

# Métodos Computacionales

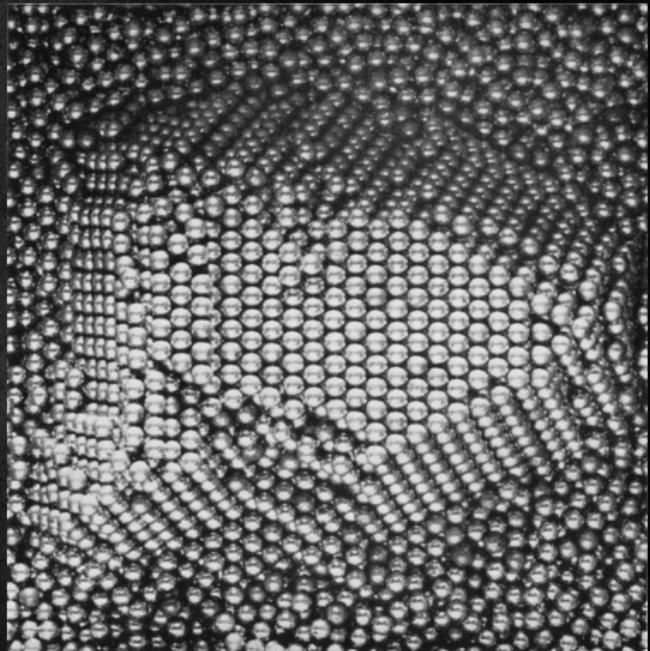
## Dinámica Molecular

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S. Alexis Paz



Departamento de  
QUÍMICA TEÓRICA  
Y COMPUTACIONAL  
Facultad de Ciencias Químicas  
Universidad Nacional de Córdoba



Bernal et al., Proc. Roy. Soc. A 280(1964)16

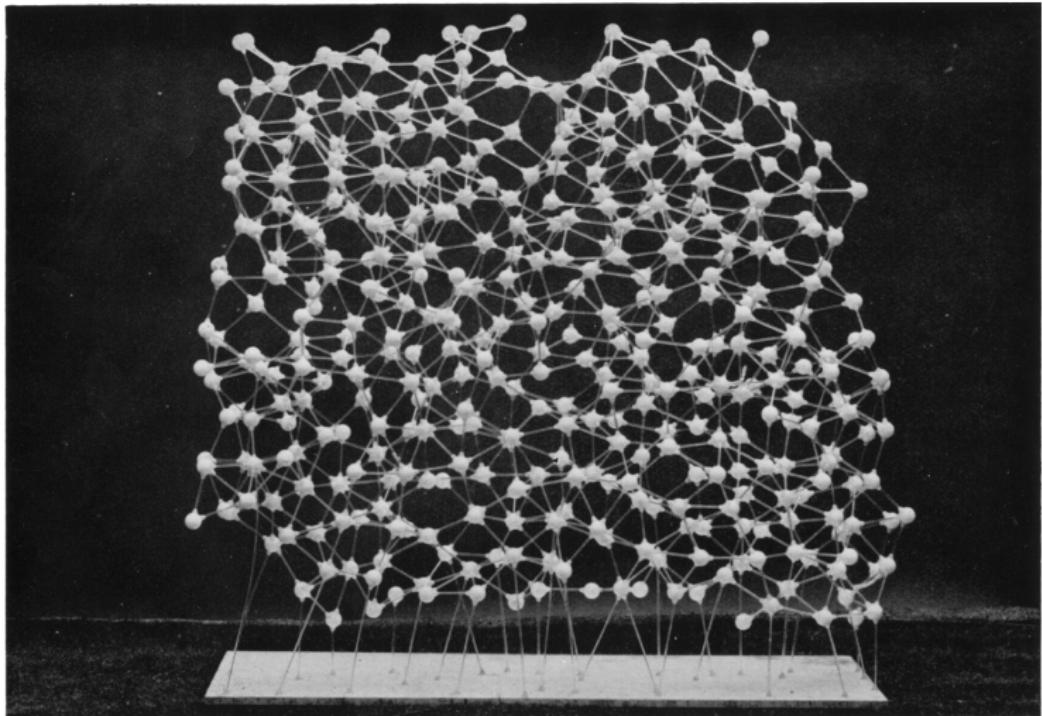


FIGURE 13. Ball and spoke model of random rigid sphere assembly. The transparency shows some of the collinearities which occur in it.

Bernal *et al.*, Proc. Roy. Soc. A 280(1964)16

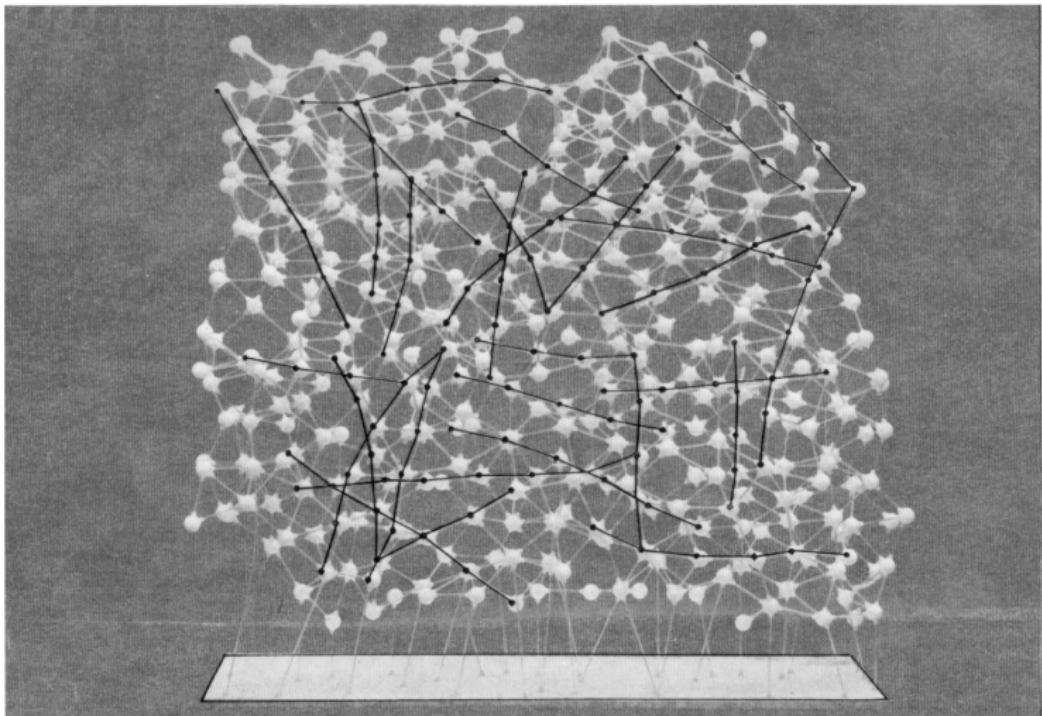


FIGURE 13. Ball and spoke model of random rigid sphere assembly. The transparency shows some of the collinearities which occur in it.

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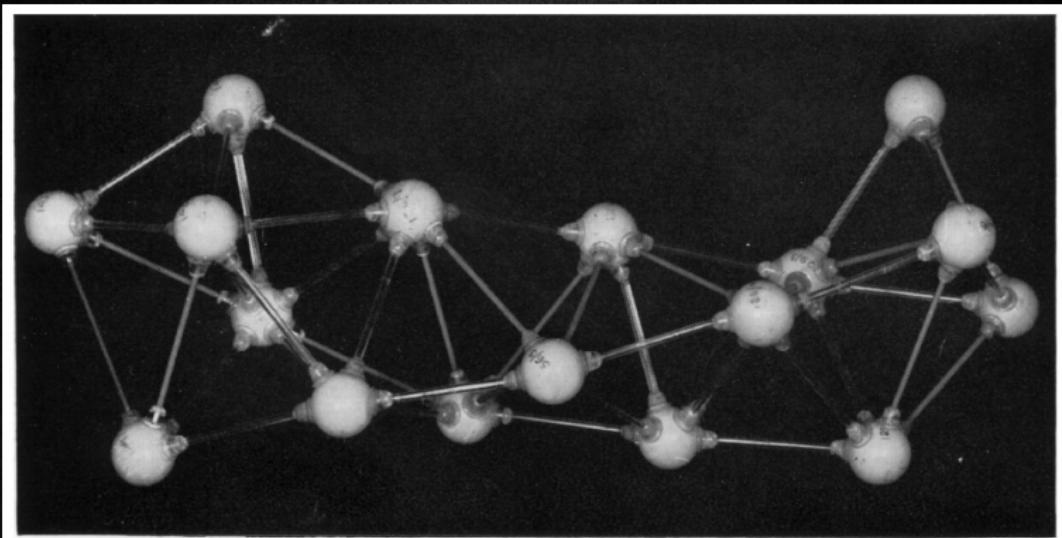


FIGURE 12. Triple helix of tetrahedra showing pseudonucleus from random rigid sphere assembly model.

Bernal et al., Proc. Roy. Soc. A 280(1964)16

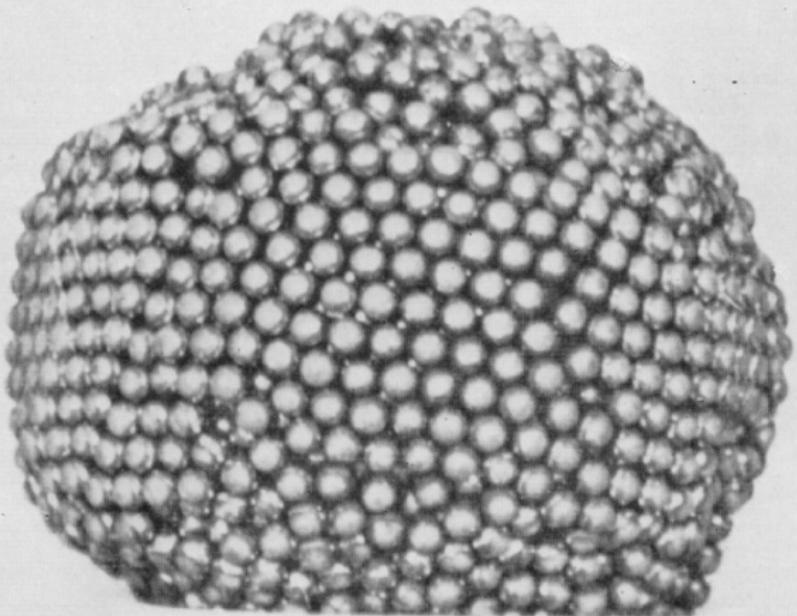


FIGURE 15. Regular crystalline patches induced on the sides of ball-bearing mass due to smoothness of balloon surface.

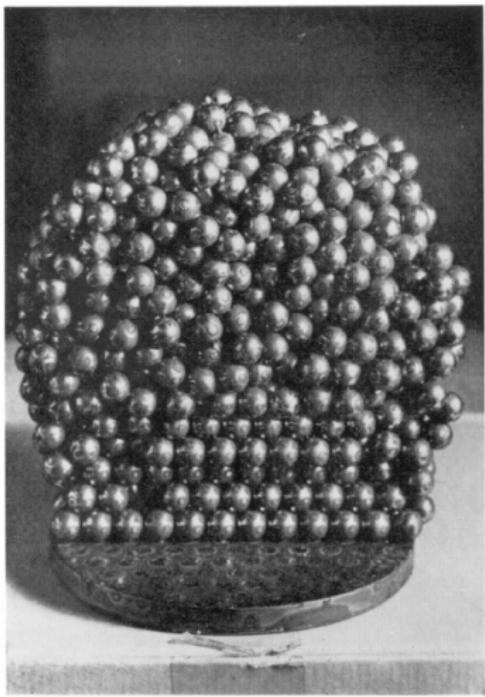


FIGURE 16. Ball-bearing assembly showing transition from random close-packing to regular crystalline array induced by inserting a flat plate.

Bernal *et al.*, Proc. Roy. Soc. A 280(1964)16

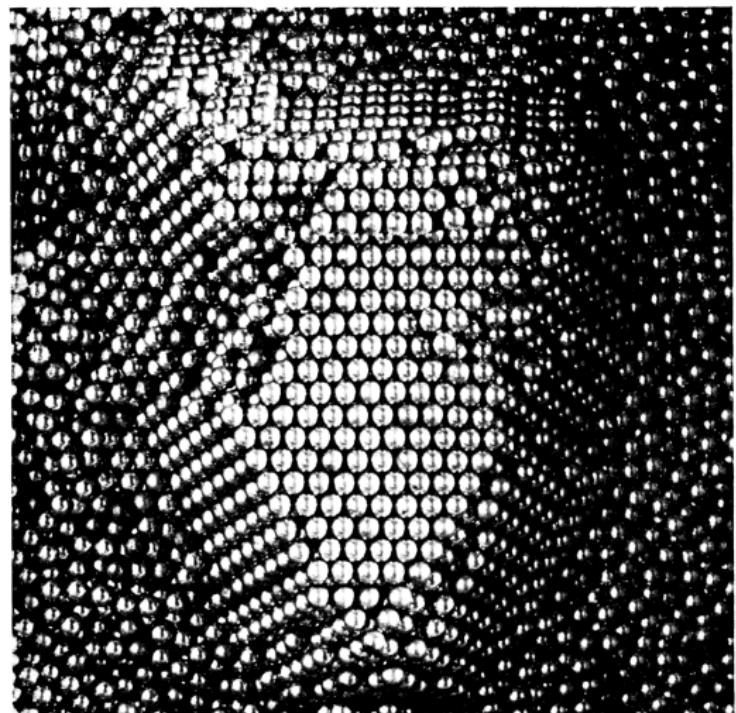


FIGURE 14. Face-centred cubic 'crystal' surrounded by 'liquid' caused by shearing ball-bearing mass. 111 face is shown at the top surface.

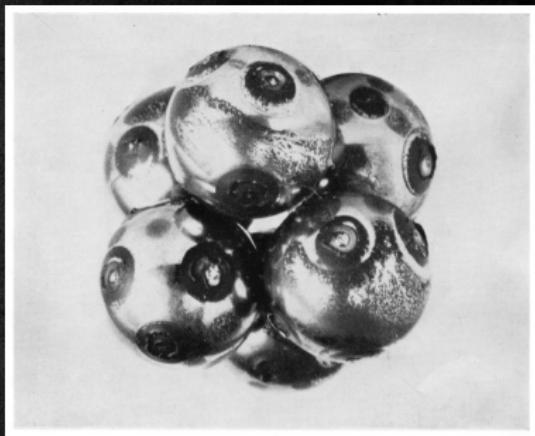


FIGURE 11. Portion of random close-packed ball assembly showing marks of further contacts.

Bernal et al., Proc. Roy. Soc. A 280(1964)16

# Dinámica

Estudia el **movimiento** en relación con las causas que lo producen.

Ecuaciones de movimiento, *por ejemplo...*:

Newton

$$m \frac{d^2\mathbf{r}}{dt^2} = -\nabla U(\mathbf{r})$$

Langevin  $m \frac{d\mathbf{v}}{dt} = -\nabla U - \gamma\mathbf{v} + \sqrt{2\gamma k_B T} \boldsymbol{\eta}$

Browniano  $\gamma\mathbf{v} = -\nabla U + \sqrt{2\gamma k_B T} \boldsymbol{\eta}$

Barostato de Andersen

$$\frac{d\mathbf{r}}{dt} = \frac{\mathbf{p}}{m} + \frac{\mathbf{r}}{3} \frac{d \ln V}{dt}$$

$$\frac{d\mathbf{p}}{dt} = -\nabla U - \frac{\mathbf{p}}{3} \frac{d \ln V}{dt}$$

$$\frac{d^2V}{dt^2} = \frac{p_0}{M} + \frac{(\mathbf{p}^2/m - \nabla U)}{3MV}$$

DM acelerada por temperatura

$$\gamma \frac{d\mathbf{r}}{dt} = -\nabla U + \sqrt{2\gamma k_B T}$$

$$-\kappa(\boldsymbol{\theta} - \mathbf{z}) \nabla \boldsymbol{\theta} \boldsymbol{\eta}^x$$

$$\bar{\gamma} \frac{d\mathbf{z}}{dt} = \kappa(\boldsymbol{\theta} - \mathbf{z}) + \sqrt{2\bar{\gamma} k_B \bar{T}} \boldsymbol{\eta}^z$$

# Distribución de equilibrio

¿Qué probabilidad de encontrar un estado del sistema generan?

Newton

$$\rho(\mathbf{r}) = \Omega^{-1}, \text{ NVE}$$

Langevin  $\rho(\mathbf{r}) = Z^{-1} e^{\beta U(\mathbf{r})}, \text{ NVT}$

Browniano  $\rho(\mathbf{r}) = Z^{-1} e^{\beta U(\mathbf{r})}, \text{ NVT}$

Barostato de Andersen

$$\rho(\mathbf{r}) = Z^{-1} e^{\beta U(\mathbf{r}) + \beta PV}$$

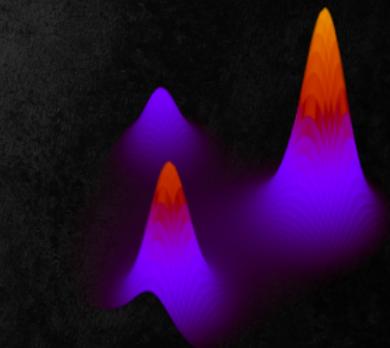
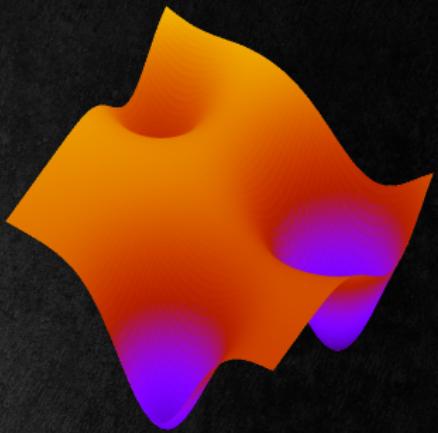
$$NPH \rightarrow NPT$$

DM acelerada por temperatura

$$\rho(\mathbf{r}) = Z^{-1} e^{\beta(U(\mathbf{r}) + \kappa(\boldsymbol{\theta} - \mathbf{z})^2/2)}$$

$$\rho(\mathbf{z}) \approx \bar{Z}^{-1} e^{\bar{\beta} F(\mathbf{z})}$$

*¡Pero la dinámica también muestrea fuera del equilibrio!*





"Rush", Tomasz 'Darkshape' Kaluzny



Horton Plains

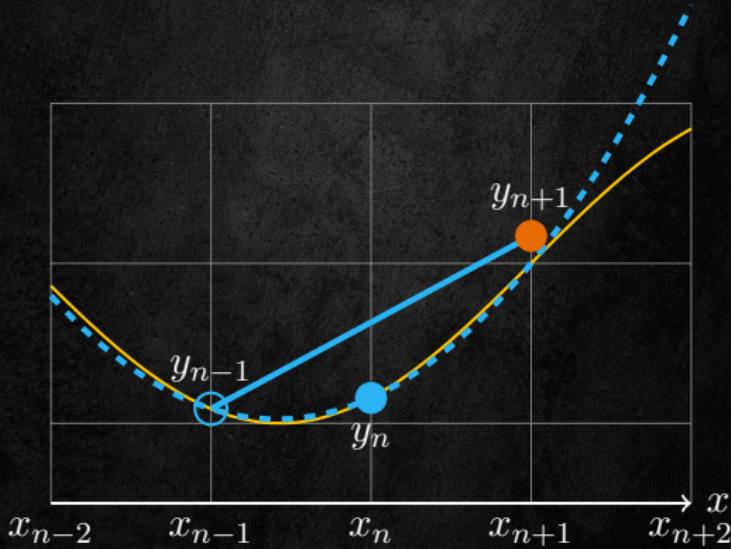


"nude"  
Shinichi Maruyama

Lo vimos con esta notación  $\frac{dy}{dx} = F(x, \vec{y})$

## Verlet

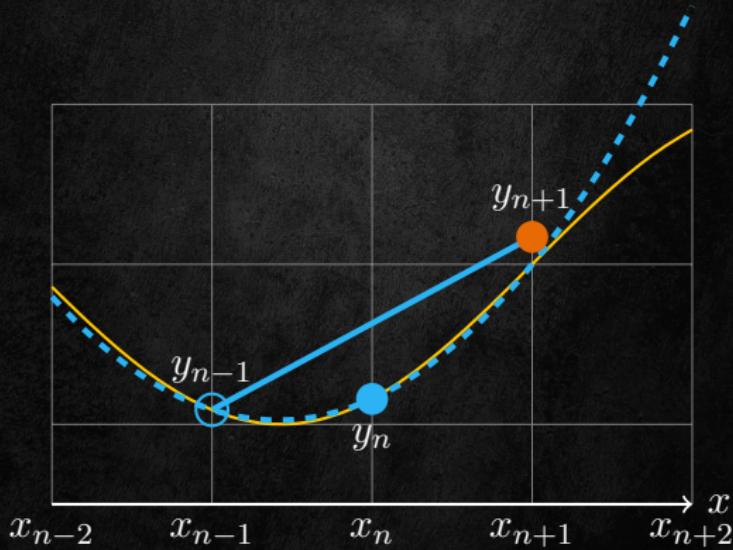
$$\mathbf{y}_{n+1} = 2\mathbf{y}_n - \mathbf{y}_{n-1} + h^2 \frac{d^2\mathbf{y}}{dx^2} + O(h^4)$$



$$\text{Newton} \quad \frac{d^2\mathbf{r}}{dt^2} = \frac{\mathbf{F}(\mathbf{r})}{m} = -\frac{\nabla U(\mathbf{r})}{m}$$

## Verlet

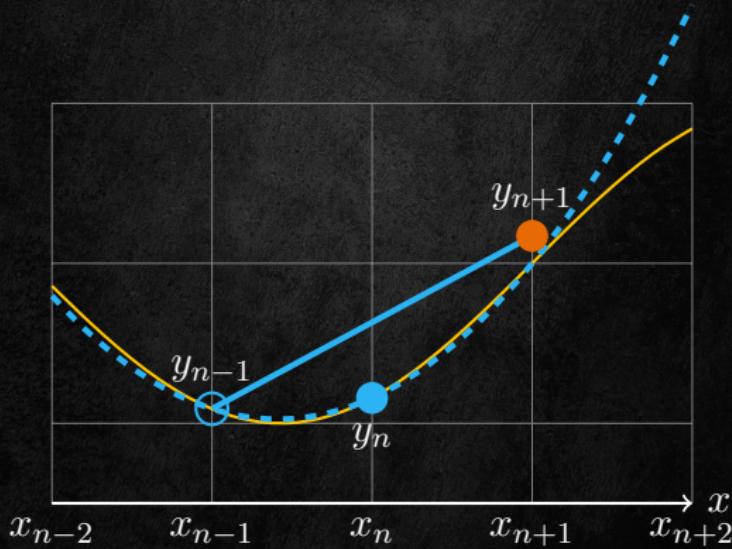
$$\mathbf{r}_{n+1} = 2\mathbf{r}_n - \mathbf{r}_{n-1} + \Delta t^2 \left. \frac{d^2\mathbf{r}}{dt^2} \right|_{\mathbf{r}_n} + O(h^4)$$



$$\text{Newton} \quad \frac{d^2\mathbf{r}}{dt^2} = \frac{\mathbf{F}(\mathbf{r})}{m} = -\frac{\nabla U(\mathbf{r})}{m}$$

## Verlet

$$\mathbf{r}_{n+1} = 2\mathbf{r}_n - \mathbf{r}_{n-1} - \Delta t^2 \frac{\mathbf{F}_n}{m} + O(h^4)$$



## Velocity Verlet / Strömer-Verlet:

$$\mathbf{r}_{n+1} = \mathbf{r}_n + \mathbf{v}_n \Delta t + \frac{\mathbf{F}_n}{2m} \Delta t^2$$

$$\mathbf{v}_{n+1} = \mathbf{v}_n + \frac{\mathbf{F}_n + \mathbf{F}_{n+1}}{2m} \Delta t \quad \text{Newton}$$

## Browniano Convencional:

$$\mathbf{r}_{n+1} = \mathbf{r}_n + \frac{\Delta t}{m\gamma} F_n + \sigma_X \mathbf{X}_G \quad \sigma_X = \sqrt{\frac{2k_B T \Delta t}{m\gamma}} \quad \text{Browniano}$$

## Ermak:

$$\mathbf{r}_{n+1} = \mathbf{r}_n + c_1 \Delta t \mathbf{v}_n + c_2 \Delta t^2 \frac{F_n}{m} + \sigma_X \mathbf{X}_G \quad \text{Langevin}$$

$$\mathbf{v}_{n+1} = c_0 \mathbf{v}_n + c_1 \Delta t \frac{F_n}{m} + \frac{c_2 \Delta t}{m} (F_{n+1} - F_n) + \sigma_V (\sigma_{V,X} \mathbf{X}_G + \sigma_{V,V} \mathbf{V}_G)$$

Born-Oppenheimer:

$$\hat{H} = - \sum_i \frac{\nabla_i^2}{2} - \sum_{i,A} \frac{Z_A}{r_{iA}} + \sum_{i>j} \frac{1}{r_{ij}} - \sum_A \frac{\nabla_A^2}{2M_A} + \sum_{B>A} \frac{Z_A Z_B}{R_{AB}}$$

$$\hat{H} = \hat{H}_e(R, r) - \sum_A \frac{\nabla_A^2}{2M_A} + \sum_{B>A} \frac{Z_A Z_B}{R_{AB}} \quad \Psi(R, r) = \psi(R, r)\phi(R)$$

Schrödinger:  $\hat{H}\Psi(R, r) = E\Psi(R, r)$

$$\psi(R, r) \left[ E_e(R) - \sum_A \frac{\nabla_A^2}{2M_A} + \sum_{B>A} \frac{Z_A Z_B}{R_{AB}} \right] \phi(R) = \psi(R, r) E \phi(R)$$

$$\left[ - \sum_A \frac{\nabla_A^2}{2M_A} + U(R) \right] \phi(R) = E \phi(R)$$

## Potencial de a pares:

$$U(\mathbf{r}) = \sum_i U_1(\mathbf{x}_i) + \sum_{i < j} U_2(\mathbf{x}_i, \mathbf{x}_j) + \sum_{i < j < k} U_3(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_k) + \dots$$

$$U(\mathbf{r}) = \sum_{i < j} U_2(r_{ij}) + \sum_{i < j < k} U_3(\mathbf{r}_{ij}, \mathbf{r}_{jk}) + \dots$$

### Lennard-Jones

$$U_2(r_{ij}) = 4\epsilon \left[ \left( \frac{\sigma}{r_{ij}} \right)^{12} - \left( \frac{\sigma}{r_{ij}} \right)^6 \right]$$

### Morse

$$U_2(r_{ij}) = D_e (1 - e^{\alpha(r_{ij} - r_e)})^2 - D_e$$

### Fuerza

$$\begin{aligned}\mathbf{F}_{r_i} &= \nabla_{r_i} U_2(r_{ij})|_{r1} \\ &= \frac{\partial U_2(r_{ij})}{\partial r_{ij}} \nabla_{r_i} r_{ij}\end{aligned}$$

$$= \frac{\partial U_2(r_{ij})}{\partial r_{ij}} \hat{\mathbf{r}}_{ij}$$

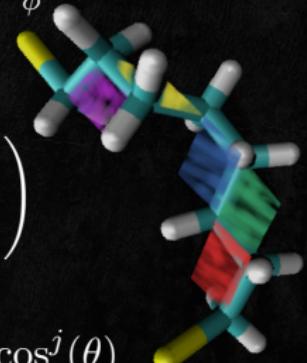
## Campo de Fuerzas:

Hill (1942)

$$U\left(\{r_{ij}\}, \{d\}, \{\phi\}\right) = U_2^{\text{LJ}}(r_{ij}) + \sum_d k_d (d - d_{eq})^2 + \sum_\phi k_\phi (\phi - \phi_{eq})^2$$

OPLS (1984)

$$\begin{aligned} U\left(\{r_{ij}\}, \{d\}, \{\phi\}, \{\theta\}\right) &= \sum_{i < j} f_{ij} \left( \frac{q_i q_j e_0^2}{r_{ij}} + U_2^{\text{LJ}}(r_{ij}) \right) \\ &+ \sum_d k_d (d - d_{eq})^2 + \sum_\phi k_\phi (\phi - \phi_{eq})^2 + \sum_\theta \sum_{j=0}^5 C_j \cos^j(\theta) \end{aligned}$$



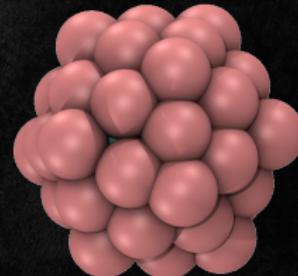
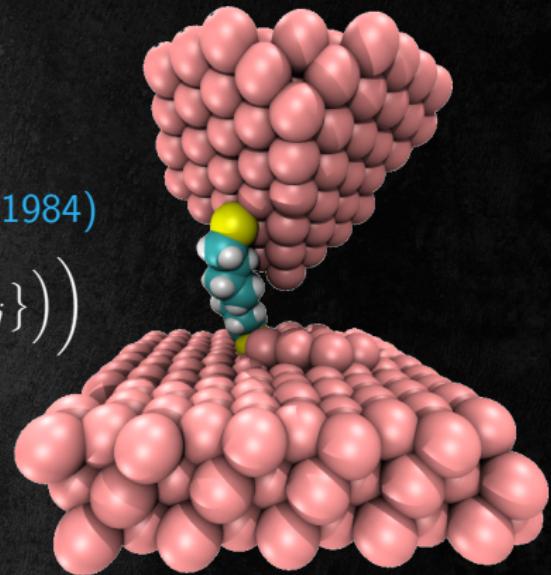
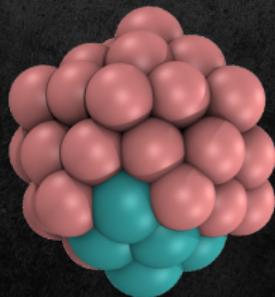
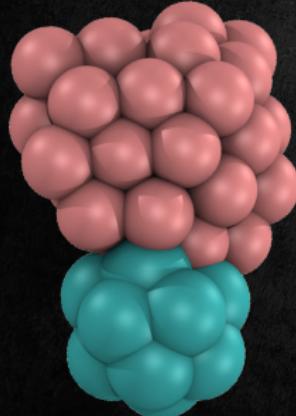
CHARMM, GROMOS, AMBER, Tersoff, ReaxFF, ...

Muchos Cuerpos:

El método del átomo embebido, EAM (1984)

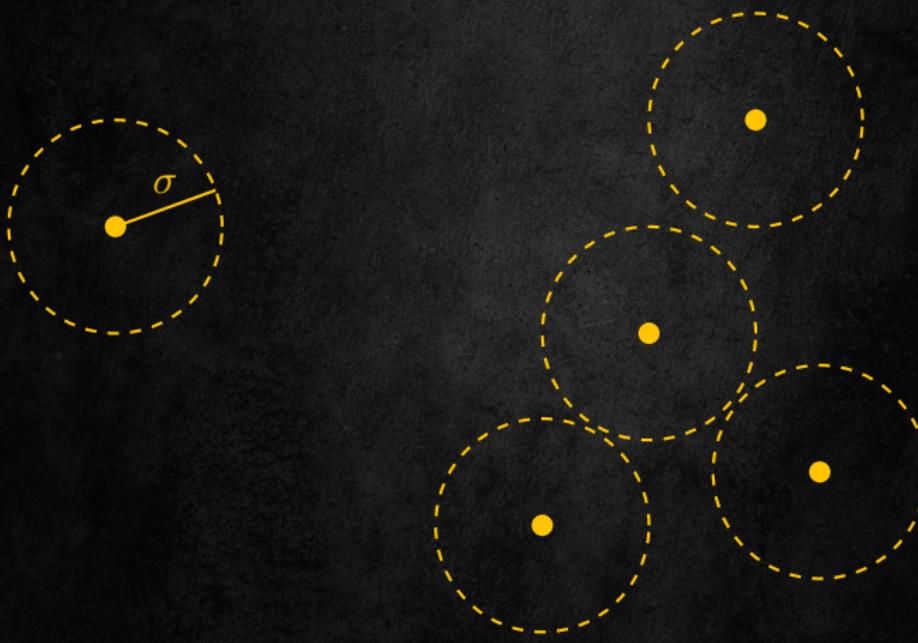
$$U(\mathbf{r}) = \frac{1}{2} \sum_{i < j} U_2(r_{ij}) + \sum_i E_i\left(\rho_i\left(\{r_{ij}\}\right)\right)$$

$$\rho_i = \sum_{j \neq i} \psi_j(r_{ij})$$

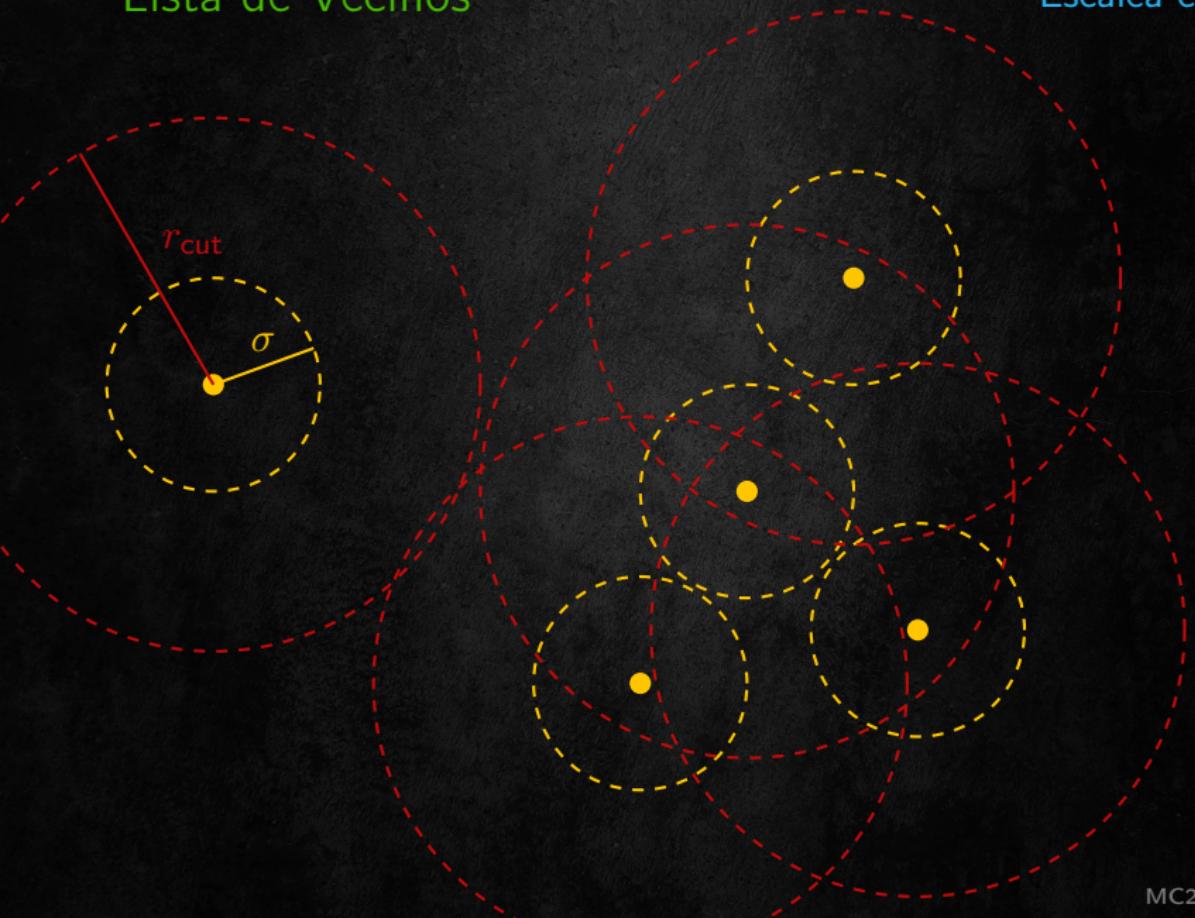


# Lista de Vecinos

Escalea con  $N^2$



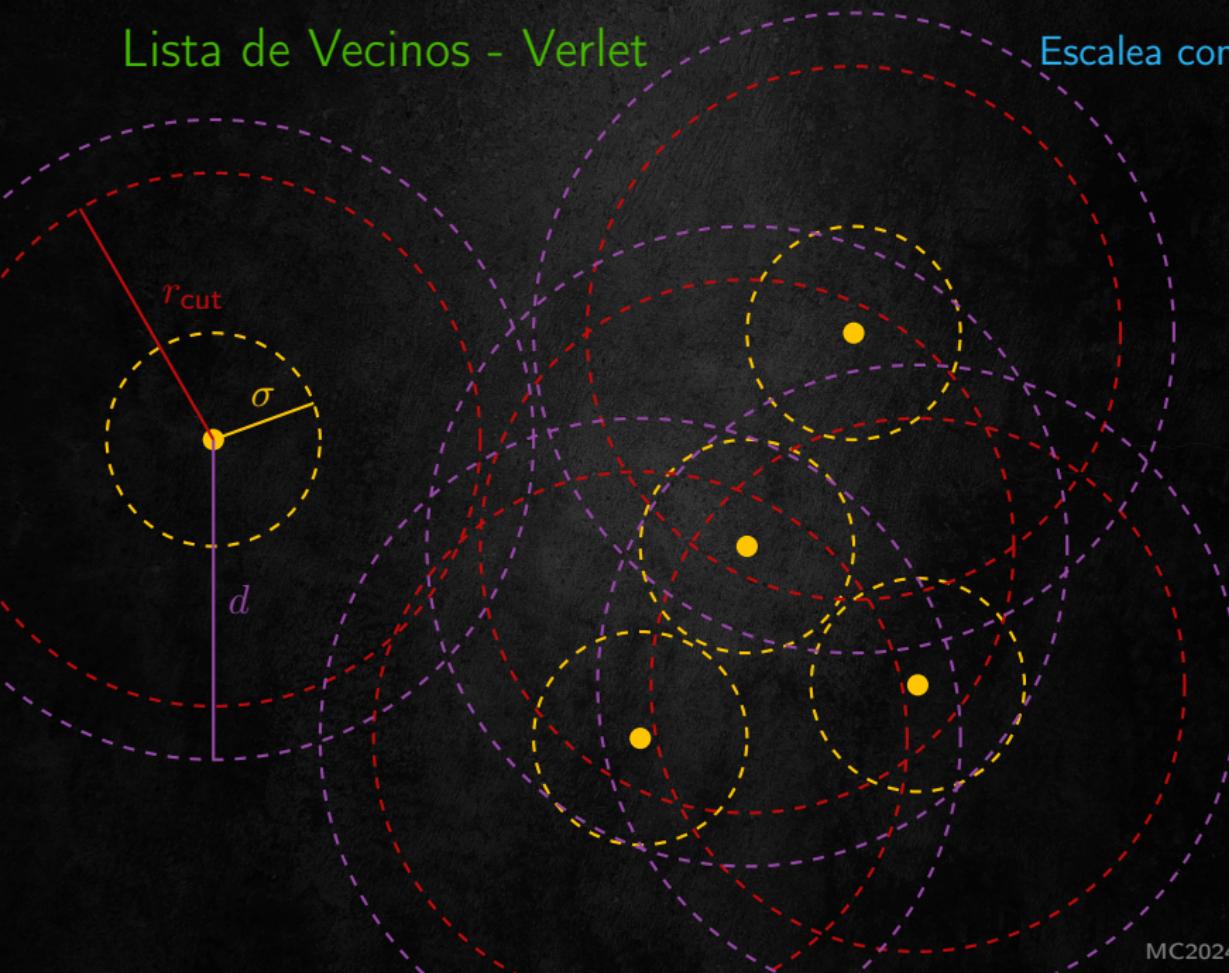
## Lista de Vecinos



Escalea con  $N^2$

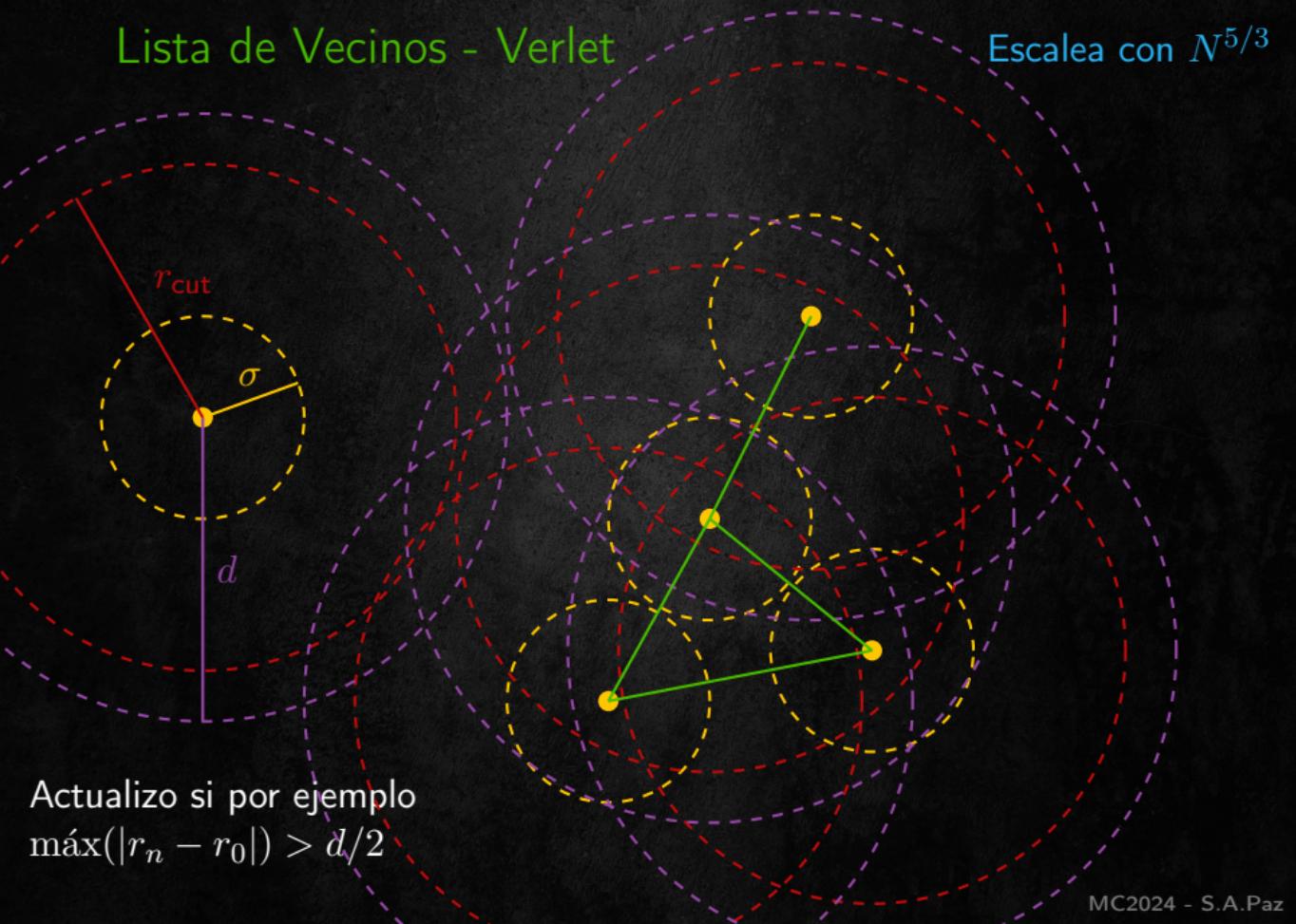
## Lista de Vecinos - Verlet

Escalea con  $N^{5/3}$



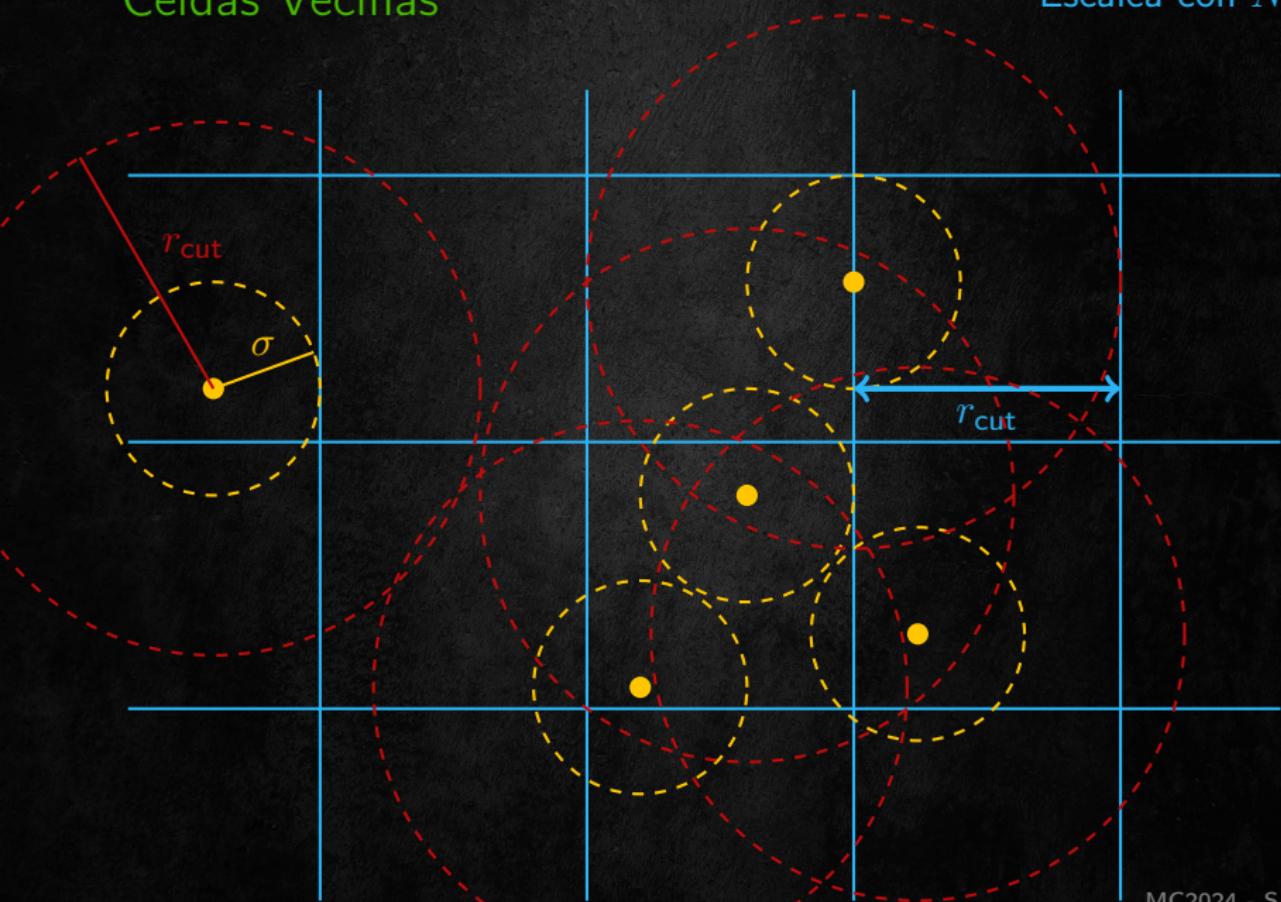
## Lista de Vecinos - Verlet

Escala con  $N^{5/3}$

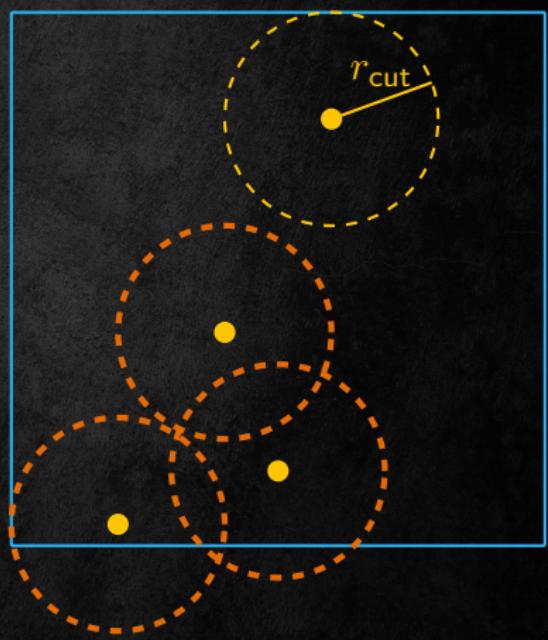
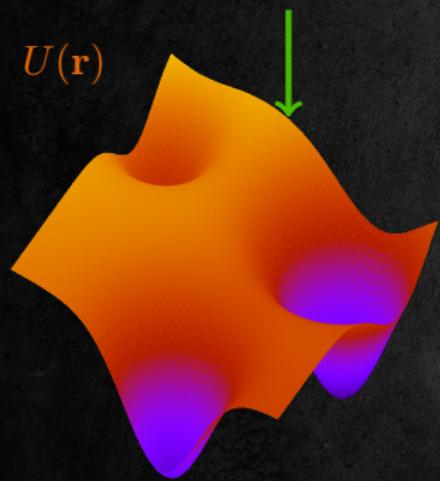


Actualizo si por ejemplo  
 $\max(|r_n - r_0|) > d/2$

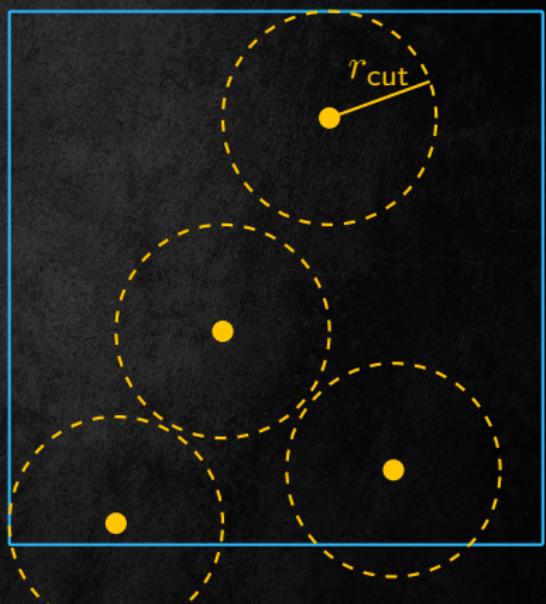
## Celdas Vecinas



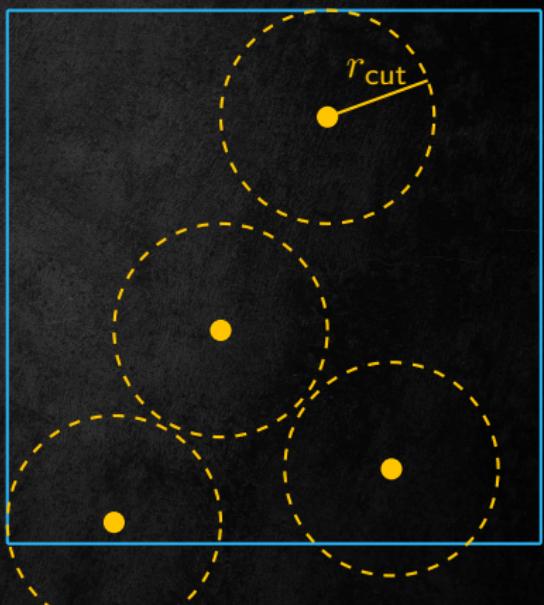
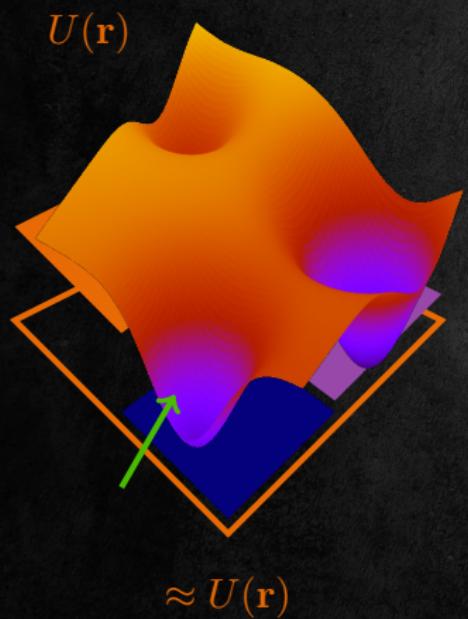
## Configuración Inicial:



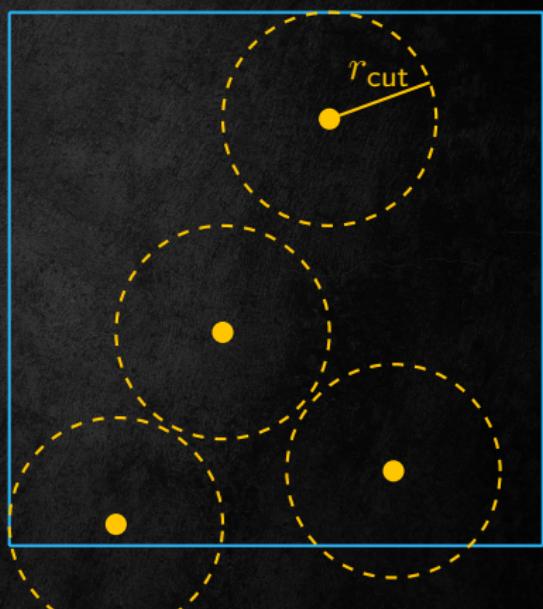
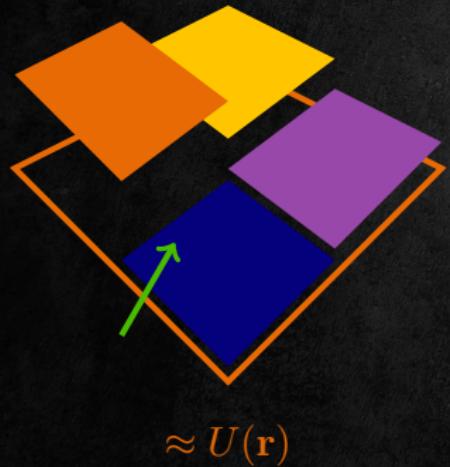
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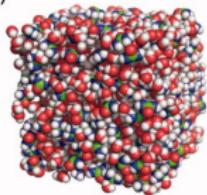




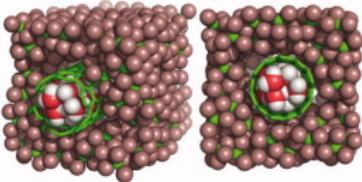
# PACKMOL

Initial configurations for Molecular Dynamics Simulations by packing optimization

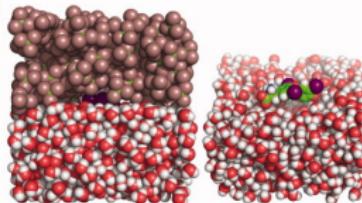
(a)



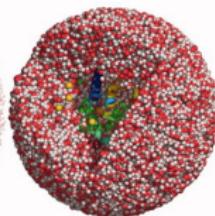
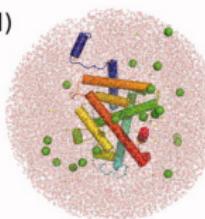
(b)



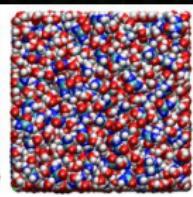
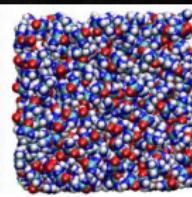
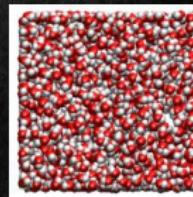
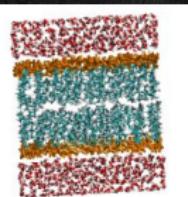
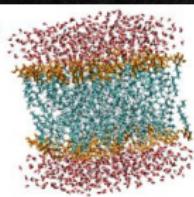
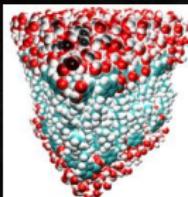
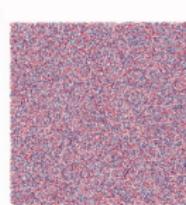
(c)



(d)



(e)



Martinez *et al.*, J. Comp. Chem. 24(2003)819  
Martinez *et al.*, J. Comp. Chem. 30(2009)2157

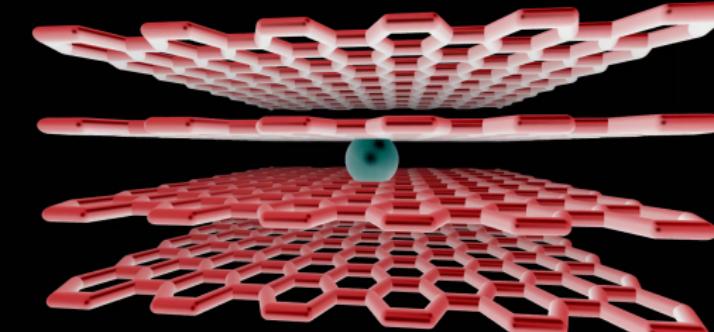
## Integracion de los momentos (NVT):

Distribución de Maxwell-Boltzmann / Espacio de configuraciones

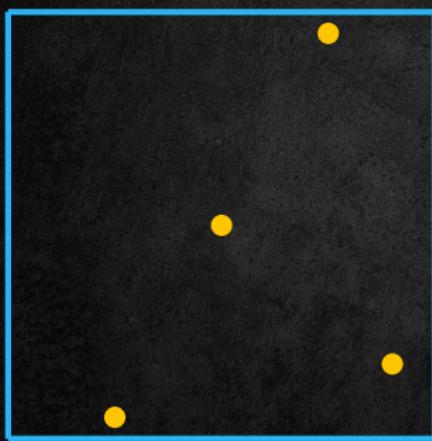
$$\begin{aligned}\rho(\mathbf{p}, \mathbf{r}) &= Z^{-1} e^{-\beta E(\mathbf{p}, \mathbf{r})} \\&= \frac{e^{-\beta \mathbf{p}^2/2m}}{\int_{\mathbf{p}} e^{-\beta \mathbf{p}^2/2m} d\mathbf{p}} \frac{e^{-\beta U(\mathbf{r})}}{\int_{\mathbf{r}} e^{-\beta U(\mathbf{r})} d\mathbf{r}} \\&= \left( \frac{\beta}{2m\pi} \right)^{\frac{3}{2}} e^{-\beta \mathbf{p}^2/2m} \frac{e^{-\beta U(\mathbf{r})}}{Q} \\&= \tilde{f}(\mathbf{p}) q(\mathbf{r}) \quad \tilde{f}(\mathbf{p}) = f_1(p_x) f_1(p_y) f_1(p_z)\end{aligned}$$

$$f(v) = \left( \frac{m\beta}{2\pi} \right)^{\frac{3}{2}} 4\pi v^2 e^{-\beta mv^2/2} \quad \langle A \rangle = \int A(\mathbf{r}) q(\mathbf{r}) d\mathbf{r}$$

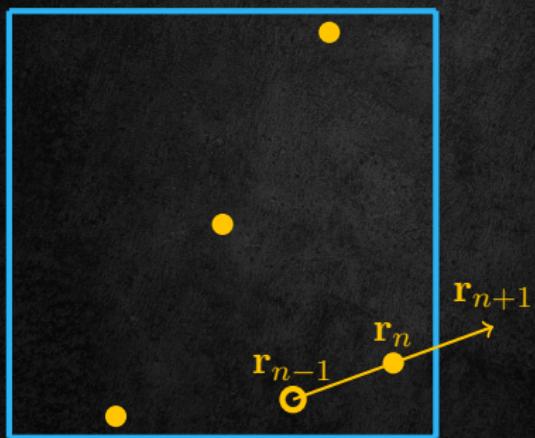
# Condiciones de Contorno Periódicas



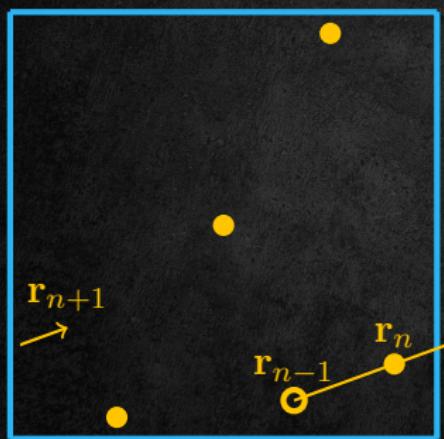
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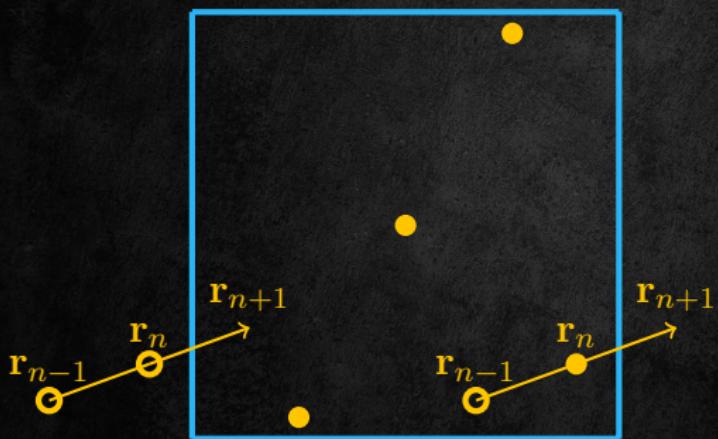
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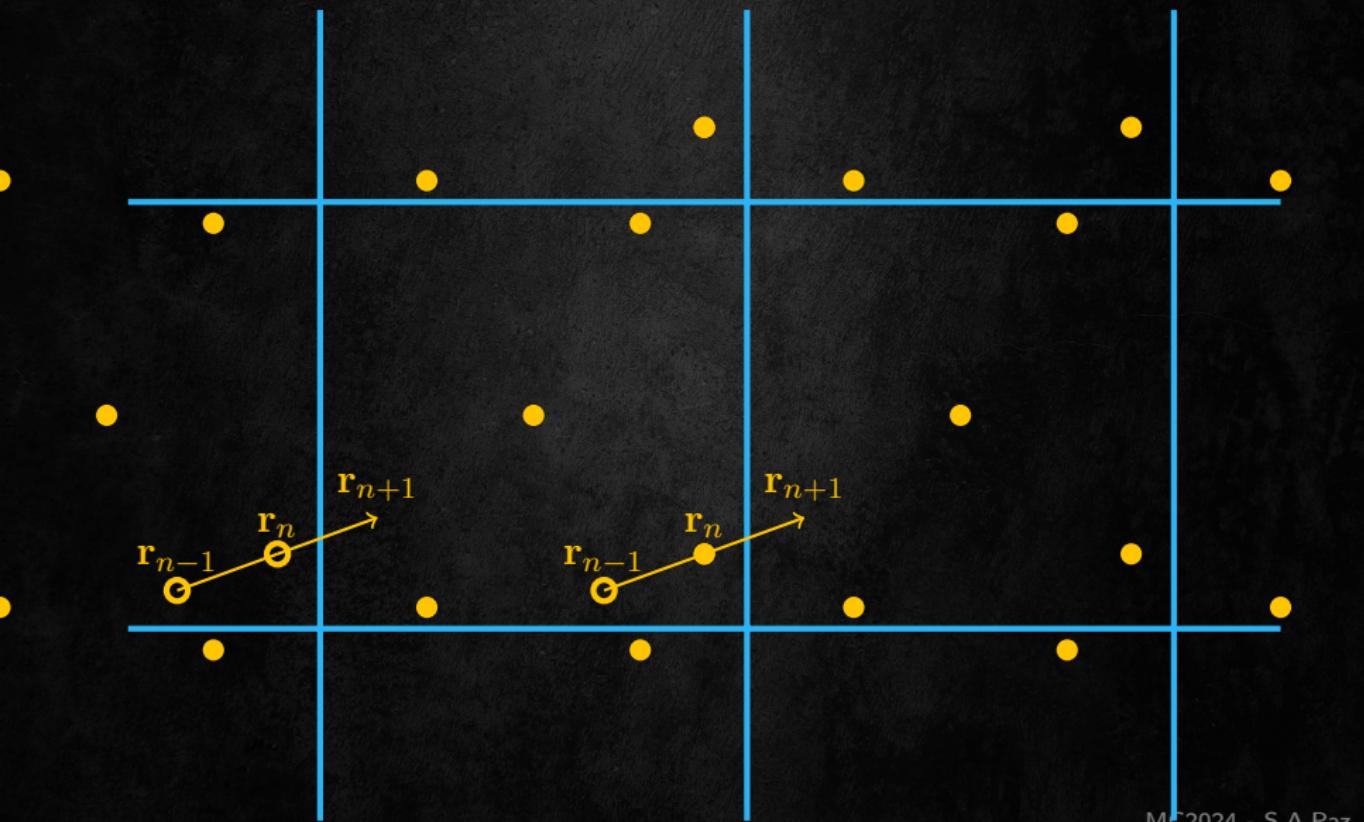
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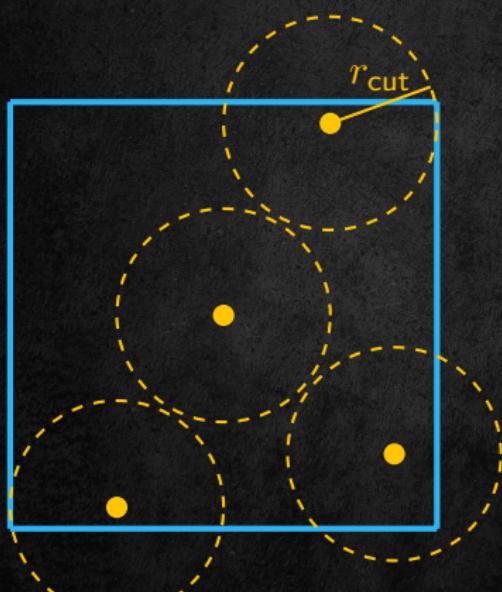
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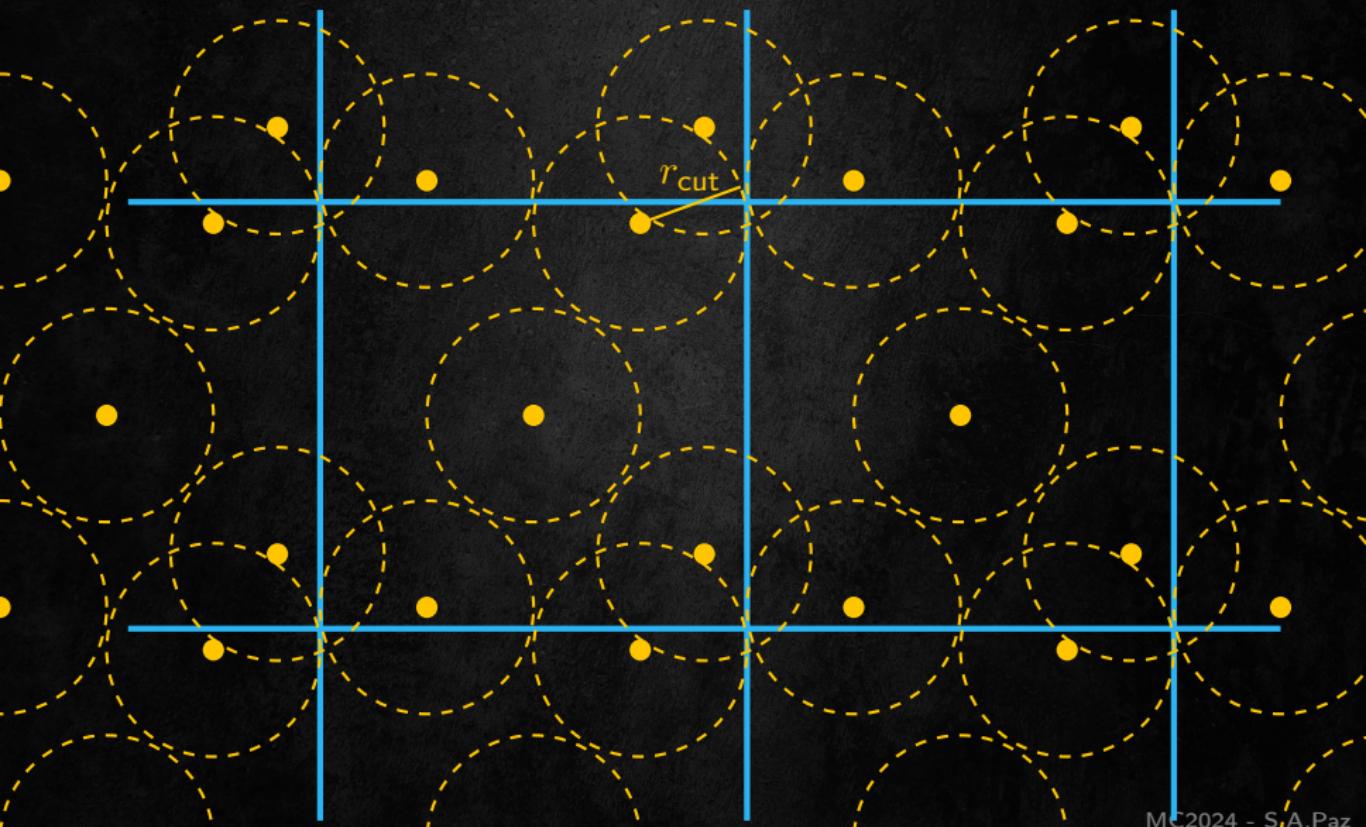
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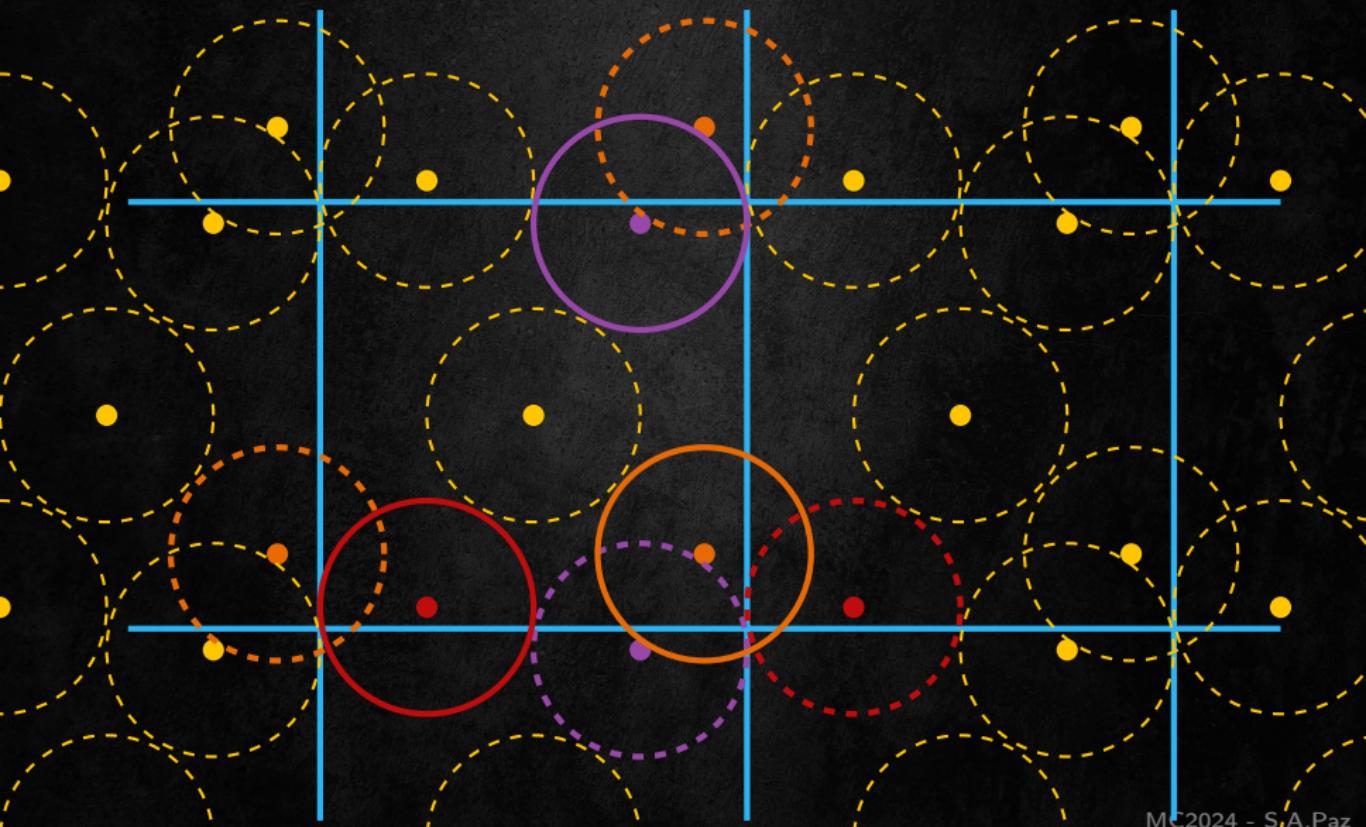
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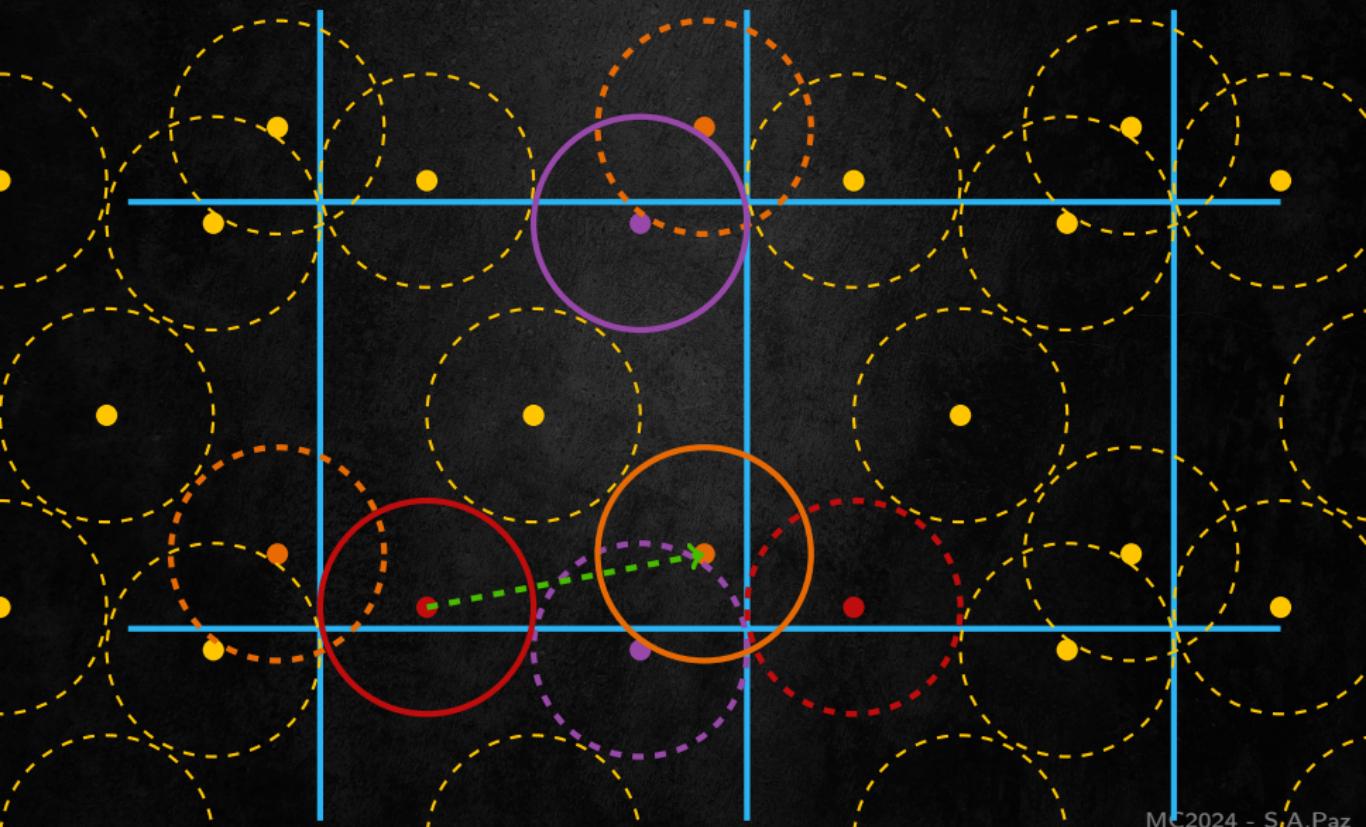
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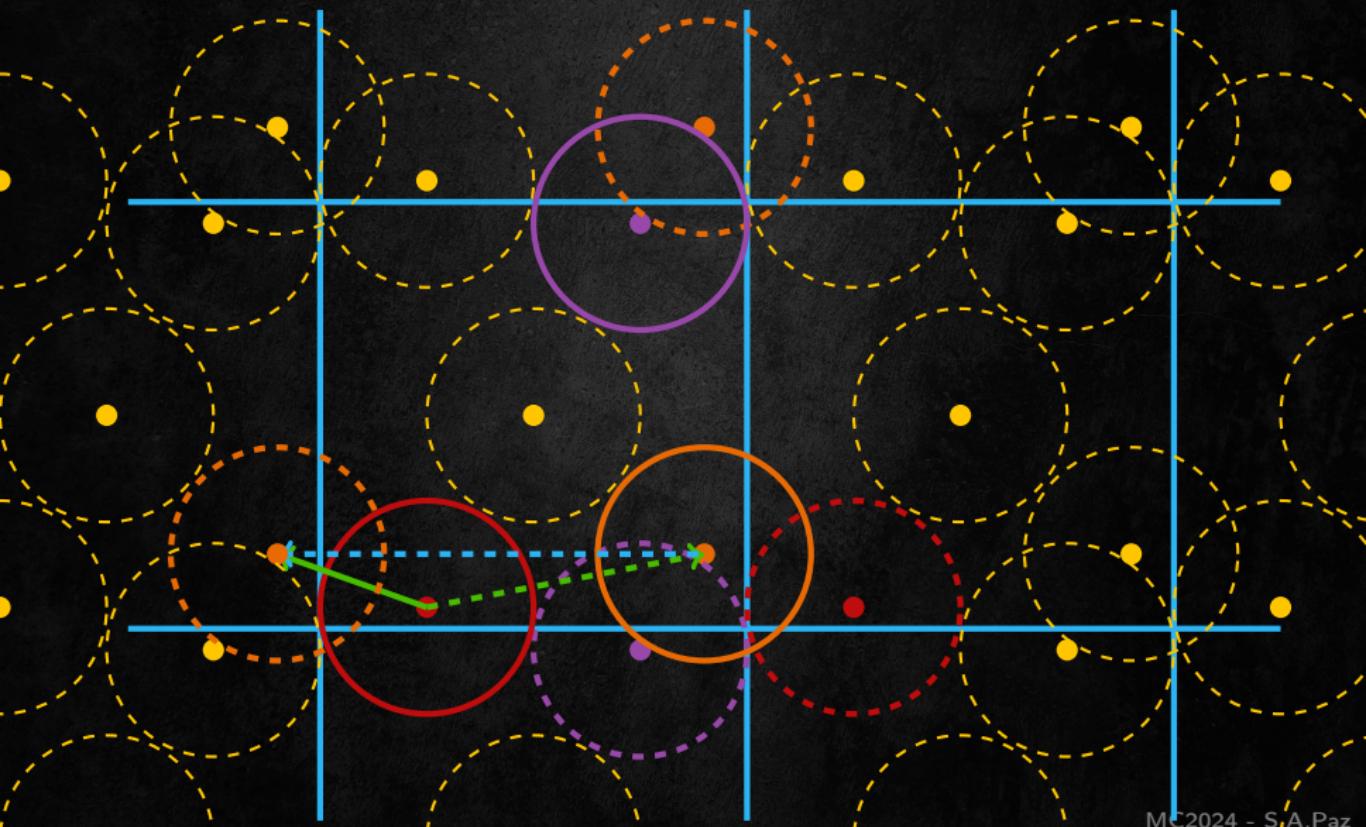
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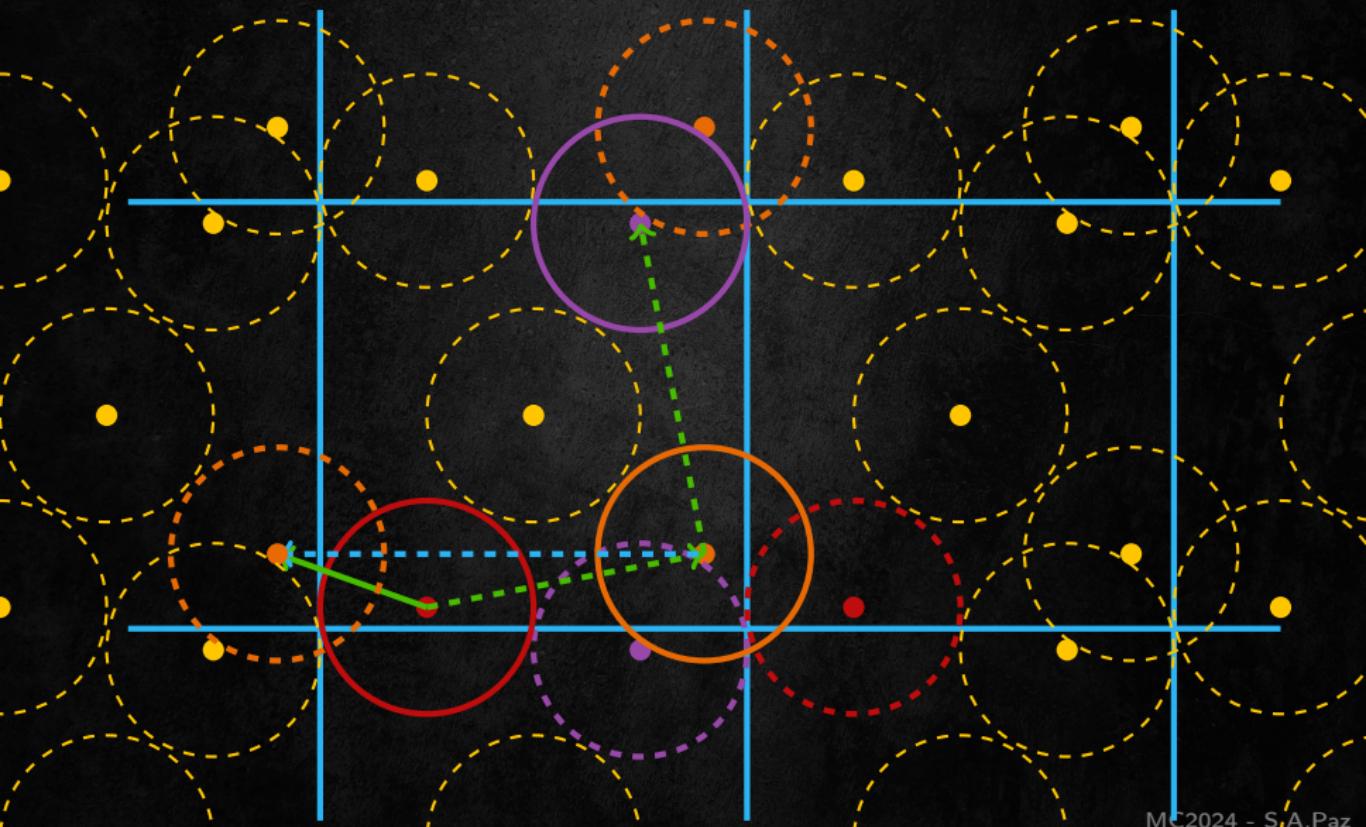
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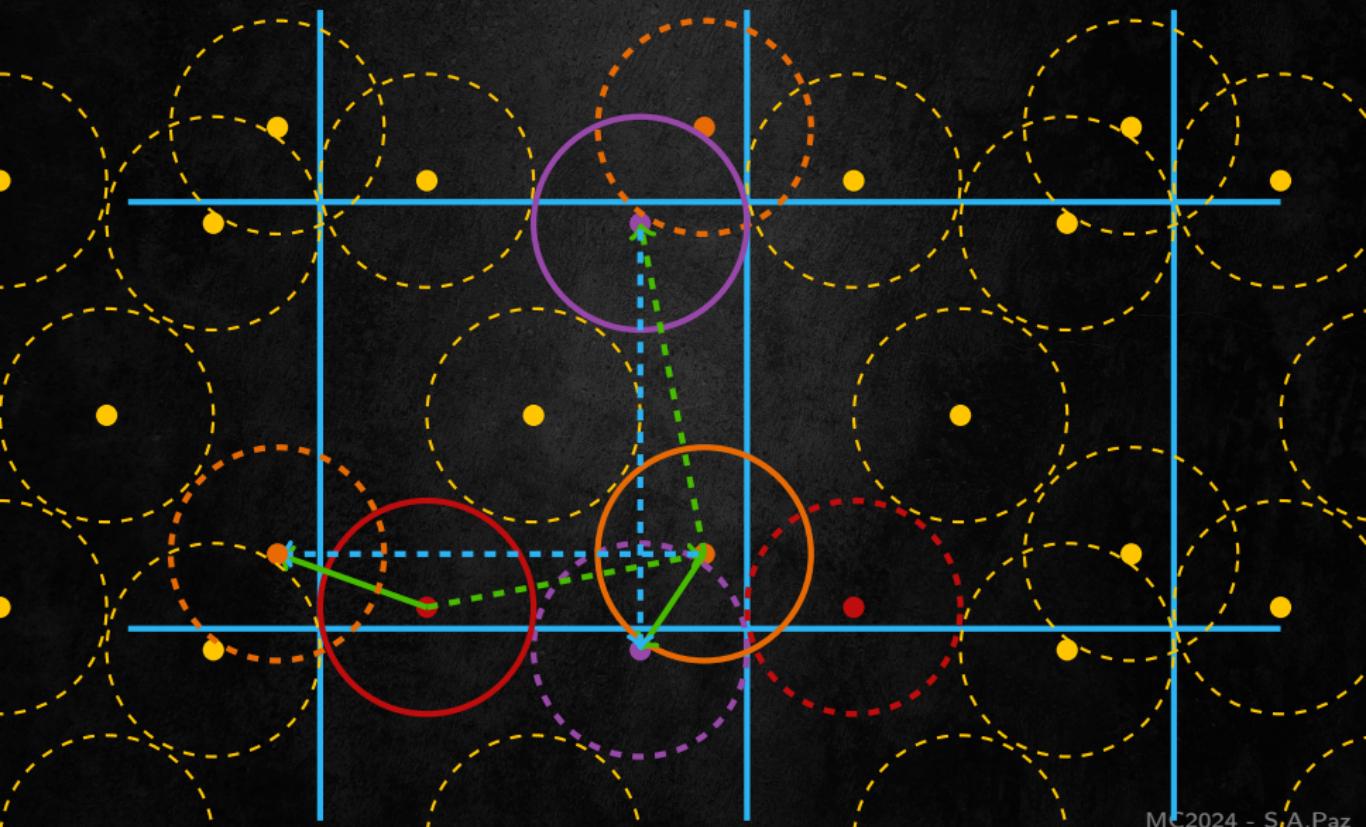
# Condiciones de Contorno Periódicas



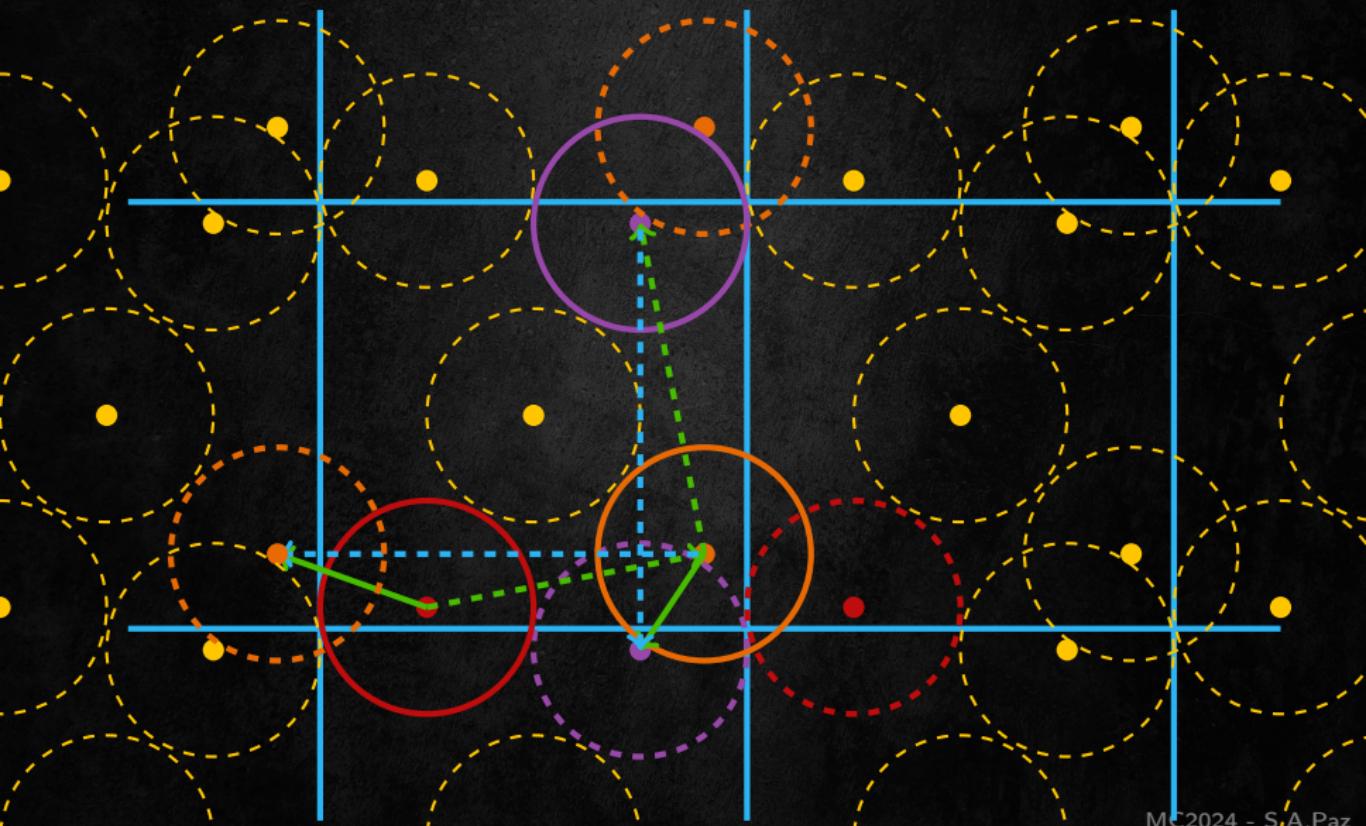
# Condiciones de Contorno Periódicas



# Condiciones de Contorno Periódicas

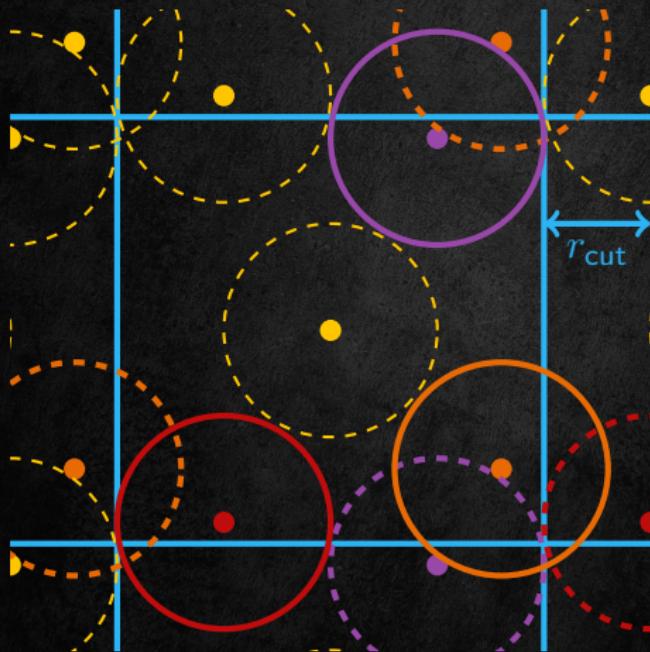


# Convección de Imagen Mínima

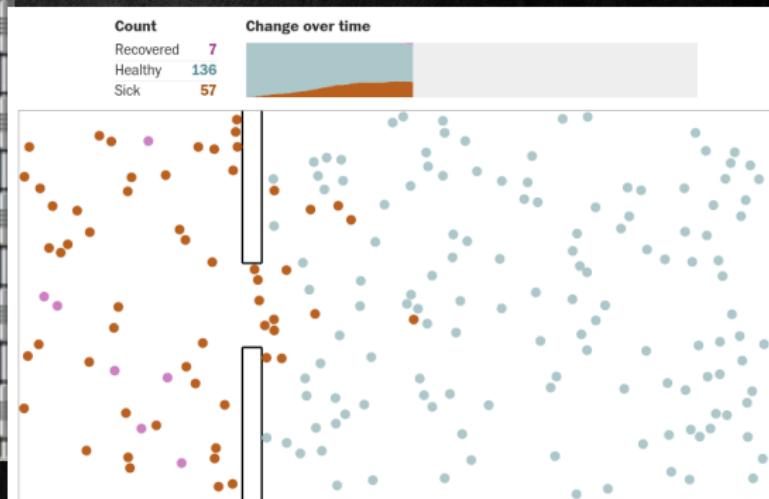
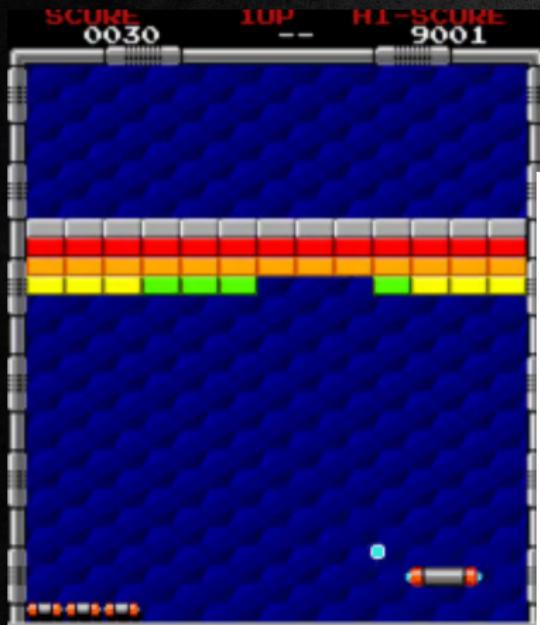


# Partículas Fantasma

¡Paralelización!



# Condiciones de Contorno Paredes Duras



Stevens, The Washington Post, March 14, 2020

# Condiciones de Contorno Reservorios

