7.5 *Equations 7.1a and 7.1b, expressions for Burgers vectors for FCC and BCC crystal structures, are of the form*

$$\mathbf{b} = \frac{a}{2}\langle uvw \rangle$$

*where a is the unit cell edge length. The magnitudes of these Burgers vectors may be determined from the following equation:*

$$|\mathbf{b}| = \frac{a}{2}\left(u^2 + v^2 + w^2\right)^{1/2} \qquad (7.11)$$

*determine the values of |**b**| for copper and iron. You may want to consult Table 3.1.*

Solution

This problem asks that we compute the magnitudes of the Burgers vectors for copper and iron. For Cu, which has an FCC crystal structure, $R$ = 0.1278 nm (Table 3.1) and, from Equation 3.1

$$a = 2R\sqrt{2} = (2)(0.1278 \text{ nm})(\sqrt{2})$$

$$= 0.3615 \text{ nm}$$

Also, from Equation 7.1a, the Burgers vector for FCC metals is

$$\mathbf{b} = \frac{a}{2}\langle 110 \rangle$$

Therefore, the values for $u$, $v$, and $w$ in Equation 7.11 are 1, 1, and 0, respectively. Hence, the magnitude of the Burgers vector for Cu is

$$|\mathbf{b}| = \frac{a}{2}\sqrt{u^2 + v^2 + w^2}$$

$$= \frac{0.3615 \text{ nm}}{2}\sqrt{(1)^2 + (1)^2 + (0)^2} = 0.2556 \text{ nm}$$

For Fe which has a BCC crystal structure, $R$ = 0.1241 nm (Table 3.1) and, from Equation 3.4

$$a = \frac{4R}{\sqrt{3}} = \frac{(4)(0.1241 \text{ nm})}{\sqrt{3}}$$

$$= 0.2866 \text{ nm}$$

Also, from Equation 7.1b, the Burgers vector for BCC metals is

$$\mathbf{b} = \frac{a}{2}\langle 111 \rangle$$

Therefore, the values for *u*, *v*, and *w* in Equation 7.11 are 1, 1, and 1, respectively. Hence, the magnitude of the Burgers vector for Fe is

$$|\mathbf{b}| = \frac{0.2866 \text{ nm}}{2}\sqrt{(1)^2 + (1)^2 + (1)^2} = 0.2482 \text{ nm}$$

7.10 *Consider a single crystal of some hypothetical metal that has the FCC crystal structure and is oriented such that a tensile stress is applied along a [112] direction. If slip occurs on a (111) plane and in a [011] direction, and the crystal yields at a stress of 5.12 MPa compute the critical resolved shear stress.*

Solution

To solve this problem, we use Equation 7.4; however, it is first necessary to determine the values of $\phi$ and $\lambda$. These determinations are possible using Equation 7.6. Now, $\lambda$ is the angle between [112] and [011] directions. Therefore, relative to Equation 7.6 let us take $u_1 = 1$, $v_1 = 1$, and $w_1 = 2$, as well as $u_2 = 0$, $v_2 = 1$, and $w_2 = 1$. This leads to

$$\lambda = \cos^{-1}\left[\frac{u_1u_2 + v_1v_2 + w_1w_2}{\sqrt{\left(u_1^2 + v_1^2 + w_1^2\right)\left(u_2^2 + v_2^2 + w_2^2\right)}}\right]$$

$$= \cos^{-1}\left\{\frac{(1)(0) + (1)(1) + (2)(1)}{\sqrt{\left[(1)^2 + (1)^2 + (2)^2\right]\left[(0)^2 + (1)^2 + (1)^2\right]}}\right\}$$

$$= \cos^{-1}\left(\frac{3}{\sqrt{12}}\right) = 30°$$

Now for the determination of $\phi$, the normal to the (111) slip plane is the [111] direction. Again, using Equation 7.6, where we now take $u_1 = 1$, $v_1 = 1$, $w_1 = 2$ (for [112]), and $u_2 = 1$, $v_2 = 1$, $w_2 = 1$ (for [111]). Thus,

$$\phi = \cos^{-1}\left\{\frac{(1)(1) + (1)(1) + (2)(1)}{\sqrt{\left[(1)^2 + (1)^2 + (2)^2\right]\left[(1)^2 + (1)^2 + (1)^2\right]}}\right\}$$

$$= \cos^{-1}\left(\frac{4}{\sqrt{18}}\right) = 19.5°$$

It is now possible to compute the critical resolved shear stress (using Equation 7.4) as

$$\tau_{\text{crss}} = \sigma_y(\cos\phi \cos\lambda)$$

$$= (5.12 \text{ MPa})\left(\frac{4}{\sqrt{18}}\right)\left(\frac{3}{\sqrt{12}}\right) = 4.18 \text{ MPa}$$

7.20 *Consider a hypothetical material that has a grain diameter of 2.1 × $10^{-2}$ mm. After a heat treatment at 600°C for 3 h, the grain diameter has increased to 7.2 × $10^{-2}$ mm. Compute the grain diameter when a specimen of this same original material (i.e., $d_0$ = 2.1 × $10^{-2}$ mm) is heated for 1.7 h at 600°C. Assume the n grain diameter exponent has a value of 2.*

Solution

To solve this problem requires that we use Equation 7.9 with $n = 2$. It is first necessary to solve for the parameter $K$ in this equation, using values given in the problem statement of $d_0$ ($2.1 \times 10^{-2}$ mm) and the grain diameter after the 3-h heat treatment ($7.2 \times 10^{-2}$ mm). The computation for $K$ using a rearranged form of Equation 7.9 in which $K$ becomes the dependent parameter is as follows:

$$K = \frac{d^2 - d_0^2}{t}$$

$$= \frac{(7.2 \times 10^{-2}\text{ mm})^2 - (2.1 \times 10^{-2}\text{ mm})^2}{3\text{ h}}$$

$$= 1.58 \times 10^{-3}\text{ mm}^2/\text{h}$$

It is now possible to solve for the value of $d$ after a heat treatment of 1.7 h using a rearranged form of Equation 7.9:

$$d = \sqrt{d_0^2 + Kt}$$

$$= \sqrt{(2.1 \times 10^{-2}\text{ mm})^2 + (1.58 \times 10^{-3}\text{ mm}^2/\text{h})(1.7\text{ h})}$$

$$= 5.59 \times 10^{-2}\text{ mm}$$

6.

① Grain size reduction.

: grain size ↓ → grain boundary ↑ : ↑ barrier to slip → strengthen (Hall-Petch eq.)

② Solid solutions.

: Impurity atoms → distort lattice : stress ↑ → barrier ↑ for dislocation motion → strengthen.

③ Precipitation strengthening

: Hard precipitate : ↑ shear stress to move toward and shear → strengthen.

④ Cold work hardening

: Room Temperature deformation (Forge, Draw, Roll, Extrusion)

→ dislocations entangle → motion difficult → strengthen.

7.19 *(a) What is the driving force for recrystallization?*

*(b) What is the driving force for grain growth?*

Solution

(a) The driving force for recrystallization is the difference in internal energy between the strained and unstrained material.

(b) The driving force for grain growth is the reduction in grain boundary energy as the total grain boundary area decreases.