

Métodos Computacionales

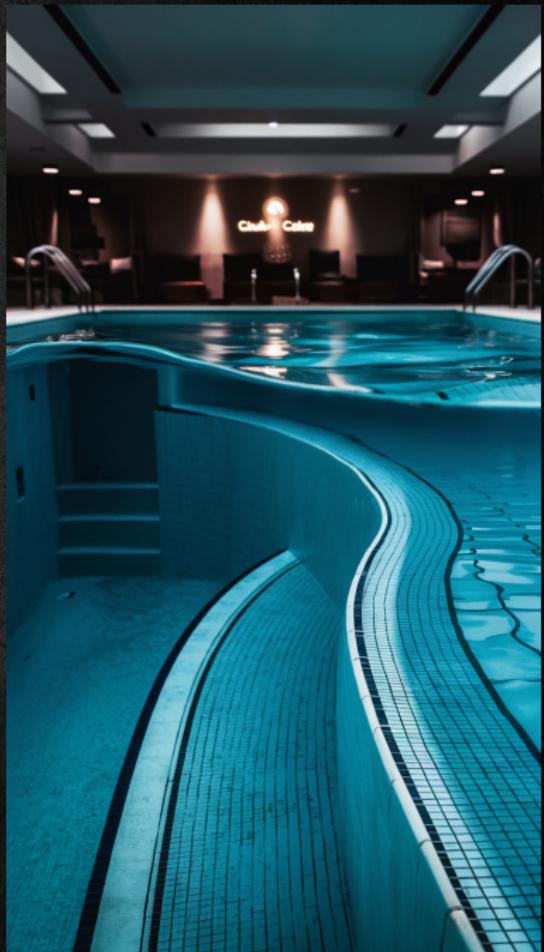
Cuadraturas

Agosto 15, 2024

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Y COMPUTACIONAL
Facultad de Ciencias Químicas
Universidad Nacional de Córdoba



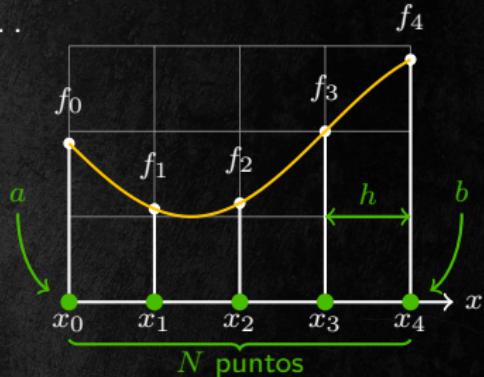
hecho con idiogram

Discretizar / Grillar / Teselar

Si discretizamos x **uniformemente** entre a y b ...

... con N puntos, entonces

$h = \frac{b - a}{N - 1}$ es el paso de la grilla.



... con un paso h , entonces

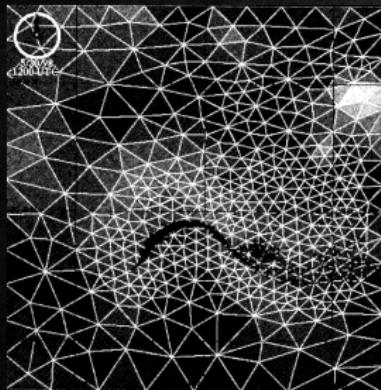
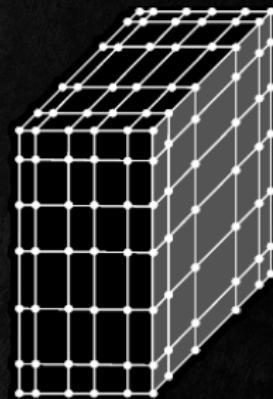
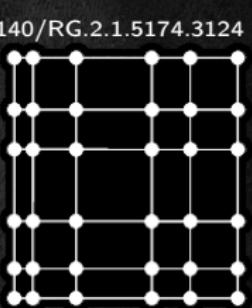
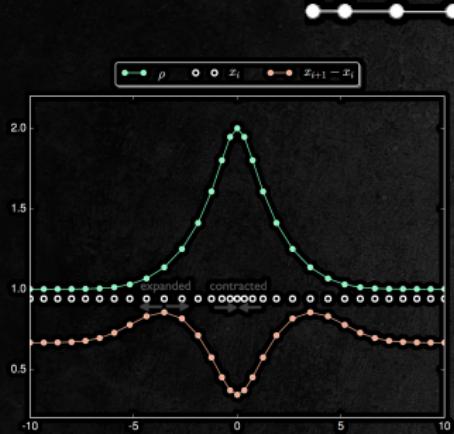
hay $N = \frac{b - a}{h} + 1$ puntos en la grilla.

Notación

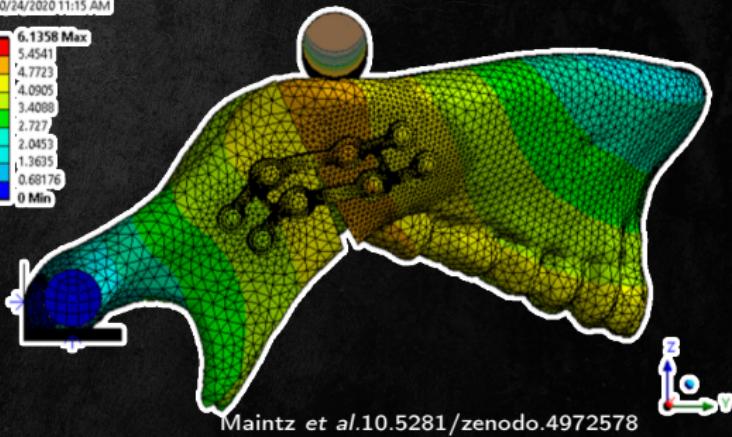
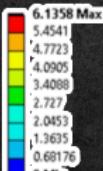
$$x_n = nh + a$$

$$f_n = f(x_n)$$

Ojo! $N \Leftarrow \text{int}\left(\frac{b - a}{h} + 1\right)$ pero entonces $b \Leftarrow (N - 1)h + a$



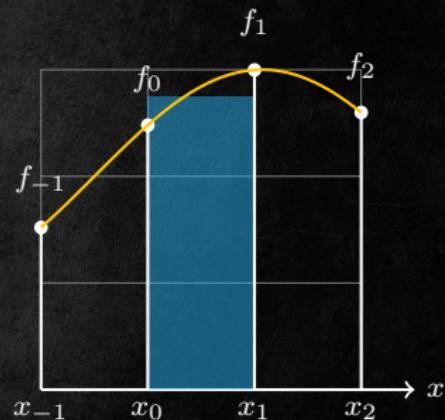
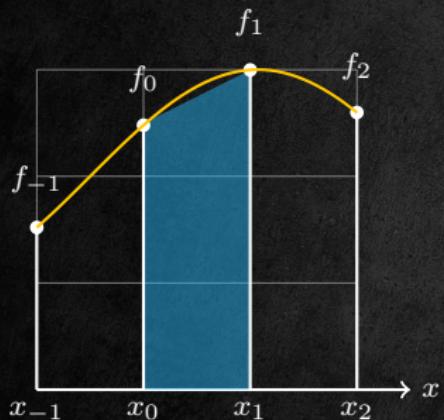
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Maintz et al.10.5281/zenodo.4972578

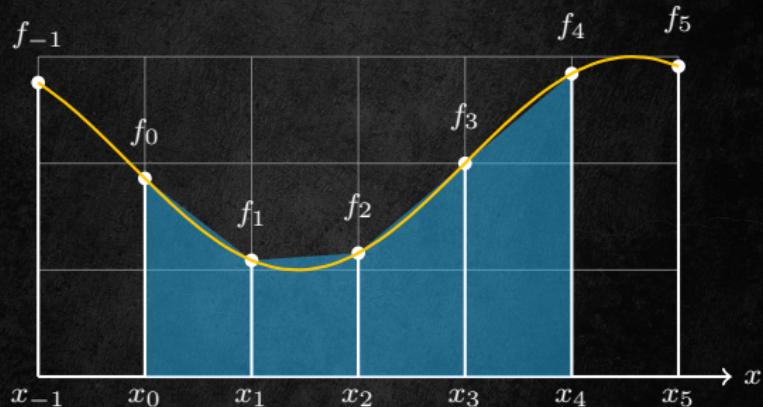
Trapecio

$$\int_{x_0}^{x_0+h} f(x)dx = \frac{h}{2}(f(x_0 + h) + f(x_0)) + O(h^3)$$



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$$\int_a^b f(\xi)d\xi = \int_{x_0}^{x_{N-1}} f(\xi)d\xi = \frac{h}{2} \sum_{i=0}^{N-2} (f_{i+1} + f_i) + O(h^2)$$

ERROR

Para un algoritmo iterativo (n es iteración)

ABSOLUTO: $\epsilon_n = |x_n - L|$

RELATIVO: $\tilde{\epsilon}_n = |x_n - L| / x_n$

Para una aproximación discreta

TRUNCAMIENTO: - LOCAL

$$\int_x^{x+h} f(\xi) d\xi = \frac{h}{2} (f(x+h) + f(x)) + O(h^3)$$

GLOBAL:

$$(N-1)O(h^3)$$

$$= \frac{b-a}{h} O(h^3)$$
$$= O(h^2)$$

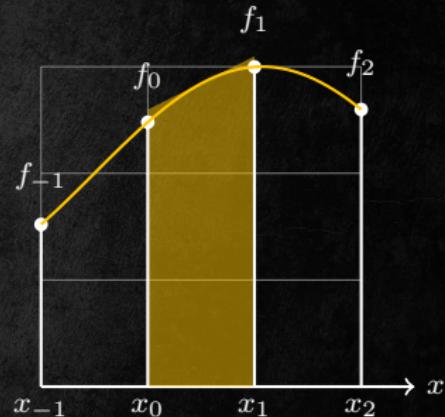
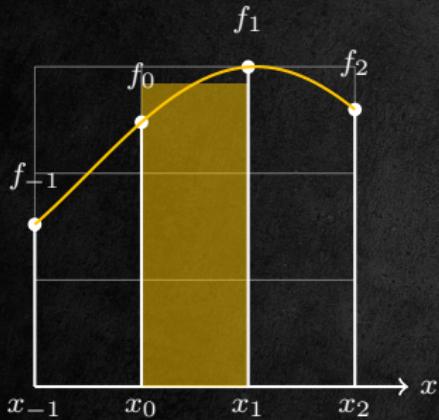
Por usar una compu

PRECISIÓN / NUMÉRICO: $\epsilon_M = 2^{-52} \approx 2.22 \cdot 10^{-16}$

$$\pi = 3.141592653589793 + O(\epsilon_M)$$

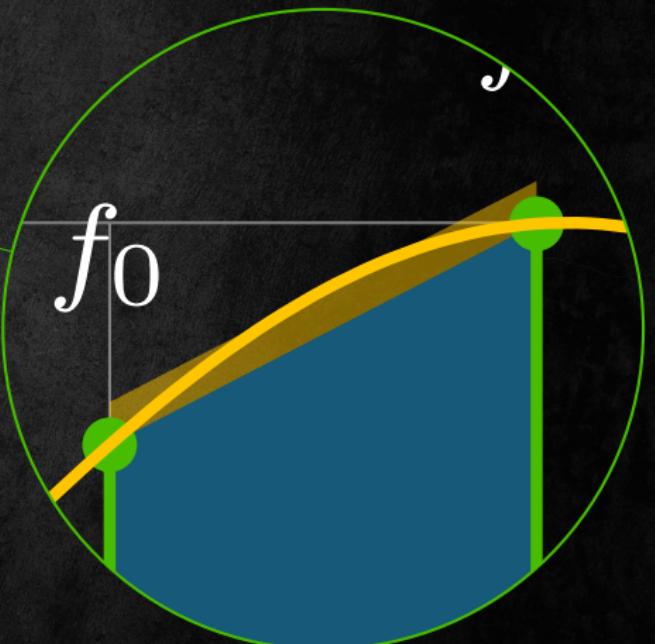
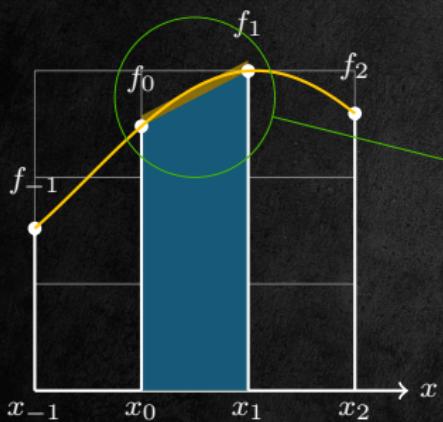
Punto Medio

$$\int_{x_0}^{x_0+h} f(x)dx = f\left(x_0 + \frac{h}{2}\right)h + O(h^3)$$



$$\int_a^b f(x)dx = h \sum_{i=0}^{N-2} f\left(\frac{x_{i+1} + x_i}{2}\right) + O(h^2)$$

Punto Medio vs Trapecios



MATHEMATICS

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Why does Trapezoidal Rule have potential error greater than Midpoint?

Asked 10 years, 8 months ago Modified 2 years, 8 months ago Viewed 35k times



I can approximate the area beneath a curve using the Midpoint and Trapezoidal methods, with errors such that:



$$Error_m \leq \frac{k(b-a)^3}{24n^2} \text{ and } Error_T \leq \frac{k(b-a)^3}{12n^2}.$$



Doesn't this suggest that the Midpoint Method is twice as accurate as the Trapezoidal Method?

Upcoming Event

2024 Comm
ends in 3 da

Featured on Me

Mathematics

<https://math.stackexchange.com/q/603830>

Wait... I get this with a few intervals, but with N intervals, the midpoint rule looks to me a shifted trapezoidal rule, with the first trapezoid drawn at $[a+h/2, a+3h/2]$, with h the step size. The half squares at the end, $[a, a+h/2]$ and $[b-h/2, b]$, can be included later, and appears to be negligible if N is large. How then the two methods give different errors if they are the same? — [alexis](#) Aug 14, 2023 at 3:37

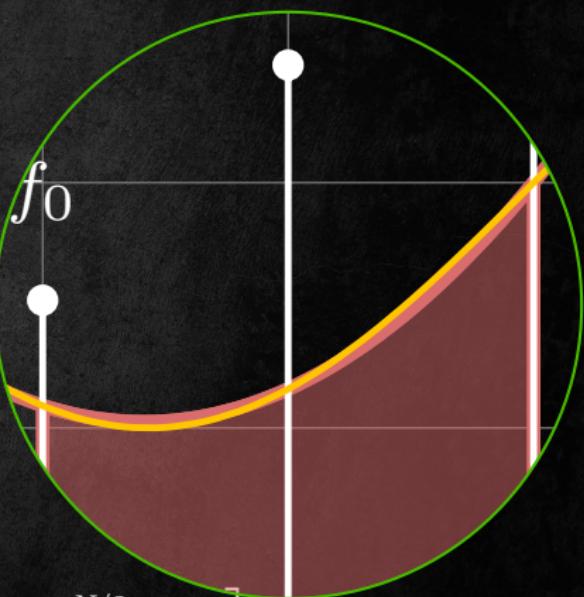
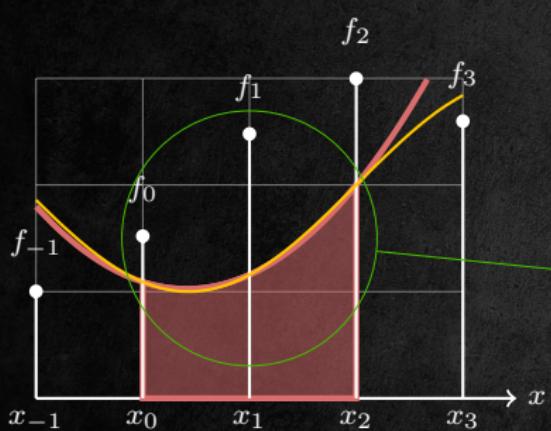
@alexis Your observation is a very interesting one, and something I hadn't thought of before. I'll have to give it more thought, but it's clear that the only possible explanation is that the half intervals at the ends are not, in fact, negligible. I've drawn some pictures and done a few small experiments confirming that that is the case, but it may take some time to write something up. — [Will Orrick](#) Aug 19, 2023 at 15:42

@alexis One example to consider: let f be increasing and convex on (a, b) . The

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Simpson

$$\int_{x_0}^{x_0+2h} f(x)dx = \frac{h}{3} [f(x_0 + 2h) + 4f(x_0 + h) + f(x_0)] + O(h^5)$$



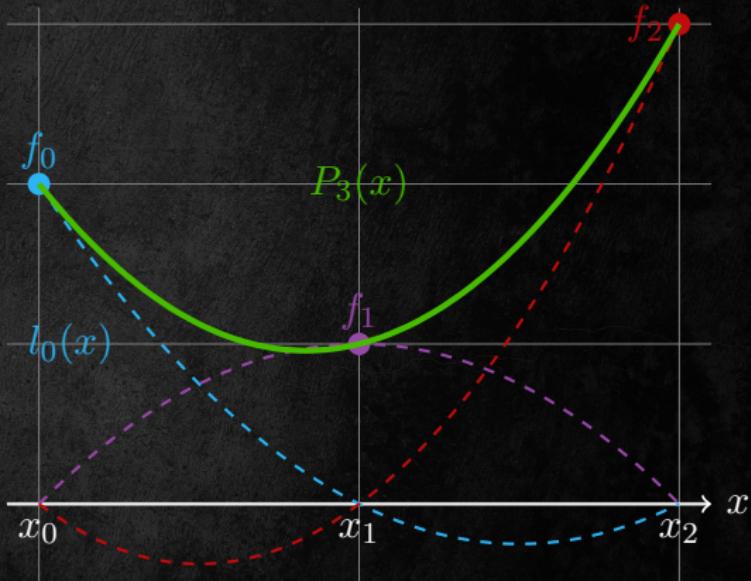
$$\int_a^b f(x)dx = \frac{h}{3} \left[f_0 + 2 \sum_{i=1}^{N/2-1} f_{2i} + 4 \sum_{i=1}^{N/2} f_{2i-1} \right] + f_{N-1} + O(h^4)$$

¿Interpretación geométrica? → Interpolación de Lagrange

$$P_k(x) = \sum_{j=0}^{k-1} f_j l_j(x)$$

$$l_j(x) = \prod_{i=0, i \neq j}^k \frac{x - x_i}{x_j - x_i}$$

El polinomio de menor grado que interpola.



$$P_3(x) = f_0 \frac{(x - x_1)(x - x_2)}{2h^2} - f_1 \frac{(x - x_0)(x - x_2)}{h^2} + f_2 \frac{(x - x_0)(x - x_1)}{2h^2}$$