Derivation of our calculations for intervals around excess death estimates

Supplement to Santos-Lozada, A.R. and Howard, J.T. (2018) Use of death counts from Vital Statistics to Calculate Excess Deaths in Puerto Rico Following Hurricane María. *JAMA*, 320(14), pp. 1491-1493.

This document explains the logic behind the confidence intervals built around the excess estimate published by Santos-Lozada and Howard (2018) in the Journal of the American Medical Association[1].

The approach we used to estimating excess deaths following Hurricane Maria was to follow a previously established method, known as the epidemiological threshold approach[2,3]. This methodology involves the calculation of a mean value of deaths based on a series of historical monthly death counts from the vital statistics system, along with the corresponding 95% confidence interval (CI) to account for random variability in monthly counts year over year.

In this methodology, excess deaths are based on the difference between the observed number of deaths for the period of interest and the value of the upper 95% CI bound, rather than defining excess as the difference in the observed value and the mean. By defining excess deaths in this manner, the uncertainty of random variation in monthly deaths is already removed from the calculation, therefore requiring no additional interval to be placed around the excess estimate. Our method is equivalent to performing a simple means comparison, placing 95% CI around the central estimate, and then using the lower bound of the interval as the definition of excess deaths (Table 1). This is why the values in column f in Table 1 are the same as column h.

Table 1. Historical Patterns of Deaths, 2017 death counts, and Estimated Excess Deaths in Puerto Rico

				Defining Excess as Observed - Mean			Defining Excess as Observed - U95% Bound		
Month	2010-2016 Historical Patterns of Deaths, Mean (95% CI)		2017 Death Counts	Excess	L 95%	U 95%	Excess	L 95%	U 95%
(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
January	2,612	(2,485-2,739)	2894						
February	2,375	(2,255-2,495)	2315						
March	2,560	(2,439-2,681)	2494						
April	2,348	(2,261-2,494)	2392						
May	2,405	(2,413-2,481)	2390						
June	2,375	(2,370-2,550)	2369						
July	2,448	(2,297-2,469)	2367						
August	2,460	(2,370-2,476)	2321						
September	2,383	(2,297-2,469)	2928	545	459	631	459	425	493
October	2,429	(2,380-2,476)	3040	611	564	660	564	531	597
November	2,428	(2,299-2,555)	2671	243	116	372	116	50	182
December	2,685	(2,543-2,824)	2820	135	-4	277			
Total	--	--	--	**1,399**	**1,139**	**1,663**	**1,139**	**1,006**	**1,272**

The total does not include December because the CIs overlap with zero.

Given this methodology, we did not include intervals around our excess death estimates in the initial submission of our paper. However, during the review process we were asked by a reviewer to provide an additional confidence interval around our excess figures, even though these figures were already at the lower bound. This equates to placing intervals around an interval, which we thought was unnecessary, but we did it anyway. We would argue strongly, however, that doing this is superfluous.

Nevertheless, we approached this request using the following logic. First, we thought about what source of variability or uncertainty might exist that was not already accounted for by our initial 95% CI bounds. Our conclusion was that there could be small differences in monthly death counts provided by the government of Puerto Rico based on data corrections, or updates to the vital records system, which we would call measurement or reporting error. In fact, we have now received five different data files, and they do exhibit small differences in some counts. At the time, we did not know what these differences would be, except that we thought they would be relatively small compared to month-to-month or year-to-year random fluctuations.

Second, we thought about the fact that in placing intervals around our excess death figures we would be comparing an observed value to an upper bound, not a mean. Therefore, a traditional statistical approach to performing a means comparison would not make sense because it would produce an interval that is at least twice as wide as it should be. Thus, performing a means comparison using the sample standard deviation as the manner of standardizing the test statistic would be inappropriate. Therefore, we used the standard error of the mean for the historical series as an estimate of the standard deviation of the potential differences in reported numbers upon repeated delivery of the data by the government agency (i.e. small differences in the monthly counts attributable to updates to the reporting system).

Therefore, we decided to calculate the additional interval requested using the one-sample t-test approach to calculating an interval around an observed difference[4].

The process follows the steps below, illustrated using data for the month of September 2017 based on the original published table:

(1) Calculate the mean difference: $\bar{d} = x - \mu$; in our case μ is the value of the upper bound of the 95% CI for the historical series, not the mean; so 2928-2469=459

(2) Calculate the t statistic, $t = \frac{\bar{d}}{s/\sqrt{n}}$, where s was the standard error of the mean not the sample standard deviation; so 459/(37.2/2.646)=30.187, with n-1 degrees of freedom yields a p-value of <0.000001

(3) Calculate 95% CI around the difference, $\bar{d} \pm t_{crit} * s/\sqrt{n}$ with 6 degrees of freedom yields for the upper bound 459 + 2.447 * (37.2/2.646) = 459 + 34 = 493, and for the lower bound 459 - 2.447 * (37.2/2.646) = 459 - 34 = 425.

(4) These steps were repeated for each month where the observed number of deaths in 2017 exceeded the upper bound of the 95% CI, from September 2017 through November 2017.

In the months following the publication of our initial results, we have received additional datasets from the government of Puerto Rico. We now have five different datasets containing 2017 death counts, for which we have now calculated the mean deaths and standard deviations. The results yields the actual observed measurement error that we were trying to estimate in our original research letter.

Using the same confidence interval process above yields narrower intervals than we originally calculated (Table 2).

(1) Calculate 95% CI around the difference, $\bar{d} \pm t_{crit} * s/\sqrt{n}$ with 4 degrees of freedom yields for the upper bound 459 + 2.776 * (24.36/2.236) = 459 + 30 = 489, and for the lower bound 459 - 2.447 * (24.36/2.236) = 459 - 30 = 429.

(2) These steps were repeated for each month where the observed number of deaths in 2017 exceeded the upper bound of the 95% CI, from September 2017 through November 2017.

Table 2. Monthly Death Counts from Five Consecutively Received Files, with Mean, SD, and resulting Confidence Intervals

	Death Count for Each File, Averages and Standard Deviations (SD)						
Month	File 1	File 2	File 3	File 4	File 5	Mean	SD
September	2,887	2,928	2,928	2,906	2,951	2,920	24.36
October	2,991	3,040	3,040	3,015	3,060	3,029	26.66
November	2,571	2,671	2,671	2,657	2,688	2,652	46.38
December	2,168	2,820	2,820	2,797	2,839	2,689	291.52

	Original Estimates and CI			Original Estimate with revised CI			
Month	**Excess**	**L 95% CI**	**U 95% CI**	**Excess**	**L 95% CI**	**U 95% CI**	
September	459	425	493	459	429	489	
October	564	531	597	564	531	597	
November	116	50	182	116	58	174	
Total	**1,139**	**1,006**	**1,272**	**1,139**	**1,018**	**1,260**	**--**

References

1. Santos-Lozada A, Howard J. Use of Death Counts from Vital Statistics to Calculate Excess Deaths in Puerto Rico Following Hurricane Maria. *JAMA.* 2018;320(14):1491-1493.

2. Eickhoff T, Sherman I, Serfling R. Observations on excess mortality associated with epidemic influenza. *JAMA.* 1961;176:104-110.

3. Serfling R. Methods for current statistical analysis of excess pneumonia-influenza deaths. *Public Health Rep.* 1963;78(6):494-506.

4. Rosner B. *Fundamentals of Biostatistics.* 7th ed. Boston: Brooks/Cole; 2011.