

Deep Hedging

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ABSTRACT: In this report we overview the idea of Deep Hedging, which uses deep reinforcement learning to hedge a portfolio of derivatives. In contrast to the traditional approach that uses the *greeks*, Deep Hedging works for general market conditions. We discuss the `Python` implementation and discuss some of the results.

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1 Introduction

Deep Hedging was proposed in Ref. [1] by Hans Bühler, Lukas Gonon, Josef Teichmann and Ben Wood. In this approach, a neural network architecture learns how to hedge a portfolio of derivatives using historical (simulated) data without computing the *greeks*. Consequently, it can be easily generalized to different market conditions. This report is based on lectures given by Josef Teichmann.

Suppose we have a portfolio of derivatives which represent our liabilities. In this work we consider a call option whose payoff depends on the underlying stock price, so we have $f(S)$ and the aim is to construct a portfolio in which this derivative is hedged.

Therefore, we want the value of the portfolio to vanish by solving the following optimization problem,

$$\min_{\delta} \mathbf{E}[(f(S_T) - p_0 - (\delta \cdot S)_T)^2] \quad (1.1)$$

where p_0 is the initial price of the derivative and

$$(\delta \cdot S)_T \equiv \sum_{k=0}^{n-1} \delta_k (S_{k+1} - S_k), \quad (1.2)$$

so δ_k corresponds to the number of shares held at time k . In other words, the agent is minimizing her loss at maturity. Transactions costs can be easily implemented by adding a new term to Eq. (1.1).

In this report we discuss a **Python** implementation of deep hedging using the **Tensorflow** library. In Section 2 we go over the parameters of the model, in Section 3 we discuss the architecture of the Neural Network and the training. Finally, we present results and conclusions in Section 4.

2 Parameters

The stock prices are simulated using the Black-Scholes model with no drift. The parameters are set to:

```
N = 20 # Time steps
S0 = 1 # Initial stock price
strike = 1 # Strike for the call option
T = 1.0 # Maturity
sigma = 0.2 # Volatility
```

The initial value for the call option price is calculated using the Black-Scholes solution.

```
# Initial price for the call option:
priceBS = BS(S0,strike,T,sigma)
```

Taking zero dividend rate, the Black-Scholes solution is given by

$$C(S, t, T) = SN(d_1) - Ke^{-r(T-t)}N(d_2), \quad (2.1)$$

where

$$d_1 = \frac{\ln(S/K) + \sigma^2(T-t)/2}{\sigma\sqrt{(T-t)}}, \quad (2.2)$$

$$d_2 = d_1 - \sigma\sqrt{(T-t)}. \quad (2.3)$$

The variable `Ktrain` determines the number of generated paths on which the model will be trained.

```
# Number of trainable paths. This number can be 1e6
Ktrain = 10**5
```

3 Training

After we have generated all the `Ktrain` paths, we use them as input to train the model. The inputs for the training are:

1. The initial stock price S_0
2. The initial hedging being 0
3. The logarithmic increments of the stock price

We use the Adam optimizer, which is an adapted version of the stochastic gradient descent, to minimize the loss function

```
model_hedge.compile(optimizer='adam',loss='mean_squared_error')
```

where the loss function is given by

$$\mathcal{L} = [f(S_T) - p_0 - (\delta \cdot S)_T]^2. \quad (3.1)$$

The number of hidden layers is $d \times N$. The strategy at the time step j will be calculated by the g_j Neural Network. The inputs are dynamical since at each time step j we use all the increments up to $j - 1$. First, we initialize the Neural Network in an abstract way, and then we call the model with the actual data.

```
# Constructing the model and implementing the loss function
# Inputs contain: 1) The initial stock price S0
# 2) The initial hedging being 0
# 3) The increments of the log price

price = Input(shape=(m,))
hedge = Input(shape=(m,))

inputs = [price]+[hedge]

# Strategy at j is the hedging strategy at j, i.e. the Neural Network g_j
for j in range(N):
    strategy = price
    for k in range(d):
        strategy = layers[k+(j)*d](strategy)

    # Update the log price of the stock
    incr = Input(shape=(m,))
    logprice = Lambda(lambda x : K.log(x))(price)
    logprice = Add()([logprice, incr])

    # Update the price at time j+1
    pricenew = Lambda(lambda x : K.exp(x))(logprice)
```

```

#Calculate the price increment
priceincr = Subtract()([pricenew, price])

# Update the value of the hedge
hedgenew = Multiply()([strategy, priceincr])

# This is only used for m > 1:
#mult = Lambda(lambda x : K.sum(x,axis=1))(mult)

# building up the discretized stochastic integral
hedge = Add()([hedge, hedgenew])
inputs = inputs + [incr]
price = pricenew

# Payoff of the call option
payoff = Lambda(lambda x : 0.5*(K.abs(x-strike) + (x-strike))
                - priceBS)(price)

# The output will be payoff-priceBS-hedge
# Loss function L = (payoff-priceBS-hedge)^2
outputs = Subtract()([payoff,hedge])

inputs = inputs
outputs = outputs

model_hedge = Model(inputs=inputs, outputs=outputs)

```

We simulate the data of the stock price paths that will be used for the training:

```

# Initial stock price
initialprice = S0

xtrain = ([initialprice*np.ones((Ktrain,m))] +
          [np.zeros((Ktrain,m))] +
          [np.random.normal(-(sigma)**2*T/(2*N),
                             sigma*np.sqrt(T)/np.sqrt(N),(Ktrain,m)) for i in range(N)])

ytrain = np.zeros((Ktrain,1))

```

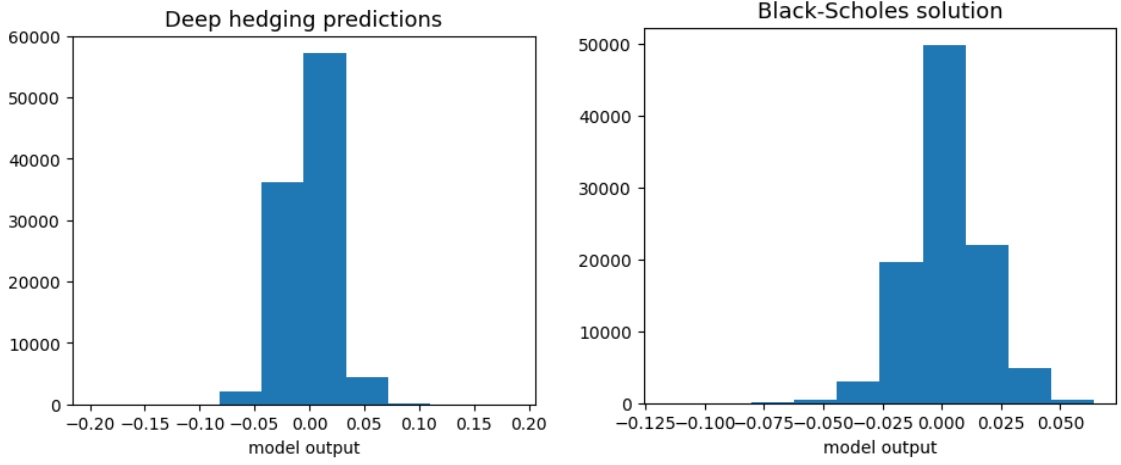


Figure 1: In the left (right) panel we show the predictions for the deep hedging model (Black-Scholes model). The results concentrate around zero as expected.

4 Results

On the left panel in Fig. 1 we show the predictions for the deep hedging model. The results are close to zero with an average value of 1.1×10^{-4} for the loss. We compare this with the analytic Black-Scholes solution, these results are shown on the right panel.

In Fig. 2 we show the Delta as a function of the stock price for a fixed time. The two lines show a very good overlap, except on the regions in the corners of the plot, since there was not enough data on these regions to generate good predictions.

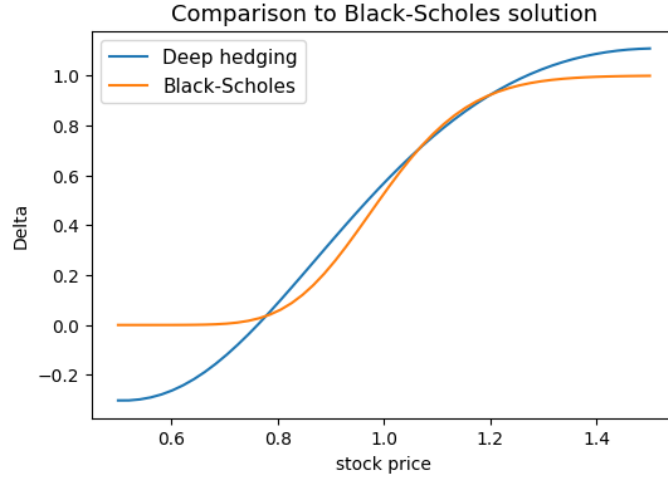


Figure 2: Comparison of the delta strategy as a function of the stock price. The blue (orange) line corresponds to the deep hedging (Black-Scholes model).

References

- [1] H. Buehler, L. Gonon, J. Teichmann and B. Wood, *Deep hedging*, *Quantitative Finance* **19** (2019) 1271–1291, [<https://doi.org/10.1080/14697688.2019.1571683>].