Finding the Effects of Differing Mass on the Damping Constant of Two Different Damped Oscillating Systems

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Lab Section 5, Table 3

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Abstract

The goal of this lab was to investigate whether the damping constant involved in decreasing the amplitude of simple harmonic motion in a vertical and horizontal spring system is a constant value, independent of the mass quantities it is attached to. In order to effectively explore this concept, we set up 2 separate spring systems where various mass quantities were combined to the Arduino setup, to then attach to a vertical spring system and horizontal spring system, utilizing the bluetooth module and ultrasonic sensor to record position and time data for each trial. The vertical spring system consists of a spring with a cardboard component to decrease the effects of air drag, while the horizontal spring system consists of an air track where the Arduino setup is placed with various masses, then connected to a spring attached to the end of the air track. The air track allows us to decrease the effects of friction. In both cases, we record the oscillations and use this data in our analysis. The data analysis requires the use of the curve fit function which is utilized to model the best fit function and we define a function, using guess parameters, to return the damping constant as a coefficient, for the different masses used. As a result, the damping constant is in fact not constant and our hypothesis is disproven as the damping constant varied throughout the different trials,with values as low as 0.6 and as high as 1.3.

Introduction

In this experiment, we are trying to determine whether or not the damping constant remains a constant value when observing damped oscillations with different mass quantities. In order to understand this experiment, it is important to understand the fundamental concepts behind the motivation of this experiment. A damped oscillation is described as the motion of an oscillating object that is sinusoidal in its motion, but the amplitude of each oscillation decreases with time until the oscillations eventually come to a halt. The equation for the damped oscillation is found below in **Equation 1.1**. We are interested in finding whether the the b term is constant because if it is, this would allow us to predict the location of an object traveling in oscillatory motion, as the only component that would vary would be the mass of the object, and we could utilize the same damping constant value throughout trials with different masses. It is also important to understand what causes damping, as this relates to the motivation of our experiment, the causes of damping could be something like friction that plays a role in the reduction of amplitude of the oscillatory motion, or air drag as well. We theorize that the damping constant will remain constant throughout each oscillation where the mass will be changed, meaning that the damping constant will be within 2 standard deviations for each trial.

Equation 1.1 Equation for Damped Oscillations

$$x(t) = Ae^{-bt/2m}\cos(\omega t + \phi_0)$$

Method

In order to effectively test this hypothesis, we set up two different systems to measure the damping constant and to effectively measure whether or not it remained constant even when the setup changed. The first system we set up was the vertical setup which consisted of the Arduino UNO, ultrasonic sensor, and bluetooth module, all integrated into one piece, which was the mounted onto a vertical spring system by attaching this system in the orientation that would allow us to take readings from the ground. (**Figure 1.1**). This Arduino setup was then attached to a cardboard holder that would contain no cubical metal mass, one mass, or two masses, which would allow us to determine whether the damping constant remained constant throughout different mass trials or not. (**Figure 1.2**) Finally, we utilized a piece of cardboard to create a standard amount of air drag throughout each of the trials. After we assembled the setup, we utilized a starting amplitude that would remain the same throughout the different mass trials in order to create a standard for our readings which was measured to be 5 cm. We recorded the data for 3 different mass sets, taking 3 datasets for each trial type, in order to effectively use data that would be the best fit for our analysis.

The horizontal setup was similar to our vertical setup in which the Arduino setup remained the same, and the ultrasonic sensor would also be responsible for recording the distance and time data for each of our trials. (**Figure 1.3**). However, the horizontal setup was set up onto an air track and it included a cart with the Arduino setup, and depending on the mass we were measuring, no mass, one mass, or two masses. The horizontal setup's spring was attached to the end of the air track to be able to oscillate the system, and we utilized a starting amplitude of 3 cm for these trials. We attached a piece of cardboard to the end of the air track that would serve as the wall from which the ultrasonic sensor would read time and distance values. We repeated the same process from the horizontal setup where we took 3 datasets for each mass quantity in order to extract the dataset from each mass that would be the best representation for our data analysis. For both of these setups, we will assume there is a component of friction and of air drag for the horizontal setup and that there is a contribution from air drag for the vertical setup that contributes to the damping of the oscillation.

For our data analysis, the idea is to create an ideal function that will model our data which takes the shape of a sinusoidal curve because of the oscillation, and this will allow us to utilize the `curve_fit` function in Python. This function **(Equation 1.1)** has one variable parameter, the x-axis data (time), and the rest are all constants to be determined by the `curve_fit` function. The output of this `curve_fit` function will be the position of the sinusoidal motion. We want to create guesses for our guess parameters for the constant values that are part of our equation which include the angular velocity, the phase shift, the offset, and the amplitude. In this damped oscillation, however there is one more parameter that did not exist in simple harmonic motion, which is the coefficient created from *e* to the power of a value that determines how the function decays. In order to make these reasonable guesses, we must clip our position/acceleration vs. time data to contain 3-4 quality oscillations, and once we plot this, then we can make a reasonable guess. To use the `curve_fit` function, we must first upload a module “scipy” which gives us access. `curve_fit` is a generalization of the `np.polyfit` function that allows us to find the ‘best-fit’ function for a sinusoidal plot such as this one as opposed to a linear plot which is when the np.polyfit function is used.

We then follow the process of plotting our data, finding guess parameters for our ideal function, and then using the `curve_fit` function to find our parameters when plotting the position vs. time data. Finally, we need to find the covariance matrix that contains the covariance for the values we get. The square root of the covariance values give us the error of each coefficient. Once we find the errors and the fitted parameters, we can obtain the damping constant by creating an equation specifically to find this in order to compare this value to the others to determine whether or not they are within two standard deviations from each other.

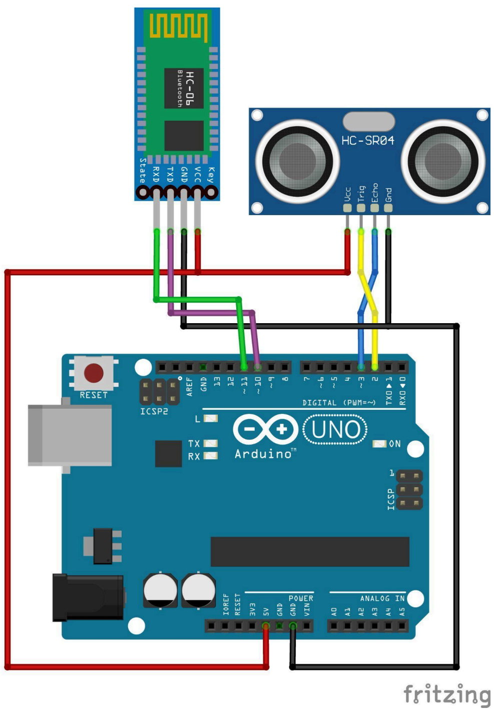

Figure 1.1 Arduino UNO, Bluetooth Module, and Ultrasonic Sensor Integration Wiring

Figure 1.2 Vertical Setup for Damped Oscillation Readings

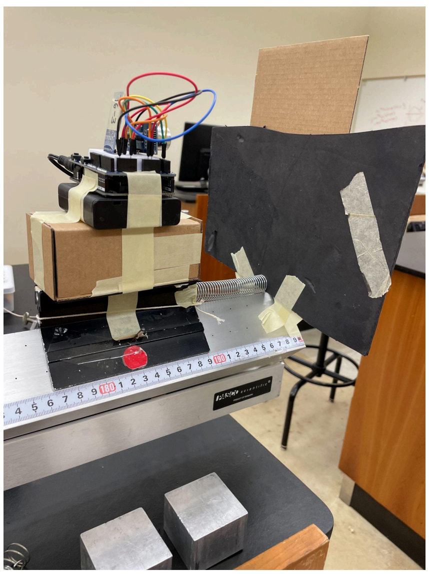

FIgure 1.3 Horizontal Setup for Damped Oscillation Readings

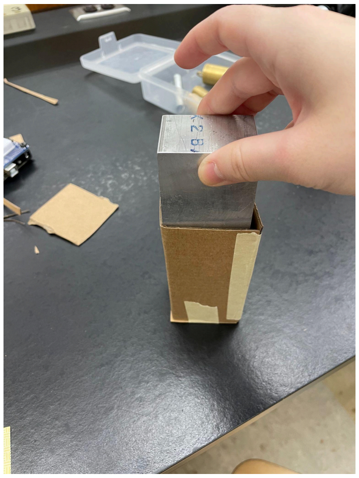

Figure 1.4 Metal Cube Masses Used in Experiment

Equipment

- Arduino Uno
- Jumper wires
- Ultrasonic sensor
- Bluetooth module
- 2 springs
- Airtrack
- Cart
- Cardboard board
- Small cardboard box
- String
- Tape
- 2 metal cubes (with mass 264-266 g)
- A power source (Laptop/Battery)

Analysis

Vertical Damped Oscillation System

Item	**Mass**
Arduino setup + empty cardboard box + battery pack	0.226 ± 0.0005 kg
Mass 1 (first metal cube)	0.265 ± 0.0005 kg
Mass 2 (second metal cube)	0.266 ± 0.0005 kg

Table 2.1 Masses and error of each individual component

Since we add the Mass 1 and Mass 2 to the oscillating setup after the Arduino is already hanging from the spring, we perform error propagation to find the error of each mass setup. The error of the Arduino setup + Mass 1 is $= \sqrt{0.0005^2 + 0.0005^2}$ and the error of the Arduino setup + Mass 1 + Mass 2 is $= \sqrt{0.0005^2 + 0.0005^2 + 0.0005^2}$.

Item	**Total Mass**
Arduino Setup + No Mass Added	0.226 ± 0.0005 kg
Arduino Setup + Mass 1	0.491 ± 0.0007 kg
Arduino Setup + Mass 1 + Mass 2	0.757 ± 0.0009 kg

Table 2.2 Masses and error of each mass setup

The table above shows the new calculated errors and masses of each mass setup for the vertical oscillation damping experiment.

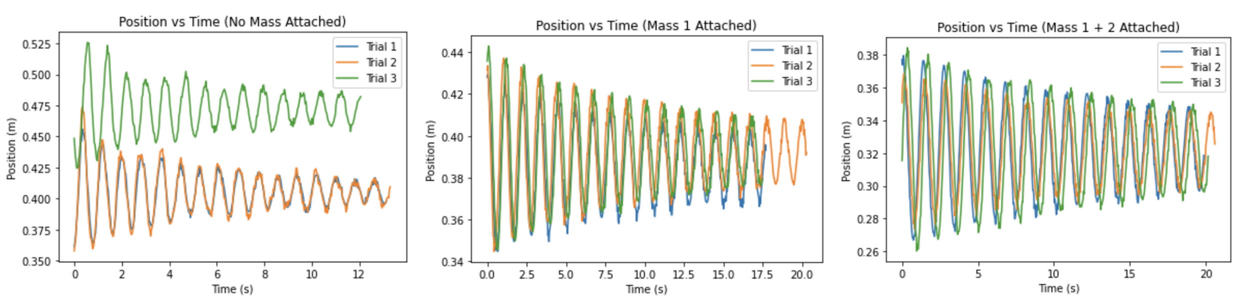

Figure 2.1 Raw data of Position vs. Time for all trials for all masses

Above we have stacked plots of raw Position vs. Time graphs for the vertical oscillations. Clearly, there is a discernible difference between each mass's oscillation patterns; the more mass there is hanging from the spring, the less damping occurs, which makes sense since according to the equation for underdamped systems $x(t) = Ae^{-\frac{b}{2m}} sin(\omega t + \phi) + C$, the more mass there is the less damping there should be if *b* does not change much between setups. We want to see if *b* is *constant* for all these masses.

We focus on 3–4 oscillations of each dataset to analyze. We first find the height and times of the first two peaks of these 3–4 oscillations to let us estimate the coefficient term $\frac{b}{2m}$ using the equation $\frac{b}{2m} = -\frac{ln(\frac{y(t_1)-C}{y(t_0)-C})}{t_1 - t_0}$. We use a sine function in the form of $x(t) = Ae^{-\frac{b}{2m}} sin(\omega t + \phi) + C$ and use guess parameters to guess a best-fitting equation for each curve, then use the Python command `curve_fit()` to obtain an actual best-fitting curve for each graph, shown below in Figure 2.

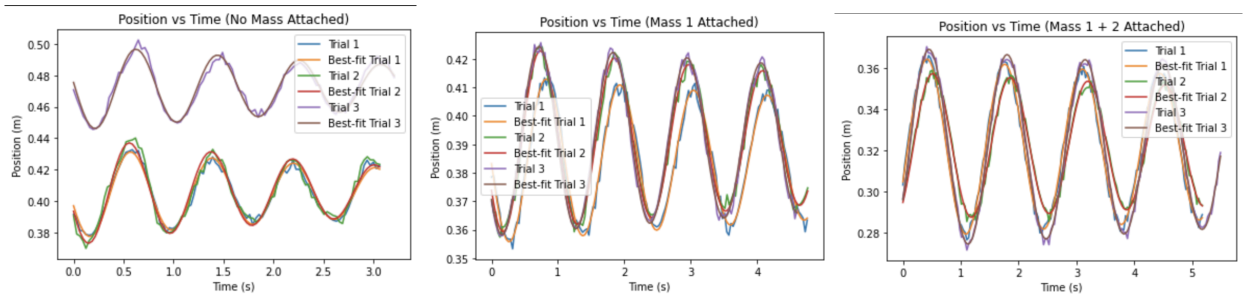

Figure 2.2 Best-fit graphs of Position vs. Time for 3–4 oscillations of all trials for all masses

The best-fit line equations for each graph are given in the table below.

Mass/Trial	**Best fit Equation:** $x(t) = Ae^{-\frac{b}{2m}}sin(\omega t + \phi) + C$
No Mass Added/Trial 1	$x(t) = (0.0290)e^{-(0.1999)}sin((7.7058)t + 3.4386) + 0.4055$
Trial 2	$x(t) = (0.0349)e^{-(0.2519)}sin((7.6879)t + 3.5300) + 0.4067$
Trial 3	$x(t) = (0.0275)e^{-(0.2061)}sin((7.6953)t + 3.0312) + 0.4725$
Mass 1 Added/Trial 1	$x(t) = (0.0296)e^{-(0.0675)}sin((5.6586)t + 3.2004) + 0.3849$
Trial 2	$x(t) = (0.0329)e^{-(0.0754)}sin((5.6282)t + 3.7177) + 0.3917$
Trial 3	$x(t) = (0.0339)e^{-(0.0577)}sin((5.6402)t + 3.8157) + 0.3918$
Mass 1 + 2 Added/Trial 1	$x(t) = (0.0441)e^{-(0.0442)}sin((4.6755)t + 5.8981) + 0.3213$
Trial 2	$x(t) = (0.0363)e^{-(0.0416)}sin((4.6892)t + 5.4340) + 0.3220$
Trial 3	$x(t) = (0.0491)e^{-(0.0411)}sin((4.6586)t + 5.7430) + 0.3212$

Table 2.3 Best fit equations of position vs. time for all trials of all masses

As we have the best-fitting position vs time functions for each trial of each mass, we can take the exponent of the *e* term, which is equivalent to $-\frac{b}{2m}$, and multiply it by 2 times the mass of the system to obtain *b* for each trial.

In order to find the error for each *b* value, we take use the covariance matrix from the `curve_fit()` function to obtain the error of the coefficient $\frac{b}{2m}$ by taking the square root of the appropriate covariance and performing further error propagation with this and the mass's error. As such, the error for each *b* value, along with the *b* value itself, is shown in the table below.

Mass/Trial	***b***	**Error**
No Mass Added/Trial 1	0.0904	0.0087
Trial 2	0.1139	0.0118
Trial 3	0.0932	0.0107
No Mass Added Average	0.0991	0.0060
Mass 1 Added/Trial 1	0.0663	0.0068
Trial 2	0.0740	0.0059
Trial 3	0.0566	0.0050
Mass 1 Added Average	0.0657	0.0034
Mass 1 + 2 Added/Trial 1	0.0838	0.0075
Trial 2	0.0790	0.0082
Trial 3	0.0779	0.0060
Mass 1 + 2 Added Average	0.0641	0.0034

Table 2.4 *b* values and errors for each trial

From the results given in Table 4., we find that *Mass 1 Added* and *Mass 1 +2 Added* average *b* values differ from one another by less than two standard deviations so we may consider *b* constant for those two masses. However, the *No Mass Added* setup's *b* value differs from the other two masses by more than two standard deviations so we may consider this *b* value to not be constant. The data is thus inconclusive for the vertical damped oscillating system with the cardboard's air resistance.

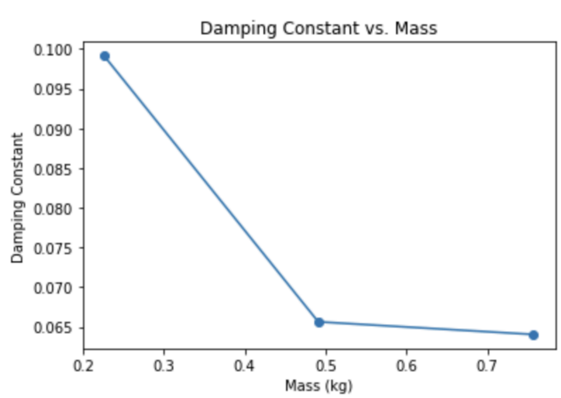

Figure 2.3. Damping constant *b* vs. Mass

The graph above shows a visual of the relationship between the average calculated damping constant and the mass of the system. While the last two data points have nearly the same damping constant, overall there is a downward trend; as mass increases, found damping constant decreases.

Horizontal Damped Oscillation System

Item	Measured Mass
Arduino setup + empty cardboard box + battery pack+ cart	0.419 ± 0.0005 kg
Mass 1 (first metal cube)	0.265 ± 0.0005 kg
Mass 2 (second metal cube)	0.264 ± 0.0005 kg

Table 2.5 Masses and error of each individual component

With the same error propagation calculations we made above in the vertical damping scenario, we can obtain the following:

Item	Total Mass
Arduino Setup + No Mass Added	0.419 ± 0.0005 kg
Arduino Setup + Mass 1	0.684 ± 0.0007 kg
Arduino Setup + Mass 1 + Mass 2	0.948 ± 0.0009 kg

Table 2.6 Masses and error of each mass setup

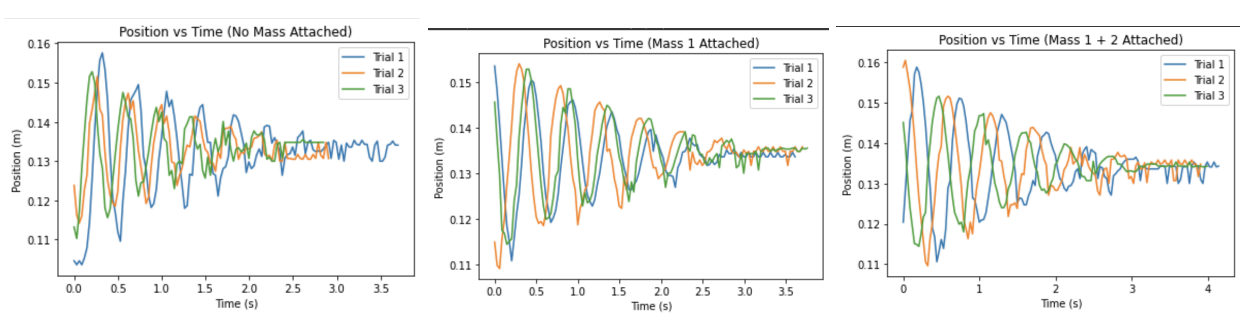

Figure 2.4 Raw data of Position vs. Time for all trials for all masses

We see above the raw position vs. time data for all trials, each having about 10–15 oscillations. We will focus on 3–4 oscillations to analyze specifically.

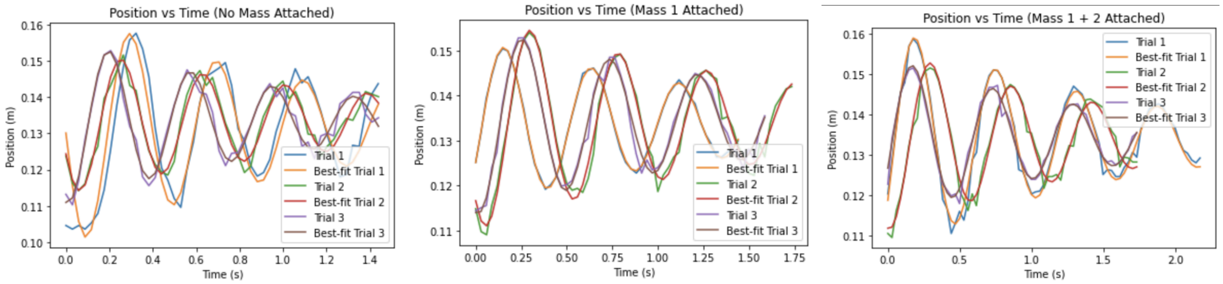

Figure 2.5 Best-fit graphs of Position vs. Time for 3–4 oscillations of all trials for all masses

As such, the best-fit equations for each mass's trial's graph of 3-4 oscillations is shown in the table below, obtained from using a sine function, guess parameters, the formula for the coefficient $\frac{b}{2m}$, and `curve_fit()`, as explained previously.

Mass/Trial	**Best fit Equation:** $x(t) = Ae^{-\frac{b}{2m}} sin(\omega t + \phi) + C$
No Mass Added/Trial 1	$x(t) = (0.0331)e^{-(0.8612)} sin((15.7385)t + 3.1944) + 0.1318$
Trial 2	$x(t) = (0.0203)e^{-(0.7156)} sin((16.6372)t + 3.6136) + 0.1336$
Trial 3	$x(t) = (0.0230)e^{-(0.9342)} sin((16.7013)t + 4.5162) + 0.1336$
Mass 1 Added/Trial 1	$x(t) = (0.0187)e^{-(0.6548)} sin((13.0825)t + 5.8025) + 0.1338$
Trial 2	$x(t) = (0.0241)e^{-(0.6025)} sin((12.9717)t + 3.9648) + 0.1343$
Trial 3	$x(t) = (0.0210)e^{-(0.5937)} sin((13.0632)t + 4.5070) + 0.1345$
Mass 1 + 2 Added/Trial 1	$x(t) = (0.0286)e^{-(0.6599)} sin((11.1658)t + 5.7284) + 0.1339$
Trial 2	$x(t) = (0.0226)e^{-(0.6498)} sin((11.1488)t + 4.5291) + 0.1342$
Trial 3	$x(t) = (0.0201)e^{-(0.6805)} sin((11.2307)t + 5.9020) + 0.1343$

Table 2.7 Best fit equations of position vs. time for all trials of all mass setups

Performing the same calculations as in the vertical oscillation to obtain each trial's *b* value and error, we get the values shown in the table below.

Mass/Trial	***b***	**Error**

No Mass Added/Trial 1	0.7217	0.1442
Trial 2	0.5997	0.0646
Trial 3	0.7829	0.0635
No Mass Added Average	0.7014	0.0568
Mass 1 Added/Trial 1	0.8957	0.0259
Trial 2	0.8242	0.0564
Trial 3	0.8122	0.0642
Mass 1 Added Average	0.8441	0.0298
Mass 1 + 2 Added/Trial 1	1.2512	0.0591
Trial 2	1.2321	0.0856
Trial 3	1.2903	0.0795
Mass 1 + 2 Added Average	1.2579	0.0436

Table 2.8 b values and errors for each trial

All three average damping constants b differ by more than two standard deviations, so for the horizontal damped oscillation setups we conclude that b is not constant for different masses.

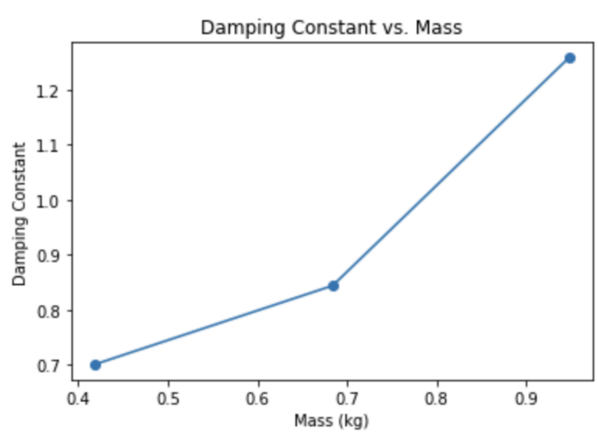

Figure 2.6 Damping constant b vs. Mass

The graph above showing the relationship between average damping constant and mass shows that *b* is not constant for different masses, and rather, there is an upward trend; as mass increases, so does the calculated damping constant.

This is the opposite of that in the vertical oscillation where the trend was downwards and an inverse relationship between damping constant and mass.

Conclusion

The goal of this experiment was to determine whether or not the damping constant was in fact constant when looking at the oscillatory motion of damped oscillations with different mass quantities. We hypothesized that the damping constant would in fact be constant, measured as being within two standard deviations from each other when looking at two different setups (vertical and horizontal) and when looking at different mass amounts (no mass, one mass, or two masses). In other words, we predicted that all the differences in oscillation patterns would be accounted for by the $2m$ term while b was constant. However, this hypothesis was not in agreement with our results; for the vertical setup for two of the mass setups, the damping constants we derived were in fact within two standard deviations of each other, but the third damping constant derived was in fact not, and for the horizontal setup, none of the three damping constants we found from using 3 different mass quantities were within 2 standard deviations from each other. This meant that our hypothesis was in fact incorrect and the damping constant was found to not be a constant value, and it would have to be solved experimentally when working with different mass amounts in damped oscillations. Potential errors that may have arisen while creating our experimental setup may have included not starting from exactly the same starting amplitude for each trial, as we could have had a starting amplitude of 5 cm for one, and, say, 4.9 cm for another, which may have resulted in errors in our data analysis and results. Another error may have also been from varying air drag quantities where the cardboard piece in the vertical setup may have had slightly different orientations throughout each trial. This experiment could have been improved by ensuring that the starting amplitude was the same by using a base object that is 5 cm, that would ensure we were not moving at the start of our oscillations that could ensure we always started from the same place. We could also have created a much sturdier setup where the cardboard piece in the vertical setup was unable to move around or change orientations throughout our different trials.

References

Young, Hugh D., and Roger A. Freedman. *University Physics with Modern Physics*. Pearson, 2019.

Physics 4AL Lab 3B:
https://docs.google.com/presentation/d/1UlEI43txo6BMnF1AtsRD9J9tdI1MgjDLgmxqv5Y3mMY/edit#slide=id.gf90efb90d1_3_714

Physics 4AL Lab 3C:
https://docs.google.com/presentation/d/162jPPFG_rHDqJKX4v37YVIlf7RABI607yRA1cWFsWDM/edit#slide=id.g1779bcf4475_0_92

Physics 4AL Error Propagation Google Colaboratory Notebook:
https://colab.research.google.com/drive/1mLezjmCS0nYG2RLJIeMcfhoU7q3DAZuu?usp=sharing