(Cheng) 7-7.3 Source-Free Fields in Simple Media

- Reference: "Field and Wave Electromagnetics", David K. Cheng 2nd ed. (1989)
- Recall
  - No charge & no current ($\rho = \boldsymbol{J} = 0$) & spatially uniform ϵ, μ
  - $\nabla^2 \boldsymbol{E}(\boldsymbol{r}) + \omega^2 \mu\epsilon \boldsymbol{E}(\boldsymbol{r}) = 0$
  - Non-conducting material was assumed (e.g. dielectric)
- Conducting media
  - $\boldsymbol{J} = \sigma \boldsymbol{E} \neq \boldsymbol{0}$ where σ is called conductivity
  - $\nabla \times \boldsymbol{H}(\boldsymbol{r}) = j\omega\epsilon \boldsymbol{E}(\boldsymbol{r}) \Rightarrow \nabla \times \boldsymbol{H}(\boldsymbol{r}) = j\omega\epsilon \boldsymbol{E}(\boldsymbol{r}) + \sigma \boldsymbol{E}$
  - $\nabla \times \boldsymbol{H}(\boldsymbol{r}) = j\omega \left(\epsilon + \frac{\sigma}{j\omega}\right) \boldsymbol{E}(\boldsymbol{r}) = j\omega\epsilon_c \boldsymbol{E}(\boldsymbol{r})$
  - $\epsilon_c \equiv \epsilon - j\frac{\sigma}{\omega} = \epsilon' - j\epsilon'' = \epsilon' \left(1 - j\frac{\epsilon''}{\epsilon'}\right)$
  - Rederivation of wave equation in a lossy media
    - $\nabla \times \left(\nabla \times \boldsymbol{E}(\boldsymbol{r})\right) = -j\omega\mu\left(\nabla \times \boldsymbol{H}(\boldsymbol{r})\right) = -j\omega\,\mu\left(j\omega\epsilon_c \boldsymbol{E}(\boldsymbol{r})\right) = \omega^2\mu\epsilon_c \boldsymbol{E}(\boldsymbol{r})$
    - $\nabla \times \left(\nabla \times \boldsymbol{E}(\boldsymbol{r})\right) = \nabla\left(\nabla \cdot \boldsymbol{E}(\boldsymbol{r})\right) - \nabla^2 \boldsymbol{E}(\boldsymbol{r}) = -\nabla^2 \boldsymbol{E}(\boldsymbol{r})$
    - $\nabla^2 \boldsymbol{E}(\boldsymbol{r}) + \omega^2\mu\epsilon_c \boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{0}$

(Cheng) 7-7.3 Source-Free Fields in Simple Media

- Plane wave in a lossy dielectric medium
  - $\epsilon_c \equiv \epsilon - j\frac{\sigma}{\omega} = \epsilon' - j\epsilon''$
  - Wave number of a plane wave k_c
    - Plane wave: $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_+ e^{-j\boldsymbol{k}\cdot\boldsymbol{r}}$
    - $\nabla^2\boldsymbol{E}(\boldsymbol{r}) = \nabla\cdot\left(\nabla\boldsymbol{E}(\boldsymbol{r})\right) \Rightarrow \partial_i\left(\nabla\boldsymbol{E}_j(\boldsymbol{r})\right)_i = \partial_i\left(\partial_i E_j\right) = \partial_i\left((-jk_i)E_j\right) = (-jk_i)\left(\partial_i E_j\right) = (-jk_i)\left((-jk_i)E_j\right) = -(k\cdot k)E_j \Rightarrow -k^2\boldsymbol{E}(\boldsymbol{r})$

    ➔ $\nabla^2\boldsymbol{E}(\boldsymbol{r}) + k_c^2\boldsymbol{E}(\boldsymbol{r}) = 0$
  - Combined with $\nabla^2\boldsymbol{E}(\boldsymbol{r}) + \omega^2\mu\epsilon_c\boldsymbol{E}(\boldsymbol{r}) = 0$

    ➔ $k_c = \omega\sqrt{\mu\epsilon_c} = \omega\sqrt{\mu(\epsilon' - j\epsilon'')}$ ➔ complex wave number
  - Loss tangent: $\tan\delta_c = \epsilon''/\epsilon' \approx \sigma/\omega\epsilon$
  - If $\sigma \gg \omega\epsilon$, good conductor.
  - If $\sigma \ll \omega\epsilon$, good insulator.

(Cheng) 8-3 Plane Waves in Lossy Media

- Plane wave in a source-free lossy media
  - ➔ $\nabla^2 \boldsymbol{E}(\boldsymbol{r}) + k_c^2 \boldsymbol{E}(\boldsymbol{r}) = 0$
  - $k_c = \sqrt{\omega^2 \mu \epsilon_c}$ ➔ complex wave number
- **Propagation constant γ**
  - generalization of wave number: $\gamma = jk_c = j\omega\sqrt{\mu\epsilon_c}$
  - Using $\epsilon_c \equiv \epsilon - j\frac{\sigma}{\omega} = \epsilon' - j\epsilon'' = \epsilon'\left(1 - j\frac{\epsilon''}{\epsilon'}\right)$

$$\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{j\omega\epsilon}\right)^{1/2} = j\omega\sqrt{\mu\epsilon'}\left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{1/2}$$

  - cf) lossless medium $\sigma = 0$ ➔ $\alpha = 0, \beta = \omega\sqrt{\mu\epsilon}$
  - $\nabla^2 \boldsymbol{E}(\boldsymbol{r}) - \gamma^2 \boldsymbol{E}(\boldsymbol{r}) = 0$ ➔ $\boldsymbol{E}(\boldsymbol{r}) = \hat{x}E_x = \hat{x}E_0 e^{-\gamma z} = \hat{x}E_0 e^{-\alpha z} e^{-j\beta z}$
  - α: attenuation constant, β: phase constant
  - $\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2}, \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2}$

(Cheng) 8-3.1 Low-loss Dielectrics

- Low-loss dielectrics
  - $\epsilon_c \equiv \epsilon - j\frac{\sigma}{\omega} = \epsilon'\left(1 - j\frac{\epsilon''}{\epsilon'}\right)$
  - Imperfect insulator with $\sigma \neq 0$ such that $\epsilon'' \ll \epsilon'$ or $\sigma/\omega\epsilon \ll 1$
  - $\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon'}\left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{1/2} \approx j\omega\sqrt{\mu\epsilon'}\left[1 + \frac{1}{2}\left(-j\frac{\epsilon''}{\epsilon'}\right) + \frac{1}{2!}\left(-\frac{1}{4}\right)\left(-j\frac{\epsilon''}{\epsilon'}\right)^2 + \cdots\right]$
    $\approx j\omega\sqrt{\mu\epsilon'}\left[1 - j\frac{\epsilon''}{2\epsilon'} + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]$
  - Attenuation constant: $\alpha \approx \frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}$
  - Phase constant: $\beta \approx \omega\sqrt{\mu\epsilon'}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]$
  - α & β are approximately linear in ω
  - Intrinsic impedance: $\eta_c = \frac{E_x}{H_y} = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu}{\epsilon'}}\left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{-1/2} \approx \sqrt{\frac{\mu}{\epsilon'}}\left(1 + j\frac{\epsilon''}{2\epsilon'}\right)$
    - Complex η_c ➔ E_x and H_y are not perfectly in-phase
  - Phase velocity: $v_p = \frac{\omega}{k} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon'}}\left[1 + \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]^{-1} \approx \frac{1}{\sqrt{\mu\epsilon'}}\left[1 - \frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]$

(Cheng) 8-3.2 Good Conductors

- Good conductors
  - $\epsilon_c \equiv \epsilon - j\frac{\sigma}{\omega} = \epsilon'\left(1 - j\frac{\epsilon''}{\epsilon'}\right)$
  - $\sigma/\omega\epsilon \gg 1$

  ➔ $\gamma = \alpha + j\beta = j\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{j\omega\epsilon}\right)^{1/2} \approx j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\sigma}{j\omega\epsilon}} = \sqrt{j}\sqrt{\omega\sigma\mu}$

  $= \frac{(1+j)}{\sqrt{2}}\sqrt{\omega\sigma\mu} = (1+j)\sqrt{\pi f\sigma\mu}$

    - $\sqrt{j} = \left(e^{j\pi/2}\right)^{1/2} = e^{j\pi/4} = (1+j)/\sqrt{2}$
  - $\alpha \approx \beta \approx \sqrt{\pi f\sigma\mu}$
  - Intrinsic impedance: $\eta_c = \frac{E_x}{H_y} = \sqrt{\frac{\mu}{\epsilon_c}} \approx \sqrt{\mu\frac{\omega}{-j\sigma}} = \sqrt{j}\sqrt{\frac{\mu\omega}{\sigma}} = (1+j)\sqrt{\frac{\pi f\mu}{\sigma}} = (1+j)\frac{\alpha}{\sigma}$
    - E_x and H_y have 45° phase difference
  - Phase velocity: $v_p = \frac{\omega}{k} = \frac{\omega}{\beta} \approx \frac{\omega}{\sqrt{\pi f\sigma\mu}} = \sqrt{\frac{4\pi f}{\sigma\mu}}$ ➔ depends on f

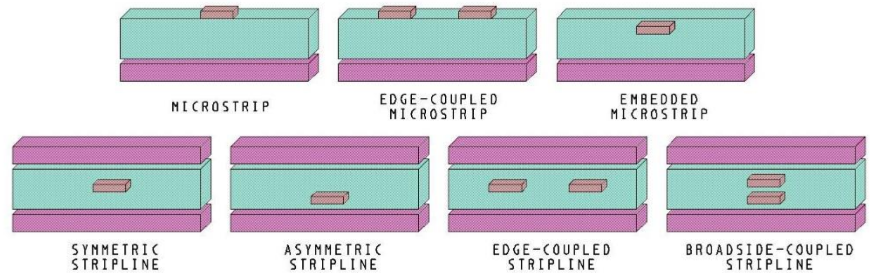
(Cheng) 8-3.2 Good Conductors

- Good conductors
  - $\alpha \approx \beta \approx \sqrt{\pi f \sigma \mu}$
  - Inside the good conductor, severe attenuation
    - $\boldsymbol{E}(\boldsymbol{r}) = \hat{x}E_x = \hat{x}E_0 e^{-\gamma z} = \hat{x}E_0 e^{-\alpha z} e^{-j\beta z}$

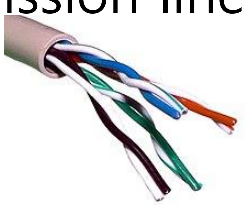
    ➔ Skin depth $\delta = 1/\alpha$: attenuation of $e^{-1} = 0.368$
    - Example) 3(MHz) & 10 (GHz) signal in copper
      - $\sigma = 5.80 \times 10^7 (\mathrm{S/m}), \mu = 4\pi \times 10^{-7}\ (\mathrm{H/m}), f = 3 \times 10^6\ (\mathrm{Hz})$ or $10^{10}\ (\mathrm{Hz})$
      - For 3 (MHz), $\alpha \approx \beta \approx \sqrt{\pi f \sigma \mu} = 2.62 \times 10^4\ (\mathrm{m}^{-1})$ ➔ $\delta = 0.038\ (\mathrm{mm})$
      - For 10 (GHz), $\alpha \approx \beta \approx \sqrt{\pi f \sigma \mu} = 1.5 \times 10^6\ (\mathrm{m}^{-1})$ ➔ $\delta = 0.66\ (\mu\mathrm{m})$
    - How to make a wire with low resistance using the same amount of conductor material?

(Cheng) 9.1 Introduction

- Common types of guiding structures
  - Parallel-plate transmission line
  - Two-wire transmission line
  - Coaxial transmission line

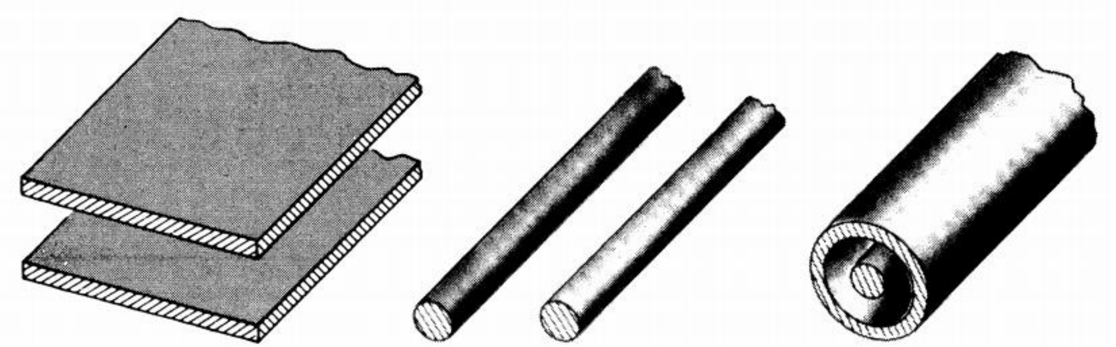


https://medium.com/@Altium/stripline-vs-microstrip-understanding-their-differences-and-their-pcb-routing-guidelines-9bad77303d2f

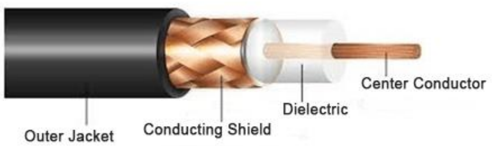


https://www.wikiwand.com/en/Twin-lead

https://electricalacademia.com/instrumentation-and-measurements/wire-gauge-sizes-circular-mils-wire-size-wire-size-chart



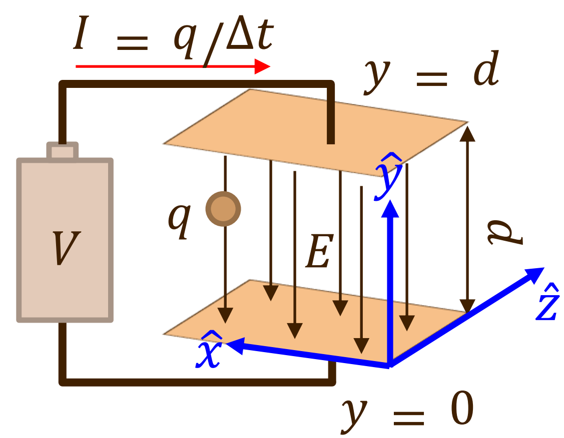
(a) Parallel-plate transmission line. (b) Two-wire transmission line. (c) Coaxial transmission line.

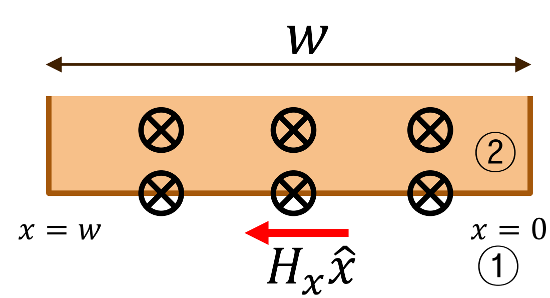


https://www.everythingrf.com/community/coaxial-cable-construction

Propagation of Voltage

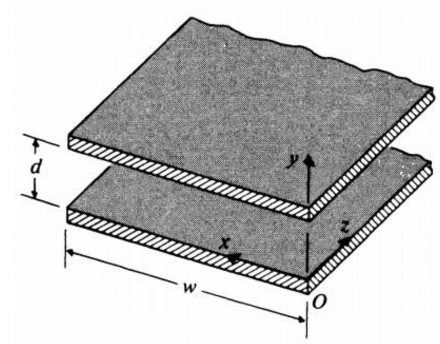
- Modelling of the electrical signal in physics
  - Voltage V ➔ $E = V/d$
    - Work done by E-field: $\Delta W = F \cdot d = qdE$
    - Voltage: amount of work to be done on a unit charge: $V = \frac{\Delta W}{q} = Ed = -\int_0^d \boldsymbol{E} \cdot d\boldsymbol{y} = -E_y d$
  - What happens if the V is changing sinusoidally?
    - Low frequency
    - High frequency ➔ How high is high?
    - Did you learn any laws for the propagation of voltage?
    - Analysis of propagation of $\boldsymbol{E}$-field ➔ the propagation of V
  - How to calculate the current?
    - Current will also change sinusoidally
    - $\oint_{\mathcal{P}} \boldsymbol{H} \cdot d\boldsymbol{l} = I_{enc}$ ➔ $I_z = H_x w = \int_0^w \boldsymbol{H} \cdot d\boldsymbol{x}$

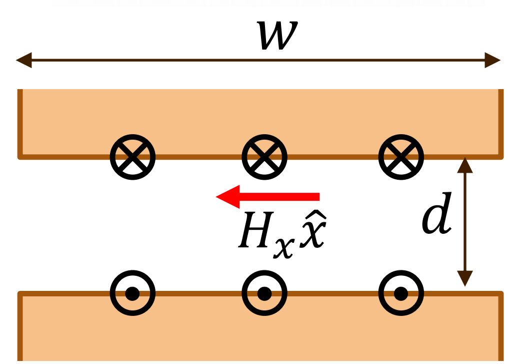


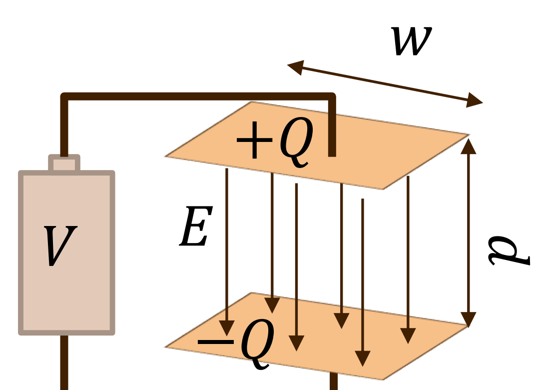


L and C per Unit Length

- L: inductance per unit length
  - Self-inductance: Φ/I where Φ is magnetic flux and I is current
  - $H_{x,u}(2w) = I$ & $H_{x,l}(2w) = I$
  - ➔ $H_x = H_{x,u} + H_{x,l} = I/w$
  - Flux for length of l: $\Phi = \mu H_x d l = \frac{\mu d I}{w} l$
  - Self-inductance: $\frac{\Phi}{I} = \mu \frac{d}{w} l$
  - Inductance per unit length: $L = \mu \frac{d}{w}$
- C: capacitance per unit length
  - Capacitance: Q/V where Q is the accumulated charge
  - From Gauss' law, $E(wl) = Q/\epsilon$
  - Using $V = Ed$, $\frac{Q}{V} = \frac{\epsilon E(wl)}{Ed} = \epsilon \frac{w}{d} l$
  - capacitance per unit length: $C = \epsilon \frac{w}{d}$

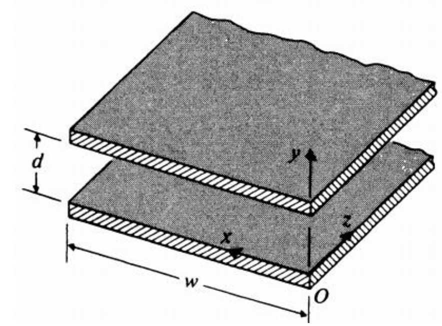


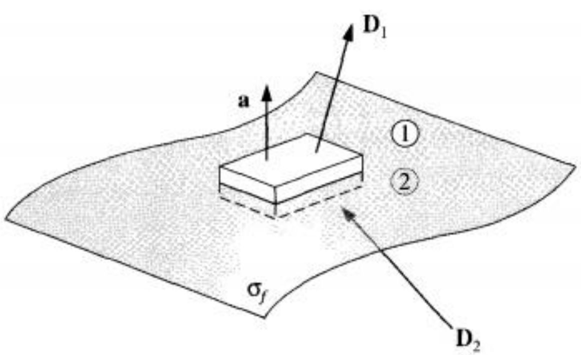


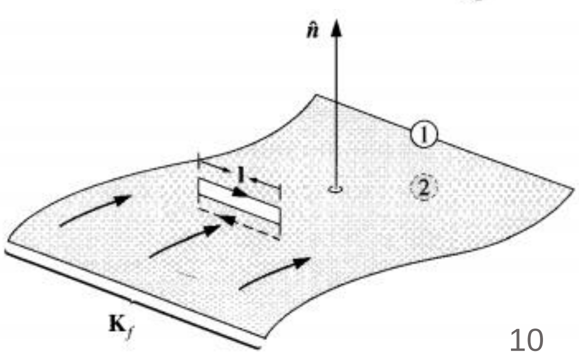


9-2 TEM Wave along a Parallel-Plate TL

- Boundary conditions between a dielectric (medium 1) and a perfect conductor (medium 2)
  - Inside the perfect conductor, no EM fields
  - $\boldsymbol{E}_1^{\parallel} - \boldsymbol{E}_2^{\parallel} = 0 \;\Rightarrow\; \boldsymbol{E}_d^{\parallel} = 0$
  - $B_1^{\perp} - B_2^{\perp} = 0 \;\Rightarrow\; H_d^{\perp} = 0$
  - $D_1^{\perp} - D_2^{\perp} = \sigma_s$
    - Recall $D_1^{\perp} = \widehat{\boldsymbol{n}}_{2\to1} \cdot \boldsymbol{D}_1$
    - $\rho_s = \widehat{\boldsymbol{n}}_{2\to1} \cdot \boldsymbol{D}_1 = \epsilon_d \widehat{\boldsymbol{n}}_{m\to d} \cdot \boldsymbol{E}_d$
  - $\boldsymbol{H}_1^{\parallel} - \boldsymbol{H}_2^{\parallel} = \boldsymbol{K}_s \times \widehat{\boldsymbol{n}}$
    - Apply $\widehat{\boldsymbol{n}}\times$ to both sides
    - $\widehat{\boldsymbol{n}} \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) = \widehat{\boldsymbol{n}} \times (\boldsymbol{K}_s \times \widehat{\boldsymbol{n}})$
      $= \boldsymbol{K}_s(\widehat{\boldsymbol{n}} \cdot \widehat{\boldsymbol{n}}) - \widehat{\boldsymbol{n}}(\boldsymbol{K}_s \cdot \widehat{\boldsymbol{n}}) = \boldsymbol{K}_s$
    - $\widehat{\boldsymbol{n}}_{2\to1} \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) = \boldsymbol{K}_s$
    - $\boldsymbol{K}_s = \boldsymbol{J}_s = \widehat{\boldsymbol{n}}_{m\to d} \times \boldsymbol{H}_d$

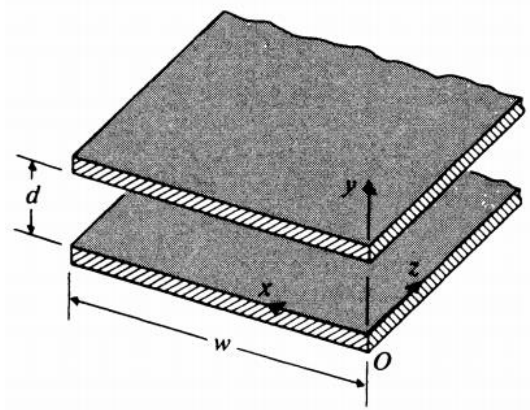






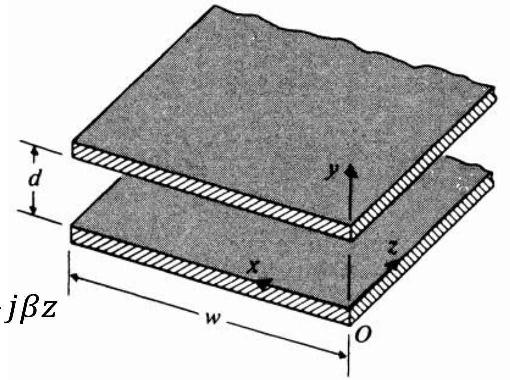
9-2 TEM Wave along a Parallel-Plate TL

- Transverse Electromagnetic (TEM) Wave along a Parallel-Plate Transmission Line
  - y-polarized TEM wave propagating in the $+z$-direction along a uniform parallel-plate transmission line
  - $\boldsymbol{E} = \hat{y}E_y = \hat{y}E_0 e^{-\gamma z}$
  - $\boldsymbol{H} = \sqrt{\frac{\epsilon}{\mu}}\hat{\boldsymbol{k}} \times \boldsymbol{E} = -\hat{x}\frac{E_0}{\eta}e^{-\gamma z} = -\hat{x}H_0 e^{-\gamma z} = \hat{x}H_x$
    - Recall: intrinsic impedance $\eta = \frac{|E|}{|H|} = \sqrt{\frac{\mu}{\epsilon}}$
  - Assumption: perfect conducting plates & lossless dielectric
    - $\gamma = j\beta = j\omega\sqrt{\mu\epsilon}$
  - Boundary conditions
    - At both $y = 0$ & $y = d$: $\boldsymbol{E}^{\parallel} = 0$ and $H^{\perp} = 0$ ➔ satisfied
    - At $y = 0$ (lower plate): $\widehat{\boldsymbol{n}}_{m \to d} = \hat{y}$
      - $\rho_{sl} = \epsilon_d \widehat{\boldsymbol{n}}_{m \to d} \cdot \boldsymbol{E}_d = \epsilon_d \hat{y} \cdot \left(\hat{y}E_0 e^{-j\beta z}\right) = \epsilon_d E_0 e^{-j\beta z}$
      - $\boldsymbol{J}_{sl} = \widehat{\boldsymbol{n}}_{m \to d} \times \boldsymbol{H}_d = \hat{y} \times \left(-\hat{x}\frac{E_0}{\eta}e^{-j\beta z}\right) = \hat{z}\frac{E_0}{\eta}e^{-j\beta z}$

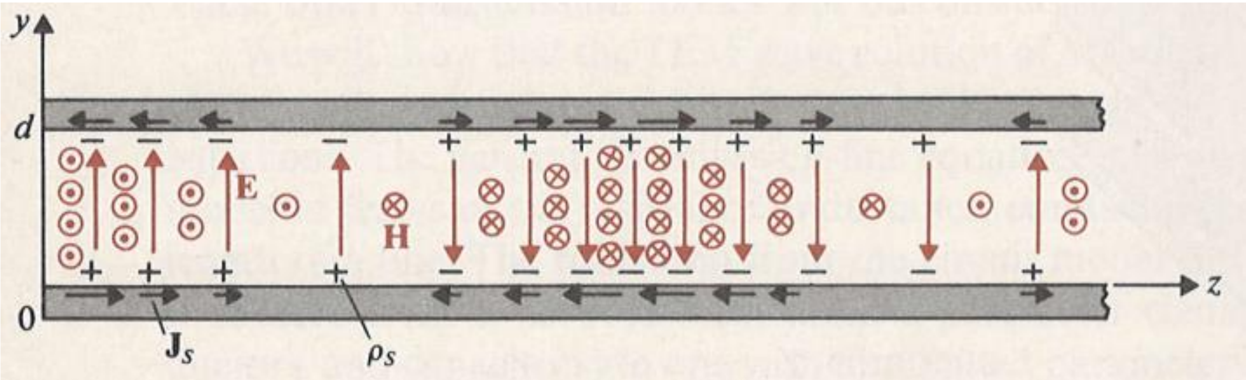


9-2 TEM Wave along a Parallel-Plate TL

- (cont'd) y-polarized TEM wave propagating in the $+z$-direction along a uniform parallel-plate transmission line

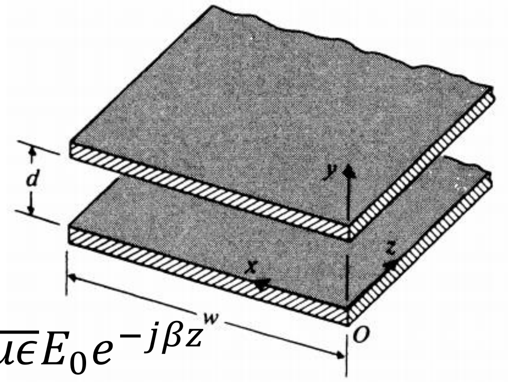


- $\boldsymbol{E} = \hat{y}E_y = \hat{y}E_0 e^{-j\beta z}$, $\boldsymbol{H} = -\hat{x}\frac{E_0}{\eta}e^{-j\beta z} = \hat{x}H_x$
- Boundary conditions
  - At $y = d$ (upper plate) : $\widehat{\boldsymbol{n}}_{m\to d} = -\hat{y}$
    - $\rho_{su} = \epsilon_d \widehat{\boldsymbol{n}}_{m\to d} \cdot \boldsymbol{E}_d = \epsilon_d(-\hat{y}) \cdot \left(\hat{y}E_0 e^{-j\beta z}\right) = -\epsilon_d E_0 e^{-j\beta z}$
    - $\boldsymbol{J}_{su} = \widehat{\boldsymbol{n}}_{m\to d} \times \boldsymbol{H}_d = (-\hat{y}) \times \left(-\hat{x}\frac{E_0}{\eta}e^{-j\beta z}\right) = -\hat{z}\frac{E_0}{\eta}e^{-j\beta z} = \hat{z}H_x$
- ρ_s and $\boldsymbol{J}_s$ change sinusoidally with z



9-2 TEM Wave along a Parallel-Plate TL

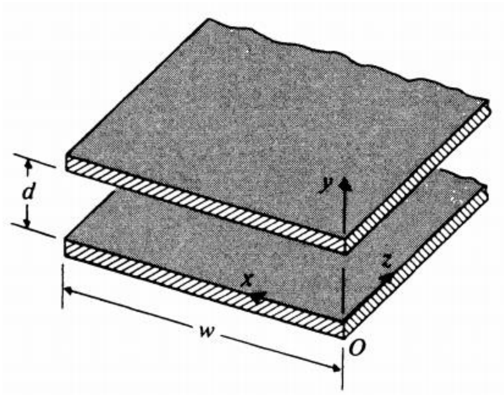
- (cont'd) y-polarized TEM wave propagating in the $+z$-direction along a uniform parallel-plate transmission line

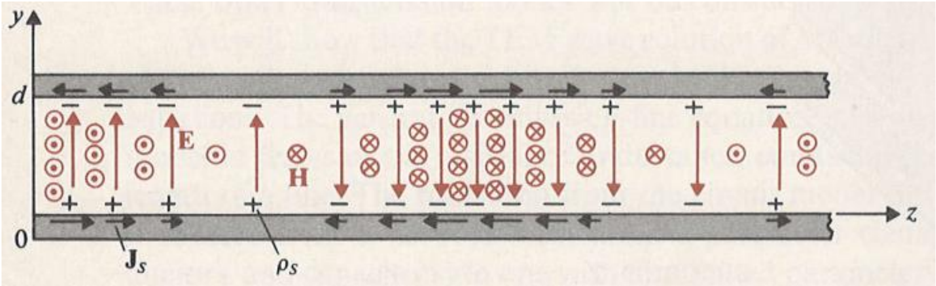


- $\boldsymbol{E} = \hat{y}E_y = \hat{y}E_0 e^{-j\beta z}$, $\boldsymbol{H} = \hat{x}H_x = -\hat{x}\frac{E_0}{\eta}e^{-j\beta z}$
- $\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}$
  - $\nabla \times \boldsymbol{E} = \hat{x}\left(-\partial_z E_y\right) = \hat{x}(j\beta)E_0 e^{-j\beta z}$
  - $-j\omega\mu\boldsymbol{H} = \hat{x}j\omega\mu\frac{E_0}{\eta}e^{-j\beta z} = \hat{x}j\omega\mu\frac{E_0}{\sqrt{\mu/\epsilon}}e^{-j\beta z} = \hat{x}j\omega\sqrt{\mu\epsilon}E_0 e^{-j\beta z}$
- $\nabla \times \boldsymbol{H} = j\omega\epsilon\boldsymbol{E}$
  - $\nabla \times \boldsymbol{H} = \hat{y}(\partial_z H_x) = -(-j\beta)\hat{y}\frac{E_0}{\eta}e^{-j\beta z} = \hat{y}j\omega\sqrt{\mu\epsilon}\sqrt{\frac{\epsilon}{\mu}}E_0 e^{-j\beta z}$
  - $j\omega\epsilon\boldsymbol{E} = j\omega\epsilon\hat{y}E_0 e^{-j\beta z}$
- $\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}$ ➔ $\frac{d}{dz}E_y = j\omega\mu H_x$
- $\nabla \times \boldsymbol{H} = j\omega\epsilon\boldsymbol{E}$ ➔ $\frac{d}{dz}H_x = j\omega\epsilon E_y$

9-2 TEM Wave along a Parallel-Plate TL

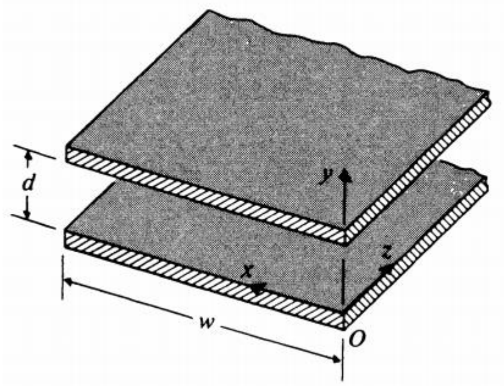
- (cont'd) y-polarized TEM wave propagating in the $+z$-direction along a uniform parallel-plate transmission line
- $\boldsymbol{E} = \hat{y}E_y = \hat{y}E_0 e^{-j\beta z}$, $\boldsymbol{H} = \hat{x}H_x = -\hat{x}\dfrac{E_0}{\eta}e^{-j\beta z}$
- $\dfrac{\boldsymbol{d}}{\boldsymbol{dz}}\boldsymbol{E_y} = \boldsymbol{j\omega\mu H_x}$, $\dfrac{d}{dz}H_x = j\omega\epsilon E_y$
- $\int_0^d \left(\dfrac{d}{dz}E_y\right)dy = \int_0^d (j\omega\mu H_x)dy$
  - $\dfrac{d}{dz}\int_0^d E_y dy = j\omega\mu\int_0^d H_x dy$
  - $\int_0^d E_y(z)dy = -V(z)$
  - H_x is independent of y and using $I_z = H_x w$
  - $-\dfrac{d}{dz}V(z) = j\omega\mu H_x d = j\omega\left[\mu\dfrac{d}{w}\right][H_x w] = j\omega LI(z)$



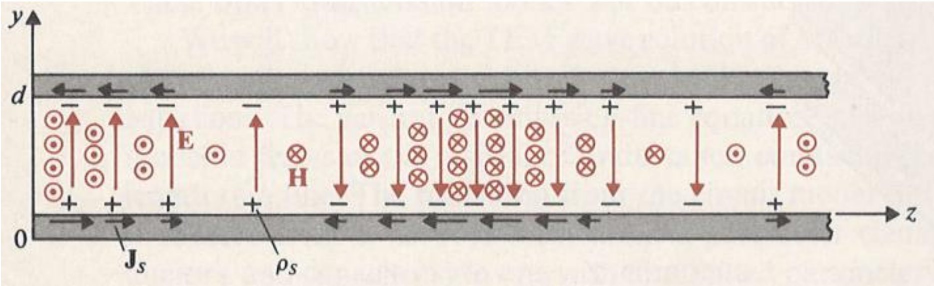


9-2 TEM Wave along a Parallel-Plate TL

- (cont'd) y-polarized TEM wave propagating in the $+z$-direction along a uniform parallel-plate transmission line

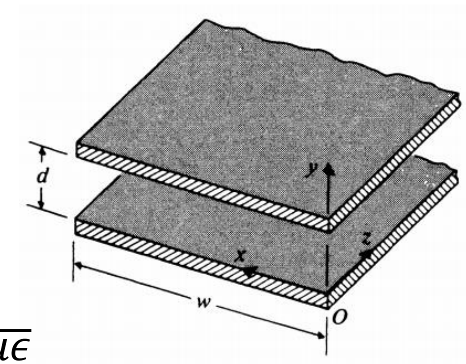


- $\boldsymbol{E} = \hat{y}E_y = \hat{y}E_0 e^{-j\beta z}$, $\boldsymbol{H} = \hat{x}H_x = -\hat{x}\frac{E_0}{\eta}e^{-j\beta z}$
- $\frac{d}{dz}E_y = j\omega\mu H_x$, $\frac{\boldsymbol{d}}{\boldsymbol{dz}}\boldsymbol{H_x} = \boldsymbol{j\omega\epsilon E_y}$
- $\int_0^w \left(\frac{d}{dz}H_x\right)dx = \int_0^w \left(j\omega\epsilon E_y\right)dx$
  - $\frac{d}{dz}\int_0^w H_x dx = j\omega\epsilon \int_0^w E_y dx$
  - $\int_0^w H_x dx = I(z)$
  - E_y is independent of x
  - $\frac{d}{dz}I(z) = j\omega\epsilon E_y w = j\omega\left[\epsilon\frac{w}{d}\right]\left[E_y d\right] = j\omega C\left(-V(z)\right)$

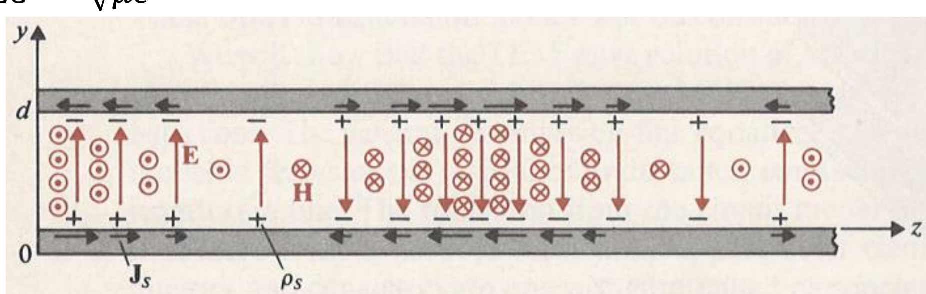


9-2 TEM Wave along a Parallel-Plate TL

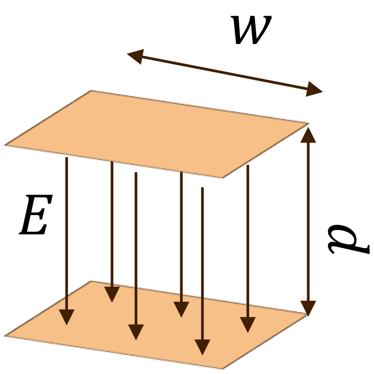
- (cont'd) y-polarized TEM wave propagating in the $+z$-direction along a uniform parallel-plate transmission line



- $\frac{d}{dz}V(z) = -j\omega LI(z)$, $\frac{d}{dz}I(z) = -j\omega CV(z)$
- $\frac{d^2}{dz^2}V(z) = -\omega^2 LCV(z)$, $\frac{d^2}{dz^2}I(z) = -\omega^2 LCI(z)$
- Solution for waves propagating in the +z-direction
  - $V(z) = V_0 e^{-j\beta z}$, $I(z) = I_0 e^{-j\beta z}$ where $\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon}$
- Characteristic impedance of the transmission line
  - $Z_0 = \frac{V(z)}{I(z)} = \frac{V_0}{I_0} = \frac{\omega L}{\beta} = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu d/w}{\epsilon w/d}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w}\eta$
  - If the transmission line is infinite, Z_0 is the impedance seen by the source
- Phase velocity of propagation along the line
  - $v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}$ ➔ the same as the TEM wave in the dielectric material



9-2.1 Lossy Parallel-Plate Transmission Lines



- Source of loss
  - Loss in the dielectric material ➔ non-zero loss tangent
    - G: conductance per unit length across the two plates
    - $G = \frac{\sigma}{\epsilon} C$
  - Loss in the plates ➔ imperfect conductor
    - R: resistance per unit length of the two plate conductors
    - $R = \frac{2}{w}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$
- Microstrip lines
  - Triplate line
    - Z_0 becomes half