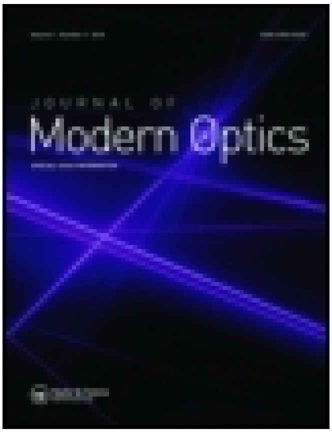

Journal of Modern Optics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmop20

Deterministic generation of a bit-stream of single-photon pulses

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Published online: 03 Jul 2009.

To cite this article: C. K. Law & H. J. Kimble (1997) Deterministic generation of a bit-stream of single-photon pulses, Journal of Modern Optics, 44:11-12, 2067-2074

To link to this article: http://dx.doi.org/10.1080/09500349708231869

JOURNAL OF MODERN OPTICS, 1997, VOL. 44, NO. 11/12, 2067–2074

Deterministic generation of a bit-stream of single-photon pulses

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(*Received 11 December 1996*)

Abstract. We report a high efficiency scheme for generating a single-photon state transmitted out of an optical cavity. For realistic cavity QED parameters, we show that the scheme can produce a single-photon pulse with a probability exceeding 99% in a user-specified time interval. By recycling the system, the scheme can be used to create a bit-stream of single-photon pulses.

Recent experimental work on the real-time detection of a single atom falling through a high-Q optical cavity provides two important breakthroughs for investigations in cavity QED [1]. In the first place, the transit time T for a laser-cooled atom to remain inside the cavity is much longer than the cavity or atomic relaxation times $(\kappa^{-1}, \gamma^{-1})$ as well as the one-photon Rabi-flipping time $(2g)^{-1}$. Indeed for the work reported in [1], $gT \sim 10^3\pi$ whereas for previous work in cavity QED $gT \sim \pi$ [2]. Secondly, by recording the light transmitted from the cavity the atomic position can be monitored in real time with high resolution. These two advances open new possibilities for exploring various applications of cavity QED, such as arbitrary quantum-state preparation [3, 4], 'one-and-the-same' atom laser [5], and the formation of mechanical binding of atom and photons [6–8].

Our focus in this paper is on the quantum control of single-photon emission from an atom in a cavity. The goal is to generate a prescribed sequence of single-photon pulses which propagates through a given transmission channel (figure 1). A key problem is to design an interaction scheme so that the (atom + cavity) system behaves as a *deterministic* single-photon source. The word 'deterministic' is emphasized here as it is defined by the property that if the source is switched 'on', then with a high degree of certainty one (and only one) photon emerges from the cavity at a known, user-specified time. By successive turning on and off the source, a deterministic bit-stream of single-photon pulses can then be generated as desired. This kind of non-classical light source would be quite important to quantum cryptography [9] and for certain quantum computation problems [10] for which a single-photon state is needed. We should point out that although parametric amplifiers can provide single-photon pulses by monitoring photons in the idler beam [11, 12], the arrival times between successive signal photons from the parametric process are stochastic and are not readily controllable.

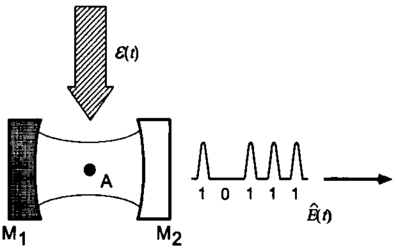

Figure 1. Illustration of the goal of the system. An external field $\mathcal{E}(t)$ drives an atom in a cavity in which M_1 is a perfect mirror. $\mathcal{E}_2$ generates a prescribed sequence of pulses transmitted through the mirror M_2. Each pulse for the output field $\hat{E}(t)$ contains exactly one-photon.

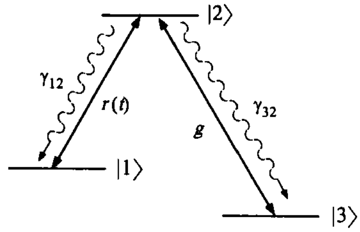

Figure 2. Energy-level diagram for a Λ-type 3-level atom interacting with a classical field $r(t)$ and a quantized field g. Here $\gamma_{12} = \gamma\cos^2\theta$ and $\gamma_{32} = \gamma\sin^2\theta$ are the atomic decay rates.

In this paper we describe an essentially deterministic single-photon source based on cavity QED interactions. Although certainly an excited atom can only emit a single photon, there are several complicated issues associated with translating this simple notion into useful sources. First, because of the intrinsic losses to the environment, an optimal scheme is required so that the emitted photon can be transmitted through a desirable channel with high probability. Second, the pumping mechanism has to be designed so that pulse shapes and time intervals for single-photon emissions can be readily controlled. Third, the system should have a high recycling efficiency for generating successive photons in a bit-stream. In this paper we shall present a realistic scheme that meets these requirements. We note that mesoscopic turnstile devices have also been suggested to be capable of generating single-photon pulses in a controllable way [13].

Consider a Λ-type three-level atom interacting with a cavity field and an external classical field, as shown in figure 2. The interaction Hamiltonian is given by

$$H = \hbar g(a\sigma_{23} + a^\dagger\sigma_{32}) + \hbar r(t)(\sigma_{21} + \sigma_{12}), \tag{1}$$

where $\sigma_{ij} = |i\rangle\langle j|$ $(i,j = 1,2,3)$ are atomic projection operators. Then a and $a^\dagger$ are annihilation and creation operators for the quantized cavity field. Here g denotes the atom–cavity coupling strength, while $r(t)$ gives the coupling between the atom and the external classical field $\mathcal{E}(t)$. Note that $r(t) \propto \mathcal{E}(t)$, and so the external field determines the time-dependence of $r(t)$. In writing equation (1), we have assumed that the cavity field frequency ω_c and the classical field frequency ω_L are on resonance with their respective atomic transitions, i.e. $\hbar\omega_c = E_2 - E_3$ and

$\hbar\omega_L = E_2 - E_1$ where E_j ($j = 1, 2, 3$) is the energy of the jth atomic level. Note that ω_L and ω_c are taken to be significantly different from each other, and so the fields can only make atomic transitions associated with their own resonant channels.

In our model, energy relaxation comes from spontaneous decay at a rate γ of the atomic level $|2\rangle$, and from photon leakage at rate 2κ out of the cavity. The lifetime of atomic levels $|1\rangle$ and $|3\rangle$ are assumed long so that their relaxation is ignored. The dynamics of the system can be modelled by the master equation for the density matrix operator ρ,

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \kappa(2a^\dagger \rho a - a^\dagger a \rho - \rho a^\dagger a) + \frac{\gamma \cos^2\theta}{2}(2\sigma_{12}\rho\sigma_{21} - \sigma_{22}\rho - \rho\sigma_{22})$$
$$+ \frac{\gamma \sin^2\theta}{2}(2\sigma_{32}\rho\sigma_{23} - \sigma_{22}\rho - \rho\sigma_{22}) \tag{2}$$

where θ determines the branching ratio of the atomic decay to levels $|1\rangle$ and $|3\rangle$.

We consider that the system is initially prepared in the state $|1\rangle$ with no photon in the cavity, and for which at most one photon can be generated by turning on the classical field $r(t)$. This is because for the assumed configuration the atom cannot emit any further photon once it reaches the state $|3\rangle$. Of course, sometimes no photon is generated in the cavity output because of spontaneous decay from $|2\rangle$ to $|3\rangle$, which may occur without giving up a photon to the cavity mode. Therefore, spontaneous atomic decay to modes other than the privileged cavity mode is the only source of loss in the model, which we shall see can be faithfully realized experimentally. We stress that the cavity decay is not considered as a loss, because our goal is to exploit the atom–cavity interaction as a source of a single-photon transmitted out of the cavity.

To measure the success of single-photon production, we examine the probability of detecting a single photon during the time interval from 0 to t, if an ideal photodetector is used,

$$P(t) = 2\kappa \int_0^t \langle n(t') \rangle \mathrm{d}t'. \tag{3}$$

Here $\langle n(t') \rangle$ is the average photon number in the cavity at the time t', and 2κ is the cavity leakage rate. The time derivative of $P(t)$ gives the rate of photons emerging from the cavity mirror M_2. As we are interested in near 'deterministic' single-photon generation, the shape of $\dot{P}(t)$ should be sufficiently narrow as to define a well-specified interval in which the photon is localized. In other words, we require that the photon be emitted as a narrow pulse which determines the 'bit interval'.

Now given an external classical excitation $r(t)$ over a time period $[0, T_0]$, the problem is to obtain a high single-photon probability $P(T_0)$. In order to minimize the loss due to atom decay, let us first consider the so-called 'one-dimensional atom' [14] in the bad cavity regime,

$$\kappa \gg g^2/\kappa \gg \gamma. \tag{4}$$

In this regime, the cavity decay rate κ sets the fastest time scale, while the rate g^2/κ of coherent coupling to the cavity mode dominates the rate r of incoherent atomic decay [15].

To investigate the quantum dynamics of the system, it is convenient to follow a quantum trajectory description [16]. The evolution of the system wavefunction is governed by a non-Hermitian Hamiltonian,

$$H' = H - i\hbar\kappa a^\dagger a - i\hbar\frac{\gamma}{2}\sigma_{22} \tag{5}$$

subject to quantum jumps at random times. The wavefunction can be written as

$$|\psi(t)\rangle = a_1(t)|1,0\rangle + a_2(t)|2,0\rangle + a_3(t)|3,1\rangle \tag{6}$$

where $|j,k\rangle$ ($j = 1,2,3, k = 0,1$) labels the atomic state $|j\rangle$ with k cavity photons, and a_j are complex amplitudes. Because H' is non-Hermitian the norm of $|\psi\rangle$ decreases with time. Once a quantum jump (either via an atomic decay or cavity decay) occurs, the wavefunction is renormalized to unity. The Schrödinger equation $i\hbar|\dot{\psi}\rangle = H'|\psi\rangle$ yields

$$i\dot{a}_1 = r(t)a_2 \tag{7}$$

$$i\dot{a}_2 = r(t)a_1 + ga_3 - \frac{i\gamma}{2}a_2 \tag{8}$$

$$i\dot{a}_3 = ga_2 - i\kappa a_3, \tag{9}$$

with an initial condition $a_1(0) = 1$, $a_2(0) = a_3(0) = 0$, and $r(0) = 0$. The $|a_j(t)|^2$ can be interpreted as the probability for the system to be found in the state $|j,k\rangle$ without making a quantum jump up to a time t.

With the assumption that the external pulse satisfies $r(t) \ll g^2/\kappa$, an adiabatic solution of (7)–(9) can be derived in the bad cavity limit as defined in (4),

$$a_1(t) \approx \exp\left(-\alpha\int_0^t r^2(t')\,dt'\right) \tag{10}$$

$$a_2(t) \approx -i\alpha r(t)a_1(t) \tag{11}$$

$$a_3(t) \approx -i\frac{g}{\kappa}a_2(t). \tag{12}$$

Here α is given by

$$\alpha = \frac{2}{\gamma + 2g^2/\kappa} \equiv \frac{\kappa}{g^2}\frac{2C_1}{1+2C_1} \tag{13}$$

where $C_1 \equiv g^2/\kappa\gamma$ is the single-atom cooperativity parameter [17]. The probability of a single photon leaking out of the cavity during the time interval $[0,t]$ *without* making a spontaneous atomic decay (jump) is given by

$$P_0(t) = 2\kappa\int_0^t |a_3(t')|^2\,dt'. \tag{14}$$

Notice that the actual single-photon probability $P(t)$ is larger than $P_0(t)$. This is because $P_0(t)$ excludes all atomic decay events for which the population in $|2\rangle$ decays back to $|1\rangle$ and is then re-excited by the classical pump. Such events have a finite probability to emit a photon, but since γ is the slowest time scale in the bad cavity limit, the occurrence of these recycling events is rare. Therefore we can make an estimation of $P(t)$ based on $P_0(t)$. From the adiabatic solution (10)–(12), we have

$$P(t) \gtrsim P_0(t) = \frac{g^2\alpha}{\kappa}\left[1 - \exp\left(-2\alpha\int_0^t r^2(\tau)\,\mathrm{d}\tau\right)\right]. \tag{15}$$

This expression specifies the time-dependence of $P(t)$ whenever a classical pulse shape $r(t)$ is given. For a sufficiently long time (or pulse area), the exponential term becomes negligible, and so

$$P(t) \approx \alpha g^2/\kappa = \frac{2C_1}{1+2C_1} \to 1 \tag{16}$$

for $C_1 \gg 1$ in the 'one-dimensional atom' limit. This suggests a remarkable result that the system is capable of generating one (and only one) photon with near certainty. We remark that the duration of the single-photon pulse can be controlled by adjusting α and the temporal profile of $r(t)$. It is possible to obtain a pulse duration shorter than an atomic lifetime γ^{-1}. The shortest pulse length achievable in our scheme depends on the physical parameter g^2/κ which limits the atomic excitation rate.

To illustrate the potential of this scheme for actual experimental implementation, we now present numerical solutions of the master equation. We specifically investigate the D1 line of atomic cesium, with the spacing of the hyperfine levels' spacings significantly larger than for D2 manifold, thus permitting a stronger driving field without exciting neighbouring hyperfine levels. The relevant hyperfine energy-level diagram is shown in figure 3. It is important to note that the Λ-type three-level configuration assumed in figure 2 is not exactly realized, because atomic population in the $6P_{1/2}$, $|F=4, m=4\rangle$ level can decay to the $6S_{1/2}$, $|F=4, m=3\rangle$ level. The Clebsch–Gordan coefficients for the dipole matrix elements give this branching ratio for the atomic decay, where for each atomic decay of $|F=4, m=4\rangle$ in $6P_{1/2}$, there is a probability 1/12 that the atom will go to $|F=4, m=3\rangle$ in $6S_{1/2}$. We have modified the master equation (2) to account for such losses. Since there is a finite turn on/off time for the classical field $r(t)$, we model the classical field envelope of length T_0 as

$$r(t) = r_0 \sin^2\left(\frac{\pi t}{T_0}\right) \qquad 0 \leqslant t \leqslant T_0, \tag{17}$$

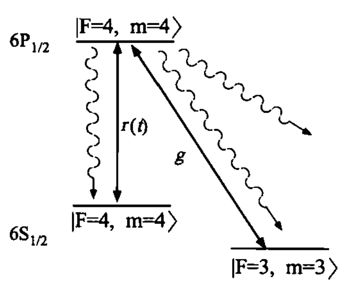

Figure 3. Hyperfine levels of a Cs atom suitable for our scheme. The dipole selection rule requires that $r(t)$ is π-polarized and g is circularly polarized. In contrast to a closed system in figure 2, there is a loss from top level to a state outside the Λ-manifold ($6S_{1/2}, F=4, m=3$)

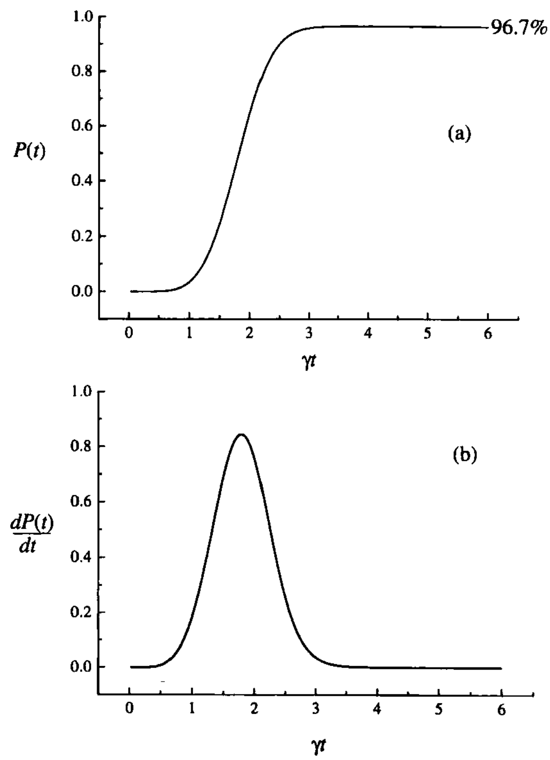

Figure 4. Numerical results for the time-dependence of (*a*) the single-photon probability $P(t)$ and (*b*) the photon counting rate $\dot{P}(t)$. The parameters are $(g, \kappa, \gamma, r_0)/2\pi = (45, 45, 4{\cdot}5, 22{\cdot}5)$ MHz and the pulse length $T_0 = 6\gamma^{-1}$

and zero otherwise. We remark that unlike equations (6)–(15), our calculations here are not limited to the 'one-dimensional atom' regime.

In figure 4 we present an example showing the high efficiency of the scheme. For realistic parameters $(g, \kappa, \gamma, r_0)/2\pi = (45, 45, 4{\cdot}5, 22{\cdot}5)$ MHz [18] and $T_0 = 6\gamma^{-1} = 210$ ns, the single photon probability $P(t)$ rises sharply to 96·7% within a few atomic lifetimes (figure 4 (*a*)). This probability is close to the adiabatic approximation (95·2%) predicted from the right side of equation (15), even though the parameters are not entirely within the 'one-dimensional atom' regime. We point out that an even higher value of $P(t)$ can be reached (99·5%) by using a larger atom–cavity coupling $g/2\pi = 120$ MHz currently obtainable in the laboratory [18]. In figure 4 (*b*), we plot the shape $\dot{P}(t)$ which is seen to be well localized on the time axis. Therefore the photon is emitted in a well-defined interval of time, and in this case the width of $\dot{P}(t)$ is about 40 ns. It should be remarked that the results given here are not sensitive to the precise value of g. We have done a numerical calculation for $g/2\pi = 22{\cdot}5$ MHz with the same values of κ, r_0, and found that even with this smaller value of g, $P(t)$ still reaches 88%. The $\dot{P}(t)$ in this case also remains narrow at a similar position.

In the example of figure 4, 99·6% of the total population is found in the state $|F = 3, m = 3\rangle$ of $6S_{1/2}$ at the end of the pulse. There is a very small population (0·003%) remaining in the initial state, and about 2×10^{-8}% will be found in the top level $|F = 4, m = 4\rangle$ of $6P_{1/2}$. Therefore the loss is mainly due to atomic decay from the $|F = 4, m = 4\rangle$ of $6P_{1/2}$ to the levels $|F = 3, m = 3\rangle$ and $|F = 4, m = 3\rangle$ of $6S_{1/2}$. The high fraction of population in $|F = 3, m = 3\rangle$ of $6S_{1/2}$ can be recycled back to the initial state so that more photons can be generated by repeating the procedure. The recycle $|F = 3, m = 3\rangle \rightarrow |F = 4, m = 4\rangle$ can be achieved by applying Raman pulses or microwaves to connect the two hyperfine levels. Optical pumping methods may also be used in order to recycle the loss population. For parameters of figure 4, we can expect to generate a few single-photon pulses in every microsecond.

To conclude, we have described a highly deterministic and efficient method to generate a single photon which will propagate in a given transmission channel. With this method one can construct a prescribed sequence of single-photon pulses in which digital information can be encoded. Perhaps the simplest scheme would be to encode a binary stream with the bit value '0' associated with $r(t) = 0$ and the bit '1' with $r(t) \neq 0$ (as in the case of figure 4). However, by employing the states $6S_{1/2}$ $F = 4$ and $m = \pm 4$, a bit stream could be constructed with 0 and 1 values encoded in photon polarizations, (i.e. prepare $F = 4, m = 4$ for σ_+ polarization ($\equiv$'0') and $F = 4, m = -4$ for σ_- polarization ($\equiv$'1')). Yet more complex encoding with more than one photon per pulse could also be implemented for quantum error correction.

Our analytical results show that the single-photon probability can appoach unity in the bad cavity limit. In our scheme, specific control of the phase and pulse area of the classical field are not required. We have tested the performance of our scheme numerically for a realistic implementation in a Cs atom. The results confirm the high probability of single-photon production and indicate the robustness of the scheme against uncertainty in the value of g. It should be noted that fundamental limitations of our scheme come from the atomic structure. Since the probability of loss due to atomic decay is of the order of $\gamma\kappa/2g^2$, increasing the value of the atom–cavity coupling g may improve the yield. However, a very large g would introduce undesirable atomic transitions to levels beyond the three-level picture. Further investigations are necessary in order to address the question of optimal parameters and the effects of recycle processes. Finally, we point out that the shape of the single-photon pulse can be 'engineered' by adjusting the temporal profile of the classical field and the parameter α. As already implied from equation (15), the functional form for $\dot{P}(t)$ can be determined with broad latitude through the specification of the classical driving field $r(t)$. This control is essential to certain applications, such as quantum state transfer in a quantum computing network [19].

Acknowledgments

CKL thanks Professor H. J. Carmichael, M. S. Chapman, H. Mabuchi, Q. A. Turchette and D. Vernooy for discussions. He also thanks the Quantum Optics group at Caltech for their hospitality. This work is partially supported by DARPA through the Quantum Information and Computation (QUIC), Institute administered by ARO, by the NSF and by the ONR. CKL is partially supported by NSF Grants No. PHY 94-08733 and No. PHY94-15583.

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