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1999 Commun. Theor. Phys. 32 189

(http://iopscience.iop.org/0253-6102/32/2/189)

Commun. Theor. Phys. (Beijing, China) **32** (1999) pp 189–194
Vol. 32, No. 2, September 15, 1999

Effective Raman Theory for a Three-Level Atom in the V-Configuration*

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(Received August 12, 1997)

Abstract *It is shown that the system of a three-level atom coupled with two modes of quantized cavity fields in the V-configuration with arbitrary detunings can be exactly reduced to a two-level system with a nonsingular effective coupling which depends nonlinearly on the intensity of the two cavity fields. By performing a unitary transformation, we obtain an exact transformed Hamiltonian in which one of the three levels is decoupled for all values of the detunings including the zero-detuning. We also present the analytical expressions of energy eigenvalues and eigenvectors, evolution operator, time-varying atomic inversion operator and photon number operators of the two modes for the effective two-level model.*

PACS numbers: 42.50.Hz, 42.50.Ar
Key words: Raman theory, quantum optics

I. Introduction

Over the last two decades there have been considerable activities in quantum-optical interactions involving one atom with a few energy levels and one or more near-resonant modes of the quantized electromagnetic fields. Interesting nonclassical effects, such as the collapse and revival of the Rabi oscillations, trapping states, sub-Poissonian photon statistics, and squeezed states, have been investigated.[1-8] Multiphoton processes in single-mode[1,2] and multimode[1,3-8] cases are of increasing interesting in recent years. Parallel progress in the field of laser cooling of single trapped atom or ion has also been reported continuously.[9-11] In dealing with multiphoton transitions, one can consider a two-level atomic system in the single-mode case[1,2,4] and a three-level atomic system in the two-mode case.[1,3-8] In the latter case, one usually reduces it to an effective two-level problem on the assumption of large detuning(s) by the approximation of either the adiabatic elimination[5] or evaluating a unitary transformation perturbatively.[6] The effective two-level Hamiltonian obtained thus has the form of the usual Jaynes–Cummings model but with the single-mode field operators replaced by products of a field operator of one mode and a field operator of the other with the effective coupling parameter $\lambda \propto g_1 g_2/\Delta$, where g_j and Δ denote the coupling parameters and detuning respectively.[5-7] Such a result obviously cannot be extrapolated to the situations of large g_j/Δ_j, and is singular for the zero-detuning case. As a matter of fact, the result may need modification in the situation when the field is strong even if the ratios g_j/Δ_j are small.[7] In view of the facts that the detunings are experimentally adjustable parameters which can be tuned to any values, that the coupling parameters can also easily be adjusted in the field of laser cooling of single trapped atom or ion,[10] and that nonlinear interactions are important when the field modes are relatively strong, it is obviously desirable to obtain effective two-level models which describe the situations where coupling constants have arbitrary relations to the detunings.

We have recently shown that the system of a two-mode three-level atom in the Λ- and Ξ-configurations[7] with arbitrary detunings can be exactly reduced to an effective two-level model with an effective Raman coupling which depends nonlinearly on the intensity of the two quantized field modes. An exact transformed Hamiltonian has obtained in which one of the

*The project partly supported by National Natural Science Foundation of China, the Foundation of State Education Commission of China, and the National Laboratory of MRAMP at Wuhan Institute of Physics, The Chinese Academy of Sciences

three levels is decoupled for any ratios of coupling parameters to detunings including the zero-detuning case. The effective Raman coupling does not show any singularity for all detunings including the zero value, and it reduces to the previous approximate result when detunings are much greater than coupling parameters. Subsequently, we[8] have developed a unified and standard method to obtain quickly the analytical solution to various linear and nonlinear two-level Jaynes–Cummings-type models, and have established the similarity between these models and the one describing a spin-$\frac{1}{2}$ particle in an external magnetic field, which generalizes the well-known conclusion by Feynman *et al.* for the usual Jaynes–Cummings model with the single-mode field treated classically and linearly to the one for the general situation of linear and nonlinear Jaynes–Cummings-type models with the quantized field(s). It is well known that in dealing with two-mode three-level atomic systems, there are three distinct atomic level configurations which are usually called respectively Λ-, Ξ-, and V-configurations.[1,4,9] In this paper, we will show that just as in the cases of the Λ- and Ξ-configurations, the system of a three-level atom interacting with two quantized electromagnetic modes in the V-configuration can also be transformed exactly to an effective two-level model with its effective coupling parameter showing no singular behavior for any values of detunings.

This paper is organized as follows. In Sec. II we present the Hamiltonian describing the two-mode three-level atomic system in V-configuration and introduce a unitary transformation. We then derive the exact expressions of all the relevant transformed quantities to obtain an exact transformed Hamiltonian in which one of the three levels is decoupled from the rest two levels for all values of the ratios of coupling constants to detunings. In Sec. III the effective two-level model is solved by the unified and standard method developed by us. We present the analytical expressions of energy eigenvalues and eigenstates, evolution operator, atomic inversion operator, atomic inversion operator, and photon number operators of the two modes for the effective two-level model. We conclude the paper with a brief summary and some discussions in Sec. IV.

II. Exact Transformed Hamiltonian

We consider a three-level system of energies E_1, E_2 and E_3 in the V-configuration interacting with two quantized cavity modes 1 and 2 as shown in Fig. 1.[4,9] The Hamiltonian of the system is written as[4]

$$H = \sum_{i=1}^{3} E_i \sigma_{ii} + \hbar\omega_1 b_1^\dagger b_1 + \hbar\omega_2 b_2^\dagger b_2 + \hbar g_1 (b_1\sigma_{13} + b_1^\dagger \sigma_{31}) + \hbar g_2 (b_2 \sigma_{23} + b_2^\dagger \sigma_{32}), \tag{1}$$

where symbols b_j ($j = 1,2$) represent the field operators of modes 1 and 2, $\sigma_{ii} = |i\rangle\langle i|$ are the level occupation numbers and $\sigma_{ij} = |i\rangle\langle j|$, $(i \neq j)$ are the transition operators from levels j to i. Levels 3 and 1 (2) are coupled by a dipole-coupling constant $g_1(g_2)$. There is no direct coupling between levels 1 and 2. The quantities Δ_1 and Δ_2 in Fig. 1 denote detunings given by $\Delta_j = (E_3 - E_j)/\hbar - \omega_j$, $j = 1,2$. Note that we have changed some notations with respect to the previous literature, and in particular have interchanged the level numberings. One of the purposes for such the change is that we want to make full use of the corresponding derivations of the Λ-configuration in Ref. [7] which will facilitate the comparison of the results in these two configurations.

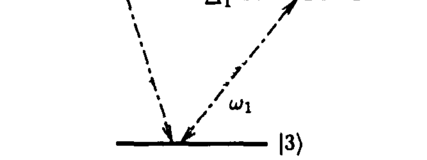

Fig. 1. Three-level atom in the V-configuration

We introduce the following unitary transformation

$$X' = \exp(S) X \exp(-S), \tag{2}$$

where X' denotes the transformed atomic and photon variables, and

$$S = \alpha(b_1\sigma_{13} - b_1^\dagger \sigma_{31}) + \beta(b_2\sigma_{23} - b_2^\dagger\sigma_{32}), \tag{3}$$

where α and β are transformation parameters to be specified later. Before calculating all the relevant unitary transformations described by Eqs (2) and (3), let us rewrite the Hamiltonian in Eq. (1) in a more convenient and symmetric form similar to what we did in $|\ll$-configuration[7] as follows:

$$H = \tfrac{1}{2}(E_1 + E_2 - \hbar\omega_1 - \hbar\omega_2) + H_1 + H_2 \tag{4}$$

with

$$H_j = \hbar\omega_j N_j + \tfrac{1}{2}\hbar\Delta_j\sigma_{jz} + \hbar g_j q_j\,, \qquad j = 1,\ 2\,, \tag{5}$$

where $q_1 = b_1\sigma_{13} + b_1^\dagger\sigma_{31}$, $q_2 = b_2\sigma_{23} + b_2^\dagger\sigma_{32}$, $\sigma_{1z} = 2\sigma_{11} - 1$, $\sigma_{2z} = 2\sigma_{22} - 1$, and

$$N_1 = b_1^\dagger b_1 + \sigma_{11}\,, \qquad N_2 = b_2^\dagger b_2 + \sigma_{22}\,. \tag{6}$$

Note that equations (4) and (5) are very similar to the corresponding results for the $|\ll$-configuration but with, besides the different numberings of atomic levels, different expressions for the quantities q_j, N_j and σ_{jz}. It is easily shown that N_j commute with the Hamiltonian and the operator S in Eq. (3). They, therefore, represent two constants of motion, and two invariant quantities under the unitary transformation (2). Consequently, we only need to calculate the transformations of the four quantities q_j, σ_{jz}, $j = 1, 2$ to obtain the transformed Hamiltonian. While doing so, the operators N_j can be treated as if they are c-numbers, since they commute with the four quantities and the operator S.

By the same routine as what we did previously,[7] we obtain

$$\begin{pmatrix} \sigma'_{1z} \\ \sigma'_{2z} \end{pmatrix} = \begin{pmatrix} \sigma_{1z} \\ \sigma_{2z} \end{pmatrix} - 2A\begin{pmatrix} \alpha q_1 \\ \beta q_2 \end{pmatrix} + B\begin{pmatrix} \sigma_{1z}^{(2)} \\ \sigma_{2z}^{(2)} \end{pmatrix}, \tag{7}$$

where $\sigma_{1z}^{(2)} = -4\bar{\alpha}^2\sigma_{1z}(1-\sigma_{22}) - 2\alpha\beta q$, $\sigma_{2z}^{(2)} = -4\bar{\beta}^2\sigma_{2z}(1-\sigma_{11}) - 2\alpha\beta q$, $\bar{\alpha} = \alpha\sqrt{N_1}$, $\bar{\beta} = \beta\sqrt{N_2}$, $q = b_1^\dagger b_2\sigma_{21} + b_1 b_2^\dagger\sigma_{12}$, and two matrices A and B are formally identical to those for Λ-configuration, i.e., Eq. (13) in Ref. [7]. The other two transformed quantities q'_1 and q'_2 relate to σ'_{1z} and σ'_{2z} again by the same formula as Eq. (19) in Ref. [7] for $|\ll$-configuration. Except for the different level numberings and different definitions of the quantities N_j, q_j, σ_{jz}, $(j = 1, 2)$ and q, the unique formally difference in the expressions of the four transformed quantities σ'_{1z}, σ'_{2z}, q'_1 and q'_2 between the V- and Λ-configurations comes from the different signs in the front of q in the expressions of $\sigma_{1z}^{(2)}$ and $\sigma_{2z}^{(2)}$. Consequently, the expressions of the four transformed quantities in the V-configuration are also given formally by Eqs (17) and (20) in Ref. [7] with their q replaced by minus q. Then, following the same steps as those for Λ-configuration,[7] we are easy to obtain the exact transformed Hamiltonian in the V-configuration as follows:

$$H' = E_0 + \hbar\omega_1 N_1 + \hbar\omega_2 N_2 + \tfrac{1}{2}\hbar\eta\sigma_{33} + \hbar\lambda(b_1^\dagger b_2\sigma_{21} + b_1 b_2^\dagger\sigma_{12}) + \tfrac{1}{2}\hbar\omega(\sigma_{22} - \sigma_{11})\,, \tag{8}$$

where

$$\begin{aligned}\eta = &-(\Delta_1 + \Delta_2)\Big(1 - \frac{3\bar{\alpha}^2\bar{\beta}^2}{2\xi^4}\Big) - \frac{3\sin 2\xi}{\xi}\Big(\frac{g_1}{\alpha}\bar{\alpha}^2 + \frac{g_2}{\beta}\bar{\beta}^2\Big) \\ &+ \frac{3(\Delta_1\bar{\alpha}^4 + \Delta_2\bar{\beta}^4)}{2\xi^4} - \frac{3(\Delta_1\bar{\alpha}^2 + \Delta_2\bar{\beta}^2)}{2\xi^2}\cos 2\xi\,,\end{aligned} \tag{9a}$$

$$E_0 = \tfrac{1}{2}(E_1 + E_2 - \hbar\omega_1 - \hbar\omega_2) - \frac{\hbar}{6}(\Delta_1 + \Delta_2 + \eta)\,, \tag{9b}$$

$$\begin{aligned}\lambda = &-\frac{\alpha\beta(1-\cos\xi)}{\xi^4}[(\Delta_2\bar{\alpha}^2 + \Delta_1\bar{\beta}^2) + (\Delta_2\bar{\beta}^2 + \Delta_1\bar{\alpha}^2)\cos\xi] \\ &- \frac{\alpha\beta\sin\xi}{\xi^3}\Big[\Big(\frac{g_1}{\alpha} - \frac{g_2}{\beta}\Big)(\bar{\alpha}^2 - \bar{\beta}^2) - 2\Big(\frac{g_1}{\alpha}\bar{\alpha}^2 + \frac{g_2}{\beta}\bar{\beta}^2\Big)\cos\xi\Big],\end{aligned} \tag{9c}$$

$$\begin{aligned}\omega = &(\Delta_2 - \Delta_1)\Big[1 + \frac{\bar{\alpha}^2\bar{\beta}^2}{2\xi^4}(8\cos\xi - 7)\Big] - \frac{4\bar{\alpha}^2\bar{\beta}^2}{\xi^4}\Big(\frac{g_1}{\alpha} - \frac{g_2}{\beta}\Big)\xi\sin\xi - \frac{(\bar{\alpha}^2 - \bar{\beta}^2)}{2\xi^4} \\ &\times\Big[(\Delta_2\bar{\beta}^2 + \Delta_1\bar{\alpha}^2)\cos 2\xi + 2\Big(\frac{g_1}{\alpha}\bar{\alpha}^2 + \frac{g_2}{\beta}\bar{\beta}^2\Big)\xi\sin 2\xi\Big] + \frac{(\Delta_1\bar{\alpha}^4 - \Delta_2\bar{\beta}^4)}{2\xi^4}\,,\end{aligned} \tag{9d}$$

where $\bar{\alpha} = \alpha\sqrt{N_1}$, $\bar{\beta} = \beta\sqrt{N_2}$, and $\xi = \sqrt{\bar{\alpha}^2 + \bar{\beta}^2}$.

We have chosen the transformation parameters $|\acute{A}$ and $|\hat{A}$ in Eqs (8) and (9) such that the terms $q_1 = b_1\sigma_{13}+b_1^\dagger\sigma_{31}$ and $q_2 = b_2\sigma_{23}+b_2^\dagger\sigma_{32}$ do not appear in the transformed Hamiltonian, that is, the transformation parameters $|\acute{A}$ and $|\hat{A}$ are determined by the following two equations

$$(\Delta_2-\Delta_1)\frac{\alpha\bar{\beta}^2}{\xi^3}\sin\xi+\frac{\alpha\bar{\beta}^2}{\xi^2}\Big(\frac{g_1}{\alpha}-\frac{g_2}{\beta}\Big)\cos\xi-\frac{\alpha(\Delta_1\bar{\alpha}^2+\Delta_2\bar{\beta}^2)}{2\xi^3}\sin 2\xi$$
$$+\Big(\frac{g_1}{\alpha}\bar{\alpha}^2+\frac{g_2}{\beta}\bar{\beta}^2\Big)\frac{\alpha}{\xi^2}\cos 2\xi=0\,, \tag{10a}$$
$$(\Delta_1-\Delta_2)\frac{\beta\bar{\alpha}^2}{\xi^3}\sin\xi+\frac{\beta\bar{\alpha}^2}{\xi^2}\Big(\frac{g_2}{\beta}-\frac{g_1}{\alpha}\Big)\cos\xi-\frac{\beta(\Delta_1\bar{\alpha}^2+\Delta_2\bar{\beta}^2)}{2\xi^3}\sin 2\xi$$
$$+\Big(\frac{g_1}{\alpha}\bar{\alpha}^2+\frac{g_2}{\beta}\bar{\beta}^2\Big)\frac{\beta}{\xi^2}\cos 2\xi=0\,. \tag{10b}$$

It is instructive to compare Eqs (8) $\sim$ (10) describing the exact transformed Hamiltonian of the V-type system with Eqs (23) $\sim$ (29) in Ref. [7] describing the exact transformed Hamiltonian of the Λ-type system. Except the differences as mentioned above, the transformed Hamiltonians for the two configurations have nearly the same form. In addition, the other parameters in the transformed Hamiltonian turn to having very similar forms for the two configurations. To be more specific, except for different N_j ($j = 1,\ 2$), the two transformation parameters α and β are determined by identical equations, and the parameters λ, η and ω only have different signs for the two configurations. This comparison facilitates the discussions about the V-type system. Particularly, as the detunings satisfy the relation $\Delta_1 = \Delta_2 \equiv \Delta$ which implies an exact two-photon resonant condition $E_2 - E_1 = \hbar(\omega_2 - \omega_1)$, equation (10) is satisfied if we choose

$$\alpha=\frac{g_1}{2\sqrt{\bar{g}_1^2+\bar{g}_2^2}}\arctan\Big(\frac{2\sqrt{\bar{g}_1^2+\bar{g}_2^2}}{\Delta}\Big),\quad \beta=\frac{g_2}{2\sqrt{\bar{g}_1^2+\bar{g}_2^2}}\arctan\Big(\frac{2\sqrt{\bar{g}_1^2+\bar{g}_2^2}}{\Delta}\Big), \tag{11}$$

where $\bar{g}_j = g_j\sqrt{N_j}$, and N_j, $j = 1,2$, are given by Eq. (6). We then find, after some manipulations, that the complicated expressions for the parameters in Eq. (9) are greatly simplified as

$$\eta=-2\Delta-3\tau\,,\quad \tau=\Big(\sqrt{(\Delta/2)^2+\bar{g}_1^2+\bar{g}_2^2}-|\Delta|/2\Big)\mathrm{sgn}\,\Delta\,,$$
$$E_0=\tfrac{1}{2}(E_1+E_2-\hbar\omega_1-\hbar\omega_2+\hbar\tau)\,,\ \omega=-[(\bar{g}_1^2-\bar{g}_2^2)/(\bar{g}_1^2+\bar{g}_2^2)]\tau\,,\ \lambda=[g_1g_2/(\bar{g}_1^2+\bar{g}_2^2)]\tau\,, \tag{12}$$

where $\mathrm{sgn}(x) = 1$ as $x \geq 0$, and otherwise, $\mathrm{sgn}(x) = -1$. Note that there is an error in Eq. (31) of Ref. [7] (all its $\sqrt{(\Delta/2)^2+\bar{g}_1^2+\bar{g}_2^2}-\Delta/2$ should be replaced by τ). Substituting Eq. (12) into Eq. (8), we explicitly arrive at the exact transformed Hamiltonian in which the level 3 decouples from the rest two levels.

Equation (8), together with its parameters determined by Eqs (9) and (10) (or by Eq. (12) in the particular case $\Delta_1 = \Delta_2 \equiv \Delta$), is the central result of this paper. Let us now discuss this result. Firstly, the λ and ω terms in this equation obviously only produce transitions between levels 1 and 2 while the other terms do not cause any transitions among the three levels. This means that the level 3 can be exactly decoupled and does not contribute to the population dynamics. Consequently, we can put $\sigma_{33} = 0$ in the transformed Hamiltonian to obtain an effective two-level model with levels 1 and 2 subject to an effective intensity-dependent coupling, i.e., the parameter λ depends on photon numbers of the two modes. We have therefore shown that the system of a three-level atom coupled to two modes of quantized cavity fields in the V-configuration with arbitrary detunings Δ_j and coupling parameters g_j can be exactly reduced to a two-level system with an effective intensity-dependent coupling λ. Secondly, the intensity-dependent coupling occurs naturally here while in previous studies it is usually introduced phenomenologically. Thirdly, the effective coupling λ is valid and nonsingular for any ratios of coupling parameters g_j to detunings Δ_j. It is easily seen from Eq. (12) that the effective coupling $\lambda \approx -g_1g_2/\Delta$ when $\Delta^2 \gg \bar{g}_1^2+\bar{g}_2^2$, and as $\Delta^2 \ll (\bar{g}_1^2+\bar{g}_2^2)$, it becomes $\lambda \approx -g_1g_2/\sqrt{\bar{g}_1^2+\bar{g}_2^2}$ which remains finite as $\Delta \to 0$. Fourthly, the absolute value of the effective coupling λ is easily seen from Eq. (12) to be a monotonically decreasing function of the detuning Δ, which means that the smaller the detuning, the stronger the effective coupling between the levels 1 and 2. This is a reasonable result that can be anticipated physically since the smaller the detuning, the stronger the direct couplings between levels 1 and 3 and between levels 2 and 3, and also the effective coupling between levels 1 and 2. This result suggests that zero-detuning or small detuning be more desirable by taking into account the fact that stronger coupling is better for realizing all kinds of nonclassical phenomena such as collapse and revivals, squeezed and trapped states experimentally.[9-11] It is emphasized that our results are still valid in these small detuning cases where those of the adiabatic elimination cease to be so.

III. Model's Solution and Dynamics

Since we have obtained the exact transformed Hamiltonian in which the level 3 decouples from the rest two levels, we can put $\sigma_{33}=0$ (or $\sigma_{11}+\sigma_{22}=1$) in the transformed Hamiltonian to obtain an effective two-level model

$$H' = E_0 + \hbar\omega_1 N_1 + \hbar\omega_2 N_2 + \hbar\lambda(b_1^\dagger b_2\sigma_{21} + b_1 b_2^\dagger\sigma_{12}) + \tfrac{1}{2}\hbar\omega(\sigma_{22}-\sigma_{11}), \tag{13}$$

where N_j are given by Eq. (6), E_0, ω and λ are given by Eq. (9) [or by Eq. (12) as $\Delta_1 = \Delta_2 \equiv \Delta$]. We will derive its eigenvalues and eigenvectors, and corresponding dynamics by the unified and standard formulas developed by us.[8] We first calculate the Rabi operator Ω_0 (or Ω) for this particular model and rewrite the effective two-level Hamiltonian as[8]

$$H' = E_0 + \hbar\omega_1 N_1 + \hbar\omega_2 N_2 + \tfrac{1}{2}\hbar\Omega\sigma_{\bar{x}}, \tag{14}$$

where

$$\Omega = \sqrt{\Omega_0^2+\omega^2}, \quad \Omega_0 = 2\lambda\sqrt{N_1N_2}, \quad \sigma_{\bar{x}} = \frac{\omega}{\Omega}\sigma_z + \frac{\Omega_0}{\Omega}\sigma_x, \quad \sigma_x = \frac{b_1b_2\sigma_{21} + b_1^\dagger b_2^\dagger\sigma_{12}}{\sqrt{N_1N_2}}. \tag{15}$$

The two constants of motion $N_j = b_j^\dagger b_j + \sigma_{jj}$, $j=1,\ 2$ (note that $b_j^\dagger b_j$ are not constants of motion) obviously have eigenvalues $N_j = 0,\ 1,\ 2,\cdots$ with $N_1+N_2 = n_1+n_2+1 \geq 1$. Let $|j;n_1,n_2\rangle$ represents a state in which the atom is in the state $|j\rangle$, while the photon state is denoted by $|n_1,n_2\rangle$ with n_1,n_2 being the photon numbers in the two modes. We shall use the symbols N_j and $n_j(=b_j^\dagger b_j)$ to denote the corresponding operators and their eigenvalues for simplicity. Obviously, the states of the effective two-level model corresponding to the zero-value of the Rabi operator Ω_0, i.e., $N_1=0$, $N_2=1$ or $N_1=1$, $N_2=0$, represent no photon states $|j;0,0\rangle$, $j=1,\ 2$. We now consider the situations where $N_1N_2\neq 0$.

Using the standard formulas,[8] we obtain the eigenvalues and eigenvectors (the energy eigenvectors are the common eigenvectors of the operators N_j and $\sigma_{\bar{x}}$) for the effective two-level model as follows:

$$E_{N_1N_2m} = E_0 + \hbar\omega_1N_1 + \hbar\omega_2N_2 + \tfrac{1}{2}\hbar\Omega m, \tag{16}$$

$$|N_1,N_2,m=1\rangle = S|2;n_1=N_1,n_2=N_2-1\rangle + C|1;n_1=N_1-1,n_2=N_2\rangle, \tag{17a}$$

$$|N_1,N_2,m=-1\rangle = C|2;n_1=N_1,n_2=N_2-1\rangle - S|1;n_1=N_1-1,n_2=N_2\rangle, \tag{17b}$$

where both E_0 and Ω are functions of N_j and are given by Eq. (9) [or by Eq. (12) as $\Delta_1=\Delta_2\equiv\Delta$] and Eq. (15) respectively, $N_j = 1,2,3,\cdots$, $m=\pm1$ and $S=\sqrt{(\Omega+|\omega|)/2\Omega}$, $C=\sqrt{(\Omega-|\omega|)/2\Omega}$. Again using the standard formulas,[8] we obtain the evolution operator $U(t)=\exp(\mathrm{i}H't/\hbar)$ and time-varying atomic inversion operator $\sigma_z(t)=\sigma_{22}(t)-\sigma_{11}(t)$ for the effective two-level model as follows:

$$\begin{aligned} U(t) &= \exp\big(\mathrm{i}H_0t/\hbar\big)\big[\cos(\Omega t/2) + \mathrm{i}\sigma_{\bar{x}}\sin(\Omega t/2)\big] \\ &= \exp(\mathrm{i}H_0t)\big\{\cos(\Omega t/2) + \mathrm{i}[(2H_{\mathrm{int}}/\Omega) + (\omega/\Omega)\sigma_z]\sin(\Omega t/2)\big\}, \end{aligned} \tag{18}$$

$$\sigma_z(t) = \Big(\frac{\omega^2}{\Omega^2} + \frac{\Omega_0^2}{\Omega^2}\cos\Omega t\Big)\sigma_z + \frac{2H_{\mathrm{int}}\sigma_z}{\Omega}\Big[\frac{\omega\sigma_z}{2\Omega}(1-\cos\Omega t) + \mathrm{i}\sin\Omega t\Big] + \frac{H_{\mathrm{int}}\omega}{\Omega^2}(1-\cos\Omega t), \tag{19}$$

where the operators on the right-hand sides of Eqs (18) and (19) denote those at the initial time [say, $\sigma_z \equiv \sigma_z(0)$], effective free and interaction Hamiltonians are respectively

$$H_0 = E_0 + \hbar\omega_1N_1 + \hbar\omega_2N_2 + \tfrac{1}{2}\hbar\omega\sigma_z, \quad H_{\mathrm{int}} = \hbar\lambda(b_1^\dagger b_2\sigma_{21} + b_1b_2^\dagger\sigma_{12}), \tag{20}$$

where ω, λ, E_0 are given by Eq. (9) [or by Eq. (12) as $\Delta_1 = \Delta_2 \equiv \Delta$], Ω and Ω_0 are determined by Eq. (15). Using the fact that $N_j = b_j^\dagger b_j + \sigma_{jj}$, $j = 1, 2$ are two constants of motion, we then easily obtain the expressions of the time-varying photon number operators of the two modes as follows:

$$n_j(t) = n_j(0) - (-1)^j[\sigma_z(t) - \sigma_z(0)]/2\,, \qquad j = 1,\ 2\,. \tag{21}$$

It is pointed out that equations (18), (19) and (21) are valid even if the operator $N_1 N_2$ taking zero-value. We have now completely solved the effective two-level model analytically.

IV. Conclusions

In this paper, we have investigated the system of a three-level atom interacting with two quantized cavity modes (the two modes can have either different or identical frequencies) in V-configuration, we have obtained the exact transformed Hamiltonian and shown that one of the three levels (level 3) can be made to decouple from the other two levels, and hence, can be eliminated from the exact transformed Hamiltonian to obtain an effective two-level Hamiltonian with a nonsingular intensity-dependent coupling between levels 1 and 2. The effective two-level Hamiltonian is, within the framework of the original Hamiltonian, valid for any magnitudes of the ratios of the coupling constants to detunings including zero-detuning. In addition, we have solved the effective two-level model by giving the corresponding analytical expressions of energy eigenvalues and eigenvectors, evolution operator, time-varying atomic inversion operator and time-varying photon number operators. These results might be of their applications in investigating the statistics of atomic and field quantities in situations of strong couplings (large g), small detunings, and intense field, where the adiabatic elimination ceases to be valid. Also, the results and the approach used here can be utilized to design two-level systems with desirable effective coupling and effective decay rate for ion or atom sideband cooling[9] particularly in the aforementioned situations.

Acknowledgment

One of the authors (Y. WU) acknowledges helpful discussions with A. Douglas Stone.

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