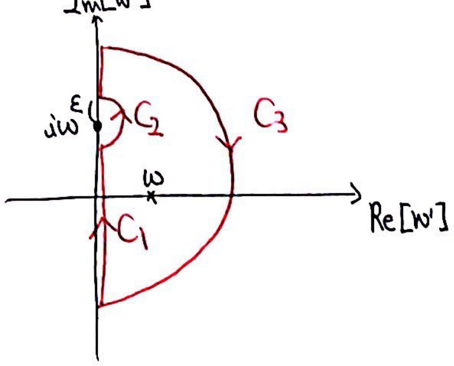
전파광 hw 4 김진재 redboy@snu.ac.kr

#1. 30점



$$\oint_C = \int_{C_1} + \int_{C_2} + \int_{C_3}$$

$$\cancel{\oint_C \frac{\chi(\omega')}{\omega'-\omega} d\omega' = 2\pi i \cdot \chi(\omega)}$$

$$\cancel{\chi(i\omega)\cdot 2\pi i = \oint_C \frac{\chi(\omega')}{\omega'-i\omega} d\omega'}$$

$$\oint_C \frac{\chi(\omega')}{\omega'-i\omega} d\omega' = 0 \quad \because \text{no pole in the } C.$$

$$\int_{C_3} \frac{\chi(\omega')}{\omega'-i\omega} d\omega' \underset{R\to\infty}{\longrightarrow} 0$$

$$\int_{C_1} = \int_{-iR}^{i\omega-i\epsilon} \frac{\chi(\omega')}{\omega'-i\omega} d\omega' + \int_{i\omega+i\epsilon}^{iR} \frac{\chi(\omega')}{\omega'-i\omega} d\omega'$$

$$\cancel{\overset{R\to\infty}{\underset{\epsilon\to 0}{=}} P\int_{-\infty}^{\infty} \omega'-}$$

set $\omega' = i\omega''$

$$\int_{C_1} = \int_{-R}^{\omega-\epsilon} \frac{\chi(i\omega'')}{\omega''-\omega} d\omega'' + \int_{\omega+\epsilon}^{R} \frac{\chi(i\omega'')}{\omega''-\omega} d\omega''$$

$$\int_{C_1} \overset{R\to\infty}{\underset{\epsilon\to 0}{=}} P\int_{-\infty}^{\infty} \frac{\chi(i\omega')}{\omega'-\omega} d\omega' \quad (\omega'' \to \omega')$$

$$\int_{C_2} \frac{\chi(\omega')}{\omega'-i\omega} d\omega' = \underbrace{\boxed{\frac{1}{2}}}_{\text{half circle}} \cdot 2\pi i \chi(i\omega)$$

$$= \pi i \chi(i\omega)$$

$$\oint_C = \int_{C_1} + \int_{C_2} + \cancel{\int_{C_3}} = 0$$

$$P\int_{-\infty}^{\infty} \frac{\chi(i\omega')}{\omega'-\omega} d\omega' + \pi i \chi(i\omega) = 0$$

$$\chi(i\omega) = +\frac{i}{\pi} P\int_{-\infty}^{\infty} \frac{\chi(i\omega')}{\omega'-\omega} d\omega'$$

$$\mathrm{Re}[\chi(i\omega)] = -\frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{\mathrm{Im}[\chi(i\omega')]}{\omega'-\omega} d\omega'$$

□

#2. 20점

Assume Lorentzian oscillator w/ damping γ.

eq. (9.170)

$$n = \frac{ck}{\omega} \simeq 1 + \frac{Nq^2}{2m\epsilon_0}\sum_j \frac{f_j(\omega_j^2-\omega^2)}{(\omega_j^2-\omega^2)^2+\gamma_j^2\omega^2}$$

assume 1 resonance,

$$\frac{ck}{\omega} = 1 + \frac{Nq^2}{2m\epsilon_0}\cdot\frac{(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+\gamma_0^2\omega^2}$$

$$ck = \omega\left[1 + \frac{Nq^2}{2m\epsilon_0}\cdot\frac{(\omega_0^2-\omega^2)}{(\omega_0^2-\omega^2)^2+\gamma_0^2\omega^2}\right]$$

$$c\cdot\frac{dk}{d\omega} = c\cdot\left(\frac{1}{v_g}\right)$$

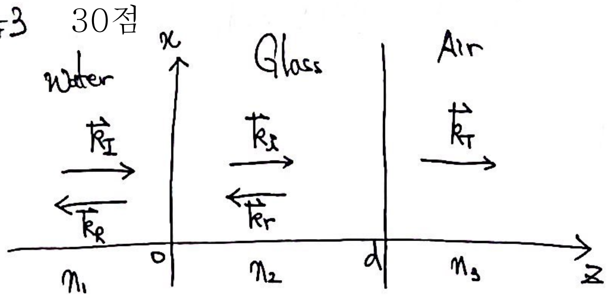
$$= 1 + \beta\omega_0^2(\omega_0^2+\omega^2)\frac{(\omega_0^2-\omega^2)^2-\gamma_0\omega^2}{[(\omega_0^2-\omega^2)^2+\gamma_0\omega^2]^2}$$

where $\beta = \frac{Nq^2}{2m\epsilon_0\omega_0^2}$

$$\therefore v_g = c\left\{1 + \beta\omega_0^2(\omega_0^2+\omega^2)\cdot\frac{(\omega_0^2-\omega^2)^2-\gamma_0\omega^2}{[(\omega_0^2-\omega^2)^2+\gamma_0\omega^2]^2}\right\}^{-1}$$

□

#3 30점



i) $z<0$

$$\begin{cases}\vec{E}_I = E_{I0}\, e^{i(k_1 z-\omega t)}\,\hat{x}\\ \vec{E}_R = E_{R0}\, e^{i(-k_1 z-\omega t)}\,\hat{x}\end{cases}$$

$$\begin{cases}\vec{B}_I = \frac{1}{v_1}\vec{k}_I\times\vec{E}_I = \frac{E_{I0}}{v_1}e^{i(k_1 z-\omega t)}\,\hat{y}\\ \vec{B}_R = \frac{1}{v_1}\vec{k}_R\times\vec{E}_R = -\frac{E_{R0}}{v_1}e^{i(-k_1 z-\omega t)}\,\hat{y}\end{cases}$$

ii) $0<z<d$

$$\begin{cases}\vec{E}_t = E_{t0}\, e^{i(k_2 z-\omega t)}\,\hat{x}\\ \vec{E}_r = E_{r0}\, e^{i(-k_2 z-\omega t)}\,\hat{x}\end{cases}$$

$$\begin{cases}\vec{B}_t = \frac{E_{t0}}{v_2}e^{i(k_2 z-\omega t)}\,\hat{y}\\ \vec{B}_r = -\frac{E_{r0}}{v_2}e^{i(-k_2 z-\omega t)}\,\hat{y}\end{cases}$$

iii) $z>d$

$$\vec{E}_T = E_{T0}\, e^{i(k_3 z-\omega t)}\,\hat{x}$$

$$\vec{B}_T = \frac{E_{T0}}{v_3}e^{i(k_3 z-\omega t)}\,\hat{y}$$

By the boundary conditions ($\rho_s=0$, $J_s=0$)

$D_\perp, B_\perp, E_\parallel, H_\parallel$: Continuous at boundary ($z=0$, $z=d$)

#3 계속

① $z=0$

$$\begin{cases} E_{I0} + E_{R0} = E_{i0} + E_{r0} \\ \frac{1}{v_1}E_{I0} - \frac{1}{v_1}E_{R0} = \frac{1}{v_2}E_{i0} - \frac{1}{v_2}E_{r0} \quad (\mu_i \simeq \mu_0) \end{cases}$$

$i=1,2,3$

$$\Rightarrow 2E_{I0} = (1+\beta)E_{i0} + (1-\beta)E_{r0} \quad \text{—©}$$

where $\beta = \frac{v_1}{v_2}$

② $z=d$

$$\begin{cases} E_{i0}e^{ik_2d} + E_{r0}e^{-ik_2d} = E_{T0}e^{ik_3d} \\ \frac{1}{v_2}E_{i0}e^{ik_2d} - \frac{1}{v_2}E_{r0}e^{-ik_2d} = \frac{1}{v_3}E_{T0}e^{ik_3d} \end{cases}$$

$$\Rightarrow 2E_{i0}e^{ik_2d} = (1+\alpha)E_{T0}e^{ik_3d} \quad \text{—Ⓐ}$$

$$\Rightarrow 2E_{r0}e^{-ik_2d} = (1-\alpha)E_{T0}e^{ik_3d} \quad \text{—Ⓑ}$$

where $\alpha = \frac{v_2}{v_3}$

Ⓐ, Ⓑ를 ©에 대입.

$$T^{-1} = \frac{1}{\alpha\beta}\left|\frac{E_{I0}}{E_{T0}}\right|^2$$

$$= \frac{1}{4\alpha\beta}\left[(1+\alpha\beta)^2\cos^2 k_2d + (\alpha+\beta)^2\sin^2 k_2d\right]$$

$$= \frac{1}{4n_1n_3}\left[(n_1+n_3)^2 + \frac{(n_1^2-n_2^2)(n_3^2-n_2^2)}{n_2^2}\sin^2 k_2d\right]$$

if set $\underset{\text{water}}{n_1=\frac{4}{3}}$, $\underset{\text{glass}}{n_2=\frac{3}{2}}$, $\underset{\text{air}}{n_3=1}$

$\sin^2 k_2d = 0$ for max $T \Rightarrow k_2d = \underset{\text{Integer}}{m}\pi$

$$\rightarrow \omega = \frac{c}{n_2d}m\pi$$

In the case of p-polarization,

Brewster's angle for the max. T

In the case of s-pol,

Normal incidence → max. T

(problem 9.17)

** -20

$$v_g = \frac{c}{n+\omega\cdot\frac{dn}{d\omega}} > c$$

if $\omega\to\infty$, $\frac{dn}{d\omega}\to 0$

Fig 1 에 따라 $v_g > c$ 인 지점을 적절하게 기술하면 정답.

B.

논문 인용

"Since there is no information carried by the peak of any limited-bandwidth signal that is not already present in its forward tail, there is no violation of causality"

↳ 이와 유사한 논리 모두 정답.