向上获取K-space 的一半
    In practice, acquire more than a half —>To provide robust phase correction.

- Advantages and disadvantages

  - Advantages

    - Time

      > Reduces the acquisition time

    - Reduce the echo time

      > When apply this in read direction

      may permit a slight reduction in repetition time per phase encode
      减少每个相位编码的重复时间

  - Disadvantages

    - SNR

      > a corresponding fall in SNR

2.2. Parallel imaging

- Background

  phased array design

- an array of independent receiver channels 7

  > using an array of independent receiver channels could increase the overall signal to noise of the image compared to a single homogeneous volume coil.

  单个均匀体积线圈—>一组独立接收器通道
  =>增加图片整体信噪比

- Coil channel sensitivities 7

  > 相控阵设计

  each of the independent channels in the array is most sensitive to the tissue nearest to that coil, and these sensitivity maps provide additional information which can be used to stabilise an undersampled image reconstruction。

  阵列中独立通道对靠近线圈的组织最为敏感

Reconstruction

- coil channel sensitivities 7

  > 线圈通道敏感度

  单独获取低分辨率扫描 从而获取线圈通道敏感度的相关信息。

- calculating sensitivity maps 7

  stabilise the reconstruction in image space

Theory/Method *2

using regular undersampling with Cartesian acquisition 7

- Missing out every alternate line of k-space in a phase encoding direction 7

  > If the k-space is undersampled in a regular manner by missing out every alternate line of k-space in a phase encoding direction, the number of repetition times required (and hence the total acquisition time) can be halved。

  相位编码方向上，每相隔一行遗漏。

with arbitrary sampling schemes and with non-Cartesian acquisitions 7

(1) Half the resolution (2) Calculate missing data (3) Standard inverse Fourier transformation 7

> fully acquire a small central portion of k-space, which corresponds to a low resolution image covering the same field-of- view as the main acquisition, and use the acquired points to calculate the missing k-space data before standard inverse Fourier transformation, as is performed in the generalized autocalibrating (GRAPPA) techniques .

采用广义自动校准

FOV 不变，图像分辨率变为原来的二分之一 4

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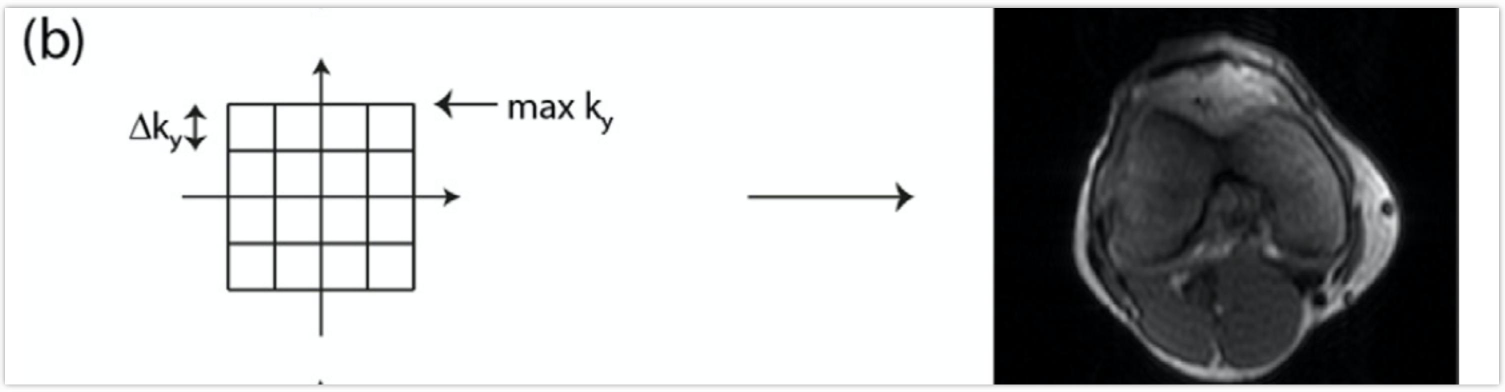
-  4

- Advantages & Disadvantages
  - Advantages
    - Robust 7

      | sufficiently robust for clinical imaging
  - Limitation factors

    Acceleration limited
    - Coil element arrangement 7

      | the arrangement of the coil elements has to provide varying sensitivity information in the same direction as the phase encodes to be omitted

      线圈元件的布置需要提供会被省略的相位编码方向上的敏感度变化信息。
      - acceleration factor 7

        | acceleration factor cannot typically be greater than the number of elements in that direction

        加速因子通常不能大于该方向上的元素数量。

        If this acceleration factor is exceeded, then
        （1）coherent artefact will become apparent in the reconstructed image as well as
        （2）enhanced noise amplification from the high acceleration factor.

        （1）相干伪像会在重构图像中变得非常明显；
        （2）高加速因子增强了噪声放大
    - 可用信噪比的减少 7

      | a reduced acquisition of energy in k-space reduces the available signal to noise
      - PI的使用条件（背景） 7

        | parallel imaging can only be used where there is sufficient signal to noise in the original fully sampled image.

3. Compressed sensing 7

| 压缩感测

3.1 . Introduction 7

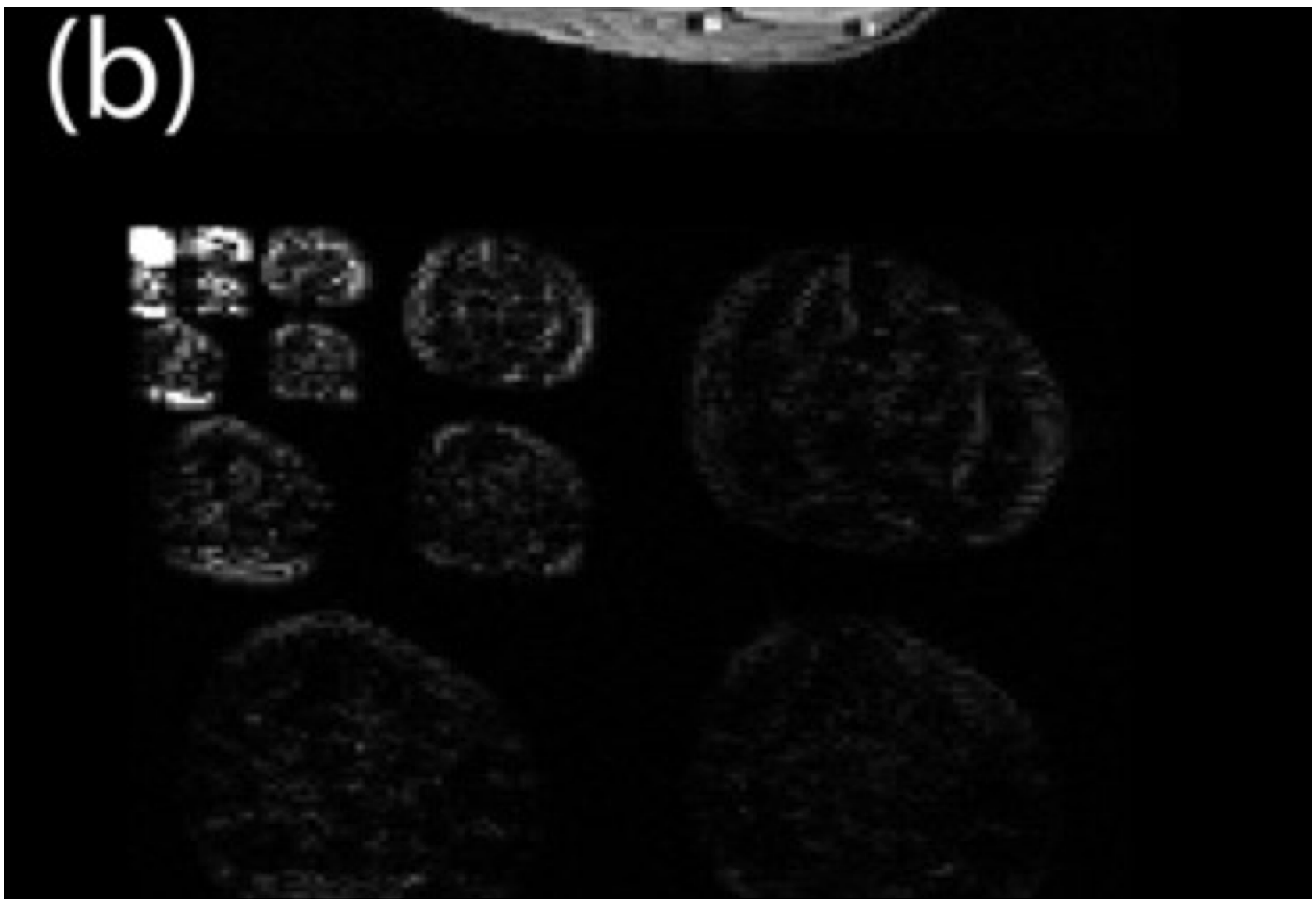
- Background
  - Example of digital camera

    The sensors within the camera detect an image and signal intensity values are assigned to each of, say, 15 megapixels.
    Storing the information for every pixel independently will lead to extremely large files
    ->The camera mathematically convert the image to JPEG format (Discrete wavelet transformation)
  - Sparse & Not sparse of Image
    - Not sparse 8

      Most of the pixels in the original image have non-zero signal intensity.
    - Sparse 稀疏 8

      | (With zero signal intensity.)Some MR images, such as angiograms. 血管造影

      ->Becomes sparse in the wavelet domain. (Small number of pixels carry the important image content)

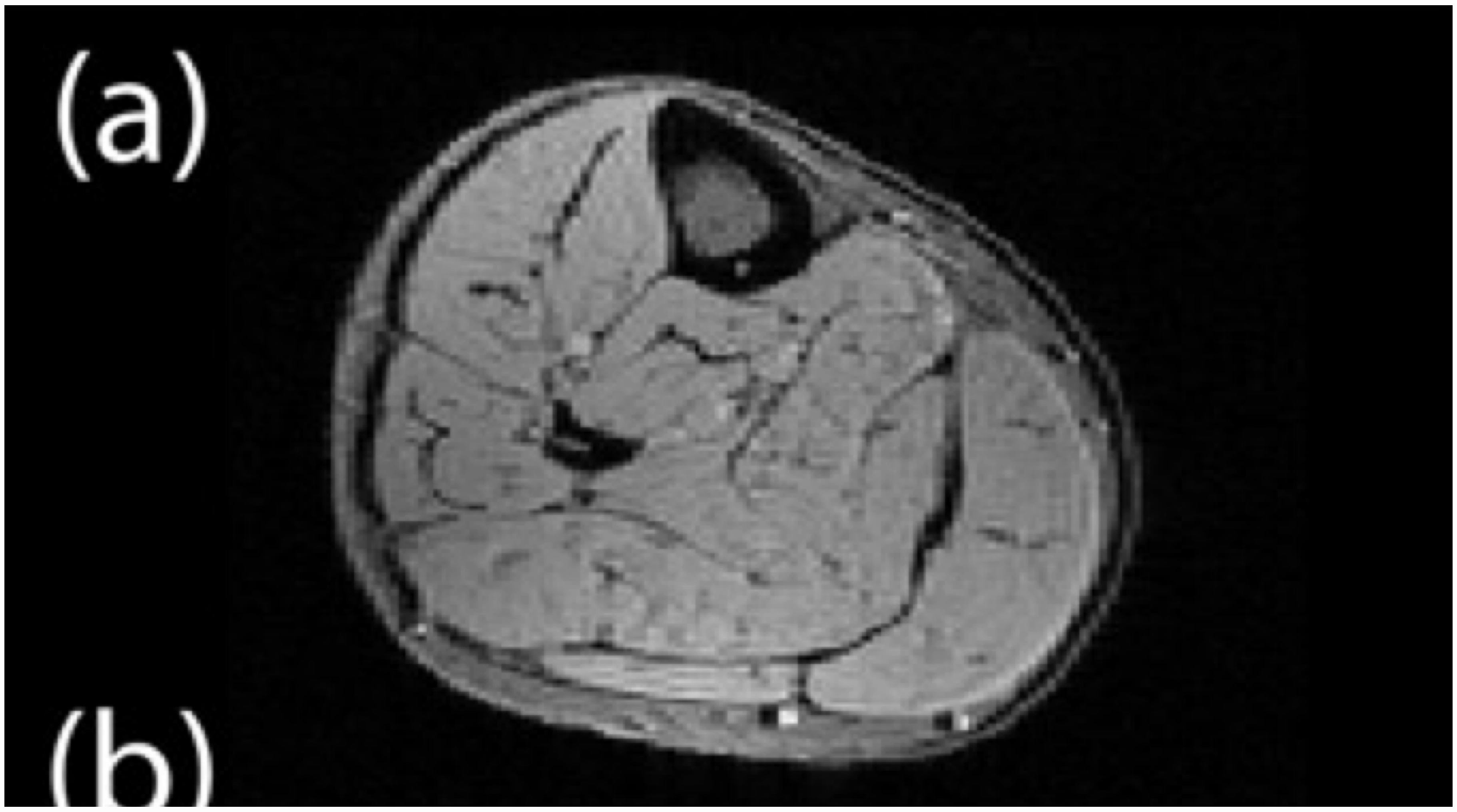
      

Discrete wavelet transform 8

离散小波变换

To compress the information for storage at a smaller size.

- The discrete wavelet transform applies a nested series of low and high pass filters (低通高通滤波器）to the image, producing a wavelet space of the same matrix size as the original image space. 产生的小波空间与原始图像空间有相同的矩阵大小。



Trade-off

- The reduction in the number of wavelet coefficients used

  所使用小波系数数量的减少

- The image fidelity required

  所需图像保真度

Theory

- If we are able to describe an MR image in a transform space in which it is sparse, then the compressed sensing theory argues that we may be able to reconstruct that image from a smaller number of measurements in k-space, provided that the k-space undersampling is performed in such a way that the artefacts in the image domain do not give rise to structured aliasing. 8

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OPEN ACCESS
IOP Publishing | Institute of Physics and Engineering in Medicine Physics in Medicine & Biology
Phys. Med. Biol. 60 (2015) R297–322 doi:10.1088/0031-9155/60/21/R297

Topical Review

Reducing acquisition time in clinical MRI by data undersampling and compressed sensing reconstruction

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Received 18 June 2015, revised 8 September 2015
Accepted for publication 11 September 2015
Published 8 October 2015

CrossMark

Abstract

MRI is often the most sensitive or appropriate technique for important measurements in clinical diagnosis and research, but lengthy acquisition times limit its use due to cost and considerations of patient comfort and compliance. Once an image field of view and resolution is chosen, the minimum scan acquisition time is normally fixed by the amount of raw data that must be acquired to meet the Nyquist criteria. Recently, there has been research interest in using the theory of compressed sensing (CS) in MR imaging to reduce scan acquisition times. The theory argues that if our target MR image is sparse, having signal information in only a small proportion of pixels (like an angiogram), or if the image can be mathematically transformed to be sparse then it is possible to use that sparsity to recover a high definition image from substantially less acquired data. This review starts by considering methods of *k*-space undersampling which have already been incorporated into routine clinical imaging (partial Fourier imaging and parallel imaging), and then explains the basis of using compressed sensing in MRI. The practical considerations of applying CS to MRI acquisitions are discussed, such as designing *k*-space undersampling schemes, optimizing adjustable parameters in reconstructions and exploiting the power of combined compressed sensing and parallel imaging (CS-PI). A selection of clinical applications that have used CS and CS-PI prospectively are considered. The review concludes by signposting other imaging acceleration techniques under present development before concluding with a consideration of the potential impact and obstacles to bringing compressed sensing into routine use in clinical MRI.

1. Abstract
Undersampling & CS 两种方法的概述
CS和Undersampling的联系
CS的实际考虑方向

2. CS

3. Theory

4. K-space undersampling
The Basic of CS method

5. Theory
局部傅立叶成像和平行成像

6. 设计K 空间欠采样方案

7. 优化重建中的可调参数

8. 利用 CS-PI 的强大功能

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Keywords: compressed sensing, acceleration, undersampling, clinical trial, MRI, hepatic steatosis, muscular dystrophy

(Some figures may appear in colour only in the online journal)

1. Introduction

MRI has revolutionized clinical medical imaging and transformed medical research by providing repeatable, non-invasive measurements of tissue structure and function. MRI is uniquely flexible as the sensitivity of the image to tissue properties can be extensively varied: this can be done by altering the timing with which MR signals are collected (the echo time, TE and repetition time, TR), or by the use of magnetization preparation or contrast agents. These techniques provide contrast in standard anatomical MRI, but can also allow measurement of function in both clinical and research settings in most of the tissues and organs of the human body. Such measurements have reduced the need for invasive techniques such as tissue biopsy or radionuclide studies. This non-invasiveness means that it is feasible to use quantitative MRI as a longitudinal biomarker in trials of therapy (Hollingsworth 2014, Macauley *et al* 2015). MR methods have come to represent gold standards of measurement for clinical research and yet the modality remains relatively unused. Why? A common refrain is that 'MRI is difficult and expensive'.

In MRI research studies, the critical factors are the cost per volunteer of the imaging (dictated principally by scan duration) and the ability of study subjects to comply with scan procedure: this is influenced by scan duration, since subjects have to lie still throughout an acquisition in order not to degrade the quality of the images. Additionally, subjects must hold their breath for abdominal/thoracic imaging, which may be difficult for children, obese individuals and those with respiratory compromise. In designing clinical study protocols, a balance must be achieved between the physiological parameters that could be measured, the demands made of the patient and the cost of the scan session. The cost of MRI can vary widely, depending on the nature of the staffing and infrastructure costs supported by the charges (Fletcher *et al* 1999, Lewis *et al* 2015). In the UK the cost of performing MRI research in a university teaching hospital is typically in the range of £350–£500 per hour of scanner occupation, depending on the staffing and infrastructure costs supported. Throughout the history of clinical MRI there have been progressive improvements in image quality and functional measurement together with reductions in scan duration, through ingenuity in hardware and software development.

The image information in MRI is not acquired directly in the image space but rather in *k*-space, which contains information about spatial frequencies within the image. The image space and *k*-space are related by Fourier transformation. Once we have prescribed the field-of-view (FOV) and spatial resolution of the image that we wish to obtain, the *k*-space information that we must then conventionally acquire is determined by the Nyquist criterion (Nyquist 1928). The distance between neighbouring points of *k*-space is inversely proportional to the field of view in each direction (figure 1), while the highest frequency acquired in each direction is inversely proportional to the desired resolution. The *k*-space is filled by acquiring data which are encoded using pulsed magnetic field gradients. In one direction, known as the read direction (usually picked to be the longest direction of the anatomy, in figure 2 right–left), we can acquire a line of *k*-space points very rapidly within one repetition time using either a spin or gradient echo. However, further directions have to be phase encoded (in figure 2, the anterior–posterior and foot-head directions), which requires one repetition time to encode one line of *k*-space points. This has then to be repeated for every combination of the number of phase encoding steps required in the anterior–posterior and the foot-head directions. Therefore, MRI acquisitions can be lengthy, particularly those prescribed with high resolution or with wide coverage.

1 1. Introduction
什么是MRI，MRI和传统医疗方法的区别?
MRI 成像 （k-space & Image space）
（一般）全采样采集 &加速全采样采集

2 可重复的

3 非侵入性的
it is feasible to use quantitative MRI as a longitudinal biomarker in trials of therapy。定量MRI作为纵向生物标记是可行的

4 MRI的局限性- Scan duration
remains relatively unused

5 cost

6 Study Object
since subjects have to lie still throughout an acquisition in order not to degrade the quality of the images
图像采集过程中被摄对象保持静止
Hold breath for abdominal/thoracic imaging

7 人员配备

8 收费支持的基础设施成本

9 K-space

10 包含图像内的空间频率信息

11 Fourier transformation

12 1： FOV
视野

13 2：Spatial Resolution
期望图像的空间分辨率

14 3： Nyquist criterion
Collect the full number of k-space points

15 K空间上相邻的点和在每一个方向上的视场成反比

16 每个方向上获得的最高频率和所需分辨率成反比

17 read direction
usually picked to be the longest direction of the anatomy 通常选择为解刨学的最长方向

18 使用自旋或者梯度回波在一个重复的时间点内非常快的获取一行k-space点

19 further directions
相位编码

20 需要重复时间来编码一行k个空间点

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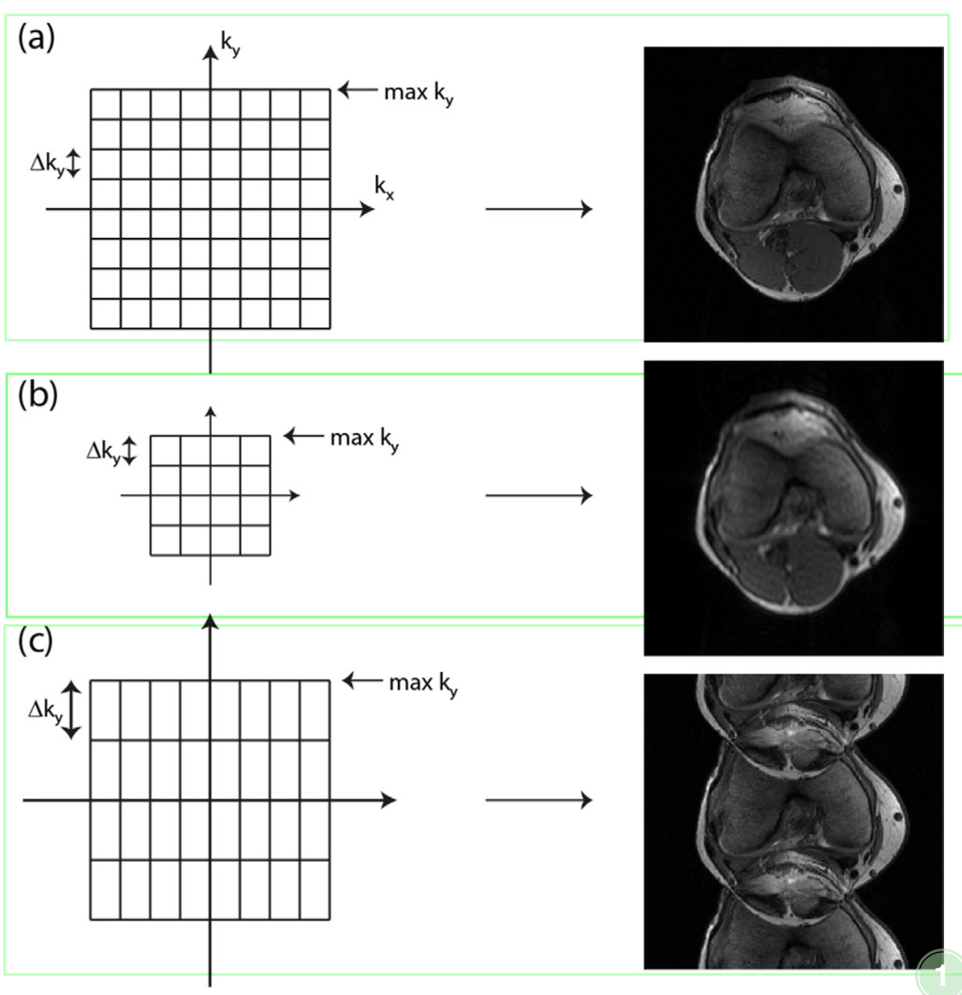

Figure 1. The relationship between k-space and image space for regular Cartesian sampling. (a) A fully sampled k_y–k_x space where max k_y α 1/(y-resolution) and Δk_y α 1/(field of view in y direction), similarly in the x direction. (b) If we half the max k_y and k_x but preserve Δk_y and Δk_x then we have halved the resolution of the resulting image while preserving its field of view. (c) By contrast, if we preserve max k_y and miss out alternate phase encoding lines, thus doubling Δk_y, we preserve the resolution but half the field of view in the y direction leading to fold-over.

Throughout the history of MRI there have been innovations which have tried to overcome the limitation of the repeated repetition times, including echo planar imaging (EPI) (Mansfield 1977), rapid acquisition with relaxation enhancement (RARE, also known as fast or turbo spin echo) (Hennig *et al* 1986), and fast low angle shot imaging and its variants (Haase *et al* 1986). Rather than the simple acquisition described above, where only one line of k-space is acquired from one gradient or spin echo per repetition time[1], these techniques use multiple rf pulses or gradient refocussings to generate multiple echoes and acquire multiple lines of k-space per

[1] For sequences with inversion preparation pulses, the definition of the repetition and inversion times can vary between vendors and acquisition time calculations need to take account of this (see McRobbie *et al* p 250).

1 导致翻折

2 Y-direction

x-direction 同理

3 Y-direction

X-direction 同理

4 FOV 不变，图像分辨率变为原来的二分之一

5 FOV 不变，图像分辨率变为原来的二分之一

6 图像分辨率不变，y方向上FOV变为原来的二分之一

7 回波平面成像

8 快速自旋转回波

9 快速低角度拍摄

10 方法

每一次重复时间，从一个梯度或者自旋回波中获取一行K-space

4

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An alternative or additional method of reducing the acquisition time is to not collect the full number of *k*-space points demanded by the Nyquist criterion. Such an acquisition is then said to be *undersampled*. In Cartesian imaging, as *k*-space points in the read direction are acquired very quickly, there is generally little advantage to undersampling in the read dimension. Rather we omit the collection of whole lines of *k*-space in one of the phase encoding directions. If a straightforward inverse Fourier transform is applied to such data then artefacts will be seen in the reconstructed image, the precise nature of the artefact depending on the undersampling pattern. However, if we can bring additional information to stabilize the reconstruction, then we may be able to produce images of acceptable quality from sub-Nyquist acquisitions. This basic premise underlies the techniques of partial Fourier imaging, parallel imaging and compressed sensing reconstruction. The first two of these methods are now routinely available on all clinical scanners and have been adopted by radiologists for routine diagnostic imaging.

In their present implementation on clinical scanners, the first two methods differ from compressed sensing in that they omit phase encoding steps in a regular fashion: compressed sensing demands an irregular undersampling pattern that will not lead to aliasing artefacts in the image domain, which we shall define further in section 3. The idea of irregular undersampling of data followed by a constrained reconstruction has been studied in general signal processing for some time (Donoho 2006), but the concept of using this within MRI was brought to the attention of the MR physics community by a number of papers including Lustig *et al* (2007) and Block *et al* (2007). Since then the number of papers with different undersampling and reconstruction schemes involving compressed sensing has burgeoned mightily, though as we shall see, only a few of these offer practical solutions for clinical scanning.

In this review, we will first examine today's most common methods of acceleration, partial Fourier imaging and parallel imaging (section 2), before going on to explain the principles of compressed sensing (section 3). We will then examine the practicalities of implementing compressed sensing (section 4), including the necessity of determining optimal reconstruction parameters, choosing the degree of acceleration to use and the computational burdens of the reconstructions. It is also critical to consider how we should validate MR data that have been reconstructed from undersampled data using compressed sensing, for both radiological and quantitative research applications. Applications which have been validated are then reviewed (section 5). There are alternatives to irregular undersampling and compressed sensing reconstruction which may be better suited to accelerate the acquisition of certain image types (section 6), and we conclude by considering the future challenges that remain to be overcome to gain widespread acceptance for acceleration by compressed sensing (section 7).

2. Acceleration by partial Fourier imaging and parallel imaging

2.1. Partial Fourier imaging

If we are principally interested in the magnitude information of an MR image, and we do not require high resolution phase information, as is often the case in diagnostic radiology, then we can exploit the property that the Fourier transformation of a purely real function has complex conjugate symmetry in *k*-space (Feinberg *et al* 1986). In theory this means that we only have to acquire half of the *k*-space in the phase encoding direction and we can reduce the number of TRs required twofold. In practice, in order to provide robust phase correction slightly more than half (commonly ~60%) of the phase encodes are acquired. This reduces the acquisition time, though there is a corresponding fall in SNR. It is also possible to apply this in the read direction to reduce the echo time, which may permit a slight reduction in repetition time per phase encode.

1 Cartesian imaging

成像方法

2 + inverse Fourier transform

（Applied to such data)

3 重建的图像中看到伪影

伪影的确切性质取决于欠采样的模式

4 + additional information

5 （1）Partial Fourier imaging

6 （2）Parallel imaging

7 （3）Compressed sensing reconstruction

8 Irregular

不规则的欠采样模式—>不会导致图像域中的混叠伪像。

9 2. Acceleration by partial Fourier imaging and parallel imaging

10 2.1 . Partial Fourier imaging

11 Fourier transformation of a purely real function has complex conjugate symmetry（共轭对称） in k-space

当主要对MR图像的幅度信息感兴趣，且不需要高分辨率的相位信息时

12 Half of the k-space in the phase encoding direction

相位编码方向上获取K-space 的一半

In practice, acquire more than a half —>To provide robust phase correction.

13 Time

14 SNR

15 Reduce the echo time

may permit a slight reduction in repetition time per phase encode

减少每个相位编码的重复时间

2.2. Parallel imaging

During the late 1990s the radiofrequency coils used to receive the signal in clinical MRI started to change in design, as it was realized that using an array of independent receiver channels could increase the overall signal to noise of the image compared to a single homogeneous volume coil (Roemer *et al* 1990). The other consequence of the phased array design is that each of the independent channels in the array is most sensitive to the tissue nearest to that coil (figure 3), and these sensitivity maps provide additional information which can be used to stabilize an undersampled image reconstruction.

If the *k*-space is undersampled in a regular manner by missing out every alternate line of *k*-space in a phase encoding direction, the number of repetition times required (and hence the total acquisition time) can be halved (figure 1(c)). The additional information about the coil channel sensitivities can be introduced by acquiring a separate low resolution scan, as is done in the sensitivity encoding (SENSE) technique (Pruessmann *et al* 1999) and calculating sensitivity maps which stabilize the reconstruction in image space, effectively by permitting the mathematical unfolding of aliased signal. The alternative is to fully acquire a small central portion of *k*-space, which corresponds to a low resolution image covering the same field-of-view as the main acquisition (figure 1(b)), and use the acquired points to calculate the missing *k*-space data before standard inverse Fourier transformation, as is performed in the generalized autocalibrating (GRAPPA) techniques (Griswold *et al* 2002).

Since the original introduction of these methods using regular undersampling with Cartesian acquisition, the techniques have been expanded to allow parallel imaging to be used with arbitrary sampling schemes and with non-Cartesian acquisitions (Pruessmann *et al* 2001, Seiberlich *et al* 2007, Lustig and Pauly 2010). In general, such techniques require iterative reconstructions and have not yet been widely used on clinical systems.

Parallel imaging methods have been found to be sufficiently robust for clinical imaging and at least one of the methods is now implemented on all clinical scanners, the choice of method being presently divided between vendors. The acceleration that can be achieved by these methods is limited by several factors. The first is that the arrangement of the coil elements has to provide varying sensitivity information in the same direction as the phase encodes to be omitted, and the acceleration factor cannot typically be greater than the number of elements in that direction (so a factor of 2 in the anterior–posterior direction for the coil in figure 3), subject to a factor related to the geometrical arrangement of the array, and thus how closely correlated are the coil sensitivities from different array elements. If this acceleration factor is exceeded, then coherent artefact will become apparent in the reconstructed image as well as enhanced noise amplification from the high acceleration factor. A detailed consideration of the possible artefacts is available elsewhere (Deshmane *et al* 2012). The second consideration, applicable to all *k*-space undersampling methods, is that a reduced acquisition of energy in *k*-space reduces the available signal to noise, and hence parallel imaging can only be used where there is sufficient signal to noise in the original fully sampled image.

3. Compressed sensing

3.1. Introduction

How then to explain how image acquisition can be accelerated by compressed sensing? First, let us imagine taking a picture with our digital camera. The sensors within the camera detect an image and signal intensity values are assigned to each of, say, 15 megapixels. However, storing the information for every pixel independently will lead to extremely large files, and

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1. 2.2. Parallel imaging

2. Coil channel sensitivities

each of the independent channels in the array is most sensitive to the tissue nearest to that coil, and these sensitivity maps provide additional information which can be used to stabilise an undersampled image reconstruction。

阵列中独立通道对靠近线圈的组织最为敏感

3. an array of independent receiver channels

单个均匀体积线圈—>一组独立接收器通道

=>增加图片整体信噪比

4. Missing out every alternate line of k-space in a phase encoding direction

相位编码方向上，每相隔一行遗漏。

5. coil channel sensitivities

单独获取低分辨率扫描 从而获取线圈通道敏感度的相关信息。

6. calculating sensitivity maps

stabilise the reconstruction in image space

7. (1) Half the resolution (2) Calculate missing data (3) Standard inverse Fourier transformation

采用广义自动校准

8. using regular undersampling with Cartesian acquisition

9. with arbitrary sampling schemes and with non-Cartesian acquisitions

10. Robust

11. Coil element arrangement

线圈元件的布置需要提供会被省略的相位编码方向上的敏感度变化信息。

12. acceleration factor

加速因子通常不能大于该方向上的元素数量。

If this acceleration factor is exceeded, then
（1）coherent artefact will become apparent in the reconstructed image as well as
（2）enhanced noise amplification from the high acceleration factor.

（1）相干伪像会在重构图像中变得非常明显；
（2）高加速因子增强了噪声放大

13. 可用信噪比的减少

14. PI的使用条件（背景）

15. 3. Compressed sensing

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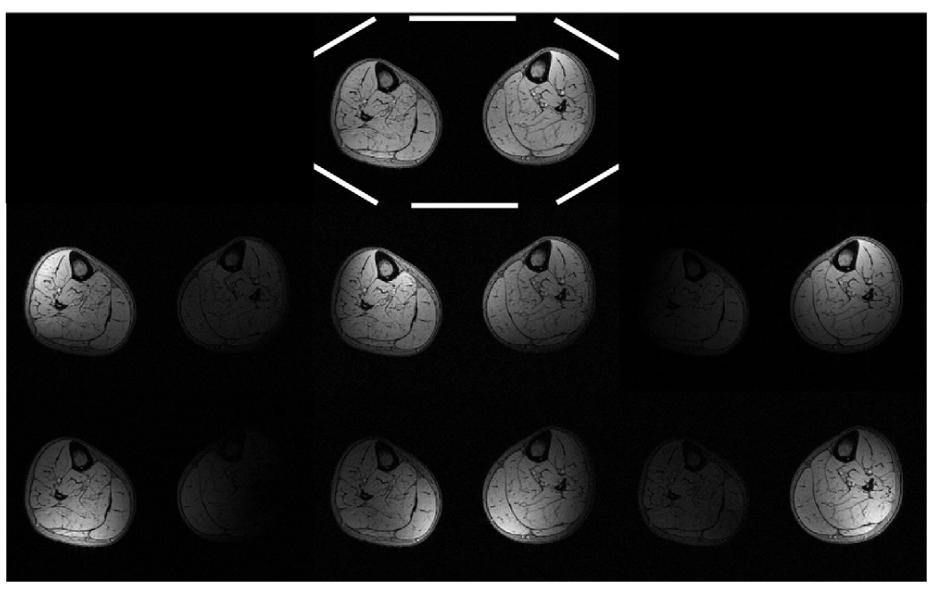


Figure 3. Parallel imaging stabilizes sub-Nyquist acquisitions, which use a reduced field of view by using the sensitivity profiles from the individual elements of the phased array coil. Here a cross-sectional image through the lower legs is acquired using a 6-channel surface coil as shown (top). Beneath, the images derived from the signal from the 6 separate channels are shown, demonstrating maximal signal intensity nearest the coil element in question, with fall off of intensity elsewhere. This information can be used to stabilize the undersampled acquisitions.

so the camera instead mathematically converts the image to JPEG format (Taubman and Marcellin 2002). The JPEG 2000 format uses a mathematical transform known as a *discrete wavelet transform* to compress the information for storage at a smaller size. The discrete wavelet transform applies a nested series of low and high pass filters to the image, producing a wavelet space of the same matrix size as the original image space (Xiong and Ramchandran 2009). This property becomes most apparent if we examine the discrete wavelet transform of an MRI cross-section through the lower leg (figure 4(a)). Most of the pixels in the original image have non-zero signal intensity: the image is therefore not *sparse*. Some MR images are naturally sparse, such as angiograms, though the majority are not. It becomes sparse in the wavelet domain, since a smaller number of pixels (which are now wavelet coefficients) carry the important image content (figure 4(b)). The difference can be illustrated by ordering the image intensities and wavelet coefficients in order of absolute size (figure 4(c)). The image can be reconstructed with high fidelity from only the most significant coefficients in the wavelet domain, whereas this is not possible in the original image domain (figure 5). For real image objects, this is not a lossless compression, that is to say that the original data is not perfectly reconstructable from the reduced number of coefficients. Therefore, there will be a trade-off between the reduction in the number of wavelet coefficients used and the image fidelity required.

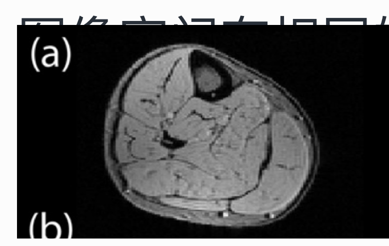
So how can we move from an issue of image storage to actually reducing the acquisition time? If we are able to describe an MR image in a transform space in which it is sparse, then the compressed sensing theory argues that we may be able to reconstruct that image from a smaller number of measurements in *k*-space, provided that the *k*-space undersampling is

1

Discrete wavelet transform

To compress the information for storage at a smaller size.

- The discrete wavelet transform applies a nested series of low and high pass filters (低通高通滤波器) to the image, producing a wavelet space of the same matrix size as the original image space. 产生的小波空间与原始图像空间有相同的矩阵大小。



2

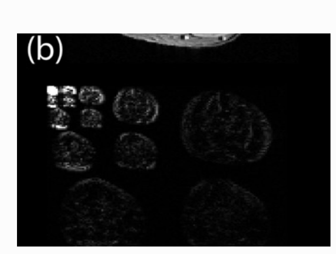
Not sparse

Most of the pixels in the original image have non-zero signal intensity.

3

Sparse 稀疏

->Becomes sparse in the wavelet domain. (Small number of pixels carry the important image content)



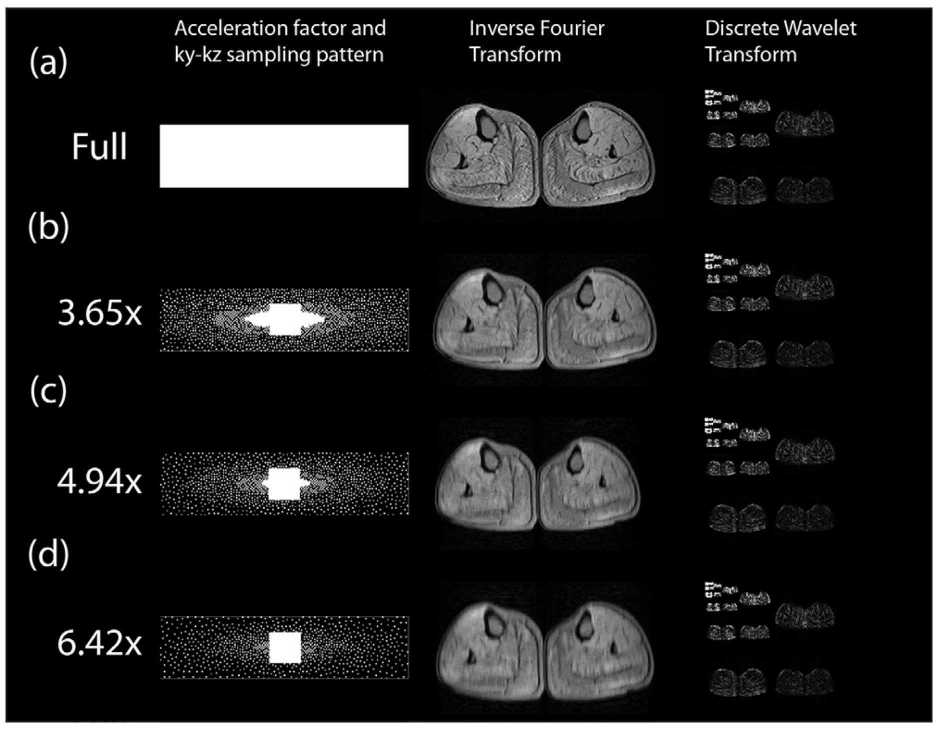


Figure 6. Examples of Cartesian variable density Poisson disk undersampling patterns with different accelerations. The phase encoding space of a 3D gradient echo (k_y–k_z plane) is shown for (a) a full Nyquist criterion acquisition and accelerations of (b) 3.65 ×, (c) 4.94 × and (d) 6.42 ×. In each case a Poisson disk with a quadratic fall off in sampling density has been designed, with a fully sampled centre portion to permit parallel imaging to be combined with compressed sensing using L1-ESPIRiT. In the middle column we see the effect of the sampling pattern if an inverse Fourier transform is performed. For (b), (c) and (d), there is no structured aliasing, but there is impaired image quality. Examining the discrete wavelet transforms of these acquisitions (right column) shows no discernible difference by eye.

Coded examples of solutions by a non-linear conjugate gradient algorithm and an iterative soft thresholding algorithm have been made available by the authors of (Lustig *et al* 2007), which allow users to reproduce some of the reconstructions performed in that paper (at www.eecs.berkeley.edu/~mlustig/Software.html).

3.2. The sparsifying transform

In this discussion, we have considered the discrete wavelet transform as an example of a sparsifying transform, but it is only one of a great array of possible transforms, which can be used alone or in combination. Common choices that have been deployed include total variation, which is a summation of the gradients in intensity between adjacent pixels (termed 'finite differences'), discrete cosine transforms (which were used in the original JPEG standard) and many others. A pictorial comparison of cosine transforms, wavelet transforms and finite differences is shown in (Lustig *et al* 2007). Total variation has proved to be a powerful constraint either when used on its own (Block *et al* 2007), or when used as an additional constraint to a discrete wavelet transform (Wiens *et al* 2014). There is also a vast array of discrete wavelet transforms, of which the Daubechies-4 wavelet is commonly used, but many others have been used and assessed.

Adding a total variation constraint to a discrete wavelet transform can avoid artefacts from using wavelets alone (such as sharp transition bands within the image) due to the Daubechies wavelet not being translation invariant. However, the use of an additional total variation constraint can be avoided by performing randomized shifting of the wavelet transform with respect to the image (Figueiredo and Nowak 2003, Murphy *et al* 2012).

There is nothing to prevent other prior knowledge being incorporated through the addition of further constraint terms to the sparsifying transform. In a compressed sensing application to measure dynamic lung volume with high temporal resolution throughout the respiratory cycle, additional priors were added to impose knowledge of the image intensity distribution at end-inspiration and end-expiration (Berman *et al* 2015). However, the more terms of constraint that are added, the more complex the determination of the optimal values of the weighting multipliers becomes.

3.3. Combining compressed sensing reconstruction with parallel imaging and/or partial Fourier imaging

Several research groups have concentrated on synergistically combining the undersampling possible using compressed sensing and parallel imaging to achieve greater acceleration factors than by either technique alone, including (Block *et al* 2007, Liu *et al* 2009, Lustig and Pauly 2010, Murphy *et al* 2012, Uecker *et al* 2014). The problem to be solved can be expressed by modifying equation (3) to incorporate the sensitivity profiles, S, of the N receiver coils in the phased array. This then yields,

$$\min_{m} \sum_{i=1}^{N} \|DFS_i m - y_i\|_2^2 + \lambda \|\Psi m\|_1. \tag{4}$$

Two reconstruction methods for parallel imaging with arbitrary undersampling were proposed by the group of Lustig, SPIRiT (Lustig and Pauly 2010) and e-SPIRiT (Uecker *et al* 2014), which operate in the *k*-space and image domains, respectively, and can incorporate an L_1-norm minimization term to additionally enforce sparsity in a transform domain. In order to derive the sensitivity map information using these methods, the undersampled *k*-space acquisition will need to either possess a fully sampled area at the centre of *k*-space, or for such data to be available from a previous acquisition. In all that follows, it will be assumed that the fully sampled centre portion of *k*-space is being acquired together with an undersampled peripheral *k*-space (figure 6), as this has been the most common way that combined compressed sensing and parallel imaging (CS-PI) has been performed.

As outlined in section 2.1, if only magnitude data are desired then partial Fourier encoding can be combined with both compressed sensing and parallel imaging (Liu *et al* 2012).

3.4. The design of undersampling schemes for compressed sensing reconstruction

How can we design the *k*-space undersampling schemes that we require for a compressed sensing reconstruction? We already know from section 2 that regular undersampling patterns will lead to structured artefact under inverse Fourier transform and are therefore unsuitable. The most desirable undersampling therefore will be quasi-random and will lack regular structure in the distance between the acquired *k*-space points and yet will not allow large regions where there are no acquired samples. In Cartesian sampling, the Poisson disk distribution achieves this by combining the random sampling of *k*-space while imposing constraints on the maximum and minimum distance between *k*-space points (Cook 1986). For signal-to-noise efficiency, the sampling pattern should also reflect the underlying signal density of *k*-space, which is not uniform but is most concentrated for the low-frequency components at the centre of *k*-space and falls away towards the periphery. Most practical Cartesian implementations of compressed sensing use a variable density Poisson disk to reflect this. A detailed algorithmic description of how such a variable density sampling pattern can be constructed for MRI can be found in (Gdaniec *et al* 2014).

Radial sampling has a particular appeal for compressed sensing as the spokes of a radial acquisition naturally lead to a dense sampling at the centre of *k*-space with decreased density towards the periphery. Undersampled radial acquisitions produce non-aliased blurring artefacts in the image space (figure 7) and therefore lend themselves well to compressed sensing reconstructions with a simple total variation constraint (Block *et al* 2007). This can be combined with golden ratio profile ordering (Winkelmann *et al* 2007), where the angle of the next radial line to be sampled is set to be 111.25° (which is 180° divided by the golden ratio 1.618) compared to the previous acquisition. This ensures that any consecutive selection of radial profiles that are retrospectively selected for reconstruction have the most favourable distribution, and that each new acquisition divides the maximum angle in *k*-space not covered by the previous profiles. Such properties not only permit robustness to anatomical motion, but provide an excellent basis for using combined compressed sensing and parallel imaging to reconstruct dynamic series of data (such as DCE-MRI) with a spatial and temporal resolution that can be selected post-acquisition according to the number of consecutive *k*-space spokes used per image (Chandarana *et al* 2013).

Spiral undersampling also possesses similar non-aliased artefact and the benefit of collecting multiple data near the *k*-space centre for motion correction. In clinical imaging it has been applied in such applications as accelerated 4D flow imaging with CS-PI reconstruction (Dyvorne *et al* 2015) and to alter the sensitivity of variable density spiral fMRI (Holland *et al* 2013). The issue of more general design strategies for optimal Cartesian and spiral undersampled *k*-space trajectories has also been addressed (Seeger *et al* 2010) using Bayesian methods.

3.5. Spatiotemporal sparsity

So far the discussion has concentrated on sparsity within static images. Clearly those image modalities where repeated images are taken with common image information, such as following the uptake in a DCE-MRI experiment using T1-w imaging, or measuring cardiac function by cine acquisition throughout the heartbeat, may have the potential for sparsity between the dynamics of the study. This means that additional constraints can be added to exploit this sparsity and that sparse sampling schemes can be spatiotemporal. If we take a slice of cardiac *k*-space data such that we see the k_y phase encode versus time, then applying a Fourier transform in both the spatial and temporal dimension gives us a compact representation in an image–frequency (usually called y–f) space (figure 8). If we then use a spatiotemporal sparse sampling pattern, which amounts to sampling different lines of k_y in different temporal phases, then we can arrange to produce non-aliased artefact in the y–f space on which a sparsity constraint (such as minimization of the L_1-norm) can be imposed to recover an artefact-free image. Examples of algorithms exploiting spatiotemporal sparsity include k–t FOCUSS (Jung *et al* 2007) and k–t SPARSE SENSE (Otazo *et al* 2010). Methods exploiting properties of compressed sensing are only a small subset of the temporal acceleration techniques available

(Tsao and Kozerke 2012): in particular using a temporal Fourier transform is not necessarily the sparsest representation of the time domain and principal component analysis in the temporal dimension has also been implemented (Pedersen *et al* 2009).

3.6. Dictionary learning

The maximum acceleration factors that can be achieved is determined by the sparsity of the images in the transformed domain, and the discrete wavelet (and other fixed basis) transforms have been successful at sparsifying many different image types. However, additional sparsity might be obtained by the use of dictionary learning (Duarte-Carvajalino and Sapiro 2009), where the basis functions are learned from the data. A greater degree of sparsity would permit greater acceleration for a given reconstruction quality and this has become an important topic of recent research (Caballero *et al* 2014). The reconstruction burden for these methods is presently large and their general applicability and efficiency in MRI has yet to be proved.

4. Practical considerations for compressed sensing

There are several important practical considerations that must be taken into account to incorporate compressed sensing to our acquisition.

4.1. Adjustable parameters in CS reconstructions

A critically important consideration is that the reconstruction formulations detailed in section 3 have adjustable regularization parameters within them, which tune the trade-off between fidelity to the undersampled raw data and sparsity in the transform domain. The avid literature reader will find that many, if not most, researchers have performed an empirical search of the parameter space and 'appropriate' values of the regularization parameter are simply quoted, or perhaps the issue is simply omitted! In certain clinical applications optimal regularization parameters have been demonstrated that by comparing metrics of image quality between a fully-sampled acquisition and a sparsely sampled acquisition: such metrics include minimizing the root mean square error (RMSE) between the images or maximizing the structural similarity index (SSIM) (Loughran *et al* 2015, Mann *et al* 2015). It is found that for a given matrix size and protocol, the optimal reconstruction parameters are stable, and can therefore be confidently predicted for new scans using that exact protocol. There have been attempts to try and automate the choice of regularization parameter in some circumstances either through the adaptation of iterative soft thresholding algorithms (Khare *et al* 2012) or by more generalized reconstruction schemes (Aelterman *et al* 2014), which claim to require no adjustable parameters. These schemes, although potentially promising, have not yet been widely adopted or tested in clinical situations. The issue of successfully predicting appropriate reconstruction parameters remains a key limitation in permitting the use of compressed sensing in the general clinical environment.

4.2. Choosing the degree of acceleration to apply and validating CS reconstructions

What is the maximum degree of acceleration that can be applied in a given situation? There is no easy answer to this question, which will depend on the signal to noise of the base acquisition sequence, the imaging matrix size and the number of receiver coils used. The signal to noise matters since each accelerated acquisition only collects a fraction of the *k*-space energy