Métodos Computacionales

Cuadraturas

Agosto 15, 2024

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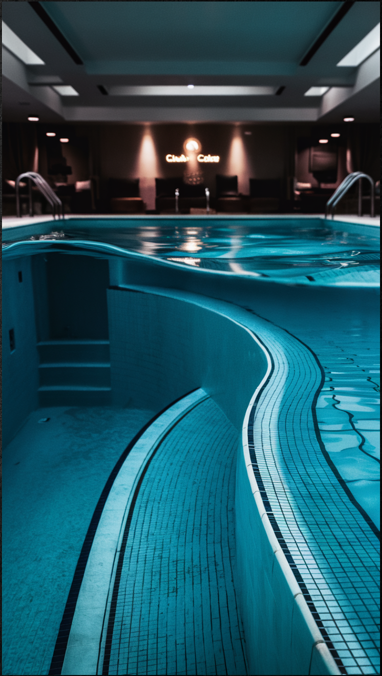
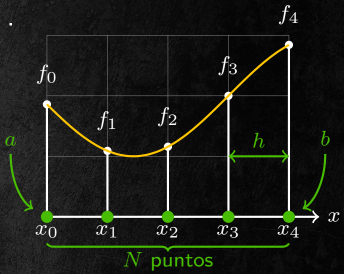


hecho con ideogram

Discretizar / Grillar / Teselar

Si discretizamos x *uniformemente* entre a y b...



...con N puntos, entonces

$h = \dfrac{b-a}{N-1}$ es el paso de la grilla.

...con un paso h, entonces

hay $N = \dfrac{b-a}{h} + 1$ puntos en la grilla.

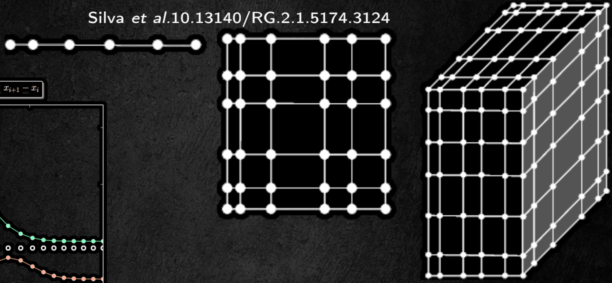
Notación

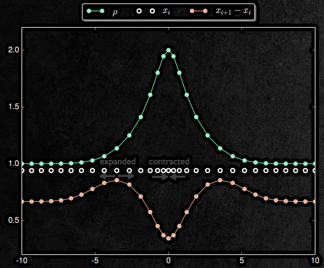
$$x_n = nh + a$$

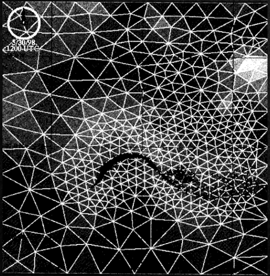
$$f_n = f(x_n)$$

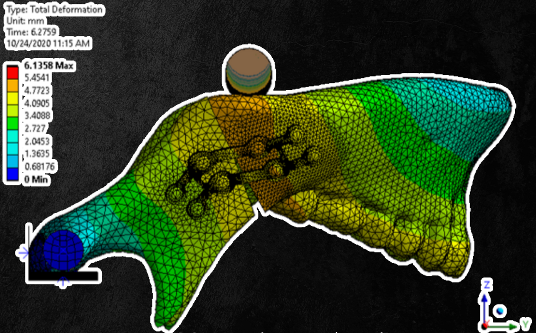
Ojo! $N \Leftarrow$ *int*$\left(\dfrac{b-a}{h} + 1\right)$ *pero entonces* $b \Leftarrow (N-1)h + a$

Silva *et al.*10.13140/RG.2.1.5174.3124







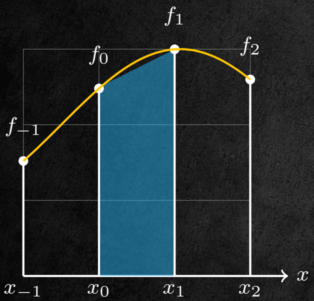


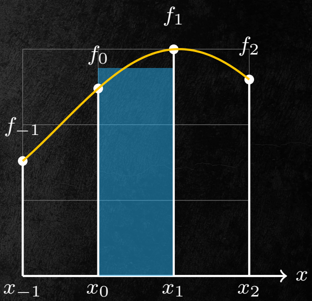
Maintz *et al.*10.5281/zenodo.4972578

Bacon *et al.*Mon. Weather Rev. 128 (2000) 2044

Trapecio

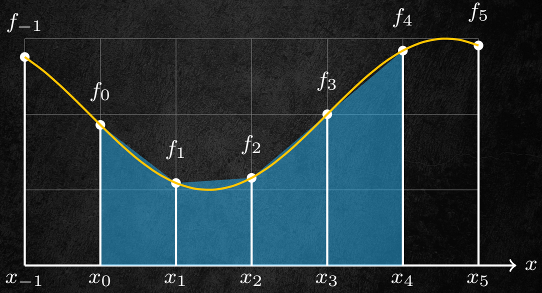
$$\int_{x_0}^{x_0+h} f(x)dx = \frac{h}{2}\big(f(x_0+h)+f(x_0)\big) + O\Big(h^3\Big)$$





Trapecio

$$\int_{x_0}^{x_0+h} f(x)dx = \frac{h}{2}\big(f(x_0+h) + f(x_0)\big) + O\Big(h^3\Big)$$



$$\int_a^b f(x)dx = \int_{x_0}^{x_{N-1}} f(x)dx = \frac{h}{2}\sum_{i=0}^{N-2}(f_{i+1} + f_i) + O\Big(h^2\Big)$$

ERROR

Para un algoritmo iterativo (n es iteración)

ABSOLUTO: $\epsilon_n = |x_n - L|$

RELATIVO: $\tilde{\epsilon}_n = |x_n - L| / x_n$

Para una aproximación discreta

TRUNCAMIENTO: - *LOCAL*

$$\int_x^{x+h} f(x)dx = \frac{h}{2}\big(f(x+h) + f(x)\big) + O\Big(h^3\Big)$$

GLOBAL:

$$(N-1)O(h^3)$$
$$= \frac{b-a}{h}O(h^3)$$
$$= O(h^2)$$

Por usar una compu

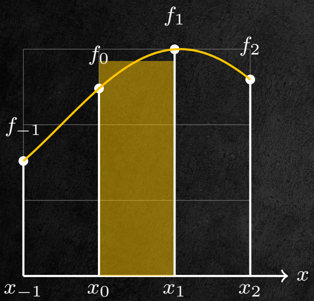
PRECISIÓN / NUMÉRICO:

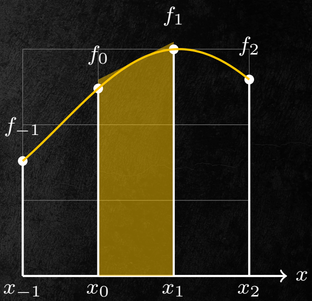
$$\epsilon_M = 2^{-52} \approx 2.22\,10^{-16}$$

$$\pi = 3.141592653589793 + O(\epsilon_M)$$

Punto Medio

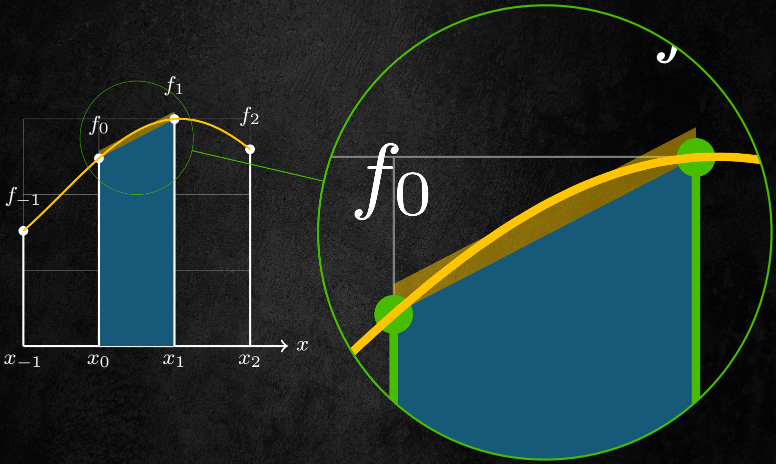
$$\int_{x_0}^{x_0+h} f(x)dx = f\left(x_0 + \frac{h}{2}\right)h + O\left(h^3\right)$$





$$\int_a^b f(x)dx = h\sum_{i=0}^{N-2} f\left(\frac{x_{i+1}+x_i}{2}\right) + O\left(h^2\right)$$

Punto Medio vs Trapecios



$$O(h^3) = \frac{f'' h^3}{24} + O(h^4)$$

$$O(h^3) = \frac{f'' h^3}{12} + O(h^4)$$

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Why does Trapezoidal Rule have potential error greater than Midpoint?

Asked 10 years, 8 months ago Modified 2 years, 8 months ago Viewed 35k times

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I can approximate the area beneath a curve using the Midpoint and Trapezoidal methods, with errors such that:

$$Error_m \leq \frac{k(b-a)^3}{24n^2} \text{ and } Error_T \leq \frac{k(b-a)^3}{12n^2}.$$

Doesn't this suggest that the Midpoint Method is twice as accurate as the Trapezoidal Method?

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https://math.stackexchange.com/q/603830

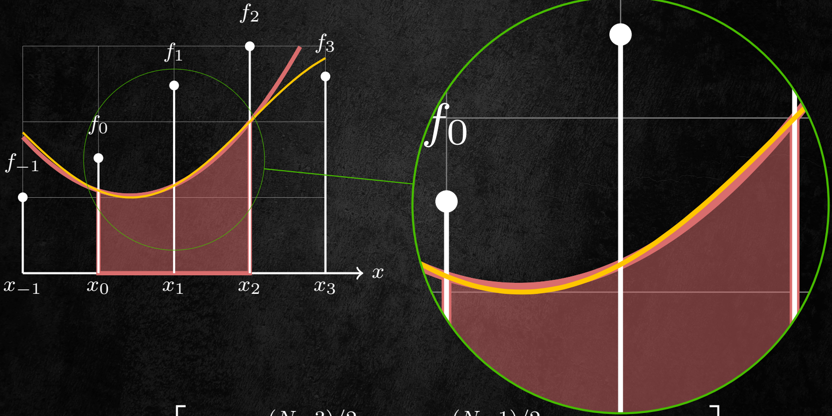
Wait... I get thisnwith a few intervals, but with N intervals, the midpoint rule looks to me a shifted trapezoidal rule, with the first trapezoid drawn at [a+h/2,a+3/2h], with h the step size. The half squares at the end, [a,a+h/2] and [b-h/2,b], can be included later, and appears to be negligible if N is large. How then the two methods gives different errors if are the same? – alexis Aug 14, 2023 at 3:37

@alexis Your observation is a very interesting one, and something I hadn't thought of before. I'll have to give it more thought, but it's clear that the only possible explanation is that the half intervals at the ends are not, in fact, negligible. I've drawn some pictures and done a few small experiments confirming that that is the case, but it may take some time to write something up. – Will Orrick Aug 19, 2023 at 15:42

@alexis One example to consider: let f be increasing and convex on (a, b). The

Simpson

$$\int_{x_0}^{x_0+2h} f(x)dx = \frac{h}{3}\left[f(x_0+2h) + 4f(x_0+h) + f(x_0)\right] + O\left(h^5\right)$$



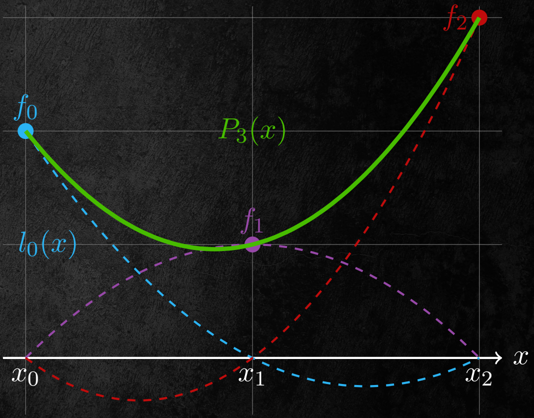
$$\int_a^b f(x)dx = \frac{h}{3}\left[f_0 + 2\sum_{i=1}^{(N-3)/2} f_{2i} + 4\sum_{i=1}^{(N-1)/2} f_{2i-1} + f_{N-1}\right] + O\left(h^4\right)$$

¿Interpretación geométrica? → Interpolación de Lagrange

$$P_k(x) = \sum_{j=0}^{k-1} f_j l_j(x)$$

$$l_j(x) = \prod_{i=0,\, i\neq j}^{k} \frac{x - x_i}{x_j - x_i}$$

El polinomio de menor grado que interpola.



$$P_3(x) = f_0 \frac{(x-x_1)(x-x_2)}{2h^2} - f_1 \frac{(x-x_0)(x-x_2)}{h^2} + f_2 \frac{(x-x_0)(x-x_1)}{2h^2}$$