과제 #5

<u>마감일:</u> 11월 21일 9시 30분

<u>제출방법:</u>

- **강의실 교탁**

주의사항:

- **숙제를 베껴 내면 관련된 모든 학생에게 불이익이 있습니다.**
- **마감일시를 반드시 준수.**

<u>Problem 1</u> Consider the ionization of atomic hydrogen, governed by the equation

$$\mathrm{H} \leftrightarrow \mathrm{p}^{+} + \mathrm{e}^{-}$$

, where p^{+} is a proton (equivalently a positively ionized hydrogen) and e^{-} is an electron.

(a) Explain why $\mu_{\mathrm{H}} = \mu_{\mathrm{p}} + \mu_{\mathrm{e}}$, where μ is the chemical potential of hydrogen, proton and electron, respectively.

(b) Using the partition function for hydrogen atoms, show that

$$-k_B T \ln \frac{Z_1^{p}}{N_p} - k_B T \ln \frac{Z_1^{e}}{N_e} = -k_B T \ln \frac{Z_1^{H}}{N_H} e^{\beta R}$$

, where Z_1^{x} and N_x are the single-particle partition function and number of particles for species x, respectively and R=13.6 eV.

(c) Show that $\frac{n_e n_p}{n_H} = \frac{(2\pi m_e k_B T)^{3/2}}{h^3} e^{-\beta R}$, where $n_x = N_x/V$ is the number density of species x. This relation is known as the Saha equation.

<u>Problem 2</u> Suppose that we have the following hypothetical chemical reaction.

$$2A_1 \rightleftharpoons 2A_2 + A_3$$

The systems of A_1, A_2, and A_3 molecules are kept at a constant temperature of T and a pressure of P. If we denote by N_i the number of molecules of type i, the Gibbs free energy of this system is then a function of these numbers so that $G=G(N_1,N_2,N_3)$. Show the following relationship in equilibrium

$$2\mu_1 = 2\mu_2 + \mu_3$$

where $\mu_i = \frac{\partial G}{\partial N_i}$ is the chemical potential of the type i.

Problem 3 Consider two identical bodies, A_1 and A_2, each characterized by a heat capacity C, which is temperature-dependent. The bodies are initially at temperature T_1 and T_2, respectively, where $T_1>T_2$. It is desired to operate an engine between A_1 and A_2 so as to convert some of their internal energy into work. As a result of the operation of the engine, the bodies ultimately will attain a common final temperature T_f.

(a) What is the total amount of work W done by the engine? Express your answer in terms of C, T_1, T_2 and T_f.

(b) Use arguments based upon entropy consideration to derive an inequality relating Tf to the intial temperature T_1 and T_2.

(c) For given initial temperature T_1 and T_2, what is the maximum amount of work obtainable from the engine?