8.1 *Estimate the theoretical fracture strength of a brittle material if it is known that fracture occurs by the propagation of an elliptically shaped surface crack of length 0.5 mm and a tip radius of curvature of 5 × 10^{-3} mm, when a stress of 1035 MPa is applied.*

Solution

In order to estimate the theoretical fracture strength of this material it is necessary to calculate σ_m using Equation 8.1 given that σ_0 = 1035 MPa, a = 0.5 mm, and $\rho_t = 5 \times 10^{-3}$ mm. Thus,

$$\sigma_m = 2\sigma_0 \left(\frac{a}{\rho_t}\right)^{1/2}$$

$$= (2)(1035\ \text{MPa})\left(\frac{0.5\ \text{mm}}{5 \times 10^{-3}\ \text{mm}}\right)^{1/2}$$

$$= 2.07 \times 10^4\ \text{MPa} = 20.7\ \text{GPa}$$

8.4 *Suppose that a wing component on an aircraft is fabricated from an aluminum alloy that has a plane-strain fracture toughness of 26.0 MPa $\sqrt{m}$. It has been determined that fracture results at a stress of 112 MPa when the maximum internal crack length is 8.6 mm. For this same component and alloy, compute the stress level at which fracture will occur for a critical internal crack length of 6.0 mm.*

Solution

This problem asks us to determine the stress level at which a wing component on an aircraft will fracture for a given fracture toughness $(26\ \text{MPa}\sqrt{\text{m}})$ and maximum internal crack length (6.0 mm), given that fracture occurs for the same component using the same alloy at one stress level (112 MPa) and another internal crack length (8.6 mm). (*Note:* Because the cracks are internal, their lengths are equal to 2a.) It first becomes necessary to solve for the parameter Y for the conditions under which fracture occurred using Equation 8.5. Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}}$$

$$= \frac{26\ \text{MPa}\sqrt{\text{m}}}{(112\ \text{MPa})\sqrt{(\pi)\left(\frac{8.6 \times 10^{-3}\ \text{m}}{2}\right)}} = 2.0$$

Now we will solve for σ_c (for a crack length of 6 mm) using Equation 8.6 as

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{26\ \text{MPa}\sqrt{\text{m}}}{(2.0)\sqrt{(\pi)\left(\frac{6 \times 10^{-3}\ \text{m}}{2}\right)}} = 134\ \text{MPa}$$

8.12 *The fatigue data for a brass alloy are given as follows:*

Stress Amplitude (MPa)	***Cycles to Failure***
170	3.7×10^4
148	1.0×10^5
130	3.0×10^5
114	1.0×10^6
92	1.0×10^7
80	1.0×10^8
74	1.0×10^9

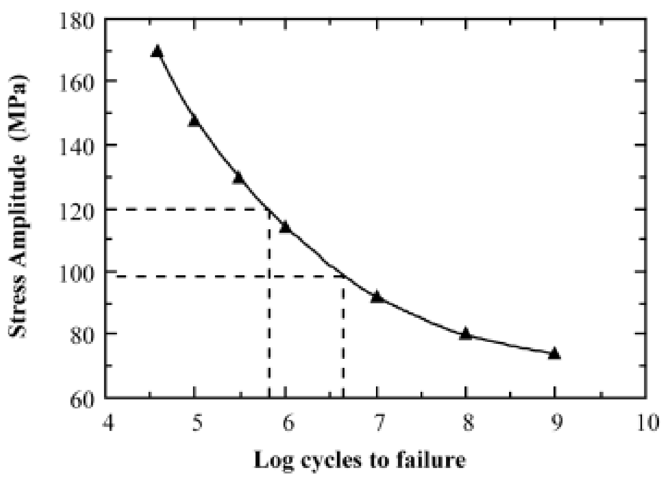
(a) Make an S–N plot (stress amplitude versus logarithm of cycles to failure) using these data.

(b) Determine the fatigue strength at 4×10^6 cycles.

(c) Determine the fatigue life for 120 MPa.

Solution

(a) The fatigue data for this alloy are plotted below.



(b) As indicated by one set of dashed lines on the plot, the fatigue strength at 4×10^6 cycles [log (4×10^6) = 6.6] is about 100 MPa.

(c) As noted by the other set of dashed lines, the fatigue life for 120 MPa is about 6×10^5 cycles (i.e., the log of the lifetime is about 5.8).