과제 #4

<u>마감일:</u> 11월 7일 9시 30분

<u>제출방법:</u>

- **강의실 교탁**

주의사항:

- **숙제를 베껴 내면 관련된 모든 학생에게 불이익이 있습니다.**
- **마감일시를 반드시 준수.**

<u>Problem 1</u> The partition function for an interacting gas is assumed to be $Z = \left(\frac{V-Nb}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3N/2} \exp\left(\frac{N^2a^2}{Vk_BT}\right)$, where a and b are constants.

(a) Show that the pressure is of the same form as van der Waals' equation.

(b) Calculate the internal energy and discuss the effects of the interaction parameter a.

(c) Show that the isobaric expansivity β_p of the van der Waals gas is given by $\beta_p = \frac{1}{T}\left(1 + \frac{b}{V-b} - \frac{2a}{pV^2+a}\right)^{-1}$.

<u>Problem 2</u> Solve the following problems.

(a) Show that the constant volume heat capacity is related to the variance of the energy fluctuations in the canonical ensemble, i.e. $C_V = \frac{1}{kT^2}(< E^2 > - < E >^2)$.

(b) The isothermal compressibility is defined as $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$. In analogy of the relation of Cv to energy fluctuations, what type of fluctuations do you think are related to κ at fixed T, P, and N?

<u>Problem 3</u> Consider a system of N free particles in which the energy of each particle can assume two and only two distinct values, 0 and E (E>0). Denote by n_0 and n_1 the occupation numbers of the energy level 0 and E, respectively. The total energy of the system is U.

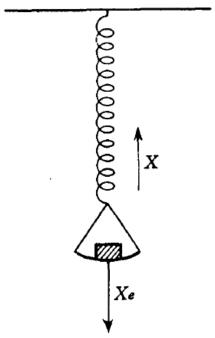
(a) Find the entropy of such a system.

(b) Find the most probable values of n_0 and n_1, and find the mean square fluctuations of these quantities.

(c) Find the temperature as a function of U, and show that it can be negative.

(d) What happens when a system of negative temperature is allowed to exchange heat with a system of positive temperature?

Problem 4 Consider a spring show in the figure which follows Hooke's law; namely, the elongation x is proportional to the tension X when it is pulled at a constant temperature. The proportionality constant (spring constant) k, is temperature-dependent. Determine the Helmhotlz free energy F, the internal energy U, and the entropy S, as a function of x. Hint: You can neglect the thermal expansion.



Problem 5 At t=0, a one-dimensional classical ideal gas is contained in the region 0<x<L. The gas has an initial temperature T_0 and a density n_0. The gas particles have mass m. The wall at x=0 reflects all particles that hit it. The wall at x=L allows any particle that hits it to pass through with a probability ε, that is much less than one and is independent of the particle's energy. Otherwise the particle is reflected with no change in its energy.

(a) Calculate the rate at which the system loses particles and energy at t=0.

(b) Assuming that the particles remaining always have a Maxwell-Boltzmann distribution, but with slowly varying density and temperature, drive and solve a differential equation for T(t).