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Master's Thesis

Object Classification based on Micro-Doppler Signatures

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Sperrvermerk

Declaration of Authorship

Dedicated to..

Contents

1. Introduction

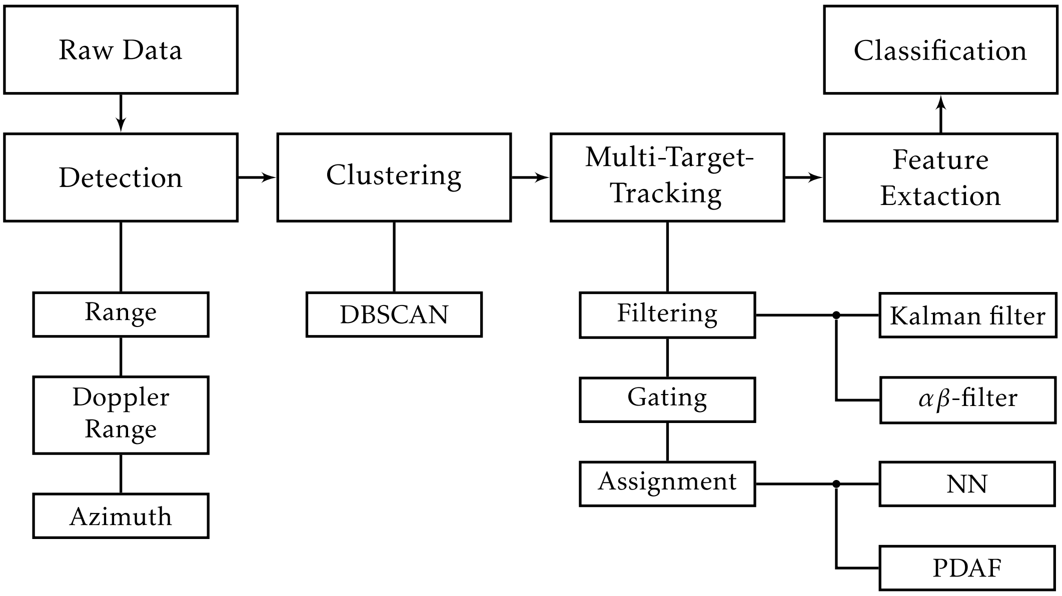

Figure 1.1.: Block Diagram of the whole process

2. Radar Fundamentals

Due two its wide spectrum of applications, radars systems have become important measurement instruments since the last century.

This introductory chapter is organized as follows. Section 2.1 explains the basic terminology used in the scientific community of radars. Section 2.3 presents a simple signal model used to explain the signal processing steps that take place to estimate the range (2.4) and doppler-range (2.5) of targets

2.1. Radar Definitions

Range

$$R = \frac{c_0 \Delta t}{2} \tag{2.1}$$

Pulse repetition interval (PRI)

$$f_r = \frac{1}{T} \tag{2.2}$$

Maximum unambiguous range R_u

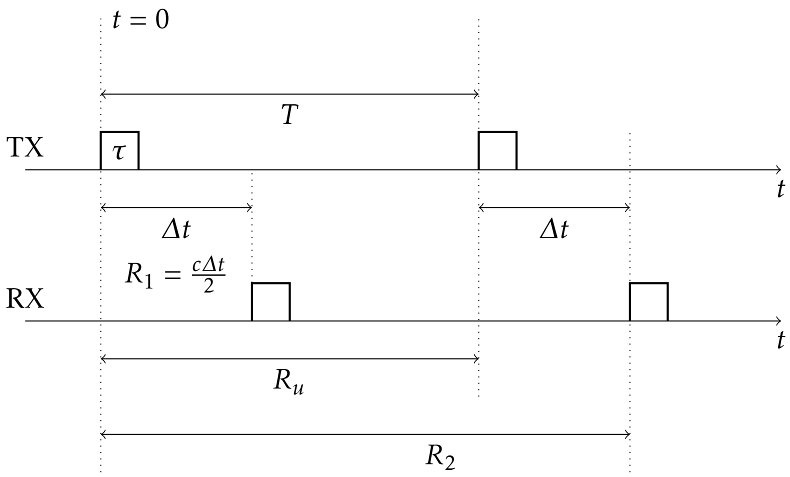

Figure 2.1.: Illustrating the unambiguous range

$$R_u = c_0 \frac{T}{2} = \frac{c_0}{2 f_r} \tag{2.3}$$

Range resolution ΔR, distance between R_{max} and R_{min} divided into M range bins.

$$M = \frac{R_{max} - R_{min}}{\Delta R} \tag{2.4}$$

$$\Delta R = \frac{c_0 \tau}{2} = \frac{c_0}{2B} \tag{2.5}$$

Doppler frequency

$$f_d = \frac{2v\cos\theta}{c_0} f_0 = \frac{2v\cos\theta}{\lambda} \tag{2.6}$$

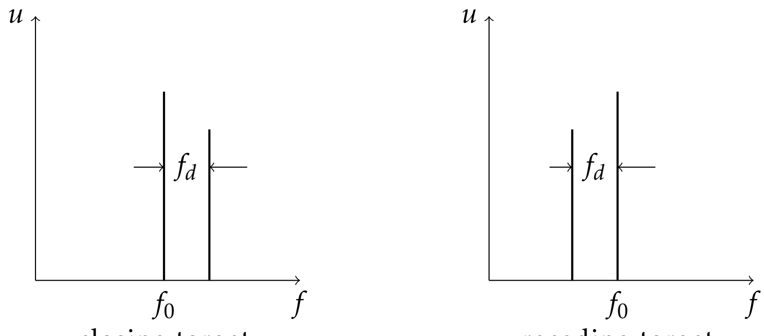

Figure 2.2.: Illustrating the doppler frequency

2.2. MIMO Array

A Multiple-Input Multiple-Output (MIMO) setup for this research has been chosen. The concepts behind the MIMO radar will be presented briefly in this section.

As the name implies, a MIMO radar consists of multiple transmit (TX) antennas and receive (RX) antennas. Given M_t transmit and M_r receive elements in an array, $M_t \times M_r$ propagation channels are obtained. This, with only $M_t M_r$ antenna elements. Furthermore, a method to define the diversity of the TX channels is required. This can be achieved by employing time division multiplexing, frequency division multiplexing, spatial coding, and orthogonal waveforms [1].

Each of the $M_t \times M_r$ propagation channels are modeled together as a virtually form array. Each element in the virtual array is placed at

$$\boldsymbol{x_{ij}} = (\boldsymbol{x}_i^{Tx} + \boldsymbol{x}_j^{Rx})/2, \tag{2.7}$$

where $\boldsymbol{x}_i^{Tx}$ is the position of the i-TX element and $\boldsymbol{x}_j^{Rx}$ of the j-RX element, as shown in fig. 2.3

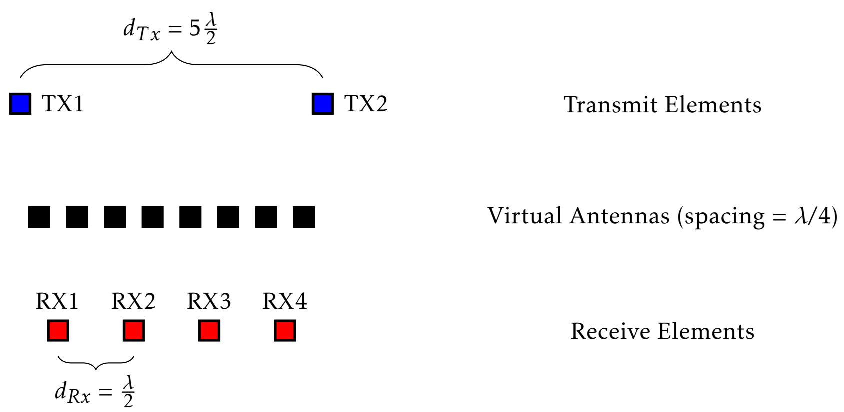

Figure 2.3.: MIMO-array

If the far-field condition is met for a given scatterer at position $\boldsymbol{p}$, the signal propagation path from a given TX element to the scatterer, plus the reflection path back to an RX element can be approximated as

$$P_{ij}(p) = |\boldsymbol{p} - \boldsymbol{x}_i^{Tx}| + |\boldsymbol{p} - \boldsymbol{x}_j^{Rx}| \approx 2|\boldsymbol{p} - \boldsymbol{x}_{ij}| \tag{2.8}$$

2.3. Signal Model

For the following analyses a Frequency-Modulated Continuous-Wave (FMCW) Radar is taken into consideration. The TX array elements transmit a FMCW chirp signal, that can be modeled in as

$$s_T(t) = \exp[\mathrm{j}(2\pi f_c t + \pi k t^2)], \tag{2.9}$$

for $-T_c/2 \leq t \leq T_c/2$, where T_c is the chirp duration, f_c the carrier frequency and the chirp rate k is defined by

$$k = \pm B/T_c. \tag{2.10}$$

Under far-field condition, the return delay between a virtual element at x_{ij} and a scatterer is given by

$$\Delta t_{ij} = \frac{2R}{c_0} + \frac{2x_{ij}\sin\theta}{c_0}, \tag{2.11}$$

where R and θ describe the location of the scatterer, R being the rang and θ the angle with respect to boresight. The received signal

$$s_R^{ij}(t) = A s_t(t - \Delta t_{ij}) \tag{2.12}$$

After the signal is received, it is down-converted by multiplying it with a replica of the transmitted signal. After low-pass filtering, the processed signal can be modeled as

$$u_{ij}(t) = s_R^{ij} s_T(t) = A\exp[\mathrm{j}(2\pi k \Delta t_{ij} t - \pi k \Delta t_{ij}^2 + 2\pi f_c \Delta t_{ij})] \tag{2.13}$$

The sampled form of the intermediate frequency (IF) signal given N samples can be expressed as

$$u_{ij}[n] = A\exp[\mathrm{j}(\phi + 2\pi \Psi_R n)], \tag{2.14}$$

with $\Psi_R = k\Delta t_{ij}\frac{T_c}{N}$,the normalized frequency containing the range information and $\phi = -\pi k \Delta t_{ij}^2 + 2\pi f_c \Delta t_{ij}$ contains the phase information. This expression can be generalized to MIMO radar, with M targets either static or dynamic, for the k-th observation interval by

$$u_k[n, n_C, n_A] = \sum_{m=0}^{M} A_m \exp[\mathrm{j}2\pi(\phi_m + \Psi_{R,m} n + \Psi_{D,m} n_C + \Psi_{\theta,m} n_A)] + w[n, n_C, n_A] \tag{2.15}$$

where $\Psi_D = \frac{2T_c f_0}{c_0} v_R$ and $\Psi_\theta = \sin(\theta)/2$ are the normalized frequencies containing the information about range-rate and azimuth. Moreover, $w[n, n_C, n_A]$ is the present additive white Gaussian measurement noise. The parameters n, n_C and n_A represent which, sample chirp and antenna is being taken into account. Please note that ϕ_m does not correspond to ϕ.

The information collected from eq. (2.15) is usually arrange into a three-dimensional data cube as presented in fig. 2.4. Here the dimensions correspond to sample number, transmit-receive channel and chirp number.

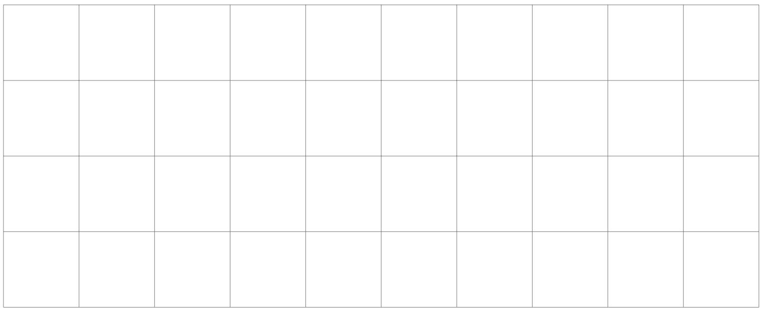

Figure 2.4.: Data Cube

In order to estimate the spectrum of the received signal and so to determine the unknown parameters R, v_R and θ for a given target, usually a three-dimensional Fast Fourier Transform (FFT) is applied to de IF signal

$$\hat{P}_k(\Psi_R, \Psi_D \Psi_\theta) = \sum_{n} \sum_{n_C} \sum_{n_A} a_R[n] a_D[n_C] a_\theta[n_A] u_k[n, n_A, n_C] \\ \times \exp(-\mathrm{j}2\pi \varPsi_R n) \exp(-\mathrm{j}2\pi \varPsi_D n_C) \exp(-\mathrm{j}2\pi \varPsi_\theta n_A), \quad (2.16)$$

however, this usually results in a bad estimation for the azimuth range. Specifics of the signal processing will be described in the following sections.

2.4. Ranging

2.5. Doppler Ranging

2.6. Azimuth Estimation

2.7. Detection Theory

3. Clustering

Note: At the moment it is not clear whether other clustering algorithms which are not density-based decompositions will be considered.

3.1. Density-Based Clustering

Density-based clustering algorithms have been developed in the context of data mining in big data bases and knowledge discovery. For radar applications, density-based clustering is of advantage since it is very efficient in a dynamic environment where insertions and deletions are present. Furthermore, no cluster-prototype has to be specified, i.e. the clusters can be of arbitrary shape. Last but not least, the number of clusters does not have to be known in advance, which is a disadvantage that most clustering algorithms present which make them unsuitable for radar-applications.

In section 3.1.1 some mathematical definitions that describe the way density-based clustering works are presented. Furthermore, section 3.1.2 presents the density-based spatial clustering of applicatios with noise (DBSCAN) algorithm.

3.1.1. Definitions

The key idea of density-based clustering is that for most points of a cluster, the ε-neighborhood for some $\varepsilon > 0$, has to contain at least a minimum numbber of points. In other words, the neighborhood has to exceed a density-threshold to be considered a cluster.

We define a distance-based ε-neighborhood N_ε of an object o on a set of points D by

$$N_\varepsilon(o) = \{o' \in D || o - o'| \leq \varepsilon\} \tag{3.1}$$

Furthermore, an object $p \in D$ is called *directly density-reachable* from an object $q \in D$ if

1. $p \in N_\varepsilon(q)$
2. $size(N_\varepsilon(q)) > MinPts$

where $size()$ returns the number of points in an ε-neighborhood and $MinPts$ is the threshold that it has to meet to be considered a cluster. Morever, the object $p \in D$ is called *density-reachable* from $q \in D$, if there is a chain of objects $p_1, ..., p_n$, with $p_1 = q$ and $p_n = p$ such that for all $i = 1, .., n-1$: p_{i+1} is directly density-reachable o p_i. These definitions are illustrated in fig. 3.1

Finally, an object p is density connected to an object q if there exist an object o such that both p and q are density-reachable from o, such as shown in fig. 3.2.Note that all these properties are dependent on ε and $MinPts$. In literature, for instance, it would be said that an object is density connected to another object with respect to ε and $MinPts$, however this statement has been omitted since it is implied.

With the previous definitions, one can now define the concepts of a *density-conenected set*, which is a subset C of a database D that satisfies the following conditions

1. Maximality: For all $p, q \in D$: if $p \in C$ and q is density-reachable from p, then $q \in C$.

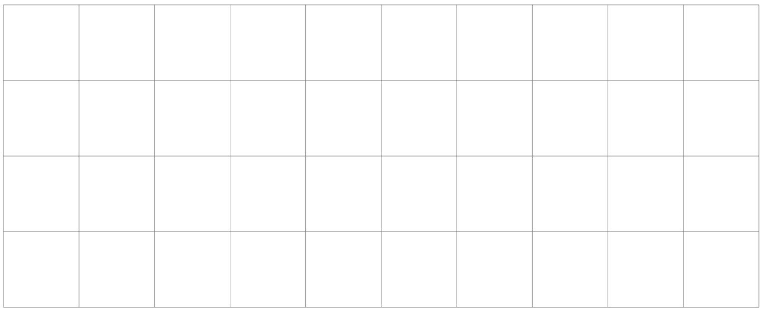

Figure 3.1.: Density-reachable example

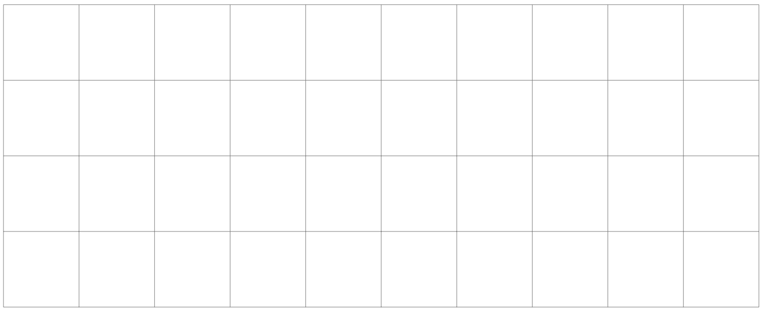

Figure 3.2.: Density-reachable example

2. Connectivity: For all $p,q \in C$: p is density-connected to q.

Now, one can define how a database should be decomposed after the clustering algorithms have been applied. The result is called density-based-decomposition (DBD) and should meet the following conditions

1. $DBD = \{S_1, ..., S_., N\}, k \geq 0$
2. $S_1 \cup ... \cup S_k \cup = D$
3. For all $i \leq k$: S_i is a *density-connected* set
4. If there exists a *density-connected* set $S \in D$, then there exists an $i \leq k$ so that $S = S_i$
5. $N = D \setminus (S_1 \cup ... \cup S_k)$ is called the noise with respect to the decomposition DBD.

This decomposition should be the output of the clustering algorithms where the subsets S_i are the clusters and N are any detections that were not assigned to any cluster.

3.1.2. DBSCAN

In this section, a MATLAB version of the DBSCAN is presented and explained.

```
        %include matlab code
```

4. Multi-Target Tracking

After a correct detection and clustering of targets, the detections have to be assigned to tracks which are to be updated over time. In this case, it is done to create a history of each of the target's range, azimuth angle and Doppler velocity. In a further step, the Micro-Doppler signature can be extracted from each of the tracks and be used as an input for the classifiers. More over, by keeping track of the measurements corresponding to each track, one can produce an estimate of future positions of the target, which result into a more accurate measurement of the targets position. This process is called Multi-Target Tracking (MTT).

Each track of a MTT-algorithm contains a Kalman Filter (section 4.2) which is a commonly used filter to predict future states and to calculate variables that cannot be measured directly. At each step, gating (section 4.3) is applied to each of the new detections to reduce the scope of detections that can be assigned to a given track at each time step k. After gating, the detections calculated in a given step are assigned to each of the tracks. For this, the assignment problem has to be solved (section 4.4). After the assignment has been done, the state of each of the tracks is updated according to pre-established rules (section 4.5). The whole process used for the MTT is illustrated in fig. 4.1. Given that usually a constant-velocity model is assumed, a special model needs to be

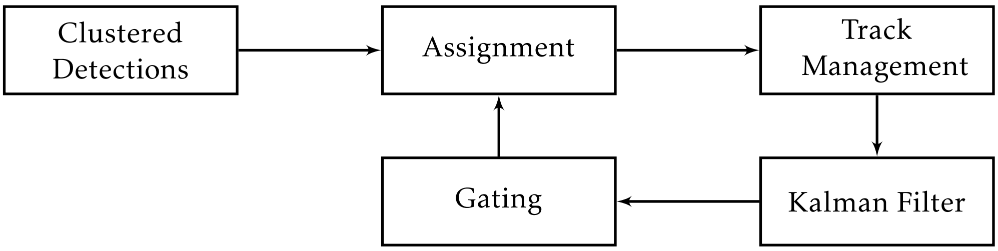

Figure 4.1.: MTT-process

implemented to consider the cases where the target has an (unknown) acceleration. This is issue is handled in section 4.6.

4.1. State Variable Representation of an LTI System

A Linear Time Invariant (LTI) system can be described by by three variables, input, output and the state variable. In the case of radar, the state can be design to contain several attributes of single targets measured by the radar such as range, range-rate and azimuth angle. Since we desire to determines a target's position and velocity in Cartesian coordinates, the state vector

$$\boldsymbol{x} = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}, \tag{4.1}$$

has been chosen. The continuous-time linear system can then be written as

$$\dot{\boldsymbol{x}}(t) = \mathbf{A}(t)\boldsymbol{x}(t) + \mathbf{B}(t)\boldsymbol{u}(t) + \tilde{\boldsymbol{v}}(t), \tag{4.2}$$

where t represents time and

$\boldsymbol{x}$	is the state vector of dimension n_x and $\dot{\boldsymbol{x}}$ its time derivative.
$\boldsymbol{u}$	is the input(or control) vector of dimension n_u
$\tilde{v}$	is the process noise
A,B	are known matrices of dimensions $n_x \times n_x$ and $n_x \times n_u$

The output of the system can be represented by

$$\boldsymbol{z}(t) = \mathbf{C}(t)\boldsymbol{x}(t) + \tilde{\boldsymbol{w}}(t), \tag{4.3}$$

where

$\boldsymbol{z}$	is the output vector of dimension n_z
$\tilde{\boldsymbol{w}}$	the output disturbance, or measurement noise and
C	is a known matrix od dimension $n_z \times n_x$

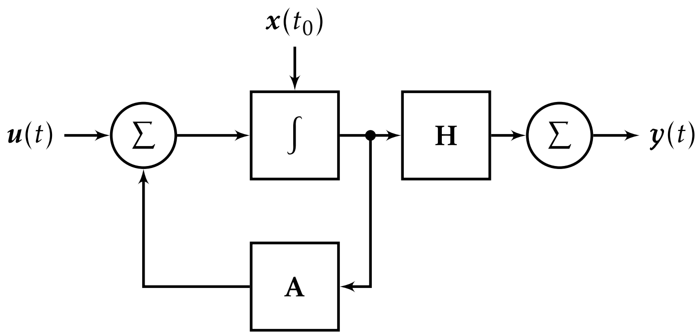

Figure 4.2.: An LTI System

Such an LTI system is depicted in fig. 4.2. Given the initial condition $\boldsymbol{x}(t_0)$, a solution to eq. (4.2) can be found as

$$\boldsymbol{x}(t) = \Phi(t - t_0)\boldsymbol{x}(t_0) + \int_{t_0}^{t} \Phi(t - \tau)\mathbf{B}(\tau)\boldsymbol{u}(\tau)\,\mathrm{d}\tau. \tag{4.4}$$

In the case of radar tracking, however, one only obtains detection at discrete time-steps. Thus, a discrete-time representation of the LTI system is to be found

$$\boldsymbol{x}[n+1] = \mathbf{A}[n]\boldsymbol{x}[n] + \mathbf{B}[n]\boldsymbol{u}[n] + \boldsymbol{v}[n] \tag{4.5}$$

$$\boldsymbol{z}[n] = \mathbf{H}[n]\boldsymbol{x}[n] + \boldsymbol{w}[n] \tag{4.6}$$

The state transition matrix for our case is the so called Newtonian matrix

$$\Phi = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{4.7}$$

4.2. The Kalman Filter

Different classes of filters (or estimators) have been used for tracking. One of the simplest class of these filters is, for instance, are the so called "Fixed-Coefficient" filters. The most implemented of these are the $\alpha\beta$ and $\alpha\beta\gamma$ trackers [cite]. These filter provide smoothed and predicted data for target position, velocity and in case of the latter, acceleration. Nonetheless, given the better estimates it provides, the Kalman filter has been implemented primarily for radar-tracking purposes. Additionaly, the Kalman filter presents the following advantages over the Fixed-Coefficient filters.

1. The gain coefficients are computed dynamically, i.e. the same filter can be used for targets of different nature
2. The filter is robust against missed detections
3. Provides an accurate measure of the covariance matrix, which is relevant for gating and association processes

An overview of the process undertaken by the Kalman filter is presented in fig. 4.3. Before pre-

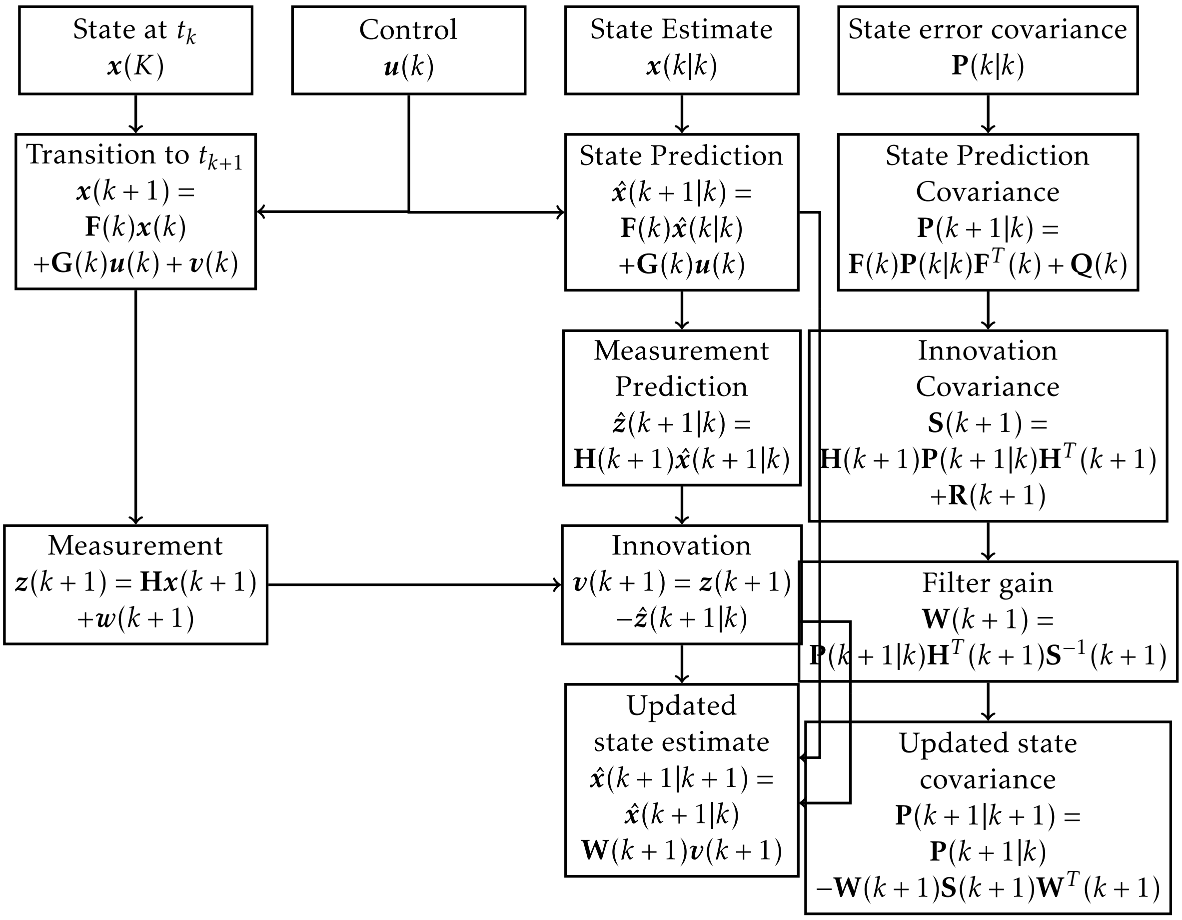

Figure 4.3.: The Kalman Filter

senting the equations that describe the Kalman filter, the notation is introduced. $\hat{\boldsymbol{x}}(n|m)$ represents the estimate during the n-th sampling interval using all data up to the m-th sampling interval, and $z(n)$ contains the measurements (or detections that have been assigned to the corresponding track) obtained at the n-th time-step.

4.2.1. Estimation in Linear Systems

The filtering equation is given by

$$\boldsymbol{x}(n|n) = \boldsymbol{x}_s(n) = \boldsymbol{x}(n|n-1) + \mathbf{W}(n)\boldsymbol{v}(n), \tag{4.8}$$

where

$\boldsymbol{x}_s(n)$ is the smoothed output of the Kalman filter

$\mathbf{W}(n)$ are the dynamically computed filter gains

$\boldsymbol{v}(n)$ is the innovation vector

The innovation vector contains information about how much the estimate $\boldsymbol{x}(n|n-1)$ differs from the newly obtained measurements $y(n)$ and can be written as

$$\boldsymbol{v}(n) = \boldsymbol{y}(n) - \mathbf{H}\boldsymbol{x}(n|n-1), \tag{4.9}$$

where $\mathbf{H}$ is called the observation matrix. In our case, the measurement vector has the form

$$\boldsymbol{y}(n) = \begin{bmatrix} x_m(n) \\ y_m(n) \end{bmatrix}, \tag{4.10}$$

so that $\boldsymbol{v}(n)$ only represent the innovation in the position measurements. Therefore, the observation matrix results in

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}. \tag{4.11}$$

The filter gains are computed at each time step by

$$\mathbf{W}(n+1) = \mathbf{P}(n+1|n)\mathbf{H}^T\mathbf{S}^{-1}(n+1) \tag{4.12}$$

where

$\mathbf{P}$ represents the predictor covariance matrix and,

$\mathbf{S}$ the measurement prediction covariance.

The state prediction covariance is first calculated recursively by

$$\mathbf{P}(n|n+1) = \mathbf{E}\{\boldsymbol{x}_s(n+1)\boldsymbol{x}_s^*\} = \mathbf{A}\mathbf{P}(n|n)\mathbf{A}^T + \boldsymbol{Q}, \tag{4.13}$$

where $\boldsymbol{Q}$ is the covariance matrix for the input $\boldsymbol{u}(n)$. After the filter gains have been calculated by eq. (4.12), the state covariance is updated by

$$\mathbf{P}(n+1|n+1) = \mathbf{P}(n+1|n) - \mathbf{W}(n+1)\mathbf{S}(n+1)\mathbf{W}^T(n+1). \tag{4.14}$$

The innovation covariance, or measurement prediction covariance is given by

$$\mathbf{S}(n+1) = \mathbf{H}\mathbf{P}(n+1|n)\mathbf{H}^T + \mathbf{R}, \tag{4.15}$$

where $\mathbf{R}$ is the measurement covariance.

4.2.2. Initialization of State Estimators

When a detection that has not been assigned to an existing track is present, a tentative track is initiated (see section 4.5) the Kalman filter is initialized. First, the state vector as

$$\boldsymbol{x}(0) = \begin{bmatrix} r_m \cos(\theta_m) \\ r_m \sin(\theta_m) \\ 0 \\ 0 \end{bmatrix}, \tag{4.16}$$

where r_m and θ_m are the measured range and azimuth respectively. Given that the measurement of the azimuth angle is not very accurate, we have chosen to initialize the velocities v_x and v_y of each target with $v_x = 0$ and $v_y = 0$. Note that, however, the velocities can also be initialized by taking two position measurements and calculating

$$v_x = \frac{x_m(0) - x_m(-1)}{\Delta} \qquad v_y = \frac{y_m(0) - y_m(-1)}{\Delta} \tag{4.17}$$

where Δ is the time between measurements, $x_m(n) = r_m(n)\cos(\theta_m(n))$ and $y_m(n) = r_m(n)\sin(\theta_m(n))$. Furthermore, the initial covariance matrix is calculated depending on the measurement matrix

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \tag{4.18}$$

as

$$\mathbf{P}(0|0) = \begin{bmatrix} R_{11} & R_{12} & R_{11}/\Delta & R_{12}/\Delta \\ R_{21} & R_{22} & R_{21}/\Delta & R_{22}/\Delta \\ R_{11}/\Delta & R_{12}/\Delta & 2R_{11}/\Delta^2 & 2R_{12}/\Delta^2 \\ R_{22}/\Delta & R_{22}/\Delta & 2R_{21}/\Delta^2 & 2R_{22}/\Delta^2 \end{bmatrix} \tag{4.19}$$

TO DO (IN RESEARCH TOO): Choosing appropiate **R** and **Q** matrices

4.3. Gating Techniques

Gating is a technique used to eliminate extremely unlikely observation-to-track pairings. A gate is calculated around the output of the Kalman filter (predicted position), and any observation outside of this gate is not further consider for that track. Furthermore if only one observation is found within the gate, then the observation is assigned directly to that track and no further processing is required (see section 4.4).

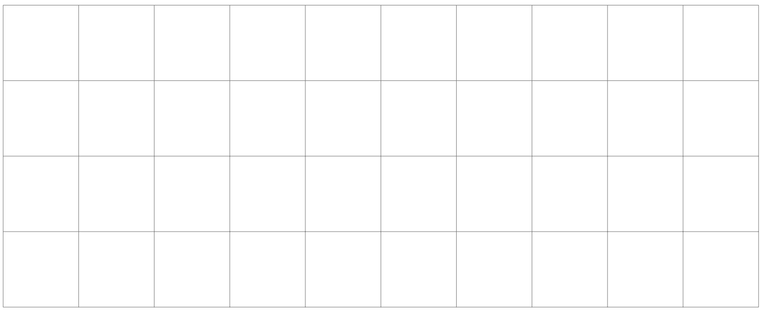

Figure 4.4.: Gating

One of the simplest gating techniques is using rectangular gates. A given observation is also said to satisfy the gates of a given track if the all the elements of the innovation vector satisfy

$$|\boldsymbol{v}(n)| \leq K_G \sigma_r \tag{4.20}$$

where σ_r is the residual standard deviation that can be expressed in terms of measurement σ_o^2 and prediction σ_p^2

$$\sigma_r = \sqrt{\sigma_o^2 + \sigma_p^2}. \tag{4.21}$$

Note that the values σ_o and σ_p can be found in the Kalman filter equations as

$$\sigma_o = R_{11} \qquad\qquad \sigma_p = P_{11} \tag{4.22}$$

The constant K_G can be chosen freely. Usually, a Gaussian error model is assumed to that choosing $K_G = 3$ leads to the probability of a valid observation satisfying the gating test to be about 99 percent.

4.4. The Assignment Problem

In a dense target environment, the gating technique is not sufficient enough to discriminate between detections. Thus a further assignment logic has to be implemented. Conflicts can occur for instance, when a detection satisfies the gating of different tracks, or when a track was multiple detections within its gate. This problem is called the assignment problem.

There are basically two types of solutions. The nearest-neighbor (NN)-approach (section 4.4.1) and the "all-neighbors" approach (section 4.4.2 and section 4.4.3). The first step of all solutions, however, is the same. First, an assignment matrix is built. For this, the norm of the innovation vector that would result if track i and detection j would be assigned is defined as

$$d_{ij}^2 \triangleq \boldsymbol{v}_{ij}(n)^T \mathbf{S}_i^{-1}(n) \boldsymbol{v}_{ij}(n), \tag{4.23}$$

where $\mathbf{S}_i(n)$ is the residual covariance matrix defined in eq. (4.15). Furthermore, it is assumed that the residual has a Gaussian distribution

$$g_{ij} = \frac{\exp(-\frac{d_{ij}^2}{2})}{(2\pi)^{M/2}\sqrt{|\mathbf{S}_i|}}, \tag{4.24}$$

where M is the measurement dimension and $|\mathbf{S}_i|$ the determinant of $\mathbf{S}_i$.

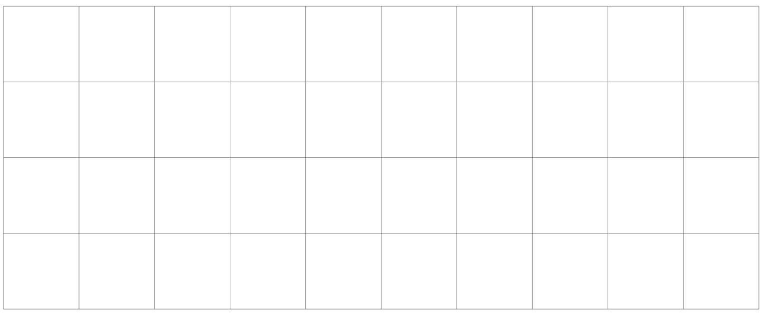

Figure 4.5.: Example of conflict situations for assignment

The basic goal is to make assignment decisions based on the maximization of g_{ij}, which is equivalent to minimizing the quantity

$$d_{Gij}^2 = d_{ij}^2 + \ln|\mathbf{S}_i|, \tag{4.25}$$

which will be used as distance function for use in the assignment problem.

4.4.1. NN-approach

For the NN approach, first an assignment matrix as the one depicted in fig. 4.6 is built. Observations that are know within the gate of a certain track, will be identified as X. Otherwise, the distance function of of the corresponding observation and track is calculated.

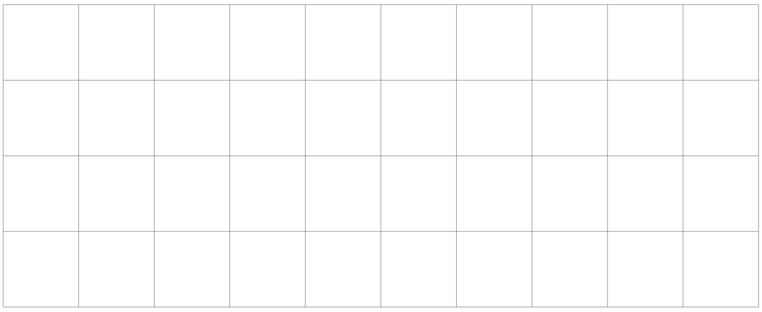

Figure 4.6.: Assignment matrix example

There are different ways to solve the assignment problem from the assignment matrix following a NN-approach.Two suboptimal solutions are presented and (LATER: OPTIMAL SOLUTION).

Suboptimal Solution one:

1. Observations that are considered for singly-validated tracks will not be considered for mutiply-validated tracks.

2. Observations that are in multiple tracks will not be considered for tracks that contain single-validated observations

3. Repeat 1. and 2. until the assignment matrix is not changed anymore

4. For the remaining tracks with multiple observations, select the observation with minimum distance

5. For the remaining observations that validate with various tracks, assign to th track with the minimum distance

Suboptimal Solution two:

1. Search the matrix for the minimum distance observation-to-track pair and assign it

2. Remove the assigned pair from the matrix and repeat 1. until all possible assignments have been made.

4.4.2. PDA-approach

4.4.3. JPDA-approach

4.5. Track Life Stages

NOTE: For now very simple methods for track management have been implemented:

Track Initiaiton: Any detection not assigned Track Confirmation: If a tentative track has at least 3 detections for 5 time steps Track Deletion: After 10 consecuitive missed detections

4.5.1. Track Confirmation

4.5.2. Track Deletion

4.6. Maneuver Detection and Adaptive Filtering

5. Micro-Doppler Signatures

6. Classification

7. Results

8. Summary and Outlook

A. Symbols and Constants

General

$\oint$	Integration over a closed curve

Latin alphabet

B	Bandwith
f_0	Center Frequency
f_d	Doppler Frequency
f_r	Pulse Repetition Frequency (PRF)
R	Range
R_u	Unambiguous Range
T	Pulse Repetition Interval (PRI)
v	Target Velocity

Greek alphabet

ΔR	Range Resolution
ΔT	Delay
λ	wavelength
τ	Pulse Width

Constants

c_0	=	299729458 m/s

B. Mathematical Formulas

Bibliography

[1] Yanchuan Huang, Paul Victor Brennan, Dave Patrick, I. Weller, Peters Roberts, and K. Hughes. FMCW Based MIMO Imaging Radar for Maritime Navigation. *Progress In Electromagnetics Research*, 115:327–342, 2011.