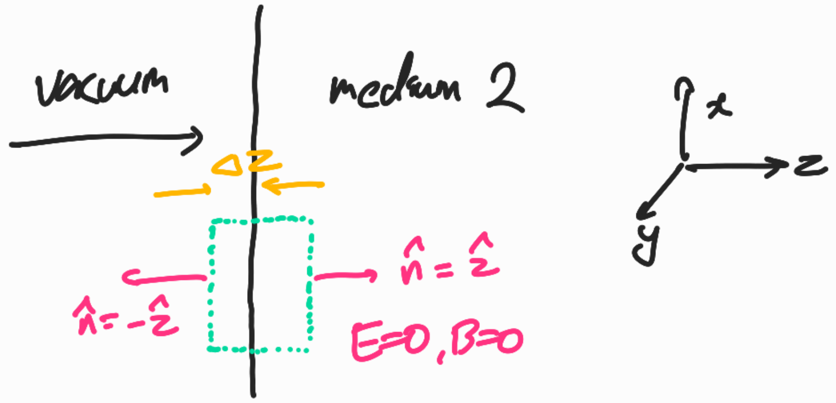
1.



- Momentum conservation

$d\vec{a} \equiv \hat{n}\, da$

$$\frac{d\mathbf{p}_{\text{mech}}}{dt} = -\epsilon_0\mu_0 \frac{d}{dt}\int_{\mathcal{V}} \mathbf{S}\, d\tau + \oint_{\mathcal{S}} \overleftrightarrow{\mathbf{T}} \cdot d\mathbf{a},$$

$$T_{ij} = \epsilon_0\left(E_i E_j - \frac{1}{2}E^2\delta_{ij}\right) + \frac{1}{\mu_0}\left(B_i B_j - \frac{1}{2}B^2\delta_{ij}\right)$$

Let $\vec{E} = E\hat{x}$, $\vec{B} = B\hat{y}$ (WLOG), xy-plane 에 대칭, 진행방향 $\parallel \hat{z}$

i) $\Rightarrow$ T_{33} 만이 기여한다. ($E_z, B_z = 0$)

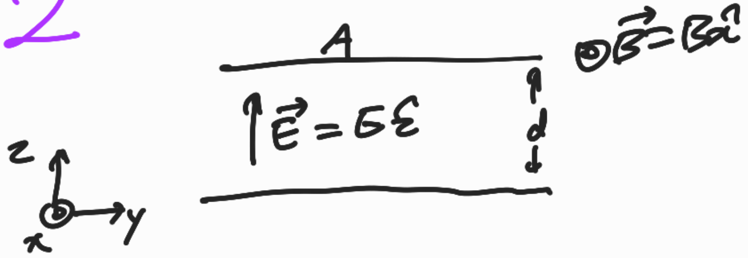
ii) & $\epsilon_0\mu_0 \frac{d}{dt}\int_V \vec{S}\, d\tau = \frac{d}{dt}\int_V \epsilon_0(\vec{E}\times\vec{B})\, d\tau \propto \Delta z \to 0$

$\therefore$ 단위 시간당 단위 면적당 plane wave가 주는 힘

$\equiv$ Radiation pressure

$= \overleftrightarrow{T}\cdot(-\hat{z}) = -T_{33} = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) =$ Field energy density $\square$

2



(a) $\vec{\mathcal{P}} = \varepsilon_0(\vec{E}\times\vec{B}) = \varepsilon_0 EB\,\hat{y}$

$\Rightarrow \vec{p} = \varepsilon_0 EBAd\,\hat{y}$

(b) discharge : $\vec{E} = E\hat{z} \longrightarrow 0$

current $\parallel \hat{z}$

$\Rightarrow \vec{F} = i\vec{l}\times\vec{B} = i\,dB\,\hat{y}$

$\rightarrow$ Impulse $= \int \vec{F}\,dt = \hat{y}\,dB\int i\,dt = dB\times(\text{total charge})\,\hat{y}$

$= dB\times\varepsilon_0\frac{A}{d}\times Ed\,\hat{y} = \varepsilon_0 EBAd\,\hat{y}$

3 (a) $\frac{dW}{dt} = \int_V (\vec{E}\cdot\vec{J}_f)\,d\tau$, $\vec{J}_f = \vec{\nabla}\times\vec{H} - \frac{\partial \vec{D}}{\partial t}$, $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\Rightarrow \vec{E}\cdot\vec{J}_f = \vec{E}\cdot(\vec{\nabla}\times\vec{H}) - \vec{E}\cdot\frac{\partial \vec{D}}{\partial t}$$

$$= \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{\nabla}\cdot(\vec{E}\times\vec{H}) - \vec{E}\cdot\frac{\partial \vec{D}}{\partial t}$$

$$= -\left(\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t} + \vec{\nabla}\cdot(\vec{E}\times\vec{H})\right)$$

$$\Rightarrow \frac{dW}{dt} = -\int_V \underbrace{\left(\vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{D}}{\partial t}\right)}_{\frac{\partial u}{\partial t}} - \oint_S \underbrace{(\vec{E}\times\vec{H})}_{\vec{S}}\cdot d\vec{a}$$

$$\therefore\ \vec{S} = \vec{E}\times\vec{H}, \quad \frac{\partial u}{\partial t} = \vec{E}\cdot\frac{\partial \vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial \vec{B}}{\partial t}\ \square$$

Linear media: $\vec{D} = \varepsilon\vec{E}$, $\vec{H} = \frac{1}{\mu}\vec{B}$, ε, μ : const. in time

$$\Rightarrow \frac{\partial u}{\partial t} = \varepsilon\vec{E}\cdot\frac{\partial \vec{E}}{\partial t} + \frac{1}{\mu}\vec{B}\cdot\frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}\left(\varepsilon E^2 + \frac{1}{\mu}B^2\right)$$

$$\Rightarrow u = \frac{1}{2}\varepsilon E^2 + \frac{1}{2\mu}B^2 = \frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H})\ \square$$

(b) $\vec{f} = \rho_f\vec{E} + \vec{J}_f\times\vec{B} = (\vec{\nabla}\cdot\vec{D})\vec{E} + \left(\vec{\nabla}\times\vec{H} - \frac{\partial \vec{D}}{\partial t}\right)\times\vec{B}$,

using $\frac{\partial}{\partial t}(\vec{D}\times\vec{B}) = \frac{\partial \vec{D}}{\partial t}\times\vec{B} + \vec{D}\times\frac{\partial \vec{B}}{\partial t}$ & adding $\overbrace{\vec{H}(\vec{\nabla}\cdot\vec{B})}^{0}$

$$\rightarrow \vec{f} = \vec{E}(\vec{\nabla}\cdot\vec{D}) + (\vec{\nabla}\times\vec{H})\times\vec{B} + \vec{H}(\vec{\nabla}\cdot\vec{B}) + \vec{D}\times\underbrace{\frac{\partial \vec{B}}{\partial t}}_{=-\vec{\nabla}\times\vec{E}} - \frac{\partial}{\partial t}(\vec{D}\times\vec{B})$$

$$= \underbrace{\vec{E}(\vec{\nabla}\cdot\vec{D}) - \vec{D}\times(\vec{\nabla}\times\vec{E}) + \vec{H}(\vec{\nabla}\cdot\vec{B}) - \vec{B}\times(\vec{\nabla}\times\vec{H})}_{\text{divergence of a stress tensor}} - \frac{\partial}{\partial t}\underbrace{(\vec{D}\times\vec{B})}_{\text{momentum density}}$$

$$\therefore\ \vec{g} = \vec{D}\times\vec{B}\ \square$$

4. $\tilde{A}_3 = \tilde{A}_1 + \tilde{A}_2$, $A_3 e^{i\delta_3} = A_1 e^{i\delta_1} + A_2 e^{i\delta_2}$

$$|A_3|^2 = \left(A_1 e^{i\delta_1} + A_2 e^{i\delta_2}\right)\left(A_1 e^{-i\delta_1} + A_2 e^{-i\delta_2}\right)$$

$$= |A_1|^2 + |A_2|^2 + 2A_1A_2\cos(\delta_1 - \delta_2)$$

$$\Rightarrow \underline{A_3 = \sqrt{|A_1|^2 + |A_2|^2 + 2A_1A_2\cos(\delta_1 - \delta_2)}}$$

$$\Rightarrow e^{i\delta_3} = \left(\frac{A_1}{A_3}\right)e^{i\delta_1} + \left(\frac{A_2}{A_3}\right)e^{i\delta_2}$$

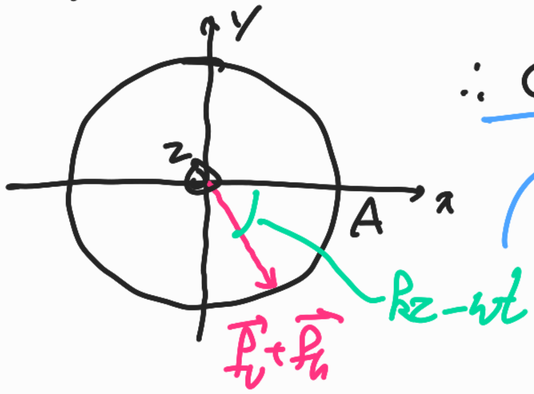
$$\Rightarrow \tan\delta_3 = \frac{\mathrm{Im}(e^{i\delta_3})}{\mathrm{Re}(e^{i\delta_3})} = \frac{A_1\sin\delta_1 + A_2\sin\delta_2}{A_1\cos\delta_1 + A_2\cos\delta_2}$$

$$\therefore \underline{\delta_3 = \tan^{-1}\left[\frac{A_1\sin\delta_1 + A_2\sin\delta_2}{A_1\cos\delta_1 + A_2\cos\delta_2}\right]}$$

5 (a) $\vec{f}_v = A\cos(kz-\omega t+\delta_v)\hat{x}$, $\vec{f}_h = A\cos(kz-\omega t+\delta_h)\hat{y}$

$\Rightarrow \vec{f}_v = A\cos(kz-\omega t)\hat{x}$, $\vec{f}_h = -A\sin(kz-\omega t)\hat{y}$

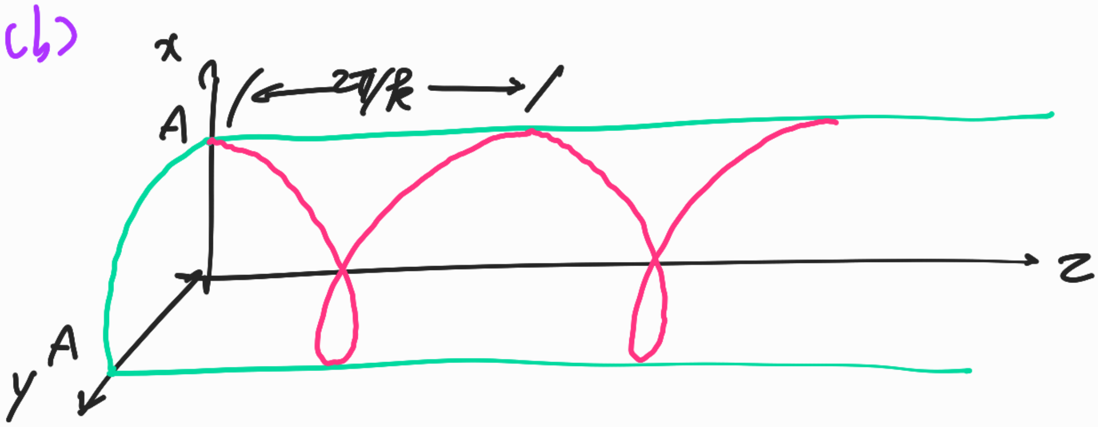
$|f_v|^2+|f_h|^2=|A|^2$ .. circular



∴ Counterclockwise

If $\delta_h-\delta_v = -90^\circ$, a wave circles clockwise

(b)



6 $\vec{E}(r,\theta,\phi,t) = A\frac{\sin\theta}{r}\left[\cos(\underbrace{kr-\omega t}_{=u}) - \frac{1}{kr}\sin(kr-\omega t)\right]\hat{\phi}$, $\frac{\omega}{k}=c$

$= E\hat{\phi}$

(a) i) $\nabla\cdot\vec{E} = \frac{1}{r\sin\theta}\frac{\partial E}{\partial\phi} = 0$ ✓

ii) $-\frac{\partial \vec{B}}{\partial t} = \nabla\times\vec{E} = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E)\hat{r} - \frac{1}{r}\frac{\partial}{\partial r}(rE)\hat{\theta}$

$$= A\left[\frac{2\cos\theta}{r^2}\left(\cos u - \frac{1}{kr}\sin u\right)\right]\hat{r}$$

$$+ A\left[\frac{\sin\theta}{r}\left[\left(k - \frac{1}{kr^2}\right)\sin u + \frac{1}{r}\cos u\right]\right]\hat{\theta}$$

$$\left(\Rightarrow \vec{B} = \frac{2A\cos\theta}{\omega r^2}\left(\sin u + \frac{1}{kr}\cos u\right)\hat{r} + \frac{A\sin\theta}{\omega r}\left(\left(-k + \frac{1}{kr^2}\right)\cos u + \frac{1}{r}\sin u\right)\hat{\theta}\right)$$

iii) $\nabla\cdot\vec{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 B_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta B_\theta)$

$$= \frac{2A\cos\theta}{\omega r^2}\left(k\cos u - \frac{1}{r}\sin u - \frac{1}{kr^2}\cos u\right)$$

$$+ \frac{2A\cos\theta}{\omega r^2}\left(\left(-k + \frac{1}{kr^2}\right)\cos u + \frac{1}{r}\sin u\right) = 0$$ ✓

iv)

$$\nabla\times\vec{B} = \frac{1}{r}\left(\frac{\partial}{\partial r}(rB_\theta) - \frac{\partial B_r}{\partial\theta}\right)\hat{\phi}$$

$$= \frac{A\hat{\phi}}{\omega r}\left[\sin\theta\left[\left(k^2 - \cancel{\frac{1}{r^2}}\right)\sin u + \frac{k}{r}\cos u - \cancel{\frac{2}{kr^3}\cos u} - \cancel{\frac{1}{r^2}\sin u}\right] + \frac{2\sin\theta}{r^2}\left(\cancel{\sin u} + \cancel{\frac{1}{kr}\cos u}\right)\right]$$

$$= \frac{A\sin\theta}{cr}\left[k\sin u + \frac{1}{r}\cos u\right]\hat{\phi}$$ ✓

$$\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} = \frac{A\sin\theta}{c^2 r}\omega\left[\sin u + \frac{1}{kr}\cos u\right]\hat{\phi}$$

(b) $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{E}{\mu_0}\left(B_r\hat{\theta} - B_\theta\hat{r}\right)$

$$= \frac{A^2\sin\theta}{\mu_0\omega r^2}\left[\frac{2\cos\theta}{r}\left(\frac{1}{kr}(\cos^2 u - \sin^2 u) + \left(1 - \frac{1}{k^2r^2}\right)\cos u\sin u\right)\hat{\theta}\right.$$
$$\left. - \sin\theta\left(\left(-k + \frac{1}{kr^2}\right)\cos^2 u - \frac{1}{kr^2}\sin^2 u + \left(\frac{2}{r} - \frac{1}{k^2r^3}\right)\cos u\sin u\right]\hat{r}\right]$$

$\vec{I} = \langle\vec{S}\rangle$, $\langle\cos^2 u\rangle = \langle\sin^2 u\rangle = \frac{1}{2}$, $\langle\cos u\sin u\rangle = 0$

$$\Rightarrow \vec{I} = \frac{A^2\sin\theta}{\mu_0\omega r^2}(-\sin\theta)\left(-\frac{1}{2}k\right)\hat{r} = \frac{A^2\sin^2\theta}{2\mu_0 c r^2}\hat{r}$$

(c) $$\int_{r=R}\vec{I}\cdot d\vec{a} = \int_{r=R} r^2 I\, d\Omega = \frac{A^2}{2\mu_0 c}\int_0^{\pi}\sin^2\theta\,(2\pi\sin\theta\, d\theta)$$

$$= \frac{4\pi}{3}\cdot\frac{A^2}{\mu_0 c}$$