2023-2학기 전자기파와 광학 HW1, Due: 11:59 pm Sep 21 (eTL upload)

Fine name: NAME_ID_HW#, e.g. 홍길동_20230101_HW#

1. Here are two vectors **A** and **B**. $\mathbf{A} = \mathbf{x} - 2\mathbf{y} + 3\mathbf{z}$ and $\mathbf{B} = \mathbf{x} + \mathbf{y} - 2\mathbf{z}$. What is a unit vector that is perpendicular both to **A** and **B**?

2. Based on the definition of the vector Laplacian from the relation,

   $\nabla \times (\nabla \times \boldsymbol{V}) = \nabla(\nabla \cdot \mathbf{V}) - \nabla \cdot \nabla\mathbf{V}$, explicitly calculate all the components of the vector Laplacian in (answers can be found here: https://mathworld.wolfram.com/VectorLaplacian.html)

   A. Cylindrical coordinate system in 3D

   B. Spherical coordinate system in 3D

3. Show the following properties of the delta function.

   A. $\delta(ax) = \frac{1}{a}\delta(x)$ for $a > 0$

   B. $\int_{-\infty}^{\infty} \delta(x - x_0) f(x) dx = f(x_0)$

   C. $\delta(g(x)) = \sum_i \frac{\delta(x - a_i)}{|g'(a_i)|}$ where a_i are simple zeros of $g(x)$ on the real axis, that is, $g(a_i) = 0$, but $g'(a_i) \neq 0$

   D. $\int_{-\infty}^{\infty} f(x)\delta'(x - x_0) dx = -f'(x_0)$

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4. **Griffiths Problem 7.19 (4th ed. Pg 325)**

Problem 7.19 A toroidal coil has a rectangular cross section, with inner radius a, outer radius $a + w$, and height h. It carries a total of N tightly wound turns, and the current is increasing at a constant rate ($dI/dt = k$). If w and h are both much less than a, find the electric field at a point z above the center of the toroid. [*Hint:* Exploit the analogy between Faraday fields and magnetostatic fields, and refer to Ex. 5.6.]

5. **Griffiths Problem 7.32 (4th ed. Pg 336)**

Problem 7.32 Two tiny wire loops, with areas $\mathbf{a}_1$ and $\mathbf{a}_2$, are situated a displacement $\boldsymbol{\imath}$ apart (Fig. 7.42).

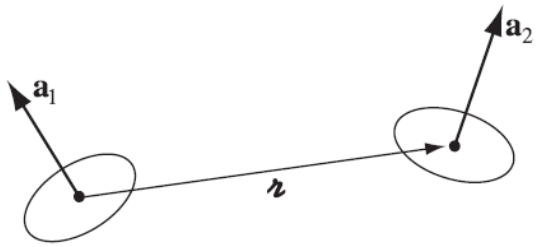

FIGURE 7.42

(a) Find their mutual inductance. [*Hint:* Treat them as magnetic dipoles, and use Eq. 5.88.] Is your formula consistent with Eq. 7.24?

(b) Suppose a current I_1 is flowing in loop 1, and we propose to turn on a current I_2 in loop 2. How much work must be done, against the mutually induced emf, to keep the current I_1 flowing in loop 1? In light of this result, comment on Eq. 6.35.

6. **Griffiths Problem 8.17 (4th ed. Pg 383) [(a) 까지만 풀으세요]**

Problem 8.17[18] Picture the electron as a uniformly charged spherical shell, with charge e and radius R, spinning at angular velocity ω.

(a) Calculate the total energy contained in the electromagnetic fields.