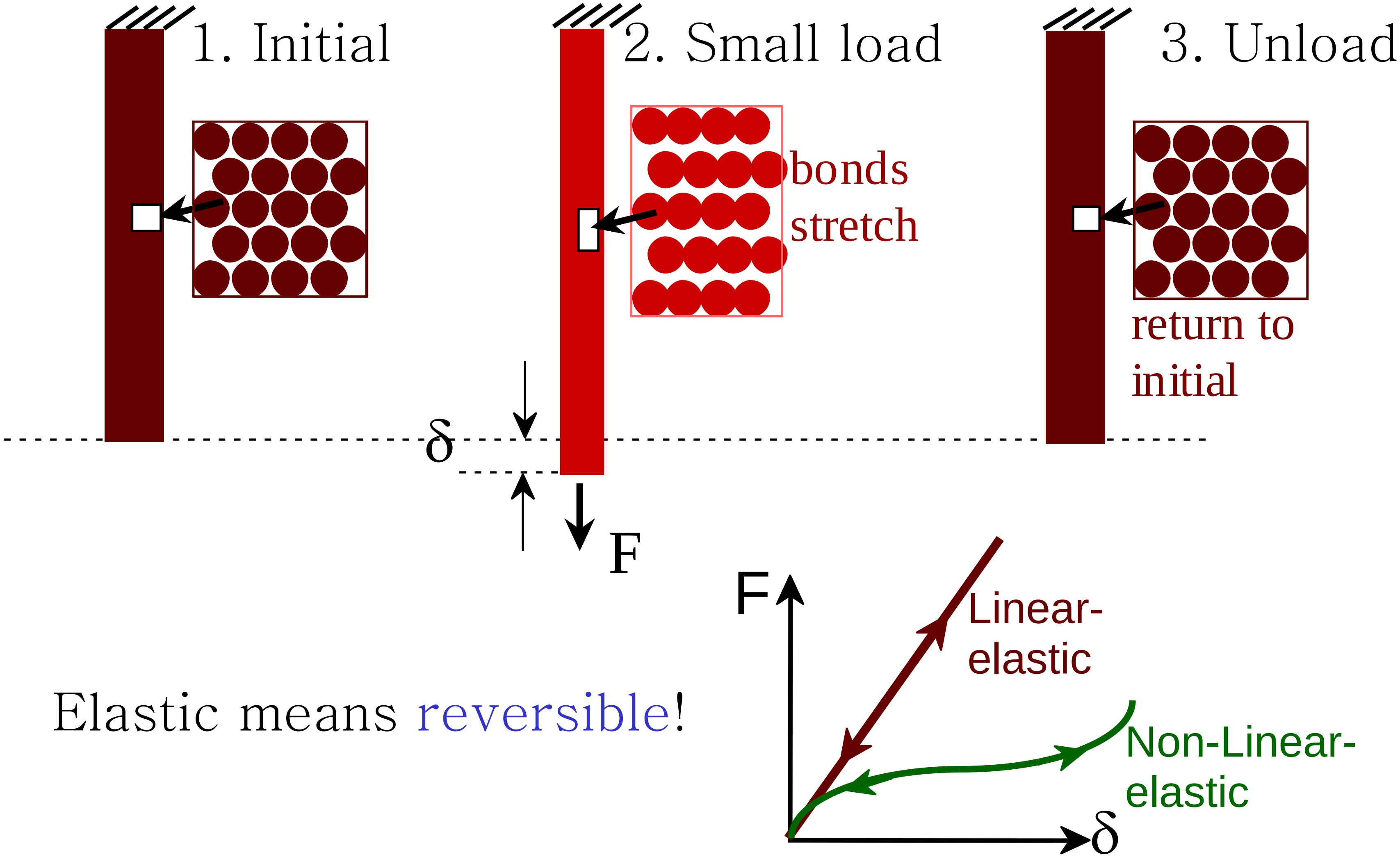
6. Mechanical Properties

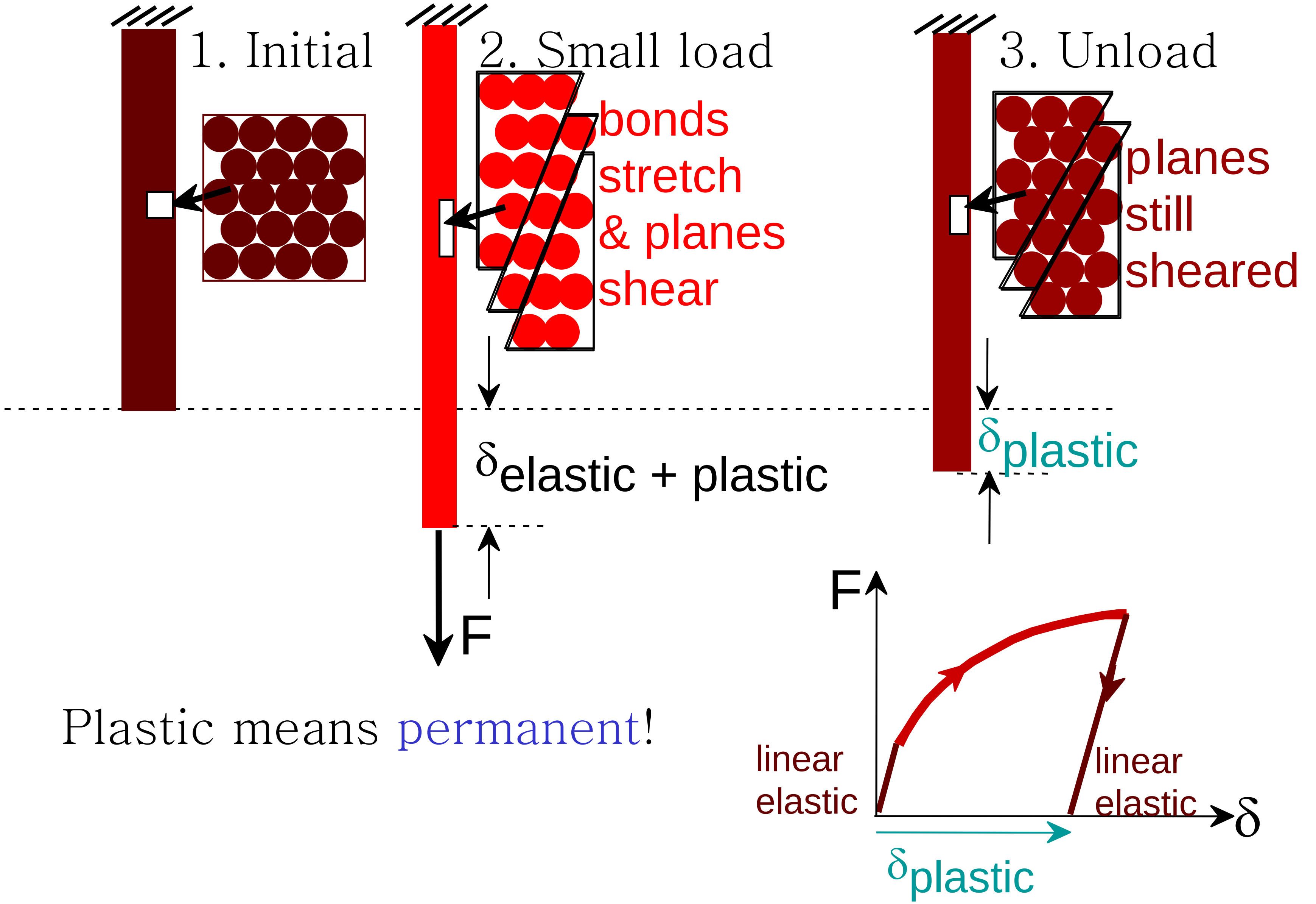
- stress and strain
- elastic deformation
- plastic deformation
- Hardness

Elastic Deformation



Elastic means reversible!

Plastic Deformation



Plastic means permanent!

Stress-Strain Test

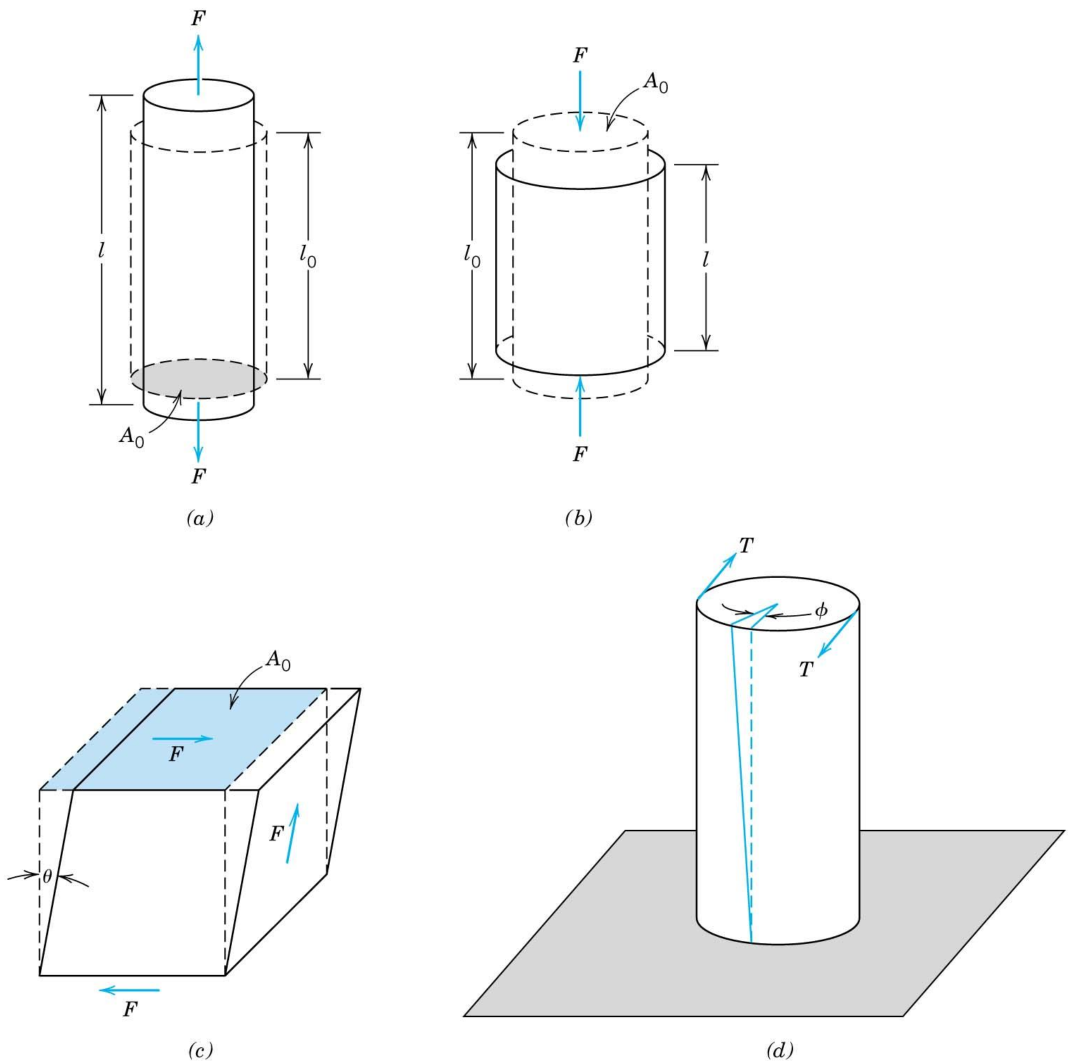
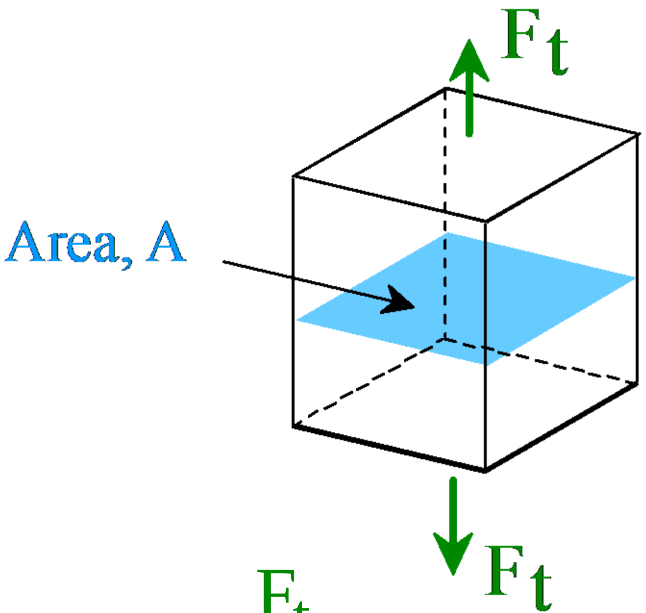


FIGURE 6.1 (*a*) Schematic illustration of how a tensile load produces an elongation and positive linear strain. Dashed lines represent the shape before deformation; solid lines, after deformation. (*b*) Schematic illustration of how a compressive load produces contraction and a negative linear strain. (*c*) Schematic representation of shear strain γ, where $\gamma = \tan\theta$. (*d*) Schematic representation of torsional deformation (i.e., angle of twist ϕ) produced by an applied torque T.

Engineering Stress

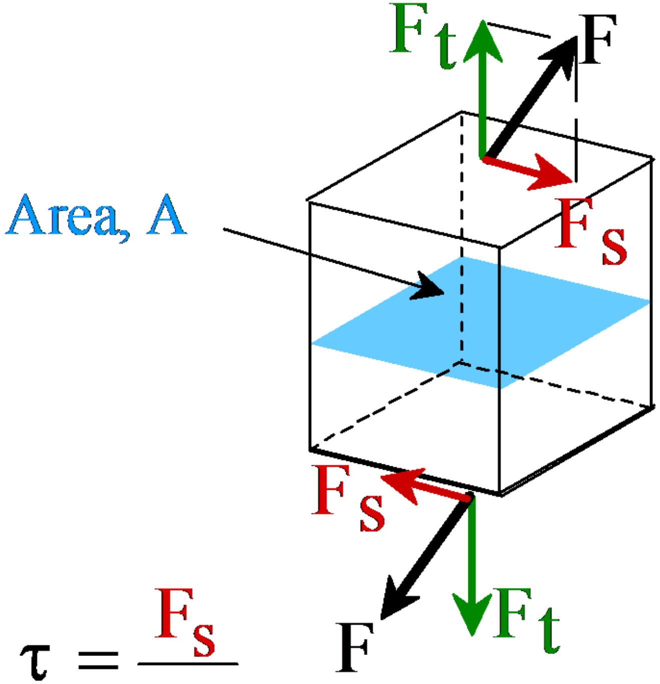
- Tensile stress, σ:



$$\sigma = \frac{F_t}{A_o}$$

original area before loading

- Shear stress, τ:



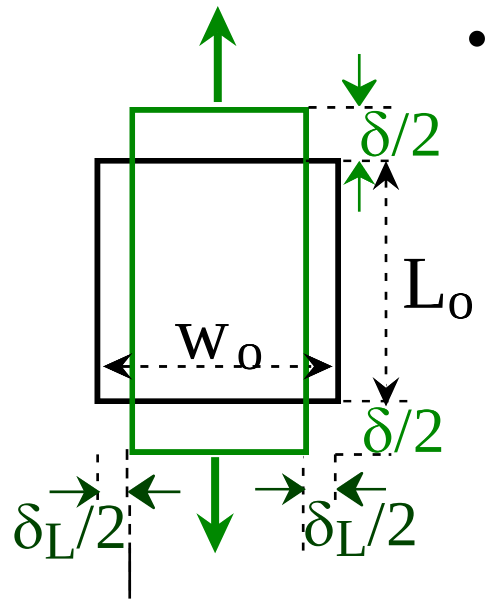
$$\tau = \frac{F_s}{A_o}$$

stress has units: N/m^2 or lb/in^2

Engineering Strain

- Tensile strain:

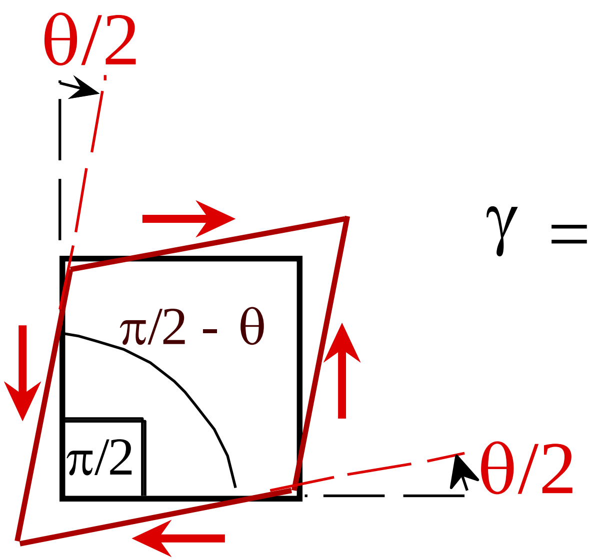
$$\varepsilon = \frac{\delta}{L_o}$$



- Lateral strain:

$$\varepsilon_L = \frac{-\delta_L}{w_o}$$

- Shear strain:

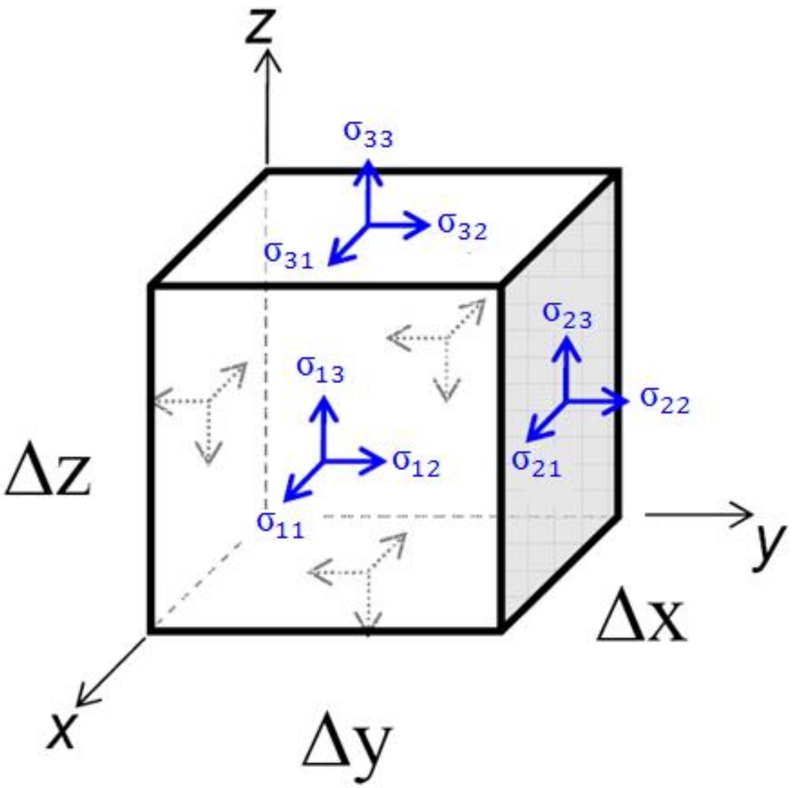


$$\gamma = \tan\ \theta$$

strain is always dimensionless.

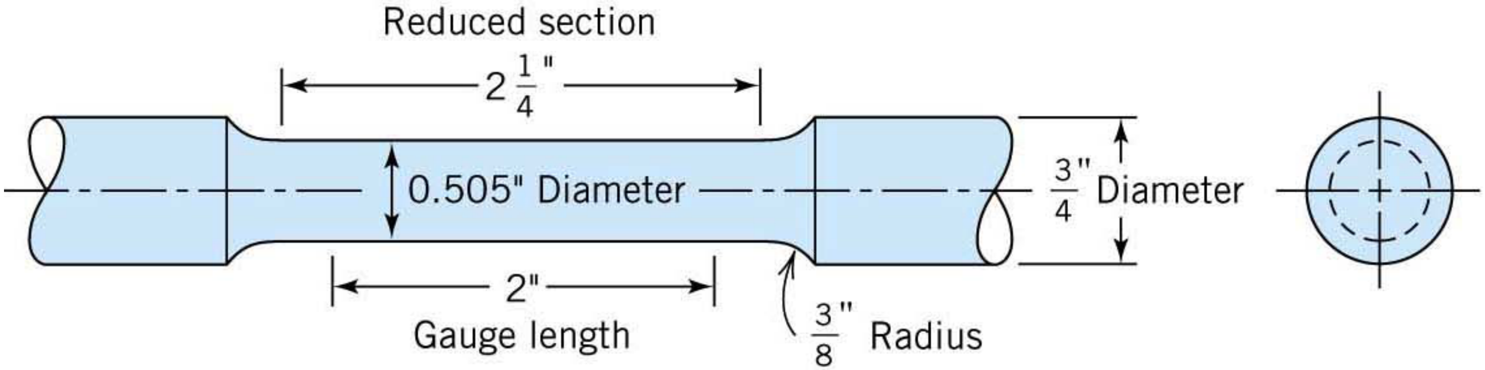
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{bmatrix}$$

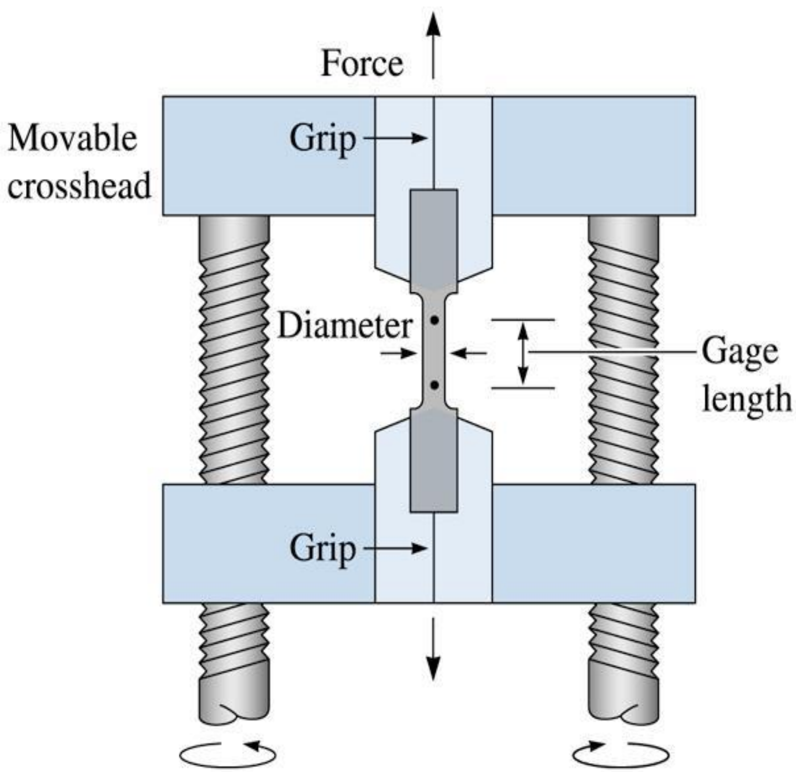


Stress-Strain Testing

- typical tensile specimen



- typical tensile test machine



Linear Elastic Properties

- Modulus of Elasticity, E:
  (also known as Young's modulus)
- Hooke's Law:

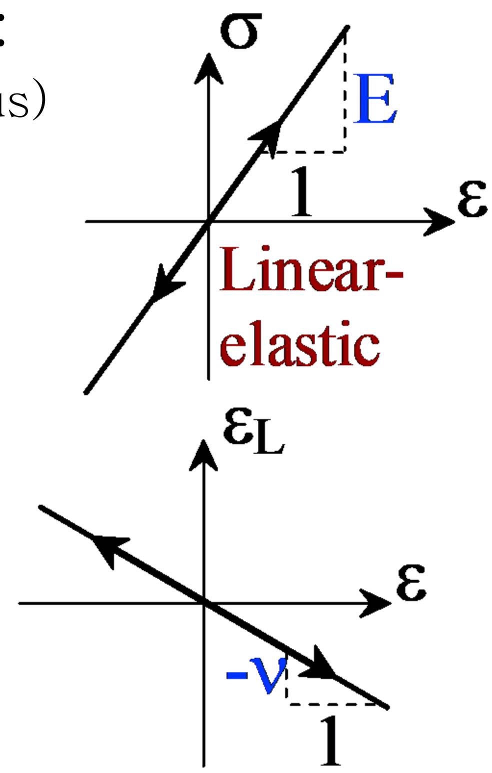
$$\sigma = E\,\varepsilon$$

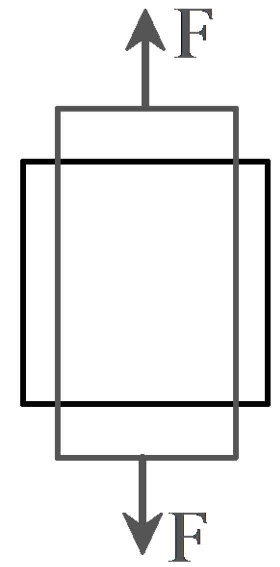
- Poisson's ratio, ν:

$$\nu = -\frac{\varepsilon_L}{\varepsilon}$$

metals: $\nu \sim 0.33$
ceramics: ~ 0.25
polymers: ~ 0.40

Units:
E: [GPa] or [psi]
ν: dimensionless

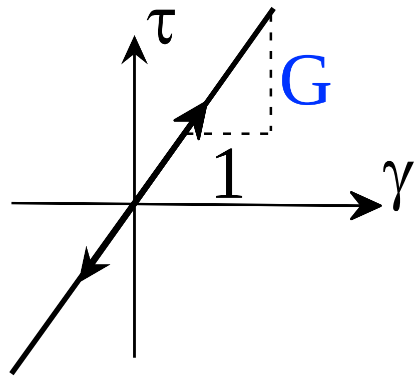




Other Elastic Properties

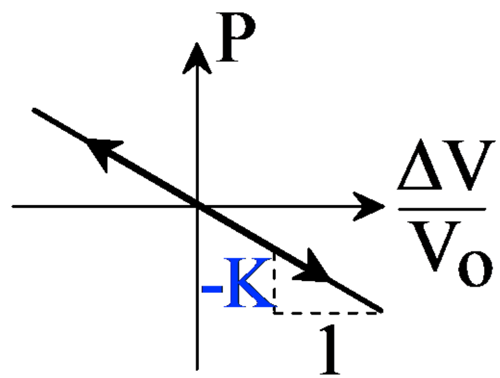
- Elastic Shear modulus, G:

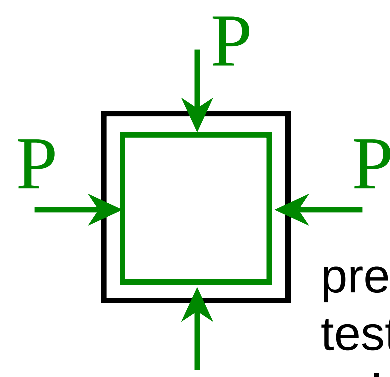
$$\tau = \mathbf{G}\,\gamma$$



- Elastic Bulk modulus, K:

$$P = -K\frac{\Delta V}{V_o}$$





pressure test: Init. vol $=V_o$. Vol chg. $=\Delta V$

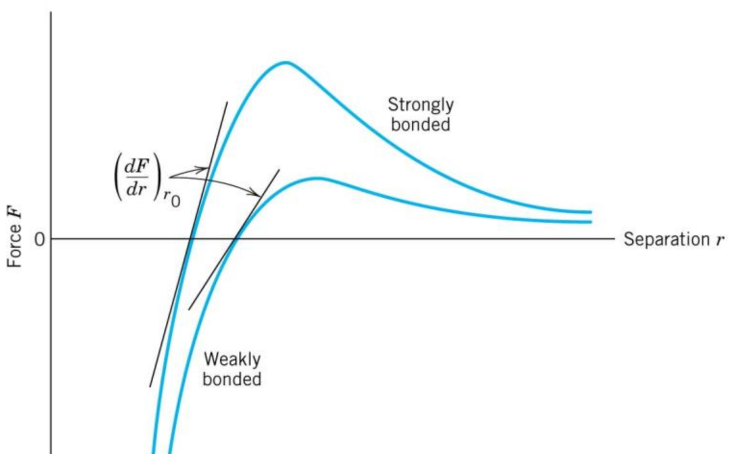
- Special relations for isotropic materials:

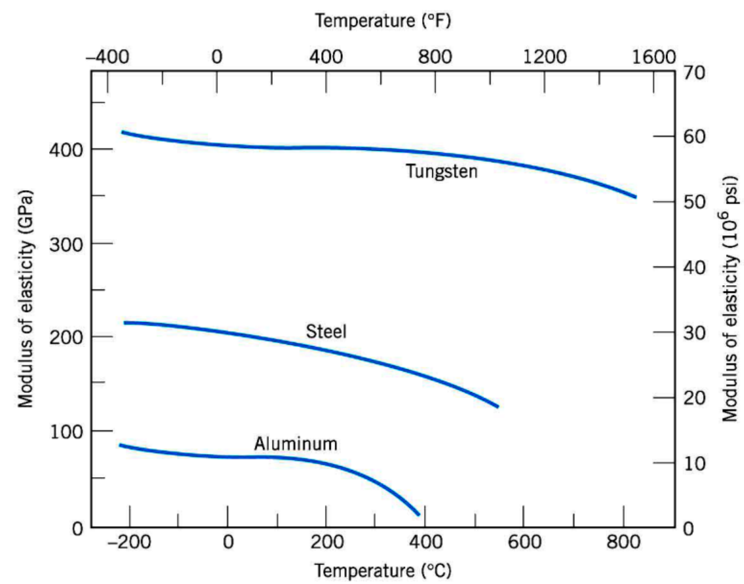
$$G = \frac{E}{2(1+\nu)} \qquad K = \frac{E}{3(1-2\nu)}$$

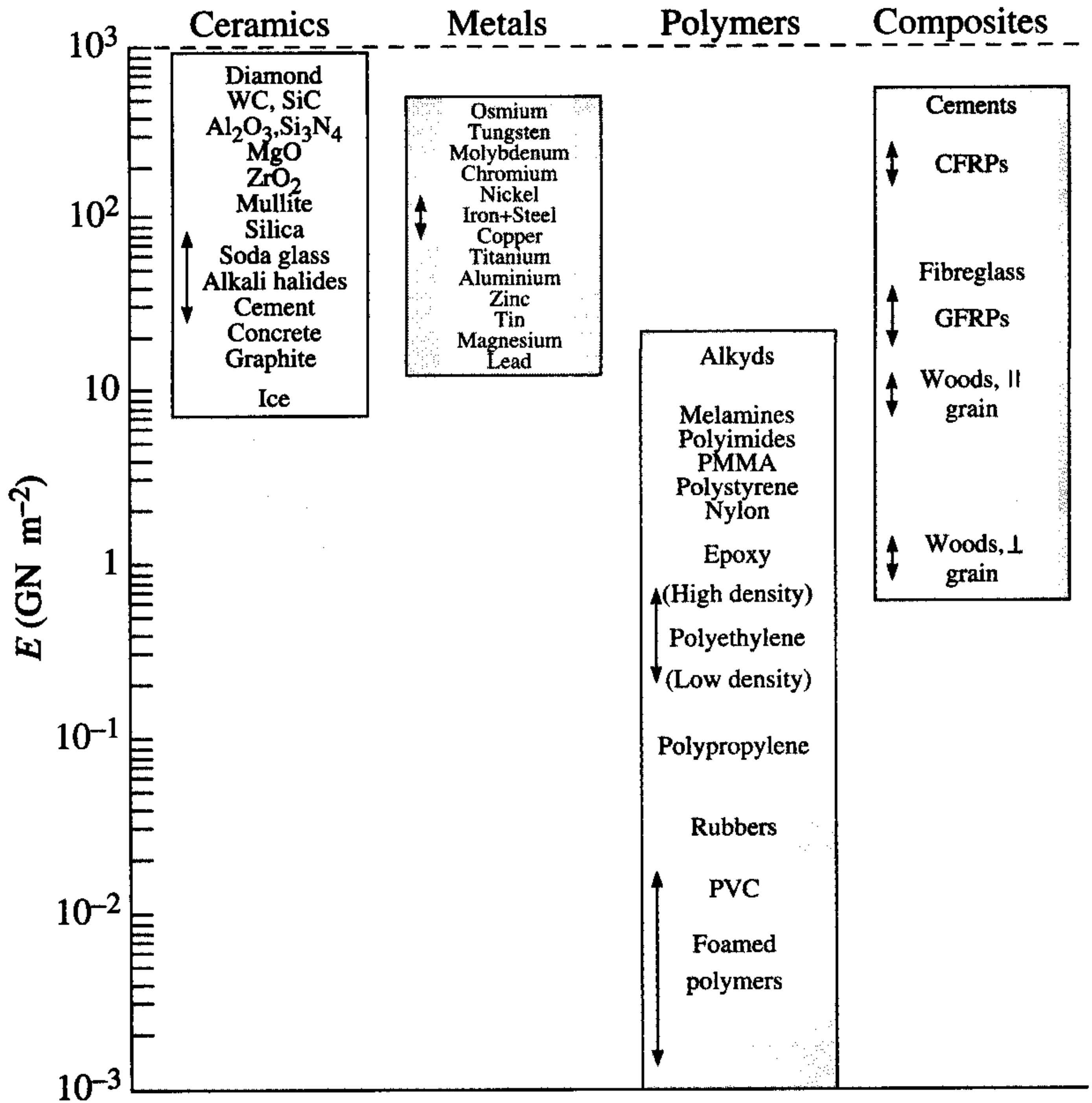
Table 6.1 Room-Temperature Elastic and Shear Moduli, and Poisson's Ratio for Various Metal Alloys

Metal Alloy	*Modulus of Elasticity* GPa	*Modulus of Elasticity* 10^6 psi	*Shear Modulus* GPa	*Shear Modulus* 10^6 psi	*Poisson's Ratio*
Aluminum	69	10	25	3.6	0.33
Brass	97	14	37	5.4	0.34
Copper	110	16	46	6.7	0.34
Magnesium	45	6.5	17	2.5	0.29
Nickel	207	30	76	11.0	0.31
Steel	207	30	83	12.0	0.30
Titanium	107	15.5	45	6.5	0.34
Tungsten	407	59	160	23.2	0.28

FIGURE 6.7 Force versus interatomic separation for weakly and strongly bonded atoms. The magnitude of the modulus of elasticity is proportional to the slope of each curve at the equilibrium interatomic separation r_0.






Ceramics
Metals
Polymers
Composites
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
E (GN m^{-2})
Diamond
WC, SiC
Al_2O_3,Si_3N_4
MgO
ZrO_2
Mullite
Silica
Soda glass
Alkali halides
Cement
Concrete
Graphite
Ice
Osmium
Tungsten
Molybdenum
Chromium
Nickel
Iron+Steel
Copper
Titanium
Aluminium
Zinc
Tin
Magnesium
Lead
Alkyds
Melamines
Polyimides
PMMA
Polystyrene
Nylon
Epoxy
(High density)
Polyethylene
(Low density)
Polypropylene
Rubbers
PVC
Foamed
polymers
Cements
CFRPs
Fibreglass
GFRPs
Woods, ∥
grain
Woods, ⊥
grain

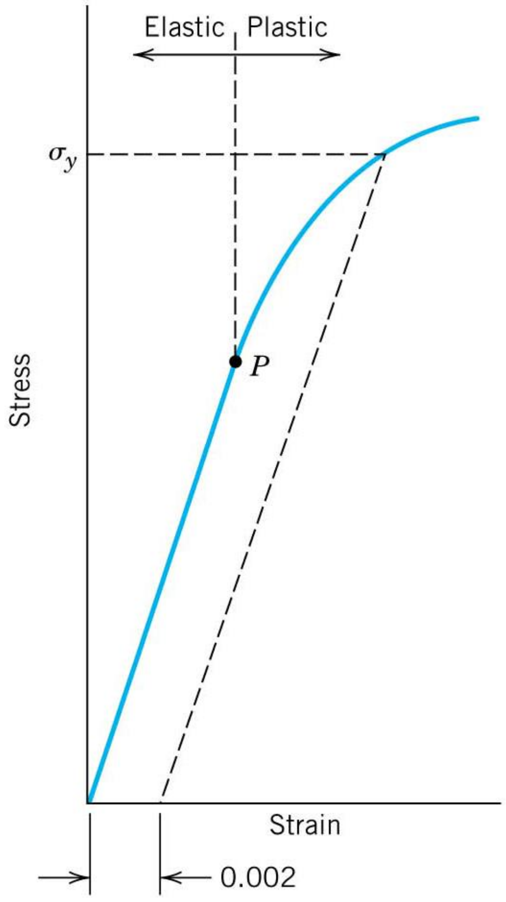
The Elastic Constants of Isotropic Elements at Room Temperature.

Element	Bulk modulus B	Young's modulus E	Shear modulus G	Poisson's Ratio ν
Li	13.6	11.5	4.2	0.36
Be	125.5	309.0	146.8	0.05
B(fibres)		379		
C(graphite fibres)		475		0.16
Na	8.16	8.92	3.38	0.32
Mg	33.25	44.3	17.35	0.29
Al	73.1	70.5	26.7	0.34
Si	316	113	39.7	
K	3.98	3.53	1.27	0.35
α -Ti	123.5	106	39.8	0.34
V	162	127	46.7	0.36
Cr	162	286		
α -Fe	166.0	208.2	80.65	0.29
Co	183	200	74.8	
Ni	192	213	81.3	0.31
Cu	137	122.5	45.5	0.34
Zn	60.5	92.2	37.2	0.29
Ge	69.7	99.0	39	0.28
Y	46.8	65.0	25.7	0.27
α -Zr	89.7	95.6	36.1	0.33
Nb	173	104	36.6	0.38
Mo	275	340	120	0.30
Pd	187	125.5	45.2	0.39
Ag	100	79	28.8	0.38
Cd	47.5	64.7	24.1	0.30
Hf	109.5	138	53	
Ta	207	184.5	68.7	0.35
W	313	388	148.5	0.29
Re		452		
Ir	371	527	210	0.26
Pt	275	171	61	0.39
Au	171	78.37	27.7	0.42
Tl	36.5	7.95	2.75	
Pb	41.4	16.2	5.6	0.44

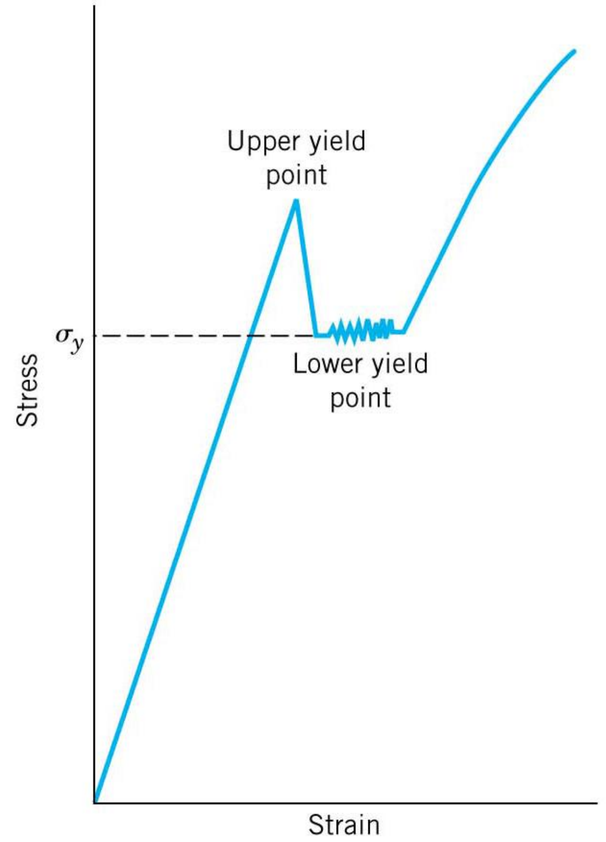
Ref : Texture and Related Phenomena, D. N. Lee, 2006

Unit : GPa

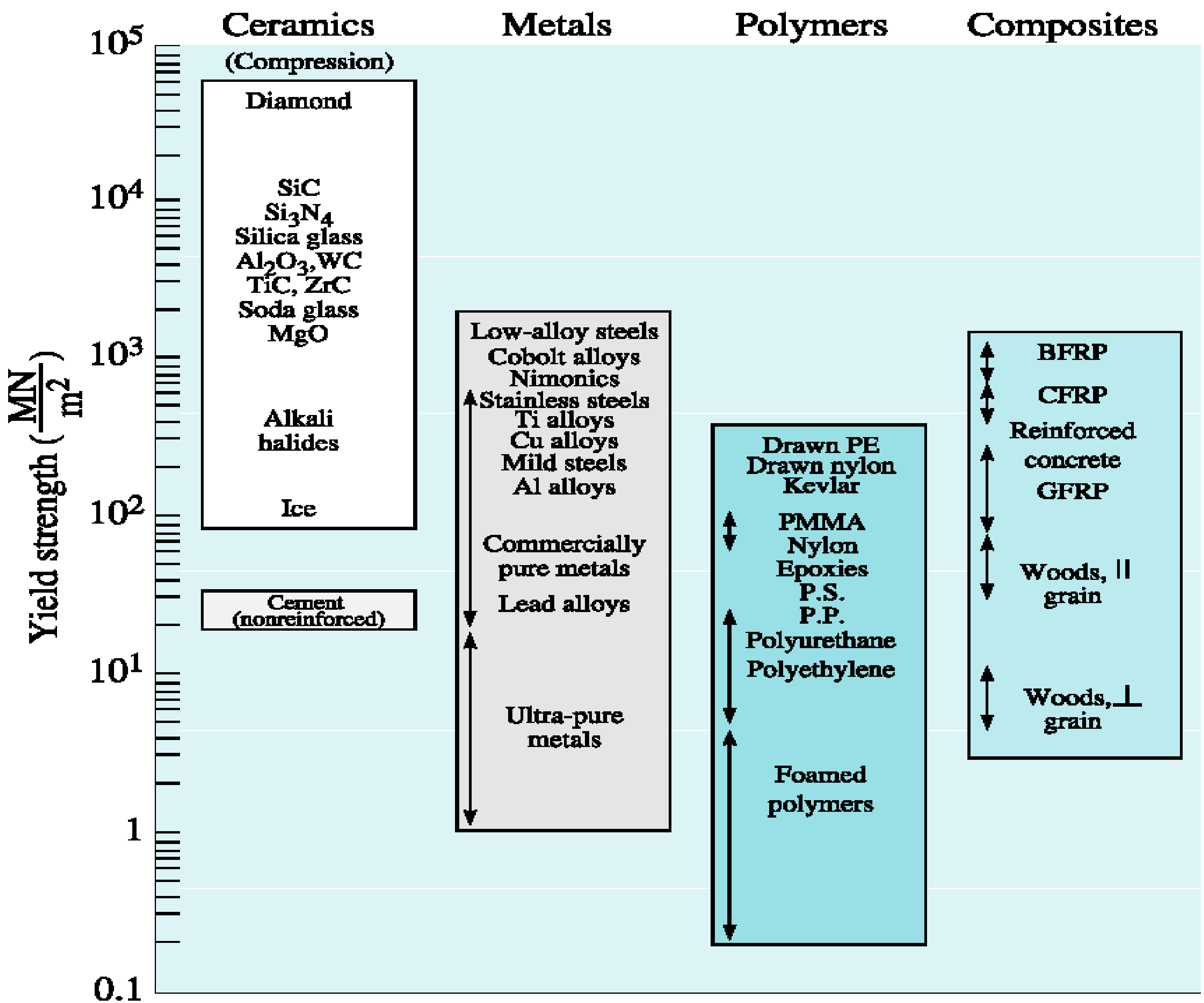
Plastic Deformation



proportional limit



yield point phenomenon

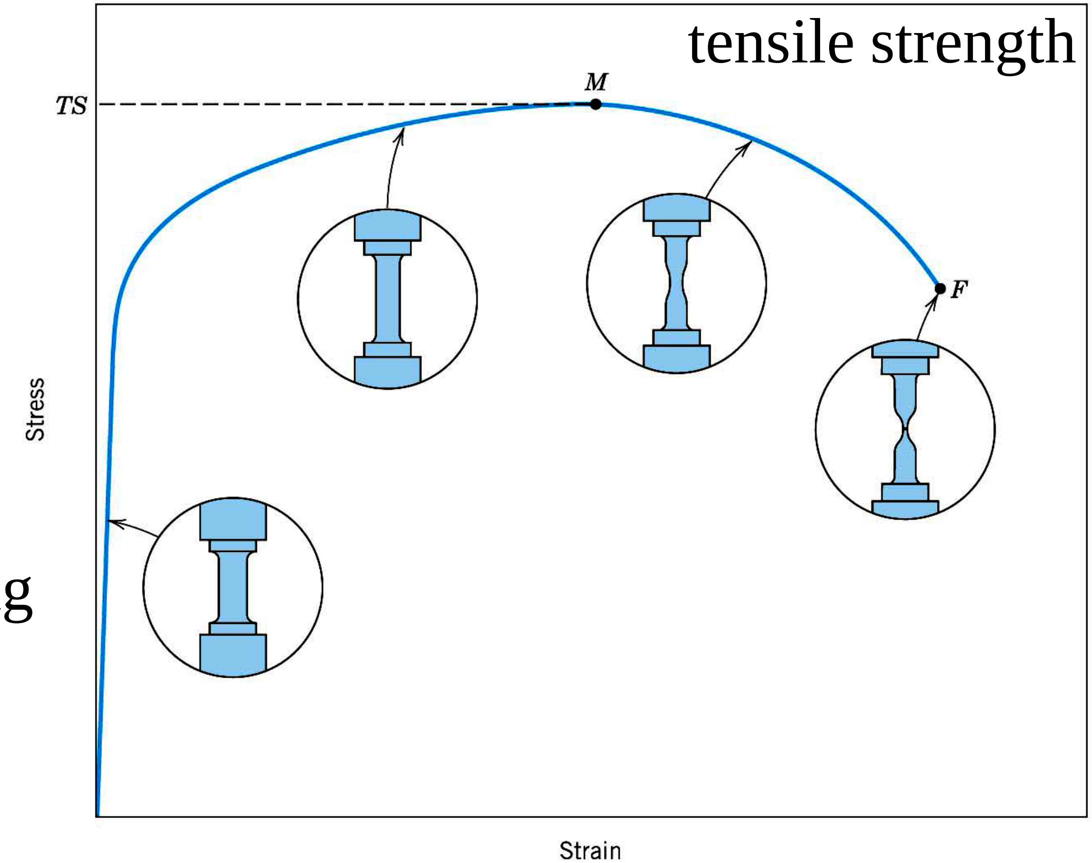


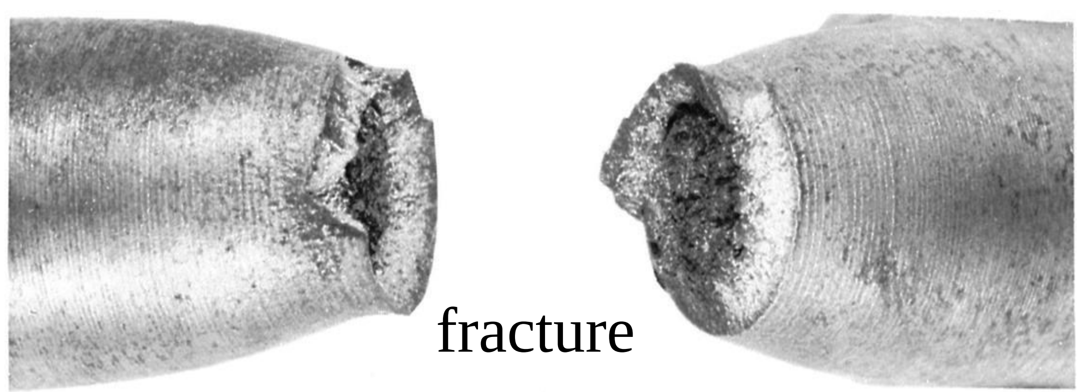
Ceramics
(Compression)
Metals
Polymers
Composites
Yield strength $(\frac{MN}{m^2})$
10^5
10^4
10^3
10^2
10^1
1
0.1
Diamond
SiC
Si_3N_4
Silica glass
Al_2O_3,WC
TiC, ZrC
Soda glass
MgO
Alkali halides
Ice
Cement (nonreinforced)
Low-alloy steels
Cobolt alloys
Nimonics
Stainless steels
Ti alloys
Cu alloys
Mild steels
Al alloys
Commercially pure metals
Lead alloys
Ultra-pure metals
Drawn PE
Drawn nylon
Kevlar
PMMA
Nylon
Epoxies
P.S.
P.P.
Polyurethane
Polyethylene
Foamed polymers
BFRP
CFRP
Reinforced concrete
GFRP
Woods, || grain
Woods, ⊥ grain

Plastic Deformation

necking

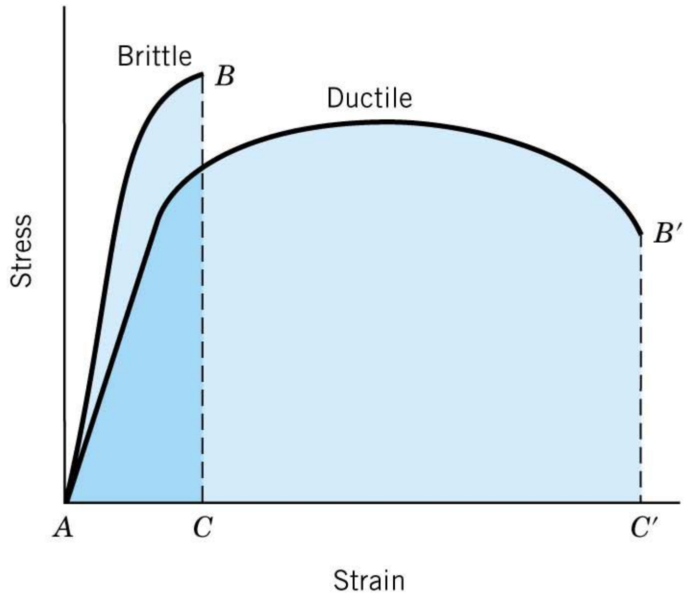
tensile strength





fracture

Plastic Deformation



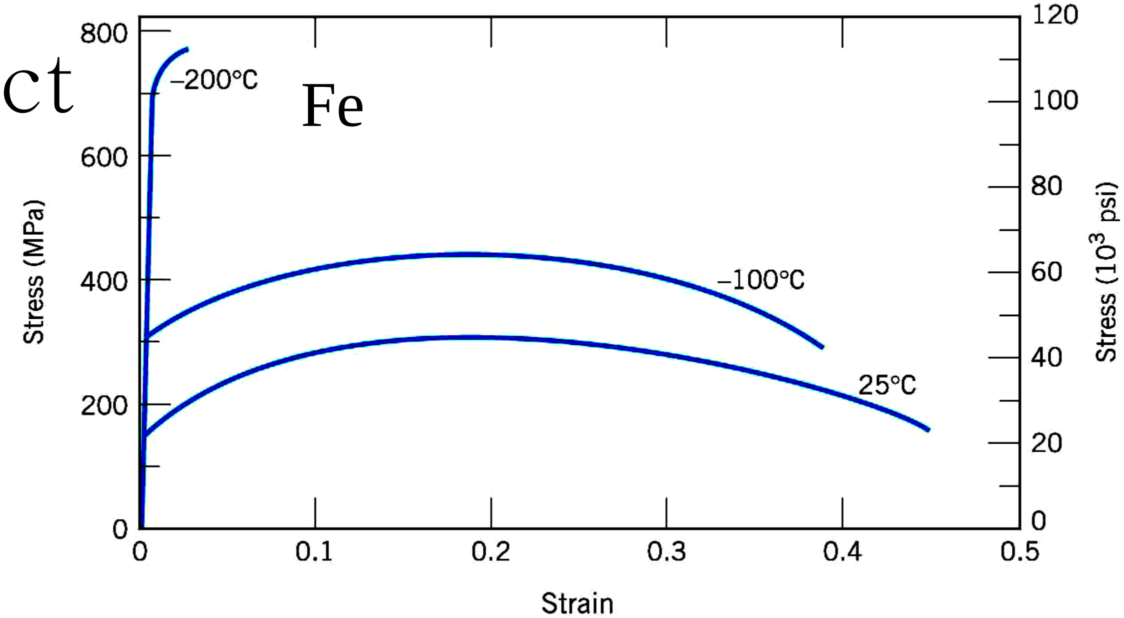
$$\%EL = \frac{L_f - L_o}{L_o} x100$$

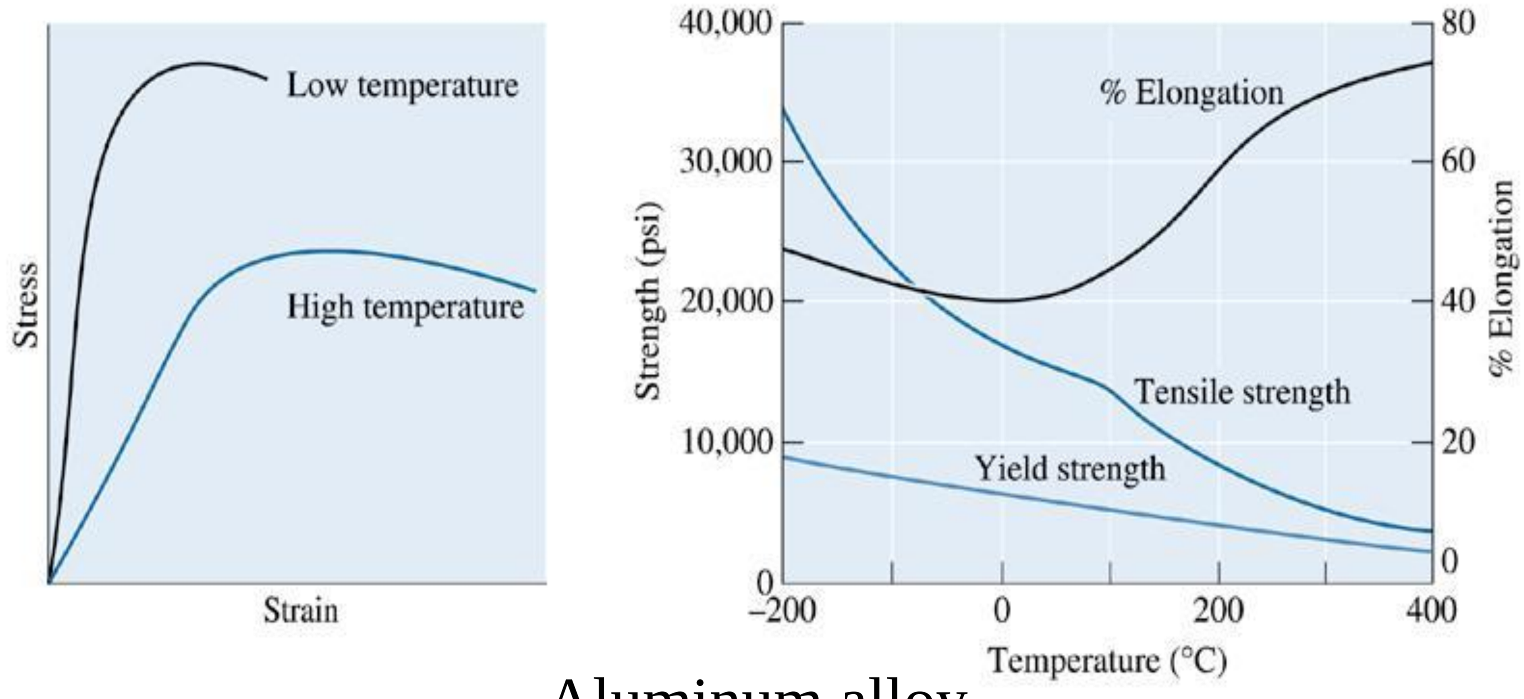
$$\%AR = \frac{A_o - A_f}{A_o} x100$$

Table 6.2 Typical Mechanical Properties of Several Metals and Alloys in an Annealed State

Metal Alloy	*Yield Strength MPa (ksi)*	*Tensile Strength MPa (ksi)*	*Ductility, %EL [in 50 mm (2 in.)]*
Aluminum	35 (5)	90 (13)	40
Copper	69 (10)	200 (29)	45
Brass (70Cu–30Zn)	75 (11)	300 (44)	68
Iron	130 (19)	262 (38)	45
Nickel	138 (20)	480 (70)	40
Steel (1020)	180 (26)	380 (55)	25
Titanium	450 (65)	520 (75)	25
Molybdenum	565 (82)	655 (95)	35

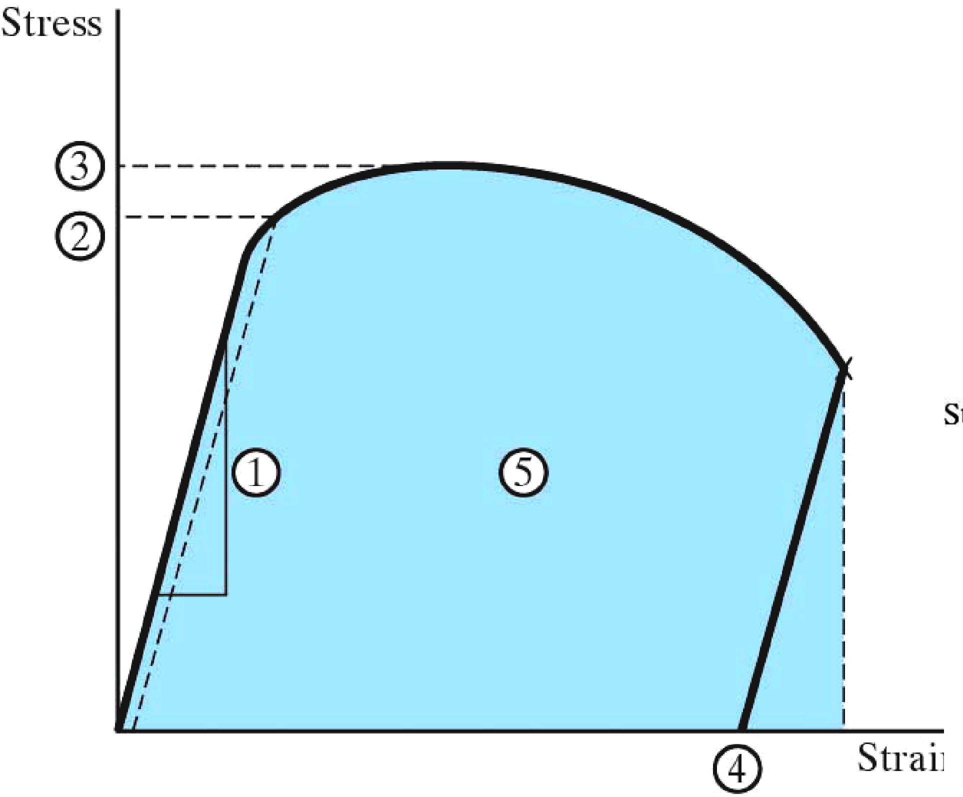
Temperature Effect



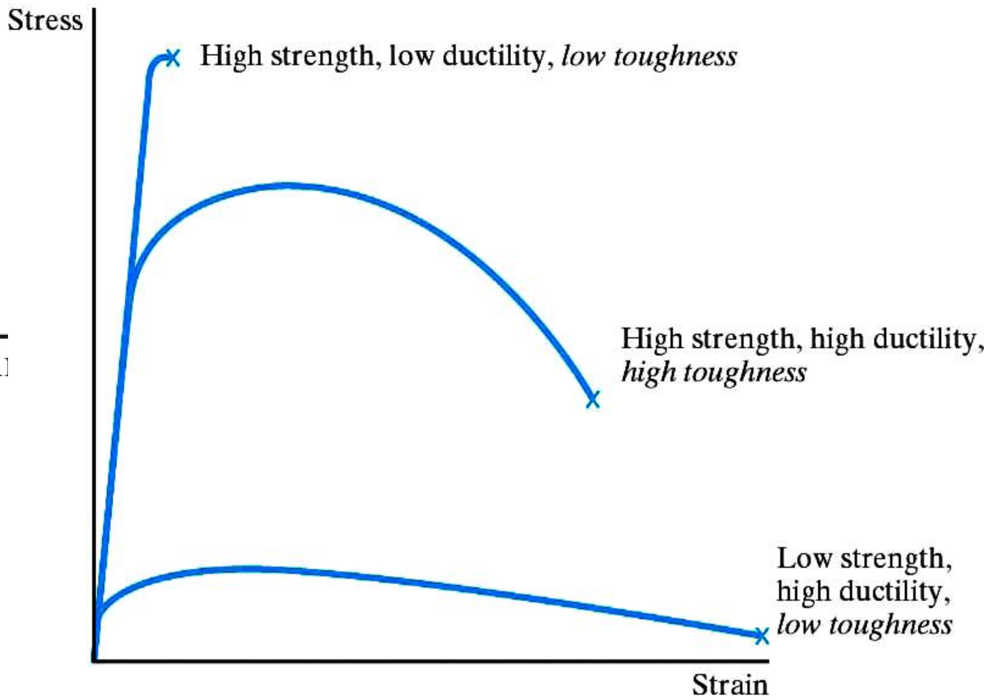


(a) Aluminum alloy (b)

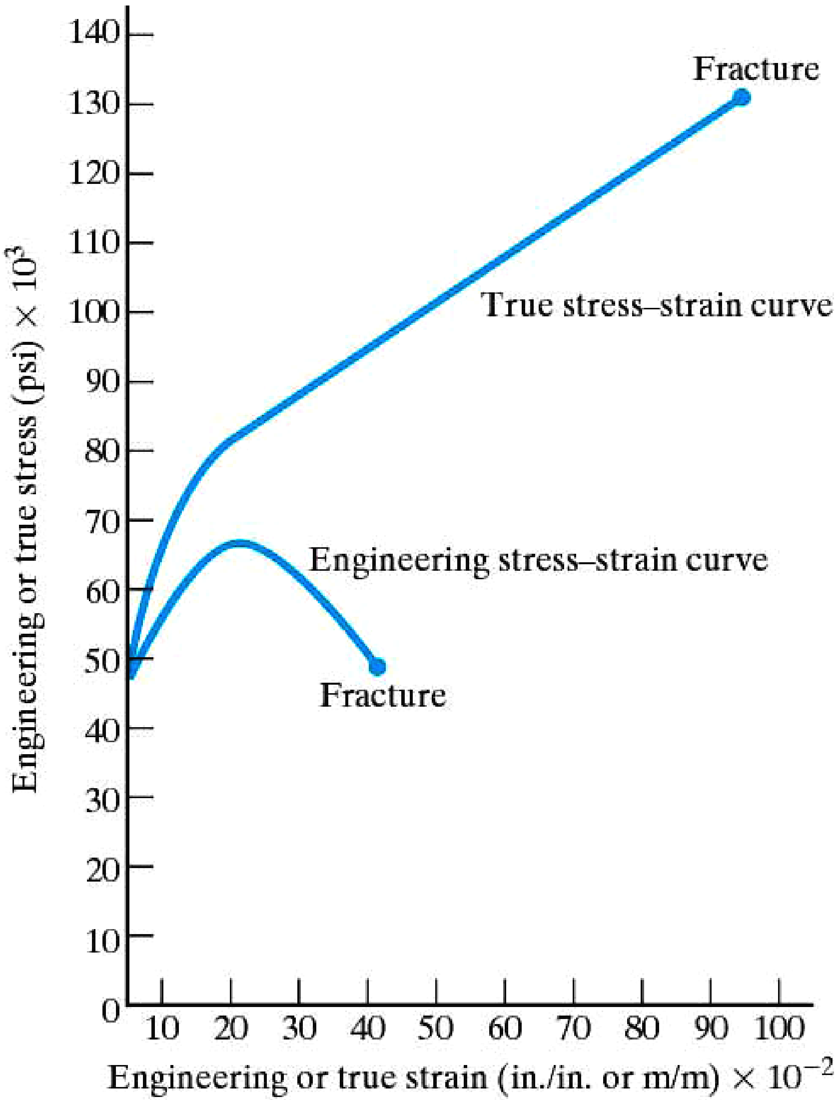
Tensile Test



1. modulus of elasticity, E
2. yield strength, Y.S.
3. tensile strength, T.S.
4. ductility, 100x $\varepsilon_{failure}$
5. toughness $= \int \sigma d\varepsilon$



True Stress and True Strain



true stress σ_t

$$\sigma_t = \frac{F}{A_i}$$

true strain ε_t

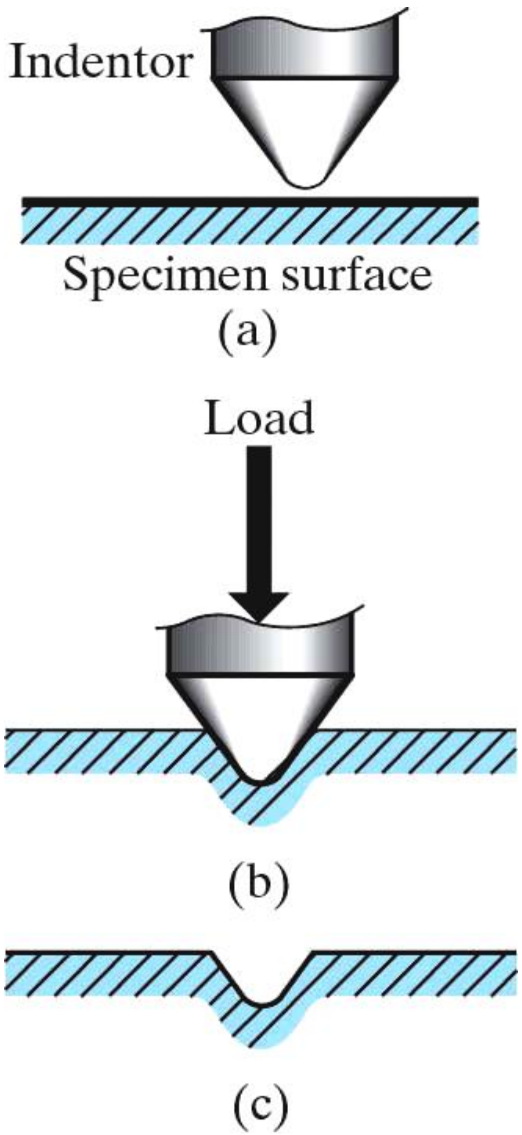
$$\varepsilon_t = \int_{l_o}^{l_i} \frac{dl}{l} = \ln\left(\frac{l_i}{l_o}\right)$$

$$A_0 l_0 = A_i l_i$$

$$\sigma_t = \sigma(1+\varepsilon)$$

$$\varepsilon_t = \ln(1+\varepsilon)$$

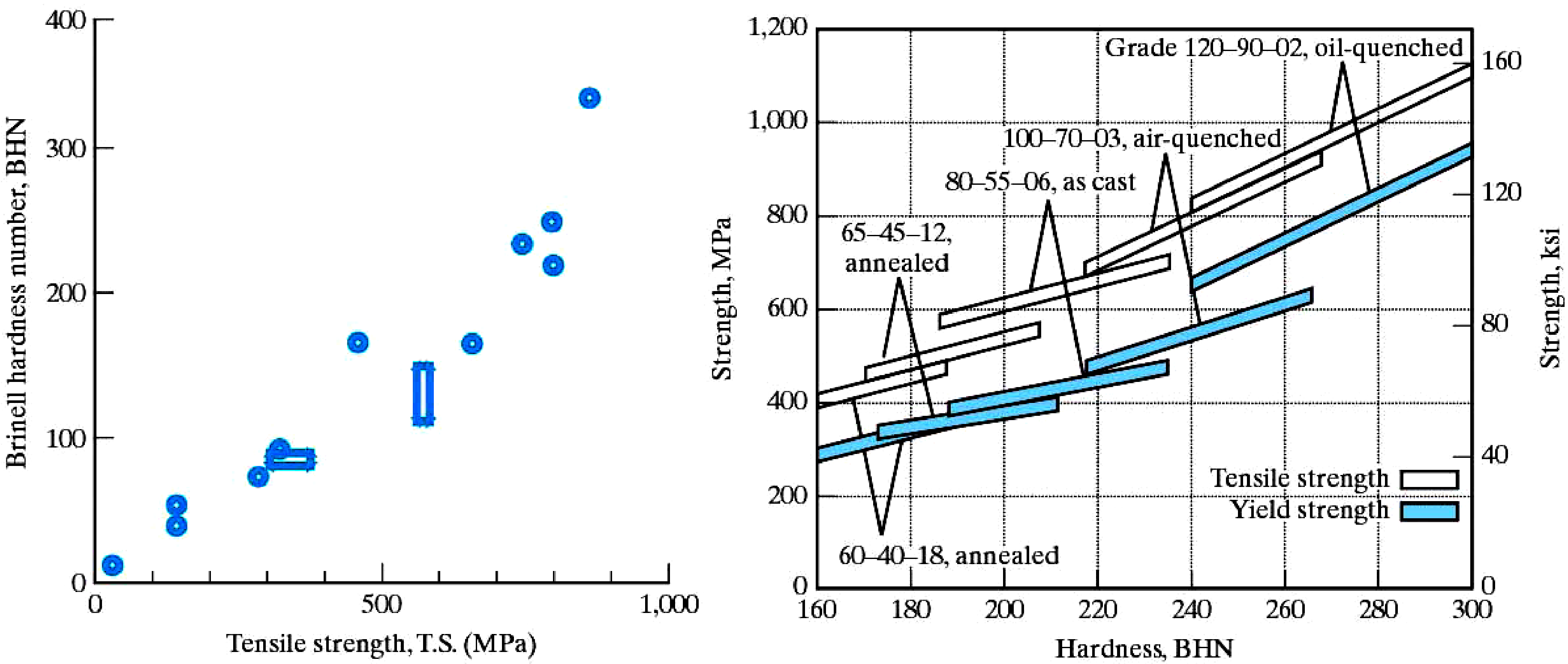
Hardness



- a measure of a material's resistance to plastic deformation

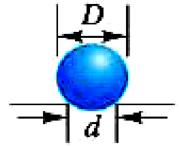
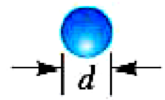
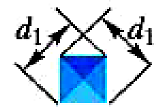
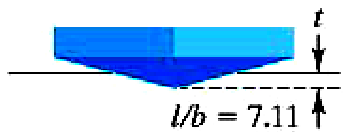
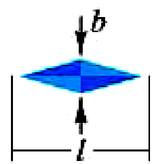
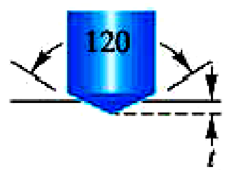
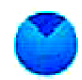
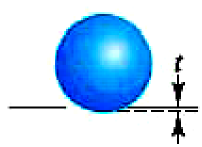
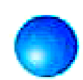
- indentation/indentor

Hardness vs. Tensile Properties

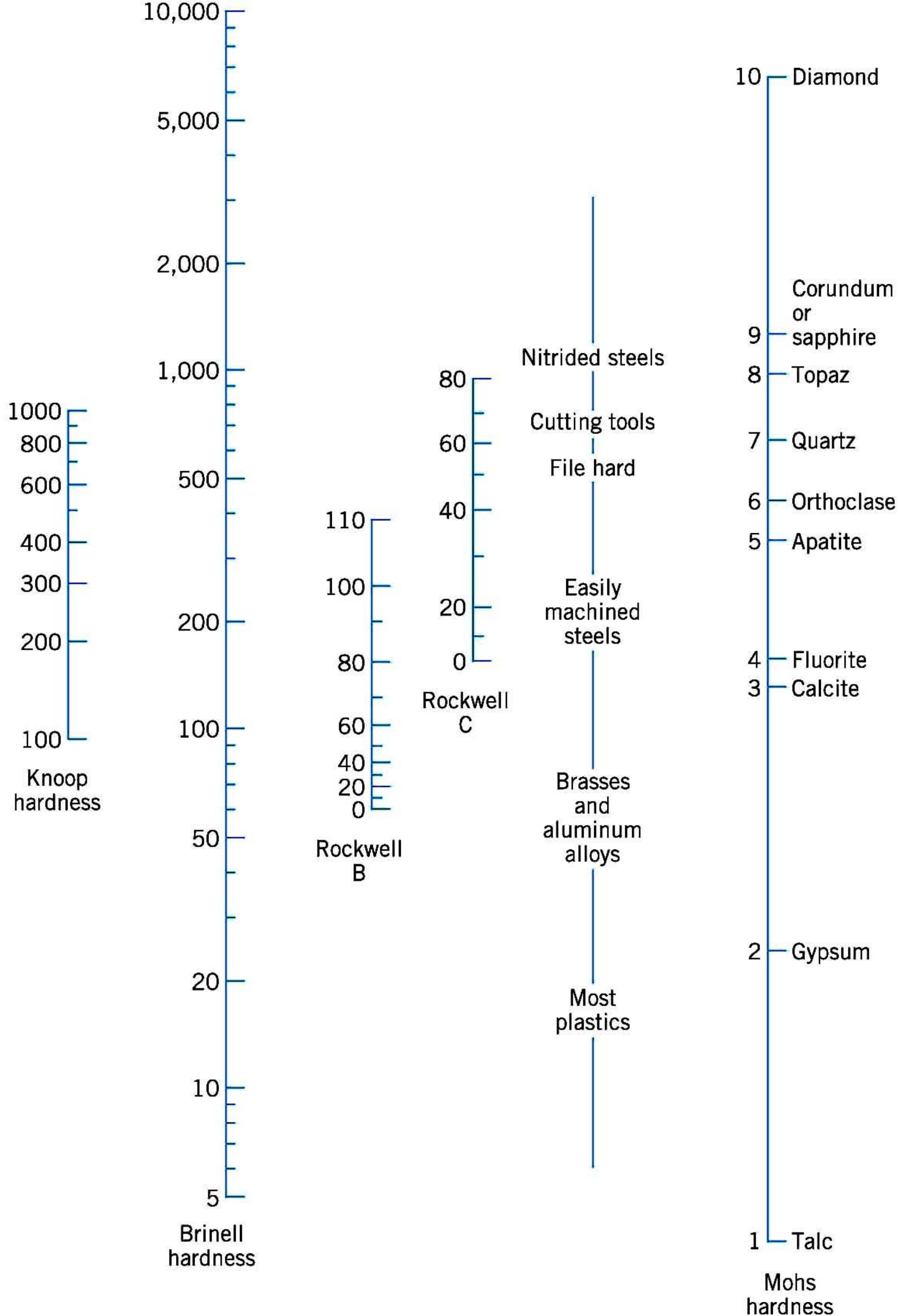


(a)

(b) Tensile properties of ductile iron versus hardness

Test	Indenter	Shape of indentation: Side view	Shape of indentation: Top view	Load	Formula for hardness number
Brinell	10 mm sphere of steel or tungsten carbide			P	$\mathrm{BHN} = \dfrac{2P}{\pi D\left[D - \sqrt{D^2 - d^2}\right]}$
Vickers	Diamond pyramid			P	$\mathrm{VHN} = 1.72P/d_1^2$
Knoop microhardness	Diamond pyramid	 $l/b = 7.11$ $b/t = 4.00$		P	$\mathrm{KHN} = 14.2P/l^2$
Rockwell					
A	Diamond cone			60 kg	$R_A =$ } $100 - 500t$
C	Diamond cone			150 kg	$R_C =$ } $100 - 500t$
D	Diamond cone			100 kg	$R_D =$ } $100 - 500t$
B	$\frac{1}{16}$ in. diameter steel sphere			100 kg	$R_B =$ } $130 - 500t$
F	$\frac{1}{16}$ in. diameter steel sphere			60 kg	$R_F =$ } $130 - 500t$
G	$\frac{1}{16}$ in. diameter steel sphere			150 kg	$R_G =$ } $130 - 500t$
E	$\frac{1}{8}$ in. diameter steel sphere			100 kg	$R_E =$ } $130 - 500t$
H	$\frac{1}{8}$ in. diameter steel sphere			60 kg	$R_H =$ } $130 - 500t$

Source: H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. 3: *Mechanical Behavior*, John Wiley & Sons, Inc., NY, 1965.



10,000
5,000
2,000
1,000
500
200
100
50
20
10
5
Brinell hardness
1000
800
600
400
300
200
100
Knoop hardness
110
100
80
60
40
20
0
Rockwell B
80
60
40
20
0
Rockwell C
Nitrided steels
Cutting tools
File hard
Easily machined steels
Brasses and aluminum alloys
Most plastics
10 Diamond
9 Corundum or sapphire
8 Topaz
7 Quartz
6 Orthoclase
5 Apatite
4 Fluorite
3 Calcite
2 Gypsum
1 Talc
Mohs hardness