전기과 2018-12432 박정현

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5+1+7+8=21

1. 5

a)

let, $(x,y)=(t,t)$ $(t\in\mathbb{R})$

$\Rightarrow f(t,t)=\dfrac{t^3-t^3+t}{t^2+t^2}=\dfrac{1}{t}$

$\Rightarrow \lim_{t\to 0}\dfrac{1}{t}\neq f(0,0)=0 \Rightarrow$ 어떤 ~~$\forall$~~ p (경로 p) 에 대해 $\lim_{(x,y)\to\vec{0}} f(x,y)=f(0,0)$ 이 아니므로

기호 활용한 표현 ..

$\{t\}\cup(t,t)$

$f(x,y)$는 $(x,y)=(0,0)$ 에서 연속이 아니다. ▨

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b)

$(a+b^2)(a+b)=a^3+b^3+ab(a+b)$ ▨

no!!!

$$|f(x,y)|=\frac{|x^3+y^3|}{|x^2+y^2|}\leq\frac{|x|^3+|y|^3}{|x|^2+|y|^2}\leq\frac{|x|^3+|y|^3+|x||y|(|x|+|y|)}{|x|^2+|y|^2}=|x||y|(|x|+|y|)$$

$$\Rightarrow \lim_{(x,y)\to(0,0)}|f(x,y)-f(0,0)|\leq\lim_{(x,y)\to(0,0)}|x||y|(|x|+|y|)=0$$

($\because$ $\varphi(x)=|x|$ 는 연속이므로 $=\lim\varphi(x)+\lim\varphi(x)$, $\lim\varphi(x)+\varphi(x)$, $\lim\varphi(x)\lim\varphi(x)$, $=\lim\varphi(x)\varphi(x)$)

$\Rightarrow$ 연속의 정의에 의해 연속이다. ▨

2. 7

let, $\varphi:\mathbb{R}^2\to\mathbb{R}^2$ ~~$g(x,y)$~~ $\varphi(1,2)=(-1,0)$

$(x,y)\to(3x^2-y^2,\ y-2x)$

~~let, $g(t)=\int_0^t\frac{1}{1+t^4}dt$~~ let, $g(x,y)=\int_0^x\frac{1}{1+t^4}dt-\int_0^y\frac{1}{1+t^4}dt$

$\Rightarrow$ 미적분학 기본정리에 의해 $g(x,y)$는 미분가능 $\Rightarrow Dg=\left(\frac{1}{1+x^4},-\frac{1}{1+y^4}\right)\Rightarrow Dg\circ\varphi(1,2)=\left(\frac{1}{2},-1\right)$

g, φ 모두 미분가능하므로 chain rule 사용가능. $f(x,y)=g\circ\varphi(x,y)$ (by chain rule)

$\Rightarrow Df=((Dg)\circ\varphi)D\varphi(x,y)$ $\quad D\varphi_{(x,y)}=\begin{pmatrix}6x & -2y\\ -2 & 1\end{pmatrix}$

$=\cancel{\frac{1}{1+x^4}}\left(\frac{1}{2},-1\right)\begin{pmatrix}6&-4\\-2&1\end{pmatrix}=\begin{pmatrix}6&-4\\-2&1\end{pmatrix}$ (for $(x,y)=(1,2)$)

great!!!

(cf) 수업시간에 이야기 한것처럼 함수 set up을 이렇게 잘 잡으면 예외를 규칙적으로 잘 다룰 수 있습니다!!!

$=(3+2,\ -2-1)=(5,-3)$

$\Rightarrow \operatorname{grad} f(1,2)=(5,-3)$ ▨

($3-4=-1$, $2-2=0$

$$\frac{\partial f}{\partial x}=\frac{1}{1+(3x^2-y^2)^4}\cdot 6x+\frac{1}{1+(y-2x)^4}\cdot 2=3+2=5,\quad \frac{\partial f}{\partial y}=\frac{-2y}{1+(3x^2-y^2)^4}-\frac{1}{1+(y-2x)^4}=-2-1=-3$$

$\Rightarrow$ 일치)

3. 8

$X(t) \in C = \{ x \in \mathbb{R}^3 : f(x) = f(p) \}$ for all $t \in \mathbb{R}$

(φ : 미분가능)

let, $\varphi : \mathbb{R} \to \mathbb{R}^3$ s.t. $f(\vec{X}(t)) = f(p)$

$t \to \vec{X}(t) = (X_1(t), X_2(t), X_3(t))$

$\Rightarrow$ $\cancel{f(\vec{X}(t))}$

$f(\vec{X}) = f(\varphi(t))$

$\Rightarrow$ by chain rule

$Df(\varphi(t)) = Df(p) = 0 \quad (\because f(p) : \text{const})$

$= ((Df) \circ \varphi(t)) \cdot D\varphi(t)$

$= (D_1 f(\vec{X}), D_2 f(\vec{X}), D_3 f(\vec{X})) \cdot \begin{pmatrix} \frac{\partial X_1}{\partial t}(t) \\ \frac{\partial X_2}{\partial t}(t) \\ \frac{\partial X_3}{\partial t}(t) \end{pmatrix}$

$= \langle Df(\vec{X}), X'(t) \rangle = 0$

$\Rightarrow$ $\forall t$에서 성립 $\Rightarrow$ let $t = 0$

$\Rightarrow \langle Df(0), X'(0) \rangle = 0 \Rightarrow$ 서로 직교한다. $\blacksquare$