Métodos Computacionales

Cuadraturas

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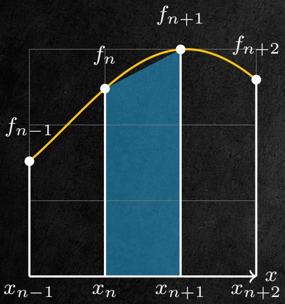
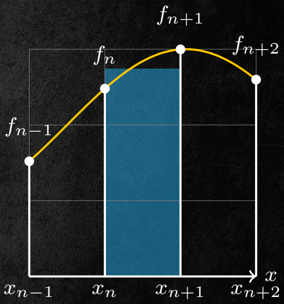
hecho con idiogram

Trapecio

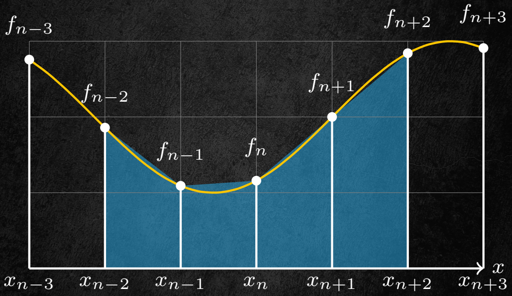
$$\int_x^{x+h} f(\xi)d\xi = \frac{h}{2}\Big(f(x+h)+f(x)\Big) + O\Big(h^3\Big)$$





Trapecio

$$\int_x^{x+h} f(\xi)d\xi = \frac{h}{2}\left(f(x+h) + f(x)\right) + O\left(h^3\right)$$



$$\int_a^b f(\xi)d\xi = \int_{x_0}^{x_{N-1}} f(\xi)d\xi = \frac{h}{2}\sum_{i=0}^{N-2}(f_{i+1} + f_i) + O\left(h^2\right)$$

ERROR

Para un algoritmo iterativo (n es iteración)

ABSOLUTO: $\epsilon_n = |x_n - L|$

RELATIVO: $\tilde{\epsilon}_n = |x_n - L| / x_n$

Para una aproximación discreta

TRUNCAMIENTO: - *LOCAL*

$$\int_x^{x+h} f(\xi) d\xi = \frac{h}{2}\big(f(x+h) + f(x)\big) + O\Big(h^3\Big)$$

GLOBAL:

$$(N-1)O(h^3)$$

$$= \frac{b-a}{h} O(h^3)$$

$$= O(h^2)$$

Por usar una compu

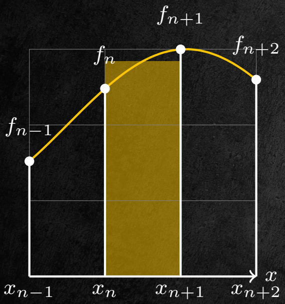
PRECISIÓN / NUMÉRICO:

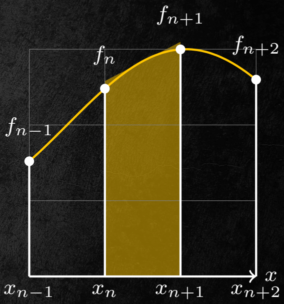
$$\epsilon_M = 2^{-52} \approx 2.22\,10^{-16}$$

$$\pi = 3.141592653589793 + O(\epsilon_M)$$

Punto Medio

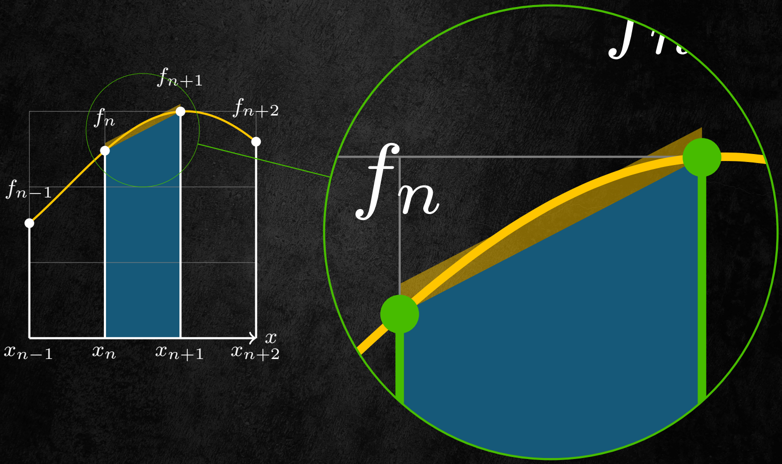
$$\int_{x}^{x+h} f(\xi)d\xi = f(x+h/2)h + O\left(h^3\right)$$





$$\int_{a}^{b} f(\xi)d\xi = h \sum_{i=0}^{N-2} f\left(\frac{x_{i+1}+x_i}{2}\right) + O\left(h^2\right)$$

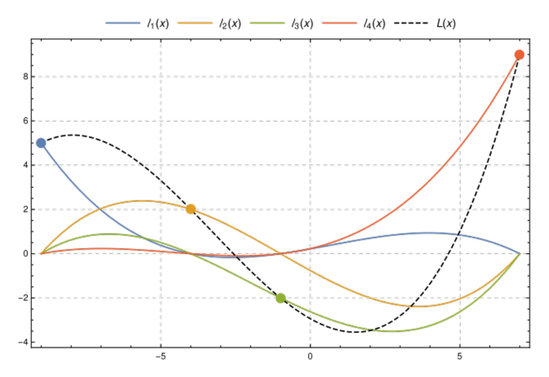
Punto Medio vs Trapecios



Interpolacion de Lagrange

$$P_k(x) = \sum_{j=0}^{k} c_j l_j(x)$$

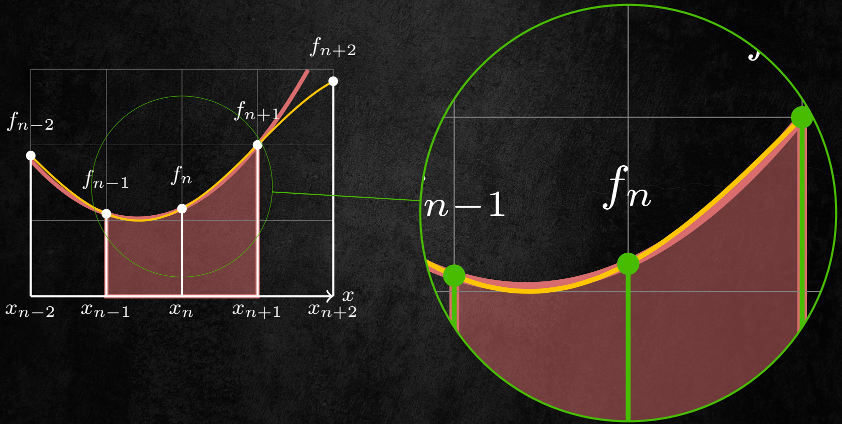
El polinomio de menor grado que interpola.



$$P_2(\xi) = \frac{f_{n-1}}{2}(\xi - x_n)(\xi - x_{n+1}) - f(\xi - x_{n-1})(\xi - x_{n+1}) + \frac{f_{n+1}}{2}(\xi - x_{n-1})(\xi - x_n)$$

Simpson

$$\int_x^{x+2h} f(\xi)d\xi = \frac{h}{3}\left[f(x+2h)+4f(x+h)+f(x)\right]+O\left(h^5\right)$$



$$\int_a^b f(\xi)d\xi = \frac{h}{3}\left[f_0 + 2\sum_{i=1}^{N/2-1} f_{2i} + 4\sum_{i=1}^{N/2} f_{2i-1}\right] + f_{N-1} + O\left(h^4\right)$$