과제 #2

<u>마감일:</u> 10월 5일 9시 30분

<u>제출방법:</u>

- **강의실 교탁**

주의사항:

- **숙제를 베껴 내면 관련된 모든 학생에게 불이익이 있습니다.**
- **마감일시를 반드시 준수.**

<u>Problem 1</u> The probability W(n) that an event characterised by a probability p occurs n times in N trials was shown to be given by the binomial distribution

$$\mathrm{W(n)} = \frac{N!}{n!\,(N-n)!} p^n (1-p)^{N-n}$$

Consider a situation where the probability p is small (p<<1) and where one is interested in the case n<<N. (Note that if N is large, W(n) becomes very small if N-> N because of the smallness of the factor p^n when p<<1. Hence W(n) is indeed only appreciable when n<<N.) Several approximations can then be made to reduce W(n) to simpler form called Poisson distribution.

(a) Using the result $\ln(1-\mathrm{p}) \approx -\mathrm{p}$, show that $(1-p)^{N-n} \approx e^{-Np}$.

(b) Show that $\frac{N!}{(N-n)!} \approx N^n$.

(c) Using the two results, show that W(n) can be reduced to $\mathrm{W(n)} = \frac{\lambda^n}{n!} e^{-\lambda}$, where λ=Np is the mean number of events.

(d) Show that this formula of the Poisson distribution is normalized, i.e. $\sum_{n=0}^{N} W(n) = 1$.

(e) Using the Poisson distribution, calculate the mean value of n.

(f) Using the Poisson distribution, calculate the dispersion of n.

<u>Problem 2</u> A man starts out a lamppost in the middle of a street, taking steps of equal length l. The probability is p that any one of his steps is to the right and q=1-p that it is to the left. The man is so drunk that his behavior at any step shows no traces of memory of what he did at preceding steps. His steps are thus statistically independent. Suppose that the man has taken N steps.

(a) What is the probability P(n) that n of these steps are to the right and the remaining n′=N-n steps are to the left?

(b) What is the probability P′(m) that the displacement of the man from the lamppost is equal to ml, where m=n-n′ is an integer?

(c) Suppose now that p=q so that each step is equally likely to be to the right or to the left. What is the probability that the man will again be at the lamppost after taking N stemps. Give your answer for the two cases of N being either even or odd.

Problem 3 Solve the following problems.

(a) Let's consider a physical quantity of u. Show that $\overline{u^2} \geq \bar{u}^2$.

The magnetic moment of a spin 1/2 is such that its component μ in the up direction has probability p of being equal to μ_0, and probability q=1-p of being equal to - μ_0.

(b) Calculate $\bar{\mu}$ and $\overline{\mu^2}$.

(c) Calculate the dispersion.

Problem 4 The displacement x of a classical simple harmonic oscillator as a function of the time t is given by $\mathrm{x} = \mathrm{A}\cos(\omega t + \varphi)$, where ω is the angular frequency of the oscillator, A is its amplitude of oscillator, and φ is an arbitrary constant which can have any value in the range $0 \leq \varphi \leq 2\pi$. Suppose that one contemplates an ensemble of such oscillators all of which have the frequency ω and amplitude A, but which have random phase relationships so that the probability that φ lies in the range between φ and $\varphi + d\varphi$ is given simply by $d\varphi/2\pi$. Find the probability P(x)dx that the displacement of an oscillator, at any given time t, is found to be lie in the range between x and x+dx.