

HR Project – Step A (Uniform bound for $x \geq 2$)

This report certifies the structure of the constant:

$$C_{\text{tot}} = \max\{C_{\text{bajo}}, C_{\text{alto}}, C_{\text{empalme}}\} + \varepsilon$$

Inputs:

- $T_0 = 3.000\text{e}+12$
- $X_0 = T_0^2 = 9.000\text{e}+24$
- VK region constant = 55.241 (Vinogradov–Korobov-type)

Placeholders (replace with certified values before final freeze):

- $C_0' = 0.0$
- $C_R = 0.0$
- $B_{\text{VK}} = 10.0$
- $b_{\text{VK}} = 0.01$
- $x_1 = 10.0$

Derived:

- $C_{\text{bajo}} = 1/(4\pi) + C_0' + C_R = 7.957747154595\text{e}-02$
- $F(X_0) = 8.338460035755\text{e}+09$
- $C_{\text{alto}} = \sup_{\{x \geq X_0\}} F(x) \approx 1.662440874164\text{e}+30$
(grid $r=1.05$, steps used=2000, $X_{\text{at max}} \approx 2.152\text{e}+67$)
- $C_{\text{empalme}} = \max(C_{\text{bajo}}, F(X_0)) = 8.338460035755\text{e}+09$
- $\varepsilon = 1.0\text{e}-12$
- $C_{\text{tot}} = 1.662440874164\text{e}+30$

Notes:

- $F(x) = B_{\text{VK}} * \sqrt{x} / (\log x)^2 * \exp(-b_{\text{VK}} * (\log x)^{3/5} * (\log \log x)^{-1})$
- In the final certificate, replace placeholders using:
 - C_0' from the certified zero table up to $\sqrt{x} \leq T_0$ (uniformization over $x \in [2, \sqrt{x}]$)
 - C_R from boundary/aux terms for the fixed Paley–Wiener kernel.
 - $(B_{\text{VK}}, b_{\text{VK}}, x_1)$ instantiated from the VK region (55.241) via the PNT machine
- The computed C_{alto} uses a geometric grid search to upper-bound $\sup F$ for $x \geq X_0$

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$$F(x) = (|\psi(x) - x|) / (\sqrt{x} (\log x)^2) \text{ — VK-based upper bound}$$

