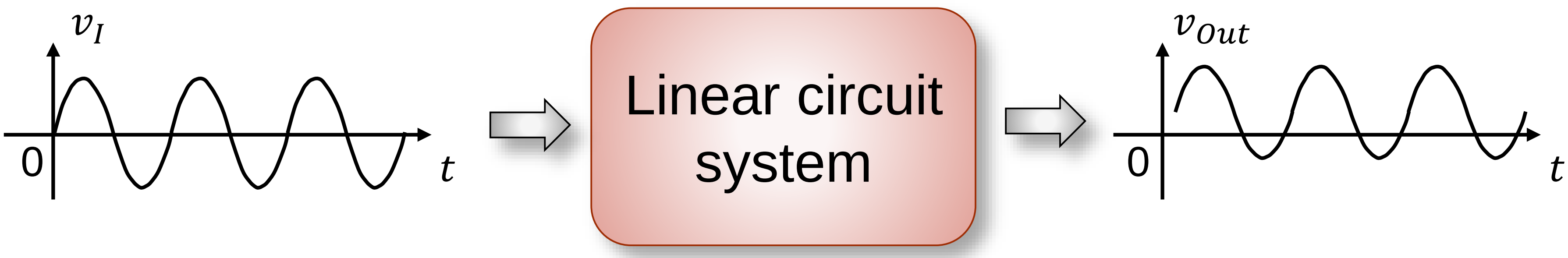
Chap. 13 Sinusoidal steady state: impedance and frequency response

- Why we are interested in the response of the circuit for the sinusoidal input?
  - Recall that the particular solution of a differential equation to a sinusoidal input is sinusoidal signal.



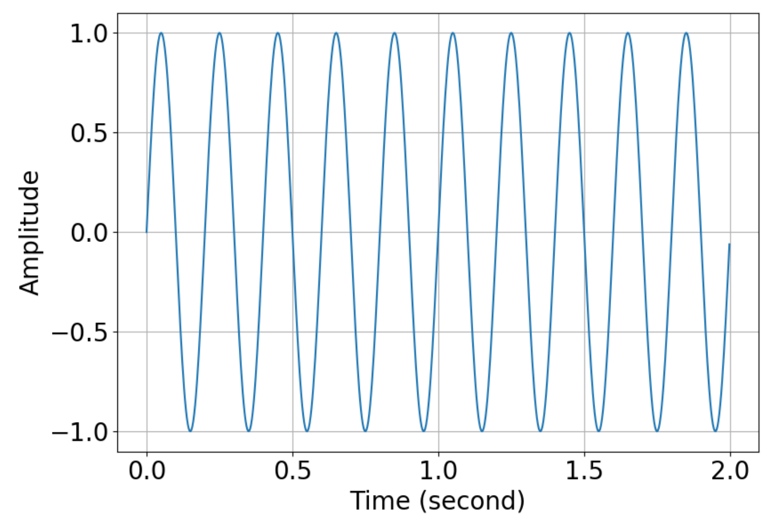
Fourier series: (loosely speaking) any arbitrary periodic signal can be represented as sum of constant, cosine, and sine only

Once we know about the response for all sinusoidal input, we basically know everything about that linear circuit.

Notations

- If period is T, frequency is $f = 1/T$, but angular frequency notation $\omega = 2\pi \cdot f$ is much simpler to deal with sinusoidal signal because sine function is always written as $\sin(2\pi \cdot f \cdot t)$ in terms of frequency f. Angular frequency notation allow us to write as $\sin(\omega t)$.

The signal on the right oscillates 10 times for 2 seconds. T=0.2 second, frequency is $f = 5\ (\mathrm{Hz})$ and $\omega = 2\pi \cdot f \approx 31.4\ (\mathrm{rad})$.



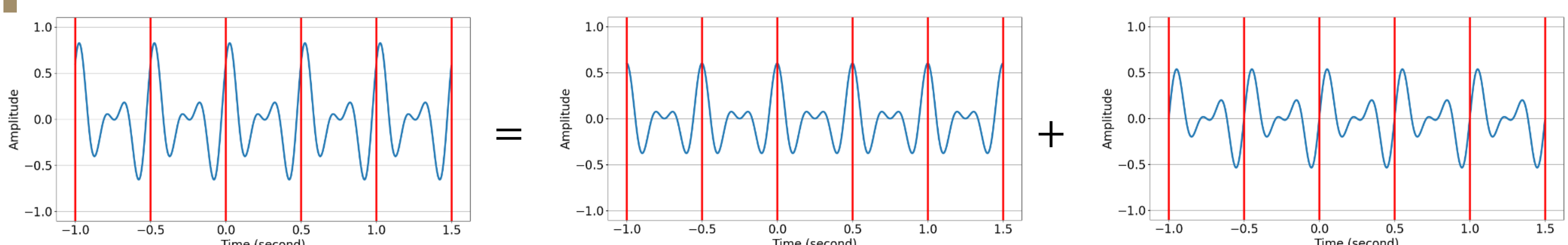
Decomposition into even and odd function

- Any arbitrary function $g(t)$ can be decomposed into even and odd function.

➔ $g(t) = even(t) + odd(t)$ where $even(t) = (g(t) + g(-t))/2$ and $odd(t) = (g(t) - g(-t))/2$

  - Cosine is an even function
  - Sine is an odd function

- Assume that we are given some arbitrary periodic function with period $T = 2\pi/\omega$. Even part of this function can be always written as sum of $\cos(n\omega t)$ and odd part of this function can be always written as sum of $\sin(n\omega t)$.



Even part

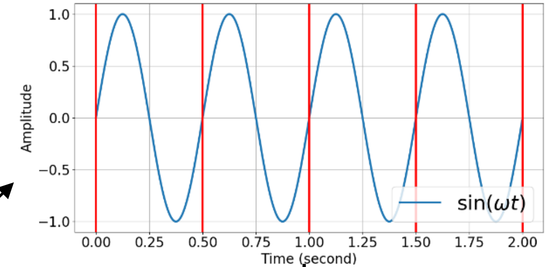
Odd part

Fundamental frequency for Fourier series

- If a periodic signal is given, the frequency of that period is called fundamental frequency. If $T = 0.5$ (sec), then the fundamental frequency is $f = 2$ (Hz) or angular frequency $\omega = 2\pi \cdot f \approx 12.56$ (rad).
- Integer-multiple of fundamental frequency ($n\omega$) are generally called as higher-order frequency and $\sin n\omega t$ are called higher-order harmonics.
- Note that higher-order harmonics can be also said that it is a periodic function with fundamental freq.
- Any arbitrary sum of higher-order harmonics will be still a periodic function with period $T = 0.5$ (sec).
- Sum of sine functions produces only odd function meaning the function is anti-symmetric w.r.t. $t = 0$ (sec) or $t = \frac{T}{2} = 0.25$ (sec).

$0.1 \times$

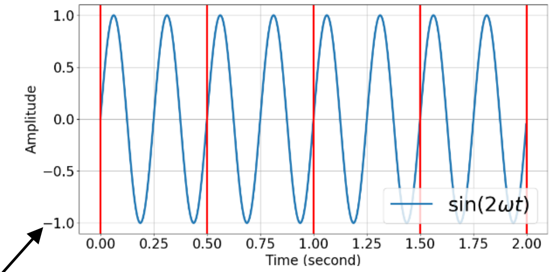
Fundamental frequency



$+$

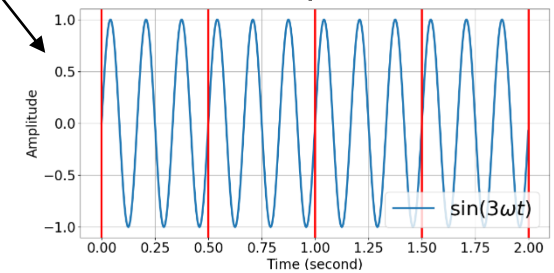
$0.3 \times$

Higher-order frequency



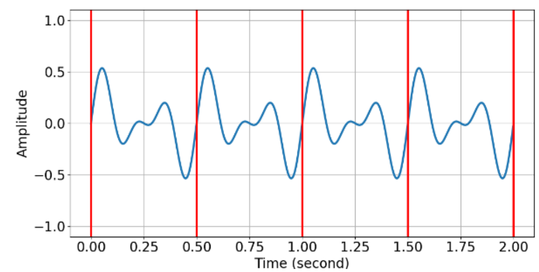
$+$

$0.2 \times$



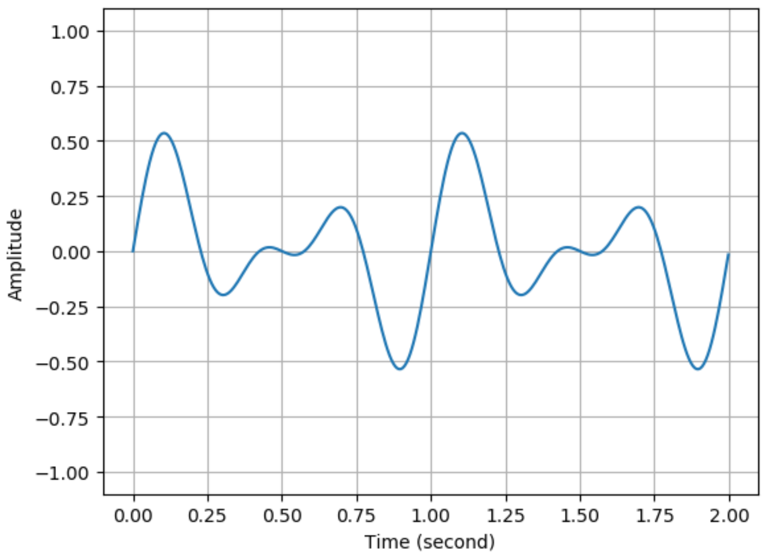
$\parallel$

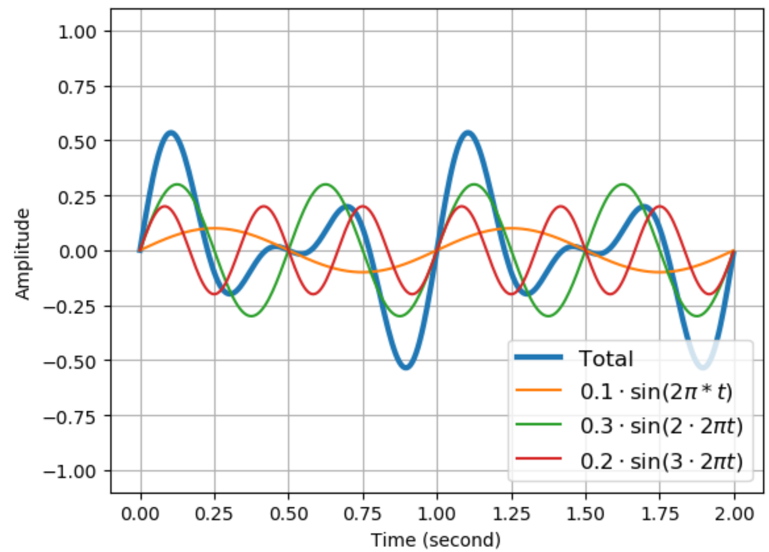
$$g(t) = 0.1 \sin \omega t + 0.3 \sin 2\omega t + 0.2 \sin 3\omega t$$



Fourier series

- Can you guess what kind of sinusoidal input is used in the following periodic signal?

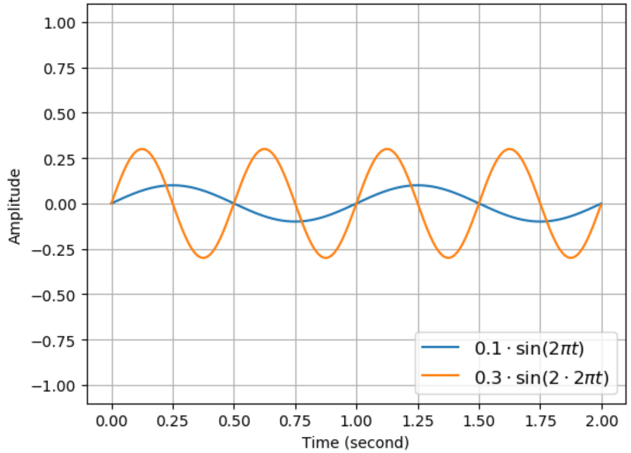


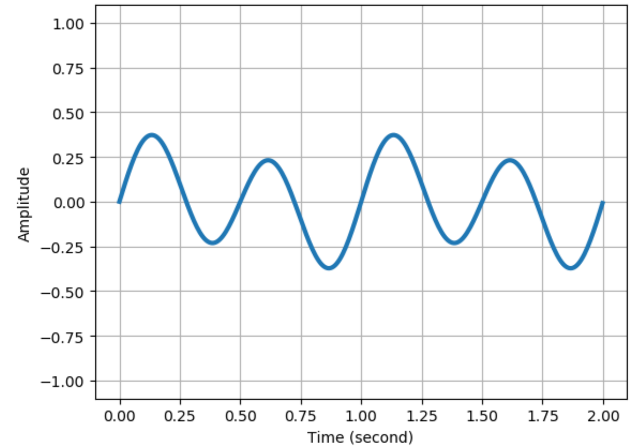


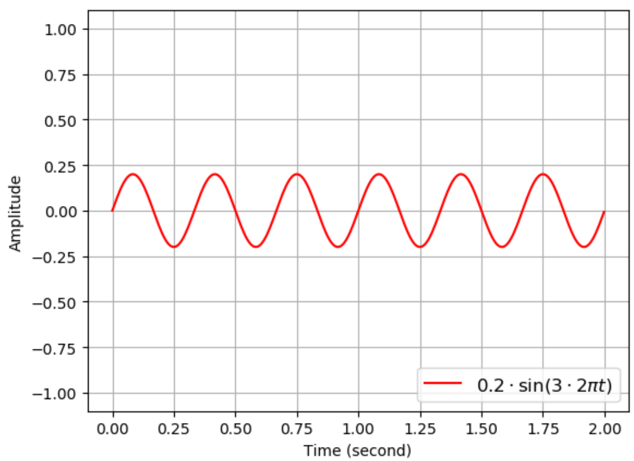
$$g(t) = 0.1 \cdot \sin(2\pi t) + 0.3 \cdot \sin(2 \cdot 2\pi t) + 0.2 \cdot \sin(3 \cdot 2\pi t)$$

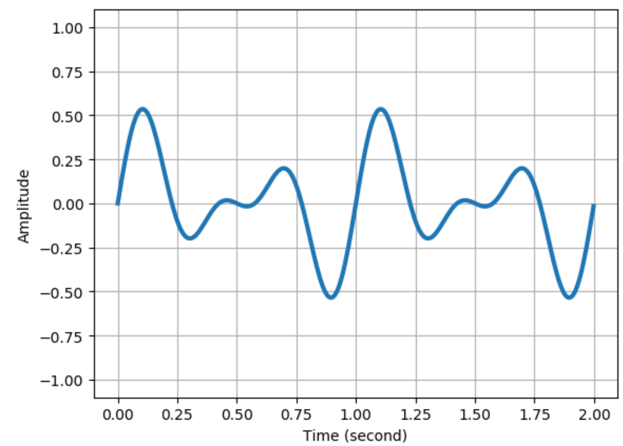
Fourier series

- Can you convince yourself?





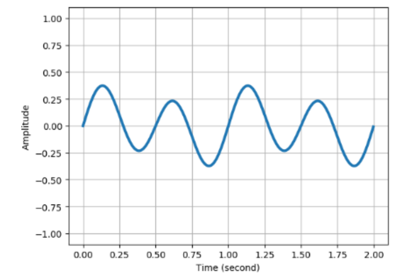




Is this the best way?

Fourier series of sine

- Assume that someone evaluated the value of the following function at each time and provide only the values, but not how they are obtained:



$$g(t) = 0.1 \cdot \sin(2\pi t) + 0.3 \cdot \sin(2 \cdot 2\pi t)$$

- If we use the following relation, we can always find the coefficient of the sine (T:period, $\omega = 2\pi/T$):
- $\frac{2}{T}\int_0^T \sin\omega t \cdot \sin\omega t\, dt = \frac{2}{T}\int_0^T \frac{1}{2}(1 - \cos 2\omega t)dt = \frac{1}{T}\left(t - \frac{1}{2\omega}\sin 2\omega t\right)\Big|_0^T = 1$
- $\frac{2}{T}\int_0^T \sin n\omega t \cdot \sin m\omega t\, dt$ ← (B.18) Appendix B. Trigonometric functions

$$= \frac{2}{T}\int_0^T \frac{1}{2}(\cos(n-m)\omega t + \cos(n+m)\omega t)dt$$

$$= \frac{1}{T}\left(\frac{1}{(n-m)\omega}\sin(n-m)\omega t + \frac{1}{(n+m)\omega}\sin(n+m)\omega t\right)\Big|_0^T = 0 \text{ when } n \neq \pm m$$

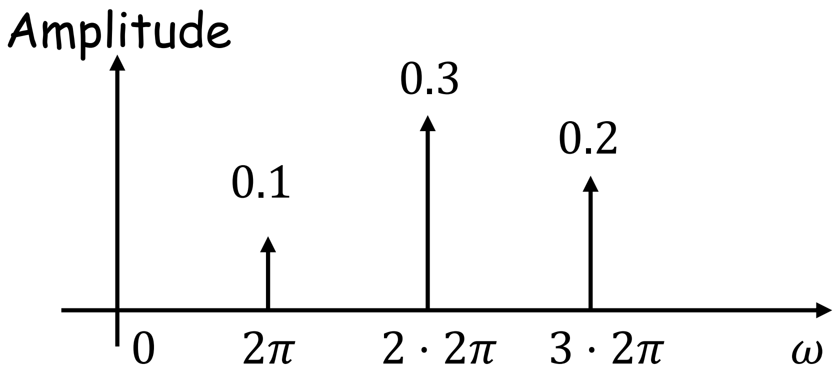
- We can use the above relation to find out 0.1 from the given data.
- $\frac{2}{T}\int_0^T \sin(2\pi t) \cdot g(t)\, dt$

$$= \frac{2}{T}\int_0^T \sin(2\pi t) \cdot \{0.1 \cdot \sin(2\pi t) + 0.3 \cdot \sin(2 \cdot 2\pi t)\}dt = 0.1$$

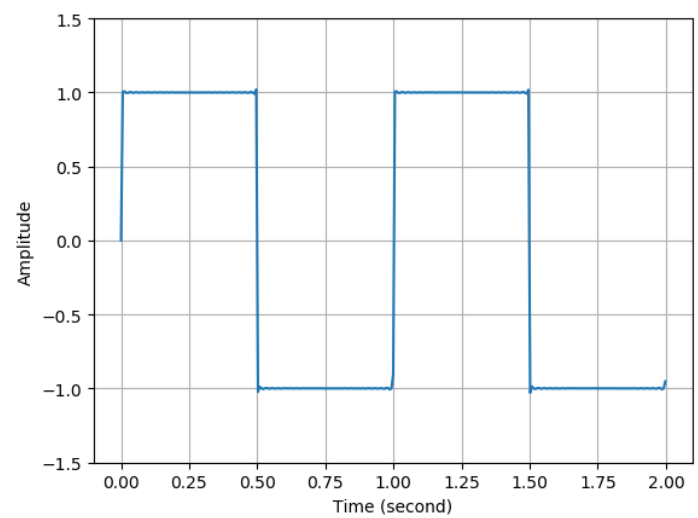
Fourier series of sine

- The following signal can be plotted in the following:

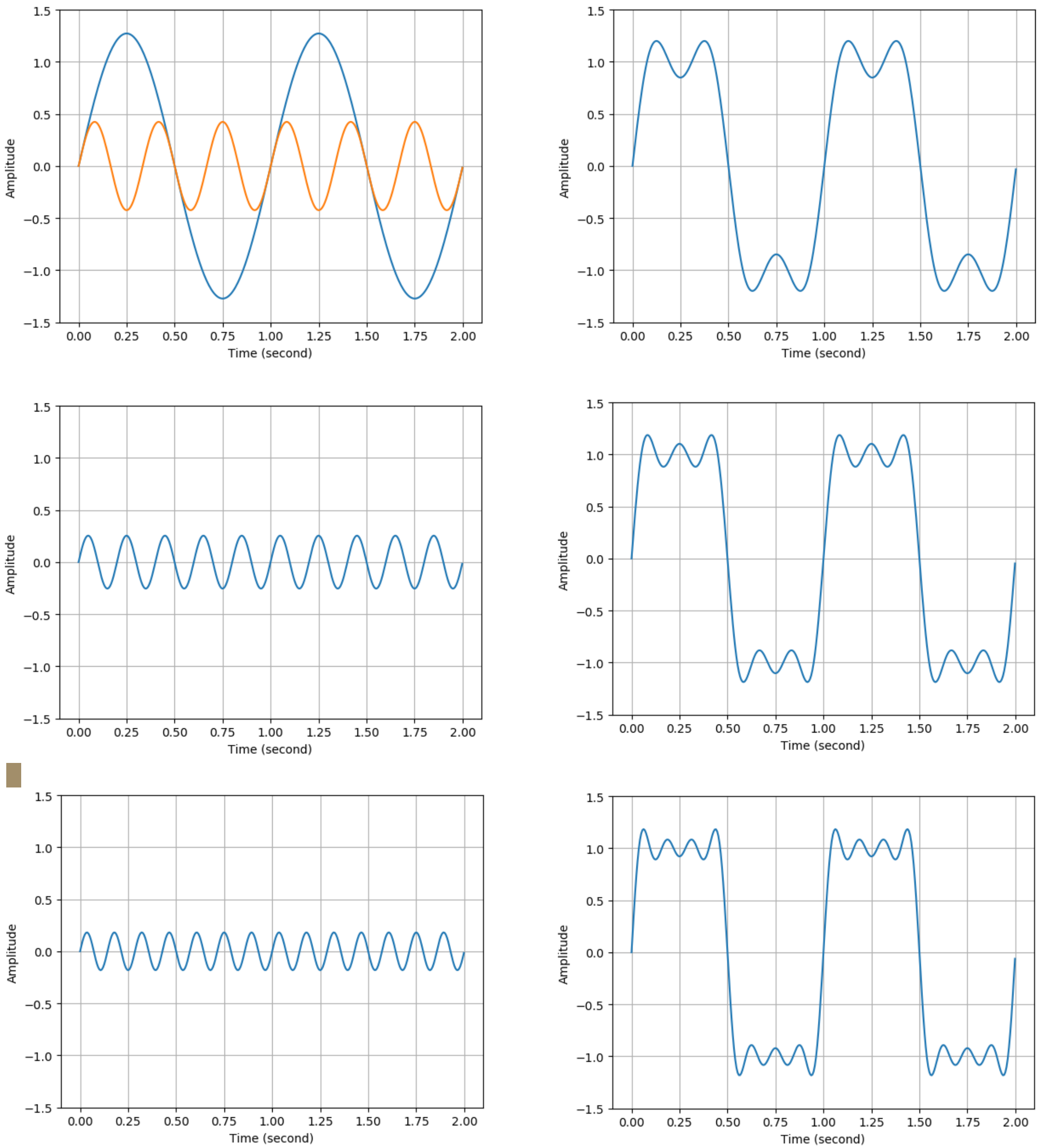
$$g(t) = 0.1 \cdot \sin(2\pi t) + 0.3 \cdot \sin(2 \cdot 2\pi t) + 0.2 \cdot \sin(3 \cdot 2\pi t)$$

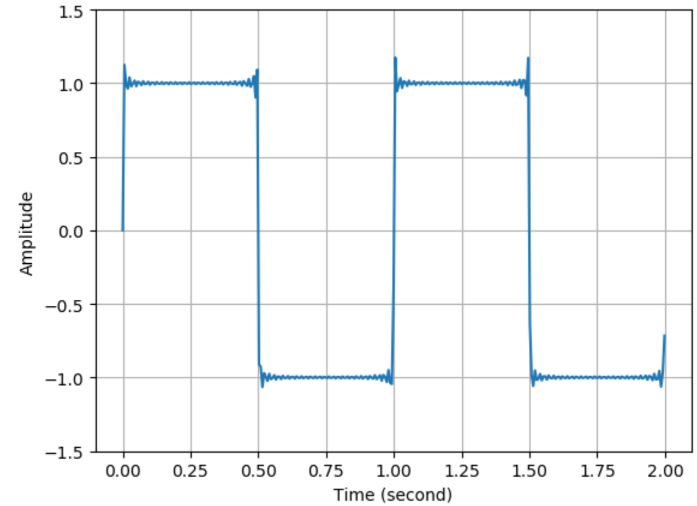


What about the square wave?

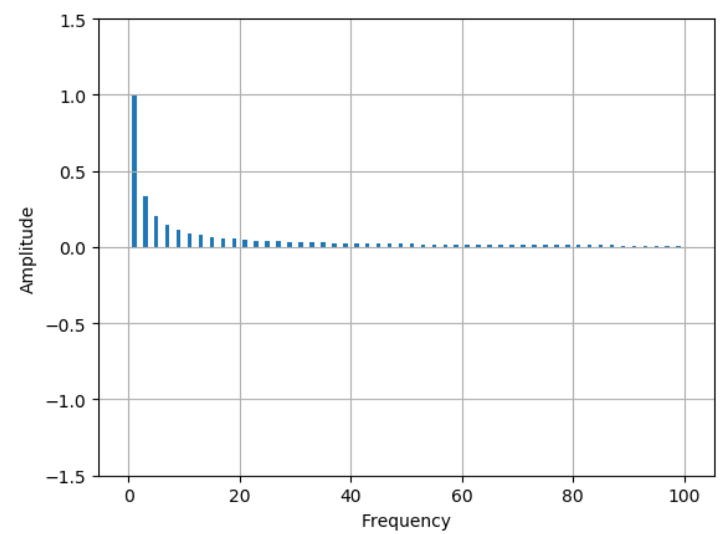


Fourier series of square wave

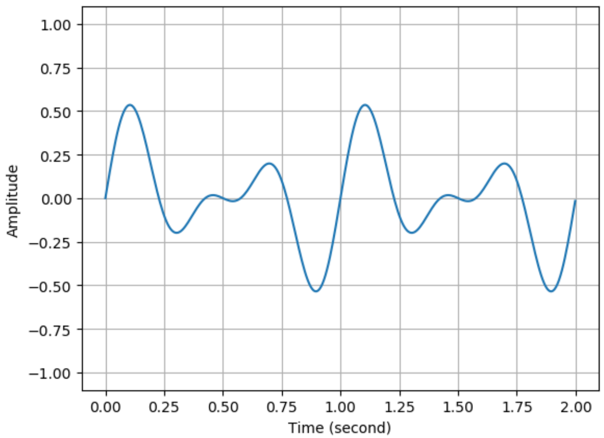


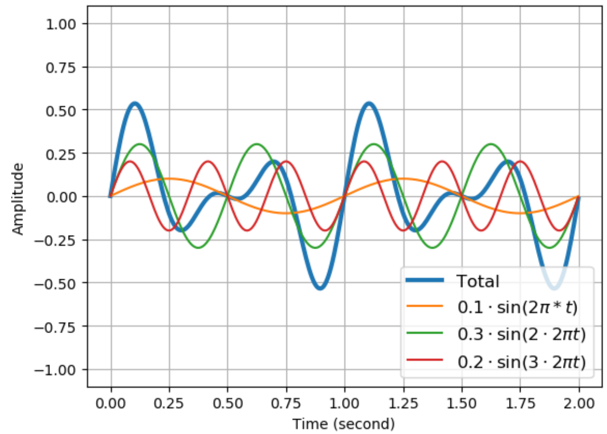


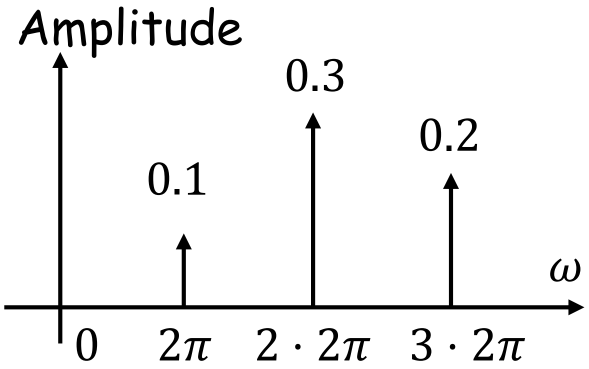
Sum of 50 sines



Frequency-domain plot

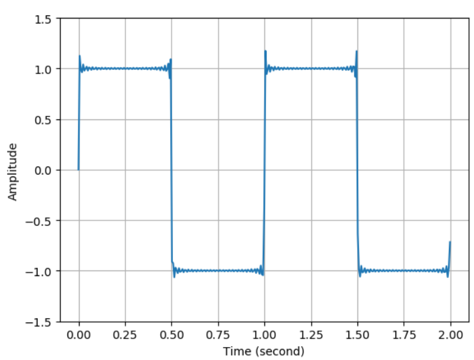


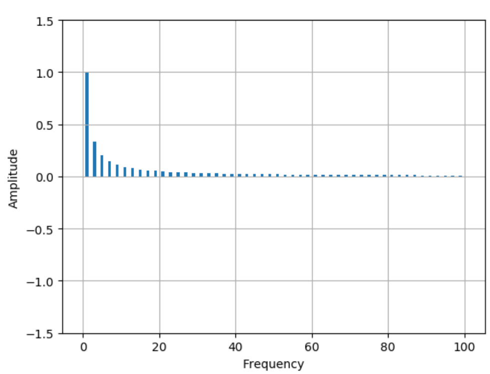




$$g(t) = 0.1 \cdot \sin(2\pi t) + 0.3 \cdot \sin(2 \cdot 2\pi t) + 0.2 \cdot \sin(3 \cdot 2\pi t)$$

When the total signal can be decomposed into multiple sinusoidal signal as shown above, we can plot the amplitude as a function of frequency.



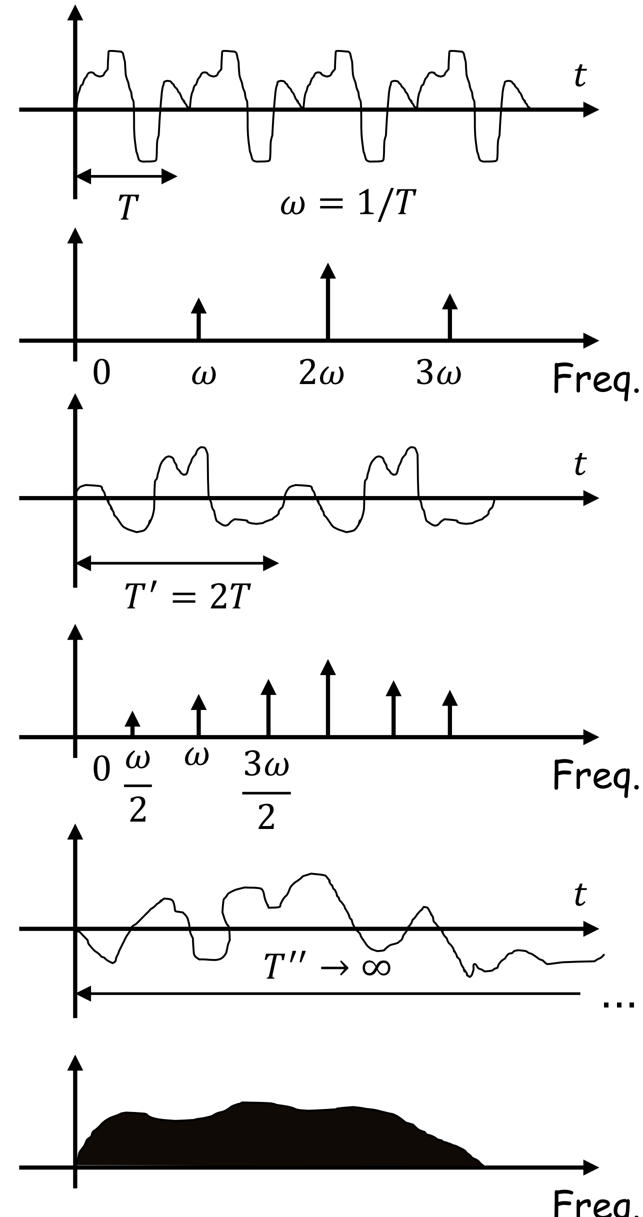


Equalizer

124
PL-02
model 537
GRAPHIC EQUALIZER
model 562 FEEDBACK SUPPRESSOR

Fourier Series vs Fourier Transform

- If the signal is periodic, the frequency components are separated by the fundamental frequency. E.g. if the period is T, fundamental frequency is $\omega = 2\pi/T$, and all the higher-order harmonics are separated by ω.
- If the period is doubled, the frequency components will be separated by $\omega/2$.
- If the period is increased infinitely, the frequency components will become continuous. ➔ Fourier transform.



Simultaneous transmission of two signals

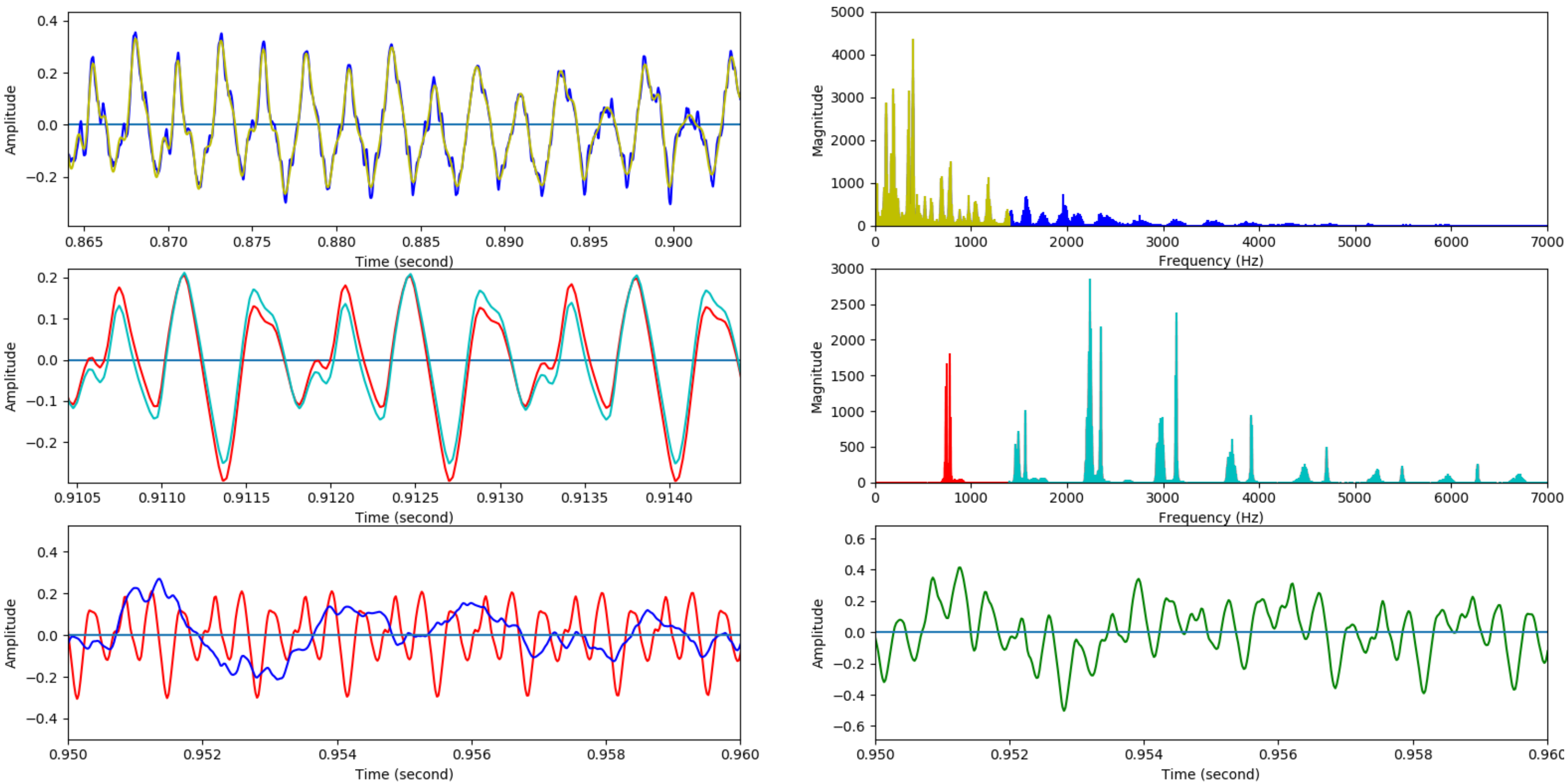
- What is the benefit of looking at the signal in frequency-domain?
- Example: low frequency signal and high frequency signal

Change of amplitude in time

Frequency components

Sound of double bass

Sound of violin



Plotted together

Sum of two signals