6.7 *In Section 2.6, it was noted that the net bonding energy E_N between two isolated positive and negative ions is a function of interionic distance r as follows:*

$$E_N = -\frac{A}{r} + \frac{B}{r^n} \qquad (6.30)$$

where A, B, and n are constants for the particular ion pair. Equation 6.30 is also valid for the bonding energy between adjacent ions in solid materials. The modulus of elasticity E is proportional to the slope of the interionic force–separation curve at the equilibrium interionic separation; that is,

$$E \propto \left(\frac{dF}{dr}\right)_{r_0}$$

Derive an expression for the dependence of the modulus of elasticity on these A, B, and n parameters (for the two-ion system), using the following procedure:

1. Establish a relationship for the force F as a function of r, realizing that

$$F = \frac{dE_N}{dr}$$

2. Now take the derivative dF/dr.

3. Develop an expression for r_0, the equilibrium separation. Because r_0 corresponds to the value of r at the minimum of the E_N-versus-r curve (Figure 2.10b), take the derivative dE_N/dr, set it equal to zero, and solve for r, which corresponds to r_0.

4. Finally, substitute this expression for r_0 into the relationship obtained by taking dF/dr.

Solution

This problem asks that we derive an expression for the dependence of the modulus of elasticity, *E*, on the parameters *A*, *B*, and *n* in Equation 6.30. It is first necessary to take dE_N/dr in order to obtain an expression for the force *F*; this is accomplished as follows:

$$F = \frac{dE_N}{dr} = \frac{d\left(-\frac{A}{r}\right)}{dr} + \frac{d\left(\frac{B}{r^n}\right)}{dr}$$

$$= \frac{A}{r^{(1+1)}} - \frac{nB}{r^{(n+1)}}$$

The second step is to set this dE_N/dr expression equal to zero and then solve for r (= r_0). The algebra for this procedure is carried out in Problem 2.10, with the result that

$$r_0 = \left(\frac{A}{nB}\right)^{1/(1 - n)}$$

Next it becomes necessary to take the derivative of the force (dF/dr), which is accomplished as follows:

$$\frac{dF}{dr} = \frac{d\left(\frac{A}{r^2}\right)}{dr} + \frac{d\left(-\frac{nB}{r^{n+1}}\right)}{dr}$$

$$= -\frac{2A}{r^3} + \frac{(n)(n + 1)B}{r^{(n+2)}}$$

Now, substitution of the above expression for r_0 into this equation yields

$$\left(\frac{dF}{dr}\right)_{r_0} = -\frac{2A}{\left(\frac{A}{nB}\right)^{3/(1-n)}} + \frac{(n)(n + 1)B}{\left(\frac{A}{nB}\right)^{(n+2)/(1-n)}}$$

which is the expression to which the modulus of elasticity is proportional.

6.9 *A cylindrical specimen of steel having a diameter of 15.2 mm and length of 250 mm is deformed elastically in tension with a force of 48,900 N. Using the data contained in Table 6.1, determine the following:*

(a) The amount by which this specimen will elongate in the direction of the applied stress.

(b) The change in diameter of the specimen. Will the diameter increase or decrease?

Solution

(a) We are asked, in this portion of the problem, to determine the elongation of a cylindrical specimen of steel. To solve this part of the problem requires that we use Equations 6.1, 6.2 and 6.5. Equation 6.5 reads as follows:

$$\sigma = E\varepsilon$$

Substitution the expression for σ from Equation 6.1 and the expression for ε from Equation 6.2 leads to

$$\frac{F}{\pi\left(\dfrac{d_0^2}{4}\right)} = E\,\frac{\Delta l}{l_0}$$

In this equation d_0 is the original cross-sectional diameter. Now, solving for Δl yields

$$\Delta l = \frac{4Fl_0}{\pi d_0^2 E}$$

And incorporating values of F, l_0, and d_0, and realizing that E = 207 GPa (Table 6.1), leads to

$$\Delta l = \frac{(4)(48{,}900\text{ N})\left(250 \times 10^{-3}\text{m}\right)}{(\pi)\left(15.2 \times 10^{-3}\text{m}\right)^2\left(207 \times 10^9\,\text{N/m}^2\right)} = 3.25 \times 10^{-4}\text{ m} = 0.325\text{ mm}$$

(b) We are now called upon to determine the change in diameter, Δd. Using Equation 6.8 (the definition of Poisson's ratio)

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d/d_0}{\Delta l/l_0}$$

From Table 6.1, for steel, the value of Poisson's ratio, ν, is 0.30. Now, solving the above expression for Δd yields

$$\Delta d = -\frac{\nu \Delta l \, d_0}{l_0} = -\frac{(0.30)(0.325 \text{ mm})(15.2 \text{ mm})}{250 \text{ mm}}$$

$$= -5.9 \times 10^{-3} \text{ mm}$$

The diameter will decrease since Δd is negative.

6.15 *Figure 6.22 shows the tensile engineering stress–strain behavior for a steel alloy.*

(a) What is the modulus of elasticity?

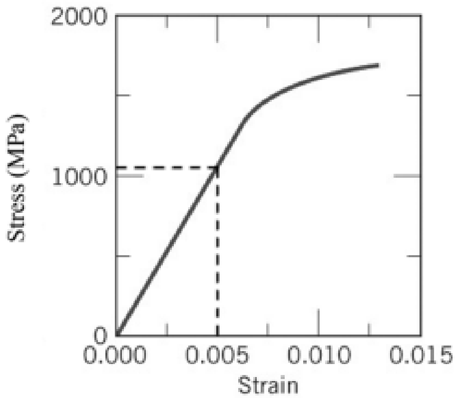
(b) What is the proportional limit?

(c) What is the yield strength at a strain offset of 0.002?

(d) What is the tensile strength?

Solution

(a) Shown below is the inset of Figure 6.22.



The elastic modulus is just the slope of the initial linear portion of the curve; or, from Equation 6.10

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1}$$

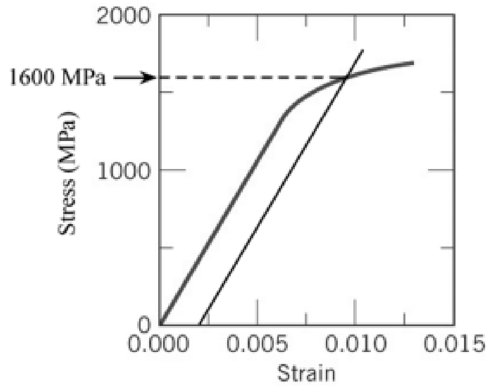
Inasmuch as the linear segment passes through the origin, let us take both σ_1 and ε_1 to be zero. If we arbitrarily take ε_2 = 0.005, as noted in the above plot, σ_2 = 1050 MPa. Using these stress and strain values we calculate the elastic modulus as follows:

$$E = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1} = \frac{(1050 - 0)\ \text{MPa}}{(0.005 - 0)} = 210 \times 10^3\ \text{MPa} = 210\ \text{GPa}$$

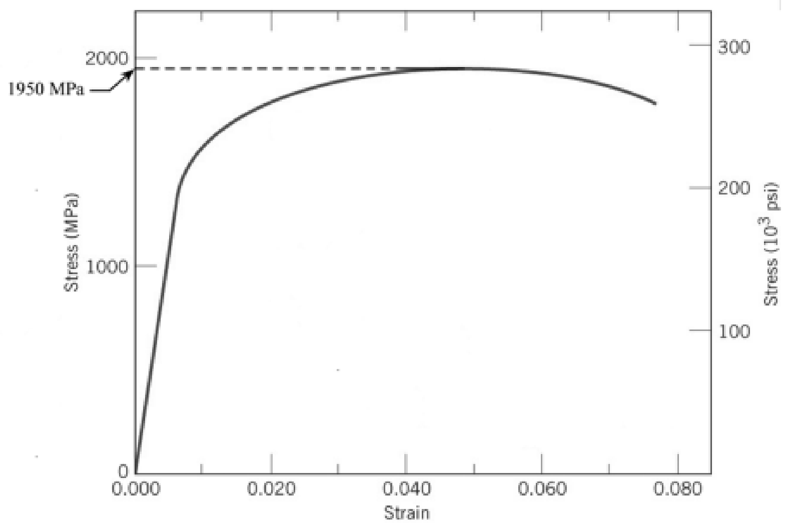
The value given in Table 6.1 is 207 GPa.

(b) The proportional limit is the stress level at which linearity of the stress-strain curve ends, which is approximately 1370 MPa.

(c) As noted in the plot below, the 0.002 strain offset line intersects the stress-strain curve at approximately 1600 MPa.



(d) The tensile strength (the maximum on the curve) is approximately 1950 MPa, as noted in the following plot.



6.22 *Show that Equations 6.18a and 6.18b are valid when there is no volume change during deformation.*

Solution

To show that Equation 6.18a is valid, we must first rearrange Equation 6.17 as

$$A_i = \frac{A_0 l_0}{l_i}$$

Substituting this expression into Equation 6.15 yields

$$\sigma_T = \frac{F}{A_i} = \frac{F}{\frac{A_0 l_0}{l_i}} = \frac{F}{A_0}\left(\frac{l_i}{l_0}\right) = \sigma\left(\frac{l_i}{l_0}\right)$$

From Equation 6.2

$$\varepsilon = \frac{l_i - l_0}{l_0} = \frac{l_i}{l_0} - 1$$

Or

$$\frac{l_i}{l_0} = 1 + \varepsilon \qquad (6.2b)$$

Substitution of this expression into the equation above leads to

$$\sigma_T = \sigma\left(\frac{l_i}{l_0}\right) = \sigma(1 + \varepsilon)$$

which is Equation 6.18a.

For Equation 6.18b, true strain is defined in Equation 6.16 as

$$\varepsilon_T = \ln\left(\frac{l_i}{l_0}\right)$$

Substitution the of the $l_i - l_0$ ratio of Equation 6.2b above leads to Equation 6.18b—viz.

$$\varepsilon_T = \ln(1 + \varepsilon)$$