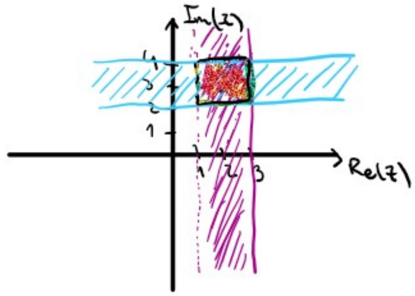
AyGA I 26/08/22

viernes, 19 de agosto de 2022 14:17

7. *a*) Representar gráficamente los siguientes conjuntos:
   1) $A_1 = \{z \in \mathbb{C} \,/\, |z| = 1\}$.
   2) $A_2 = \{z \in \mathbb{C} \,/\, arg(z) = \frac{\pi}{6}\}$.
   3) $A_3 = \{z \in \mathbb{C} \,/\, |z| = 2,\ \frac{\pi}{4} \leq arg(z) \leq \frac{\pi}{2}\}$.
   4) $A_4 = \{z \in \mathbb{C} \,/\, 1 < Re(z) \leq 3,\ 2 \leq Im(z) \leq 4\}$.
   5) $A_5 = \{z \in \mathbb{C} \,/\, |z - i| = |z + i|\}$.

   b) Dar en cada uno de los casos anteriores dos números complejos que pertenezcan y dos que no pertenezcan al conjunto indicado.

4)



5) Sea $z = a+bi$, $z \in \mathbb{C}$, $a, b \in \mathbb{R}$

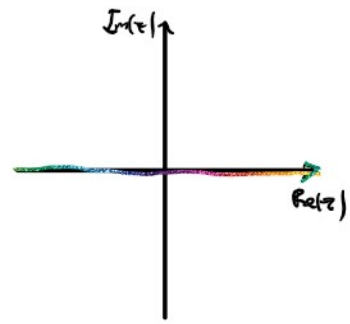
$$\begin{cases} z+i = a+bi+i = a+(b+1)i \\ z-i = a+bi-i = a+(b-1)i \end{cases}$$

$\boxed{|z+i| = |z-i|} \iff \sqrt{a^2+(b+1)^2} = \sqrt{a^2+(b-1)^2} \iff \cancel{a^2}+(b+1)^2 = \cancel{a^2}+(b-1)^2 \iff$

$a^2 \geq 0$, $(b+1)^2 \geq 0$, $(b-1)^2 \geq 0$

$\iff \cancel{b^2}+2b+\cancel{1} = \cancel{b^2}-2b+\cancel{1} \iff$

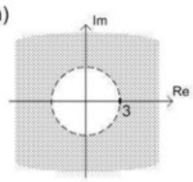
$\iff 2b = -2b \iff 4b = 0 \iff b=0$



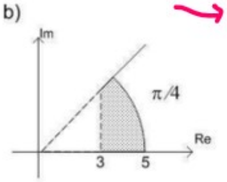
$A_5 = \{z \in \mathbb{C} : |z+i| = |z-i|\} = \{z = a+bi \in \mathbb{C} : b = 0\} = Re(z)$

8. Caracterizar las siguientes regiones graficadas mediante un subconjunto de $\mathbb{C}$.

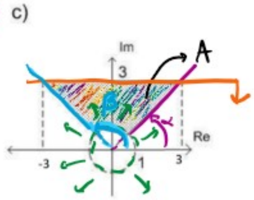
a)



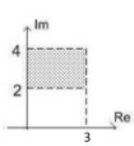
b)



c)



d)



¿A?

$A = \{z \in \mathbb{C} : \ldots\}$

- $|z| > 1$
- $Im(z) \leq 3$

$$\begin{cases} \cdot\ tg\,\alpha = \frac{3}{3} = 1 \Rightarrow \alpha = arctg\,1 \Rightarrow \alpha = \frac{\pi}{4} \\ \cdot\ \beta = \pi - \frac{\pi}{4} = \frac{3}{4}\pi \end{cases} \Rightarrow \frac{\pi}{4} \leq arg(z) \leq \frac{3}{4}\pi$$

$\therefore A = \{ z \in \mathbb{C} : |z| > 1, \ Im(z) \leq 3, \ \frac{\pi}{4} \leq arg(z) \leq \frac{3}{4}\pi \}$

9. Hallar las $\underbrace{\text{soluciones reales}}_{x, y \in \mathbb{R}}$ de cada una de las ecuaciones lineales con dos incógnitas a coeficientes en $\mathbb{C}$:

a) $x + iy = 1$

b) $ix + y = 1 + i$

c) $(1+i)x + (2-i)y = 7$

d) $(3+i)(x+iy) = 6 + 2i$

$(1+i)x + (2-i)y = 7 \iff$

$\iff 1x + ix + 2y - iy = 7 + 0i \iff$

$\iff (x+2y) + (x-y)i = 7 + 0i \iff$

$\iff \begin{cases} x + 2y = 7 \ (*) \\ x - y = 0 \end{cases} \rightarrow x = y \underset{(*)}{\rightarrow} x + 2x = 7 \rightarrow 3x = 7 \Rightarrow x = \frac{7}{3}$

$\therefore \boxed{x = y = \frac{7}{3}}$

10. Hallar las $\underbrace{\text{soluciones complejas}}_{z \in \mathbb{C}}$ de cada una de las ecuaciones lineales con una incógnita a coeficientes en $\mathbb{C}$:

a) $z = 1$

b) $(3+i)\,z = 4i$

c) $(3+i)\,z = 6 + 2i$

d) $4iz = 7 + 2i - 6z$

$4iz = 7 + 2i - 6z \iff 6z + 4iz = 7 + 2i \iff z \cdot (6 + 4i) = 7 + 2i \iff$

+8

$i^2 = -1$

$\iff \boxed{z} = \frac{7+2i}{6+4i} = \frac{7+2i}{6+4i} \cdot \frac{6-4i}{6-4i} = \frac{42 - 28i + 12i - 8i^2}{\underbrace{6^2}_{36} + \underbrace{4^2}_{16}} = \frac{50 - 16i}{52} =$

$= \frac{\cancel{50}^{25}}{\cancel{52}_{26}} - \frac{\cancel{16}^{4}}{\cancel{52}_{13}} i = \boxed{\frac{25}{26} - \frac{4}{13} i}$

11. Calcular

a) $\sqrt{2i}$

b) $\sqrt[3]{-27}$

c) $\sqrt[5]{-\sqrt{2} - \sqrt{2}i}$

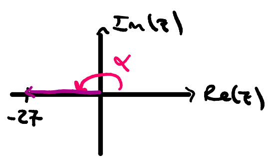
d) $\sqrt[4]{1}$

e) $\sqrt[3]{-1}$

f) $\sqrt[6]{-i}$

b) $\sqrt[3]{-27}$

$z = -27 = 27_\pi$ forma polar



$k = 0, 1, 2$

(*) Fórmula de Moivre: $z = \rho_\varphi \Rightarrow$

$\sqrt[n]{z} = \sqrt[n]{\rho}_{\frac{\varphi + 2k\pi}{n}}$ con $k = 0, 1, \ldots, n-1$

Luego, $w_k = \sqrt[3]{z} \underset{(*)}{=} \sqrt[3]{27}_{\frac{\pi + 2k\pi}{3}}$

$= 3_{\frac{\pi + 2k\pi}{3}} \quad k = 0, 1, 2$

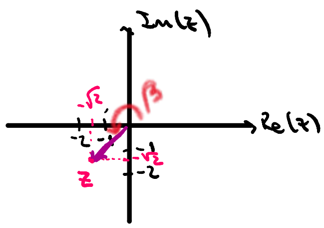
- $k = 0 \rightarrow w_1 = 3_{\frac{\pi}{3}}$
- $k = 1 \rightarrow w_2 = 3_{\frac{\pi + 2k}{3}} = 3_\pi$
- $k = 2 \Rightarrow w_3 = 3_{\frac{\pi + 4\pi}{3}} = 3_{\frac{5\pi}{5}}$

c) $\sqrt[5]{-\sqrt{2}-\sqrt{2}i}$

$z = -\sqrt{2}-\sqrt{2}\,i$, ¿$\sqrt[5]{z}$?

Escribimos z en su forma polar :

$* \; |z| = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{2+2} = 2$



$* \; tg\,\alpha = \dfrac{Im(z)}{Re(z)} = \dfrac{-\sqrt{2}}{-\sqrt{2}} = 1 \Rightarrow \alpha = \dfrac{\pi}{4} \rightarrow \beta = \pi + \dfrac{\pi}{4} = \dfrac{5}{4}\pi$

$\therefore z = 2_{\frac{5}{4}\pi} \Rightarrow w_k = \sqrt[5]{z} = \sqrt[5]{2}_{\,\frac{\frac{5}{4}\pi + 2k\pi}{5}}$ con $k = 0,1,2,3,4$

⋮

TERMINAR!