Phase-Modulated Decoupling and Error Suppression in Qubit-Oscillator Systems

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We present a scheme designed to suppress the dominant source of infidelity in entangling gates between quantum systems coupled through intermediate bosonic oscillator modes. Such systems are particularly susceptible to residual qubit-oscillator entanglement at the conclusion of a gate period that reduces the fidelity of the target entangling operation. We demonstrate how the exclusive use of discrete shifts in the phase of the field moderating the qubit-oscillator interaction is sufficient to both ensure multiple oscillator modes are decoupled and to suppress the effects of fluctuations in the driving field. This approach is amenable to a wide variety of technical implementations including geometric phase gates in superconducting qubits and the Molmer-Sorensen gate for trapped ions. We present detailed example protocols tailored to trapped-ion experiments and demonstrate that our approach has the potential to enable multiqubit gate implementation with a significant reduction in technical complexity relative to previously demonstrated protocols.

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Quantum mechanical entanglement is an important resource for a new generation of quantum-enabled technologies, most notably quantum-information processing (QIP) [1]. A key requirement for scalable QIP is the ability to controllably produce high-fidelity multiparticle entanglement on demand. This is accomplished in experimental systems using a variety of techniques, but a prominent approach relies on the realization of an indirect interaction between basic quantum systems (here qubits) mediated by bosonic oscillator modes [2–7]. A significant source of infidelity in these experiments is the presence of residual qubit-oscillator entanglement at the conclusion of an interaction period, leading to decoherence and a degradation of the fidelity of entanglement generation.

In this Letter, we describe a simple technique to *decouple* qubits from multiple intermediary bosonic modes in order to improve entangling-gate fidelity. The technique is based solely on technologically simple, discrete shifts in the *phase* of the field that mediates the qubit-oscillator coupling. In addition to ensuring that all excited modes are decoupled under ideal operating conditions, we demonstrate how the same framework allows decoupling of each mode to *arbitrary order* when noise leads to imperfect evolution of the composite system. We first present a generic description of the method and then demonstrate its application in the context of Molmer-Sorensen (MS) gates for trapped-ion qubit pairs embedded in a linear chain [8], where residual coupling of ion internal states to multiple modes of motion leads to reduced gate fidelity. This method complements existing optimal-control techniques [9,10], but reduces technical complexity in gate implementation, permits suppression of noise in the drive, and has the potential to achieve the same decoupling operation in a shorter time.

We model the dynamical evolution of a compound system of N qubits ($\mathcal{S}$) coupled to M bosonic oscillator modes ($\mathcal{B}$) via the interaction Hamiltonian

$$H_{SB}(t) = i\hbar \sum_{\mu=1}^{N} \sigma_{\varsigma}^{\mu} \sum_{k=1}^{M} (\gamma_k^{\mu}(t) a_k^{\dagger} - \gamma_k^{\mu}(t) a_k). \tag{1}$$

Here, σ_{ς}^{μ} is a Pauli spin operator acting on the state of the μth qubit, in a "direction" defined by the subscript $\varsigma \in \{x, y, z\}$, while $a_k^{\dagger}$ (a_k) acts on $\mathcal{B}$ creating (annihilating) a single bosonic excitation of the kth oscillator mode. Each of the complex-valued functions $\gamma_k^{\mu}(t)$ has the general form $\gamma_k^{\mu}(t) = f_k^{\mu} e^{i\delta_k t} r(t;\tau)$, where δ_k is the excitation frequency of the kth mode and the coupling constant f_k^{μ} quantifies the strength of its interaction with the μth qubit. The function $r(t;\tau) = \Theta[t]\Theta[\tau - t] e^{-i\phi(t)}$ represents an externally controlled temporal modulation of the coupling phase $\phi(t)$, implemented over an interval $t \in [0, \tau]$. During this time, the Hamiltonian (1) is effectively "switched on," generating the unitary operation

$$U(\tau) = \exp\left\{ \sum_{\mu=1}^{N} \sigma_{\varsigma}^{\mu} B_{\mu}(\tau) + i \sum_{\mu,\nu=1}^{N} \varphi_{\mu\nu}(\tau) \sigma_{\varsigma}^{\mu} \sigma_{\varsigma}^{\nu} \right\}. \tag{2}$$

For $N > 1$, $\varphi_{\mu\nu}(\tau) \equiv \sum_k \mathrm{Im} \int_0^{\tau} dt_1 \int_0^{t_1} dt_2 \gamma_k^{\mu}(t_1) \gamma_k^{\nu}(t_2)$ represents an effective coupling between qubits μ and ν ($\mu \neq \nu$) that arises due to the state-dependent displacement of the oscillator system in phase space, given by $B_{\mu}(\tau) \equiv \sum_{k=1}^{M} [f_k^{\mu} \alpha_k(\tau) a_k^{\dagger} - f_k^{\mu} \alpha_k^{*}(\tau) a_k]$, where $\alpha_k(\tau) \equiv \int_0^{\infty} dt e^{i\delta_k t} r(t;\tau)$.

While the coupling interaction presented here is generic, it is frequently implemented using a controlled periodic driving field (DF). In this context, the excitation frequency

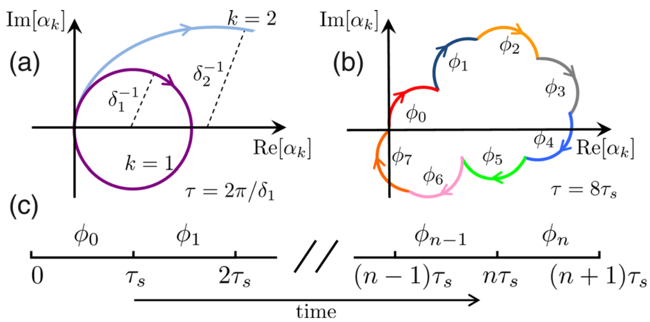

FIG. 1 (color online). (a) Schematic plots of $\alpha_k(t)$, $0 \le t \le \tau$, for two modes, $k = 1, 2$. Each path represents N phase-space trajectories $\alpha_k^\mu(t) = f_k^\mu \alpha_k(t)$, for $\mu = 1, \ldots, N$. Here $\tau = 2\pi/\delta_1$, so that the path labeled $k = 1$ closes. (b) An illustrative example of a single closed path, generated by an eight-interval, piecewise-constant phase-modulation sequence. (c) Timing schematic for piecewise-constant phase-modulation sequence, comprising a total of n instantaneous phase shifts. For the general M mode decoupling sequence (5) $n = 2^M - 1$.

δ_k is realized via a detuning between the frequency of the DF and a sideband associated with the kth motional mode (formally due to a transformation to the interaction picture with respect to the free Hamiltonian). The coupling strength f_k^μ is controlled by the magnitude of the field, and $\phi(t)$ by its phase. If this phase is fixed, the off-resonant DF simply excites the kth mode, increasing the amplitude of oscillation, until the accumulated phase difference between the oscillator and the DF reaches the point at which they are "out-of-phase" ($\delta_k t = \pi$). Continued application of the DF then acts to dampen or "deexcite" the mode until it returns to its initial state ($\delta_k t = 2\pi$). This cycle of excitation and deexcitation maps to a circular path in phase space, the radius of which is inversely proportional to the detuning δ_k [Fig. 1(a)].

Any residual qubit-oscillator entanglement at the conclusion of the control operation (at time $t = \tau$) will result in qubit decoherence and must, therefore, be suppressed in order to achieve a high-fidelity entangling operation. Complete qubit-oscillator decoupling occurs if

$$\alpha_k(\tau) \equiv \int_0^\infty dt e^{i\delta_k t} r(t;\tau) = 0 \tag{3}$$

for $k = 1, \ldots, M$. Each of the time-parametrized functions $\alpha_k(t)$, $0 \le t \le \tau$, defines a set of N phase-space trajectories $\alpha_k^\mu(t) = f_k^\mu \alpha_k(t)$, for $\mu = 1, \ldots, N$, associated with the kth oscillator mode (Fig. 1). These trajectories vary in extent and orientation, according to the complex coupling constant f_k^μ. However, by satisfying the condition (3) all trajectories are closed at $t = \tau$.

Decoupling from any particular mode k can be achieved by fixing the control phase at a constant value (which can be taken to be zero) and setting the total operation time and coupling-drive detuning such that $\delta_k \tau = 2\pi j$, for $j \in \{1, 2, \ldots\}$ [Fig. 1(a)]. Simultaneous decoupling from any of the remaining modes is possible only if the associated detunings are commensurate with δ_k. This can be difficult to engineer, even approximately, for more than one additional mode without the need to resort to undesirably long gate times [9,10].

Modulation of the relevant control, as demonstrated recently [10], provides a path to simultaneously realizing the decoupling condition for multiple modes, and any of the parameters of the DF may, in principle, be varied in time in order to achieve the necessary condition. Unlike previous work, our method fixes both the frequency of the DF (hence, δ_k) as well as the coupling strength f_k^μ during the interaction period, and treats only the drive phase $\phi(t)$ as a tunable parameter. In the following, we demonstrate that the freedom to modulate $\phi(t)$ via discrete shifts, easily implemented using state-of-the-art digital frequency synthesis technology, may be used to impose a commensurate periodicity on the full set of phase-space paths so that they all close simultaneously at an (in principle) arbitrary time τ.

The key to the method lies on the dependence of each phase-space trajectory on the accumulated phase difference between the DF and the relevant oscillator. Instantaneous shifts in the phase of the DF change this relationship and may, therefore, be used to "redirect" the trajectory to achieve a desired outcome. In particular, any path may be closed by simply repeating the control sequence that produced it, with an appropriately chosen *overall* phase shift. An additional trajectory may then be closed, without affecting the closure of the first, by applying the required overall phase shift only when the first passes through the origin. This procedure may then be repeated until all M modes have been accounted for. To describe this process of "phase-compensated" concatenation mathematically, we define a family of *nonlinear* operators R_δ, parametrized by the real number δ, that act to extend any phase-modulation sequence $r(t;\tau')$, defined over the interval $[0,\tau']$, to the interval $[0, 2\tau']$ in the following way:

$$R_\delta r(t;\tau') = r(t;\tau') + e^{-i(\delta\tau' - \pi)} r(t - \tau';\tau'). \tag{4}$$

The function $r(t; 2\tau') \equiv R_{\delta_k} r(t;\tau')$ then describes a sequence for which $\alpha_k(2\tau') = 0$.

The qubit system may be simultaneously decoupled from all M modes by implementing the piecewise-constant phase-modulation sequence $r_{\delta_M \cdots \delta_1}(t; 2^M \tau_s) \equiv R_{\delta_M} \cdots R_{\delta_1} r_0(t;\tau_s)$, starting with the trivial "no-operation" base sequence $r_0(t;\tau_s) = \Theta[t]\Theta[\tau_s - t]$, for which $\phi(t) \equiv 0$ over an arbitrary interval $[0, \tau_s]$. This sequence, applied over discrete time steps of duration τ_s, indexed by ℓ, may be written explicitly as

$$r_{\delta_M\cdots\delta_1}(t;2^M\tau_s) = \sum_{\ell=0}^{2^M-1} r_0(t-\ell\tau_s;\tau_s)e^{-i\phi_\ell}, \qquad (5)$$

where

$$\phi_\ell = \sum_{j=0}^{q} \varepsilon_j(\ell)2^j\delta_{j+1}\tau_s - s(\ell)\pi \qquad (6)$$

is the requisite phase value for the time interval $[\ell\tau_s,(\ell+1)\tau_s]$ [see Fig. 1(c)]. In this expression, $\varepsilon_q(\ell)\varepsilon_{q-1}(\ell)\cdots\varepsilon_0(\ell)$ is the binary representation of ℓ and $s(\ell)\equiv\sum_{j=0}^{q}\varepsilon_j(\ell)$ is its Hamming weight. As an example, the binary representation of $\ell=6$ is 110, so that $q=2$, $\varepsilon_0(\ell)=0$, $\varepsilon_1(\ell)=1$, $\varepsilon_2(\ell)=1$, and $s(\ell)=2$. The resulting "entangling phases" $\varphi_{\mu\nu}$ can then be calculated for these phase values (see Supplemental Material [11]), and the strength of the DF adjusted to generate target values $\varphi_{\mu\nu}^{(0)}$, for $\mu,\nu=1,\ldots,N$ and $\mu\neq\nu$.

Ideally, the decoupling condition (3) should also be met in the presence of time-domain variations in the coupling parameters defining phase-space trajectories. When only a single oscillator mode is coupled to the qubit system, discrete bivalued phase modulation in the form of simple binary (±1) concatenated dynamical decoupling (CDD) sequences [12,13] has been shown to suppress errors due to thermal dissipation [14] and static detuning offsets [15]. We observe that by relaxing the binary-valued constraint on $\phi(t)$, phase-compensated CDD sequences, targeting noise associated with particular modes, may be realized.

Specifically, we consider noise that may be represented by a function $\beta_k(t)$ that modifies the spin-oscillator coupling via $\gamma_k^\mu(t)\to\gamma_k^\mu(t)\beta_k(t)$ (the subscript k allows for the possibility of mode dependence.) This model encompasses a number of important noise sources, including: frequency (δ_k) errors, thermal dissipation of oscillator modes and fluctuations in the strength of the qubit-oscillator interaction. We suppose that in the weak or slowly varying noise limit $\beta_k(t)$ may be approximated by a pth-order polynomial $\beta_k^{(p)}(t)=\sum_{j=0}^{p}\beta_{k,j}t^j$. In the presence of such noise, the decoupling condition for the kth mode to order $p+1$ is

$$\alpha_k^{(p+1)}(\tau)\equiv\int_0^\infty dte^{i\delta_k t}r(t;\tau)\beta_k^{(p)}(t)=0, \qquad (7)$$

for $p\geq0$.

Mathematically, binary CDD sequences are essentially time-domain representations of finite iterations of the infinite Thue-Morse (TM) sequence [16,17]. From the perspective of noise suppression, the most interesting property of the TM sequence is that the $(p+1)$th iteration $r(t;2^{p+1}\tau_s)\equiv R_0^{p+1}r_0(t;\tau_s)$ is orthogonal to any pth degree polynomial $\beta^{(p)}(t)=\sum_{j=0}^{p}\beta_jt^j$, i.e., $\int_0^\infty dtr(t;2^{p+1}\tau_s)\beta^{(p)}(t)=0$ [18]. Using this insight, and the fact that we have relaxed the binary-value restriction on $\phi(t)$, it can be shown (see Supplemental Material [11]) that the phase-compensated TM or CDD sequence $r[t;(n+1)\tau_s]\equiv R_{\delta_k}^{p+1}r_0(t;\tau_s)=\sum_{\ell=0}^{n}r_0(t-\ell\tau_s;\tau_s)e^{-i\phi_\ell}$, where $n=2^{p+1}-1$ and $\phi_\ell=\ell\delta_k\tau_s-s(\ell)\pi$, will achieve $\alpha_k^{(p+1)}(2^{p+1}\tau_s)=0$.

The general properties of concatenated control sequences may now be brought to bear in providing simultaneous high-order mode decoupling in the presence of time-varying coupling parameters. To suppress noise across multiple modes, one may construct a phase-modulation sequence of the general form $R_{\delta_{k_q}}\cdots R_{\delta_{k_2}}R_{\delta_{k_1}}r_0(t;\tau_s)$, where the order of error suppression associated with mode k_i is determined by the number of times k_i appears in the sequence indices $(k_1,k_2,\ldots,k_q)$. For example, $R_{\delta_3}R_{\delta_2}R_{\delta_3}R_{\delta_1}r_0(t;\tau_s)$ will close all trajectories associated with the first three modes, providing additional error suppression to second order for mode $k=3$. The order in which the operators are best applied will depend on the properties of the noise; in particular, the extent to which the noise varies with k. Vitally, in the presence of any such modulation protocol it remains possible to analytically calculate the entangling phase $\varphi_{\mu\nu}(\tau)$ for arbitrary qubit-pair $\mu-\nu$ and to adjust $f_k^{\mu(\nu)}$ appropriately.

For concreteness, we now consider the task of entangling the internal states of a pair of adjacent trapped ions embedded in an $N>2$ ion chain. Under certain simplifying assumptions, the effect of a state-dependent force generated by a bichromatic light field is well described by the Hamiltonian (1) [3,19,20]. Effective spin-1/2 manifolds realized within the electronic states of each of the ions comprise the system of qubits $\mathcal{S}$, and the shared vibrational modes of the ions in a confining potential constitute the oscillator system $\mathcal{B}$ [21]. The qubit-oscillator coupling

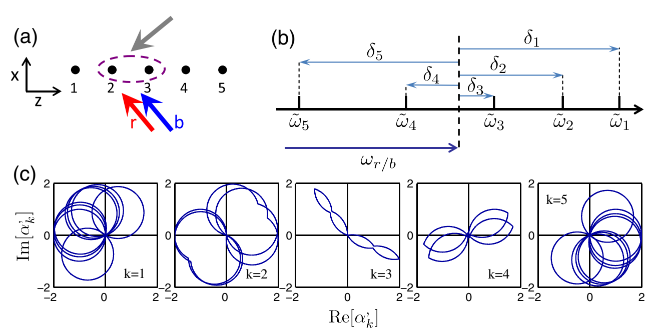

FIG. 2 (color online). (a) Raman laser geometry for a MS gate applied to 2 ions in a 5 ion chain in which only transverse (x-direction) phonon modes are excited. The red (r) and blue (b) Raman fields have frequencies $\omega_{r/b}$ and phases $\phi_{r/b}$. (b) Detuning diagram for 5 excited TP modes, $\tilde\omega_k=\omega_0\pm\omega_k$ (+ for ω_b and − for ω_r), where ω_0 is the hyperfine qubit level splitting. (c) Closed paths, $\alpha_k'\equiv|\delta_k|\alpha_k(t)$, $0\leq t\leq\tau$, (normalized by $|\delta_k|$) for the detunings shown in (b), generated by 7 discrete phase shifts.

strength f_k^μ is proportional to the amplitude of the Raman fields (see Supplemental Material [11] for further details). The relevant control phase $\phi(t)$ is determined by the difference between the phases of the red and blue Raman fields, and can be varied without altering the spin dependence of the entangling gate. Most importantly, in this setting, $\phi(t)$ inherits all the flexibility and precision provided by modern laser control systems. In particular, the discrete phase shifts required for the basic decoupling sequence (5) can be implemented quasi-instantaneously and with high accuracy using standard optical modulators driven by radiofrequency sources.

Figure 2 shows closed mode trajectories representing the complete decoupling of a pair of $^{171}\mathrm{Yb}^+$ hyperfine qubits from five excited transverse phonon modes [22], using phase shifts derived from Eq. (5). In this illustrative example, we set the laser frequencies so that two modes have commensurate detunings and choose $\tau_s = 2\pi/\delta_{1,5}$ to match the period of the associated phase-space evolution. In this way, a sequence of only $n = 7$ phase shifts is required to decouple the qubits from all 5 modes, rather than the more general sequence of $n = 31$ phase shifts. Assuming equivalent physical parameters to recent demonstrations of multimode decoupling using optimized amplitude modulation [10], the resulting phase-modulate gate has duration $\tau \sim 140\ \mu$s which compares favorably with the reported value of $\tau = 190\ \mu$s, while obviating considerations of nonlinear amplitude responses in optical modulators and rf amplifiers. Faster gate times may be achieved, at the expense of a greater number of phase shifts, by allowing τ_s to vary arbitrarily. This freedom to vary the step time may also be used to "tune" the phase-modulation protocol with the aim of mitigating additional decoherence effects that, while not accounted for in the model Hamiltonian, will nonetheless negatively impact gate fidelity in practice.

We also demonstrate the effectiveness of phase-modulation sequences in suppressing decoupling errors induced by laser amplitude noise, a prominent time-dependent gate error. We assume that a pair of qubits to be entangled is initially in an "evenly weighted" separable pure state and then quantify the extent of residual entanglement between the internal and vibrational ion states by calculating the linear entropy $\bar{P} \equiv 1 - \mathrm{Tr}[\rho_S^2(\tau)]$, where $\rho_S(\tau)$ is the final qubit state [23,24]. The laser amplitude instability is modeled as a time-dependent contribution to the Rabi rate $\Omega(t) = \Omega_0 + \Omega_e(t)$, where Ω_0 is the ideal value and $\Omega_e(t)$ represents the noise (we assume the Rabi rate is the same for the two adjacent ions). As $\Omega_e(t)$ is a stochastic process, we average over the ensemble of noise realizations ($\mathbb{E}[\ldots]$) to obtain the final metric (see Supplemental Material [11] for a derivation)

$$\mathbb{E}[\bar{P}] \approx \frac{1}{8\pi}\int_{-\infty}^{\infty} d\omega S_{\Omega_e}(\omega)F(\omega), \tag{8}$$

where $S_{\Omega_e}(\omega)$ represents the power spectral density of amplitude fluctuations as a function of frequency ω. In deriving this expression, we've assumed the weak noise limit in which only the first-order effect of amplitude fluctuations is significant, and that phase-modulation results in complete decoupling in the *absence* of noise. When these conditions are met, Eq. (8) provides a simple analytical tool for quantifying the extent of qubit-oscillator decoupling in the presence of time-varying noise.

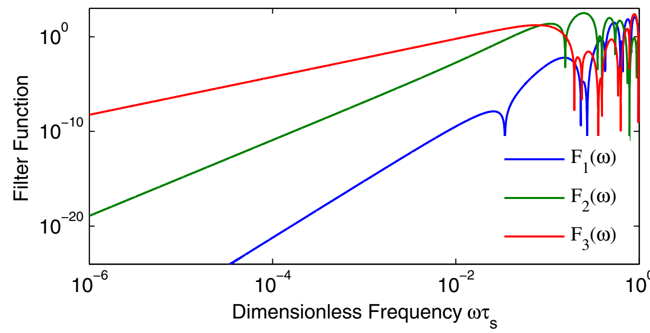

FIG. 3 (color online). Laser amplitude noise filter functions for 3 excited oscillator modes $k = 1, 2, 3$. The effect of the noise on the coupling to each mode is suppressed to third order for $k = 1$ [$F_1(\omega) \propto \omega^3$ as $\omega \to 0$], second order for $k = 2$ [$F_2(\omega) \propto \omega^2$], and first order for $k = 3$ [$F_3(\omega) \propto \omega$].

The effect of the phase modulation is captured by the expression $F(\omega) \equiv \sum_k D_k F_k(\omega)$, with D_k a constant. The "modal filter function" $F_k(\omega) \equiv |\int_0^\infty dt e^{i(\omega+\delta_k)t} r(t;\tau)|^2$ represents the phase-modulation sequence $r(t;\tau)$ in the frequency domain, following insights presented in [25,26]. It captures the effectiveness of the phase-modulation protocol in suppressing laser amplitude noise for different modes; a higher "slope" on a log-log plot indicates higher-order (p) suppression of temporal fluctuations. This general filter-function framework provides a formalism for assessing the robustness of complex control protocols against arbitrary time-dependent noise. The approach has recently been validated by a range of single-qubit experiments [27,28], and has been extended here to qubit-oscillator and qubit-qubit interactions.

In Fig. 3 we plot $F_k(\omega)$ calculated for specific, but arbitrarily chosen, orders associated with each of three modes, $k = 1, 2, 3$. By increasing the level of concatenation for specific modes we are able to improve qubit-oscillator decoupling through the suppression of low-frequency amplitude fluctuations while simultaneously ensuring all modes are efficiently decoupled. In general, D_k depends on the initial qubit state $|\phi_0\rangle$ and the effective temperature, in addition to the frequency of the kth mode ω_k. Here, where we consider only the collective zero temperature limit and the particular initial state $|\phi_0\rangle = |11\rangle_z$.

In summary, we have presented a unified framework for improving entangling gate fidelity in qubit-oscillator systems by ensuring efficient decoupling of intermediary bosonic modes. The framework permits simultaneous decoupling of multiple oscillator modes using only sequences of discrete phase shifts, and may also be combined with concatenation procedures to include robustness against time-dependent control noise. The number of phase shifts required grows exponentially with the number of oscillator modes, and with increasing order of error suppression, so that, in practice, accumulated control errors will limit the number of oscillator modes that may be effectively decoupled (the precise limit will depend on the particular technology). Nonetheless, within these bounds, the approach complements and provides substantial benefits relative to existing techniques, leveraging the technical simplicity of phase modulation in rf electronics. It also mitigates the need to account for nonlinearities in the response of modulating hardware (such as acousto-optic modulators for laser beams) or variations in residual light shifts endemic to Raman-mediated trapped-ion gates. In addition, the required DF amplitude is time independent and expressible as a simple analytical function of the time step τ_s, DF frequency, and the number and order of phase shifts. By varying these parameters, the required constant DF amplitude may be maximized to optimize total operation time. Finally, we note that the decoupling capacity of the sequences is unaffected by any constant phase offset, so that spatial uniformity across the qubit system is not a prerequisite.

Overall, we hope that this phase-modulation approach to improving entangling gate fidelities will prove useful in a range of quantum-information settings across different qubit-oscillator systems. We are also excited by the possibility that similar approaches may be employed to effectively modify the entangling phase between different qubits in a multipartite system, or be employed to improve the performance of quantum-enhanced sensors [29].

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[11] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.120502 contains an explicit expression for the entangling phase generated by a phase-modulation sequence, a brief description of the operation of the Molmer-Sorensen gate, and a list of parameter values for an example decoupling sequence. It also includes a derivation of the ensemble average linear entropy for an entangling gate operation affected by weak amplitude noise.
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