Measurement of Sheet Resistivities with the Four-Point Probe

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Correction factors are evaluated for the measurement of sheet resistivities on two-dimensional rectangular and circular samples with the four-point probe. Diffused surface layers can be treated as two-dimensional structures, but the factors are also useful in obtaining body resistivities on thin samples.

I. INTRODUCTION

The "four-point probe" has proven to be a convenient tool for the measurement of resistivities. For a description of the method see a paper by L. Valdes,[1] which gives the functional relationship between the resistivity, ρ, and the voltage and current readings for various geometries. Later, A. Uhlir evaluated functions[2] which give the relationship for additional geometries. All these treatments are concerned with three-dimensional structures infinite in at least one direction.

For diffused layers, a similar relationship is needed for the evaluation of sheet resistivities on various sample shapes. This is a two-dimensional problem which is treated here for various finite sample sizes. With the solution, however, it also is possible to obtain body resistivities on thin slices of the same finite geometries.

II. METHOD

A current source in an infinite sheet gives rise to the logarithmic potential

$$\varphi - \varphi_0 = -\frac{I\rho_s}{2\pi} \ln r,$$

where φ is the potential, I the current, ρ_s the sheet resistivity and r the distance from the current source.

In particular, the potential for a dipole (+ source and − source) becomes

$$\varphi - \varphi_0 = \frac{I\rho_s}{2\pi} \ln \frac{r_1}{r_2}.$$

In the case of a four-point probe on a sheet, the two outside (current) points represent the dipole. Therefore, the potential difference between the two inner points is, for an infinite sheet,

$$\Delta\varphi = V = \frac{I\rho_s}{\pi} \ln 2$$

(only equal point spacing is considered). Thus, the sheet resistivity is obtained as

$$\rho_s = \frac{V}{I} \frac{\pi}{\ln 2} = \frac{V}{I} 4.5324 \cdots .$$

To obtain the sheet resistivity on a finite sample, the method of images can be applied.[3] Only nonconducting boundaries are considered; the boundary (edges) of a diffused layer must be etched or else the layer on the back side of the sample would act as a shunting path.

III. RECTANGULAR SAMPLE

We consider a four-point probe on a rectangular sample with the dimensions a and d. The probe is arranged symmetrically with point spacing s according to Fig. 1. To obtain the voltage between the two center points 1 and 2, an infinite arrangement of dipoles must be considered, as shown in Fig. 2. All contribute to the voltage between points 1 and 2.

F. Ollendorff[4] gives the potential distribution for an infinite number of current sources, arranged in a line and equally spaced. With a coordinate system as in Fig. 3 the potential is

$$\varphi - \varphi_0 = -\frac{I\rho_s}{2\pi} \ln 2 \sqrt{\sin^2 \frac{\pi}{d} x + \sinh^2 \frac{\pi}{d} y}.$$

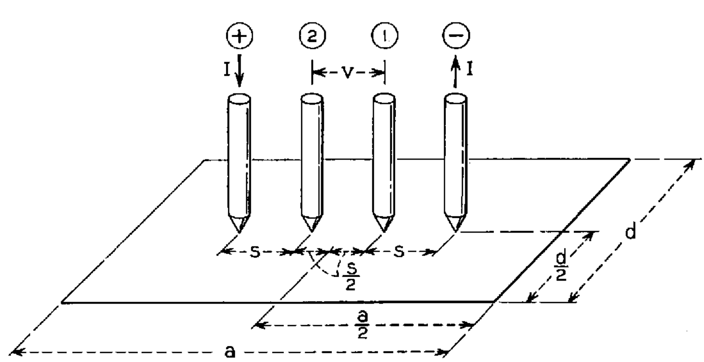


Fig. 1 — Arrangement of a four-point probe on a rectangular sample.

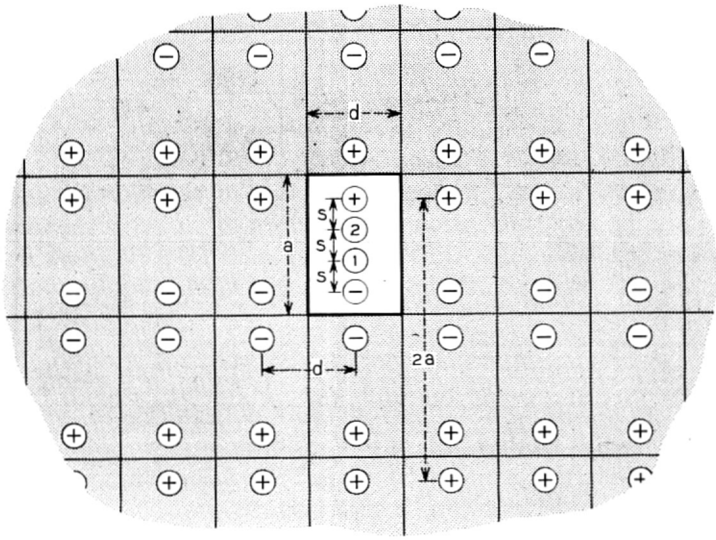


Fig. 2 — System of images.

With this expression the present problem is reduced to a summation of lines of current sources with alternating sign in only one direction. In the coordinate system for every line of sources the points 1 and 2 have the x-coordinate zero. Thus the expression simplifies to:

$$\varphi - \varphi_0 = -\frac{I\rho_s}{2\pi} \ln \left(e^{\pi y/d} - e^{-\pi y/d}\right).$$

Each line of sources thus contributes to the voltage V the amount

$$\Delta\varphi_n = -\frac{I\rho_s}{2\pi} \ln \left(\frac{e^{\pi(y_n+s)/d} - e^{-\pi(y_n+s)/d}}{e^{\pi y_n/d} - e^{-\pi y_n/d}}\right)$$

where y_n is the distance from point 1 to the center source in line n. The dimension a is involved in the values y_n. The total voltage between the

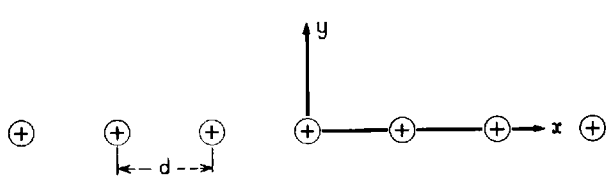


Fig. 3 — Coordinate system for a linear arrangement of current sources.

points 1 and 2 is therefore

$$V = \Sigma \Delta \varphi_n \equiv \rho_s I \frac{1}{C\left(\frac{a}{d}; \frac{d}{s}\right)}$$

where $C(a/d;\ d/s)$ is a constant defined by this equation. As shown in the Appendix, the summation can be done easily by expanding the logarithm and summing each term as a geometrical series. In almost all cases, the first term gives four-place accuracy. The sheet resistivity is thus given by

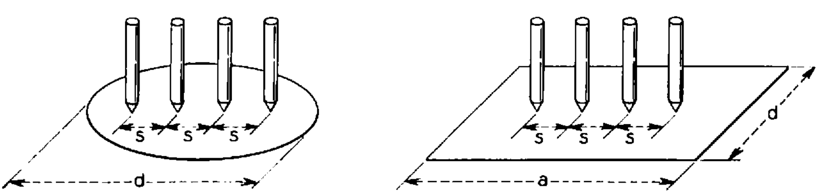
$$\rho_s = \frac{V}{I} C\left(\frac{a}{d}; \frac{d}{s}\right).$$

Table I gives this factor C for various geometries.

For small d/s the quantity $C' = (s/d)C$ is close to unity. In these cases the sheet resistivity can be expressed as:

$$\rho_s = \frac{V}{I}\frac{d}{s}C' \approx \frac{V}{I}\frac{d}{s}.$$

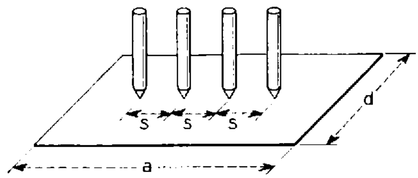
Table I — Correction Factor C for the Measurement of Sheet Resistivities with the Four-Point Probe



$$\rho_s = \frac{V}{I} C$$

d/s	circle diam d/s	$a/d = 1$	$a/d = 2$	$a/d = 3$	$a/d \geq 4$
1.0				0.9988	0.9994
1.25				1.2467	1.2248
1.5			1.4788	1.4893	1.4893
1.75			1.7196	1.7238	1.7238
2.0			1.9454	1.9475	1.9475
2.5			2.3532	2.3541	2.3541
3.0	2.2662	2.4575	2.7000	2.7005	2.7005
4.0	2.9289	3.1137	3.2246	3.2248	3.2248
5.0	3.3625	3.5098	3.5749	3.5750	3.5750
7.5	3.9273	4.0095	4.0361	4.0362	4.0362
10.0	4.1716	4.2209	4.2357	4.2357	4.2357
15.0	4.3646	4.3882	4.3947	4.3947	4.3947
20.0	4.4364	4.4516	4.4553	4.4553	4.4553
40.0	4.5076	4.5120	4.5129	4.5129	4.5129
∞	4.5324	4.5324	4.5324	4.5325	4.5324

Table II — Correction Factor C' for the Measurement of Sheet Resistivities with the Four-Point Probe on Narrow Structures



$$\rho_s = \frac{V}{I}\,\frac{d}{s}\,C'$$

d/s	$a/d = 1$	$a/d = 2$	$a/d = 3$	$a/d \geqq 4$
1.0			0.9988	0.9994
1.25			0.9973	0.9974
1.5		0.9859	0.9929	0.9929
1.75		0.9826	0.9850	0.9850
2.0		0.9727	0.9737	0.9737
2.5		0.9413	0.9416	0.9416
3.0	0.8192	0.9000	0.9002	0.9002
4.0	0.7784	0.8061	0.8062	0.8062

Table II gives C' for various geometries. The table may be used to determine the error one makes by just approximating with $C' = 1$.

IV. CIRCULAR SAMPLE

In this case, only one image is necessary to fulfill the boundary condition. The image is obtained by "reflecting" the dipole on the circle.[5] For a four-point probe centered in the sample, the voltage becomes:

$$V = \frac{I\rho_s}{\pi}\left[\ln 2 + \ln\left(\frac{\left(\dfrac{d}{s}\right)^2 + 3}{\left(\dfrac{d}{s}\right)^2 - 3}\right)\right] = I\rho_s\,\frac{1}{C\left(\dfrac{d}{s}\right)},$$

where d is the diameter of the circle. The correction factor is also given in Table I.

V. MEASUREMENT OF BODY RESISTIVITIES ON THIN SAMPLES

On a slice of finite thickness w, the four-point probe will introduce voltage gradients perpendicular to the surface. Insofar as these gradients are negligible, the slice can be treated in the same way as an infinitely thin slice and the proper sheet resistivity can be obtained, thus giving the body resistivity ρ with the relation $\rho = \rho_s w$.

As mentioned, for a four-point probe on an infinite sheet the following relation holds:

$$\rho_s = \frac{V}{I} \frac{\pi}{\ln 2}.$$

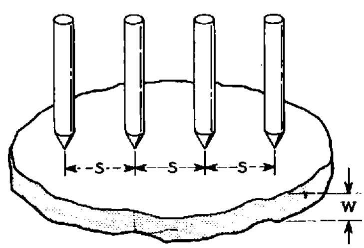
For an infinite slice of finite thickness w, one can express the resistivity as

$$\rho = \rho_s w = \frac{V}{I} w \frac{\pi}{\ln 2} F\left(\frac{w}{s}\right),$$

where $F(w/s)$ is a correction factor approaching unity as w approaches zero.

From Uhlir's paper[2] $F(w/s)$ can be obtained; it is tabulated in Table III. This table may serve to evaluate the error one makes by treating thin samples as an infinitesimal sheet. For a finite sample, $F(w/s)$ could be used as a first order correction, thus giving the resistivity of a slice as $\rho = \rho_s w = (V/I)w\, C\, F$. The exact treatment would require the summation of the potential from an infinite three-dimensional array of dipoles.

Table III — Measurement of Body Resistivities ρ on Thin Samples of Thickness w



$$\rho = \rho_s \cdot w = \frac{V}{I} \cdot w \cdot \frac{\pi}{\ln 2} \cdot F\left(\frac{w}{s}\right)$$

w/s	$F(w/s)$
0.4	0.9995
0.5	0.9974
0.5555	0.9948
0.6250	0.9898
0.7143	0.9798
0.8333	0.9600
1.0	0.9214
1.1111	0.8907
1.25	0.8490
1.4286	0.7938
1.6666	0.7225
2.0	0.6336

APPENDIX

Summation of $\Delta\,\varphi_n$

Without loss of generality the following substitutions are made:

$$d \text{ is used for } \frac{d}{s}, \qquad a \text{ for } \frac{a}{s}, \qquad \text{and} \qquad y \text{ for } \frac{y}{s}.$$

From symmetry reasons, it follows that each source in the upper half of the plane (Fig. 2) contributes the same $\Delta\,\varphi_n$ as does the corresponding source in the lower half. Thus only the sources in the lower half are to be considered and the result must be multiplied by 2.

The terms

$$-\ln\,(e^{\pi y/d} - e^{-\pi y/d})$$

are first written in the form

$$-\ln e^{\pi y/d}(1 - e^{-2\pi y/d}) = -\pi\frac{y}{d} - \ln\,(1 - e^{-2\pi y/d}).$$

With this one obtains

$$\frac{2\pi}{|I|\,\rho_s}\,\Delta\varphi_n = \pm\left[-\frac{\pi}{d} - \ln\,(1 - e^{-2\pi(y_n+1)/d}) + \ln\,(1 - e^{-2\pi y_n/d})\right].$$

with $+$ standing for a positive source and $-$ for a negative source. Summing the first term gives π/d.

To sum the ln terms the logarithm can be expanded:

$$-\ln\,(1 - x) = x + \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 + \cdots.$$

With this, each term in n becomes

$$\pm\sum_{m=1}^{\infty}\frac{1}{m}\,[e^{-2\pi(y_n+1)m/d} - e^{-2\pi y_n m/d}].$$

The y_n can be expressed as

$$y_n = A_i + n2a$$

with four different A_i. With this, each term in m becomes

$$a_m = \sum_{i=1}^{4}\sum_{n=0}^{\infty} \pm\frac{1}{m}\,(e^{-4\pi a m/d})^n e^{-2\pi A_i m/d}[e^{-2\pi m/d} - 1].$$

This is a geometrical series in n. Forming $\sum_{n=0}^{\infty}$ gives

$$a_m = \sum_{i=1}^{4} \pm\frac{1}{m}\,\frac{(e^{-2\pi m/d} - 1)}{(1 - e^{-4\pi a m/d})}\,e^{-2\pi A_i m/d}.$$

For reasons of convergence, $y = 1$ is treated separately. The A_i thus become

for + sources:

$$A_2 = a + 1$$

$$A_4 = 2a - 2$$

for − sources:

$$A_1 = 2a + 1$$

$$A_3 = a - 2.$$

Forming $\sum_{i=1}^{4} (-1)^i e^{-2\pi A i m/d}$ and inserting into the expression for a_m gives

$$a_m = \frac{1}{m} e^{-2\pi(a-2)m/d} \frac{(1 - e^{-6\pi m/d})(1 - e^{-2\pi m/d})}{(1 + e^{-2\pi a m/d})}.$$

The total voltage between points 1 and 2 is therefore

$$V = I\rho_s \frac{1}{\pi}\left[\frac{\pi}{d} + \ln(1 - e^{-4\pi/d}) - \ln(1 - e^{-2\pi/d}) + \sum_{m=1}^{\infty} a_m\right].$$

In all but three cases the first term in m gave four-place accuracy.

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