

과제 #5

마감일: 11월 21일 9시 30분

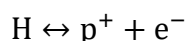
제출방법:

- 강의실 교탁

주의사항:

- **숙제를 베껴** 내면 관련된 모든 학생에게 **불이익**이 있습니다.
- 마감일시를 반드시 준수.

Problem 1 Consider the ionization of atomic hydrogen, governed by the equation



, where p^+ is a proton (equivalently a positively ionized hydrogen) and e^- is an electron.

(a) Explain why $\mu_{\text{H}} = \mu_{\text{p}} + \mu_{\text{e}}$, where μ is the chemical potential of hydrogen, proton and electron, respectively.

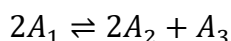
(b) Using the partition function for hydrogen atoms, show that

$$-k_{\text{B}}T \ln \frac{Z_1^{\text{p}}}{N_{\text{p}}} - k_{\text{B}}T \ln \frac{Z_1^{\text{e}}}{N_{\text{e}}} = -k_{\text{B}}T \ln \frac{Z_1^{\text{H}}}{N_{\text{H}}} e^{\beta R}$$

, where Z_1^x and N_x are the single-particle partition function and number of particles for species x , respectively and $R=13.6$ eV.

(c) Show that $\frac{n_{\text{e}}n_{\text{p}}}{n_{\text{H}}} = \frac{(2\pi m_{\text{e}}k_{\text{B}}T)^{3/2}}{h^3} e^{-\beta R}$, where $n_x = N_x/V$ is the number density of species x . This relation is known as the Saha equation.

Problem 2 Suppose that we have the following hypothetical chemical reaction.



The systems of A_1 , A_2 , and A_3 molecules are kept at a constant temperature of T and a pressure of P . If we denote by N_i the number of molecules of type i , the Gibbs free energy of this system is then a function of these numbers so that $G=G(N_1, N_2, N_3)$. Show the following relationship in equilibrium

$$2\mu_1 = 2\mu_2 + \mu_3$$

where $\mu_i = \frac{\partial G}{\partial N_i}$ is the chemical potential of the type i .

Problem 3 Consider two identical bodies, A_1 and A_2 , each characterized by a heat capacity C , which is temperature-dependent. The bodies are initially at temperature T_1 and T_2 , respectively, where $T_1 > T_2$. It is desired to operate an engine between A_1 and A_2 so as to convert some of their internal energy into work. As a result of the operation of the engine, the bodies ultimately will attain a common final temperature T_f .

- (a) What is the total amount of work W done by the engine? Express your answer in terms of C , T_1 , T_2 and T_f .
- (b) Use arguments based upon entropy consideration to derive an inequality relating T_f to the initial temperature T_1 and T_2 .
- (c) For given initial temperature T_1 and T_2 , what is the maximum amount of work obtainable from the engine?