Lecture

- Electrodynamics and relativity

The special theory of relativity
Relativistic mechanics
Relativistic electrodynamics

Chap. 12

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Relativity mechanics

- **Relativistic dynamics**

$p^{\mu} \equiv m\eta^{\mu}$ Recall relativistic momentum → spatial part of 4-vector

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$ Minkowski space?

First, start with an easy example, e.g. constant force

Relativity mechanics

- **Relativistic dynamics**

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$ Newton's second law is valid if use the relativistic momentum

Example 12.10. Motion under a constant force. A particle of mass m is subject to a constant force F. If it starts from rest at the origin, at time $t = 0$, find its position (x), as a function of time.

$$\frac{dp}{dt} = F \quad \Rightarrow \quad p = Ft + \text{constant},$$

$$p = \frac{mu}{\sqrt{1 - u^2/c^2}} = Ft.$$

$$u = \frac{(F/m)t}{\sqrt{1 + (Ft/mc)^2}}.$$

Relativity mechanics

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$$x(t) = \frac{F}{m}\int_0^t \frac{t'}{\sqrt{1+(Ft'/mc)^2}}\,dt'$$
$$= \frac{mc^2}{F}\sqrt{1+(Ft'/mc)^2}\Big|_0^t = \frac{mc^2}{F}\left[\sqrt{1+(Ft/mc)^2} - 1\right].$$

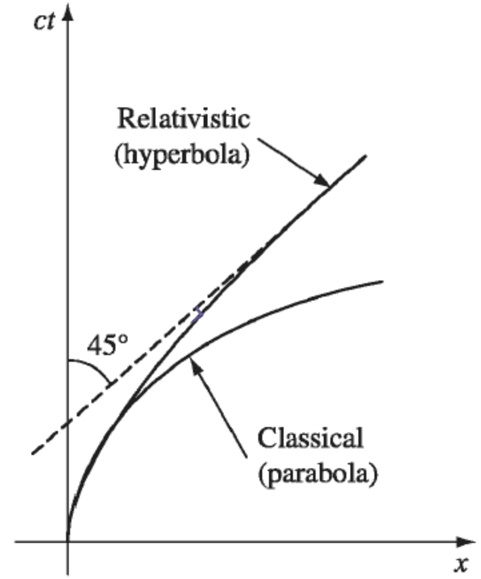

FIGURE 12.30

Relativity mechanics

- **Relativistic dynamics**

Work, energy, (Minkowski) force

Work $W \equiv \int \mathbf{F} \cdot d\mathbf{l}$ Work, as always, is the line integral of the force:

work-energy theorem $$W = \int \frac{d\mathbf{p}}{dt} \cdot d\mathbf{l} = \int \frac{d\mathbf{p}}{dt} \cdot \frac{d\mathbf{l}}{dt}\, dt = \int \frac{d\mathbf{p}}{dt} \cdot \mathbf{u}\, dt.$$

$$\frac{d\mathbf{p}}{dt} \cdot \mathbf{u} = \frac{d}{dt}\left(\frac{m\mathbf{u}}{\sqrt{1-u^2/c^2}}\right) \cdot \mathbf{u}$$

$$= \frac{m\mathbf{u}}{(1-u^2/c^2)^{3/2}} \cdot \frac{d\mathbf{u}}{dt} = \frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-u^2/c^2}}\right) = \frac{dE}{dt}$$

$$W = \int \frac{dE}{dt}\, dt = E_{\text{final}} - E_{\text{initial}}$$

Relativity mechanics

- **Relativistic dynamics**

Work, energy, (Minkowski) force

$$\bar{F}_y = \frac{d\bar{p}_y}{d\bar{t}} = \frac{dp_y}{\gamma\, dt - \dfrac{\gamma\beta}{c}\, dx} = \frac{dp_y/dt}{\gamma\left(1 - \dfrac{\beta}{c}\dfrac{dx}{dt}\right)} = \frac{F_y}{\gamma(1 - \beta u_x/c)}$$

$$\bar{F}_z = \frac{F_z}{\gamma(1 - \beta u_x/c)}$$

$$\bar{F}_x = \frac{d\bar{p}_x}{d\bar{t}} = \frac{\gamma\, dp_x - \gamma\beta\, dp^0}{\gamma\, dt - \dfrac{\gamma\beta}{c}\, dx} = \frac{\dfrac{dp_x}{dt} - \beta\dfrac{dp^0}{dt}}{1 - \dfrac{\beta}{c}\dfrac{dx}{dt}} = \frac{F_x - \dfrac{\beta}{c}\left(\dfrac{dE}{dt}\right)}{1 - \beta u_x/c}$$

$$\bar{F}_x = \frac{F_x - \beta(\mathbf{u}\cdot\mathbf{F})/c}{1 - \beta u_x/c}$$

$$\bar{\mathbf{F}}_\perp = \frac{1}{\gamma}\mathbf{F}_\perp, \quad \bar{F}_\parallel = F_\parallel$$

the component of $\mathbf{F}$ *parallel* to the motion of $\bar{S}$ is unchanged

perpendicular components are divided by γ

Relativity mechanics

- **Relativistic dynamics**

Work, energy, (Minkowski) force

$$\bar{\mathbf{F}}_{\perp} = \frac{1}{\gamma}\mathbf{F}_{\perp}, \quad \bar{F}_{\|} = F_{\|}$$

Issue :

Q: How to define 4-force vector? General vectors follows Lorentz transformation.

Relativity mechanics

- **Relativistic dynamics**

Work, energy, (Minkowski) force

using proper force (like proper velocity) derivative of momentum with respect to *proper* time:

Minkowski force $K^{\mu} \equiv \dfrac{dp^{\mu}}{d\tau}$ 4-vector, since p^{μ} is a 4-vector and proper time is invariant.

Inspection:

Next question : $\tilde{F}$ and K^{μ} relationship?

Relativity mechanics

- **Relativistic dynamics**

Work, energy, (Minkowski) force

using proper force (like proper velocity)

Minkowski force $K^\mu \equiv \dfrac{dp^\mu}{d\tau}$ 4-vector, since p^μ is a 4-vector and proper time is invariant.

spatial components of K^μ are related to the

$$\mathbf{K} = \left(\frac{dt}{d\tau}\right)\frac{d\mathbf{p}}{dt} = \frac{1}{\sqrt{1-u^2/c^2}}\mathbf{F},$$

$$K^0 = \frac{dp^0}{d\tau} = \frac{1}{c}\frac{dE}{d\tau}$$

$\mathbf{F} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$, or should it rather be $\mathbf{K} = q(\mathbf{E} + \mathbf{u} \times \mathbf{B})$?

Relativity mechanics

- **Relativistic dynamics**

Work, energy, (Minkowski) force

Example 12.11. The typical trajectory of a charged particle in a uniform magnetic field is **cyclotron motion** (Fig. 12.31). The magnetic force pointing toward the center,

$$F = QuB$$

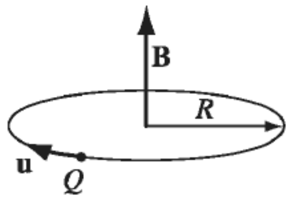

FIGURE 12.31

$$dp = p\,d\theta \qquad F = \frac{dp}{dt} = p\frac{d\theta}{dt} = p\frac{u}{R}$$

$$QuB = p\frac{u}{R} \rightarrow p = QBR.$$

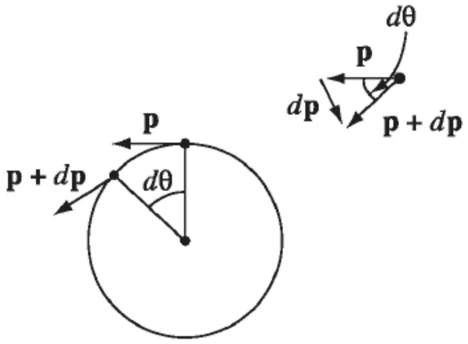

FIGURE 12.32

Relativity mechanics

- **Relativistic dynamics**

Q: what if there are many particles?
center of mass momentum?

In classical mechanics, the total momentum (**P**) $\mathbf{P} = M\frac{d\mathbf{R}_m}{dt}$

the center-of-mass ($\mathbf{R}_m = \frac{1}{M}\sum m_i \mathbf{r}_i$) is replaced by the **center-of-energy** ($\mathbf{R}_e = \frac{1}{E}\sum E_i \mathbf{r}_i$, where E is the total energy), and M by E/c^2:

$$\mathbf{P} = \frac{E}{c^2}\frac{d\mathbf{R}_e}{dt}$$