1

(a) $\ln(1-p) \simeq -p$, note that it is valid when $p \ll 1$

$e^{\ln(1-p)} \simeq e^{-p} \Rightarrow (1-p) \simeq e^{-p}$

$\Rightarrow (1-p)^N \simeq e^{-pN} \Rightarrow (1-p)^N/(1-p)^n \simeq e^{-pN}/e^{-pn}$

$\simeq e^{-pN}$, for $n \ll N$

(b) $\frac{N!}{(N-n)!} = \underbrace{N*(\cdots)*(N-n+1)}_{n \text{ terms}} = N^n*[1*\frac{N-1}{N}*(\cdots)*\frac{N-n+1}{N}]$

$\simeq N^n$, valid when $n \ll N$

(c) $W(n) = \frac{1}{n!}\overbrace{\frac{N!}{(N-n)!}}^{N^n}p^n\overbrace{(1-p)^{N-n}}^{e^{-Np}} = \frac{1}{n!}\lambda^n e^{-\lambda}$

(d) $\Sigma_n W(n) = 1$ original form: $(1+(1-p))^N = \Sigma_n W(n) = 1$

approx. form: $\Sigma_{n=0}^{\infty}\frac{1}{n!}\lambda^n\exp(-\lambda)$

$= (1+\lambda+\frac{\lambda^2}{2!}+\cdots)/(1+\lambda+\lambda^2/2+\cdots) = 1$

(e) $\mu = \langle n\rangle = \Sigma_n n W(n) = \Sigma_{n=0}^{\infty}\frac{n}{n!}\lambda^n\exp(-\lambda)$

$= \lambda\,\underbrace{\Sigma_{n=0}^{\infty}\frac{n}{n!}\lambda^{n-1}}_{\frac{d}{d\lambda}\exp(\lambda)}\exp(-\lambda) = \lambda$

(f) $\langle(n-\langle n\rangle)^2\rangle = \langle n^2\rangle - \langle n\rangle^2$

$= \Sigma_n n^2 W(n) - \lambda^2 = \Sigma_n \frac{n^2}{n!}\lambda^n/\exp(\lambda) - \lambda^2$

$= \Sigma_n n*\frac{\lambda^n}{(n-1)!}/\exp(\lambda) - \lambda^2 = \lambda(\Sigma_n n*\frac{\lambda^{n-1}}{(n-1)!})/\exp(\lambda) - \lambda^2$

$= \frac{\lambda}{\exp(\lambda)}[\underbrace{\Sigma_n (n-1)\frac{\lambda^{n-1}}{(n-1)!}}_{\lambda\Sigma_n\frac{\lambda^{n-2}}{(n-2)!}} + \underbrace{\Sigma_n\frac{\lambda^{n-1}}{(n-1)!}}_{\exp(\lambda)}] - \lambda^2 = \cancel{\lambda^2} + \lambda - \cancel{\lambda^2}$

$\langle n^2 - \langle n\rangle^2\rangle = \lambda$

2

(a) $P(n) = \frac{N!}{n!(N-n)!}p^n(1-p)^{N-n}$, $\Sigma_n P(n) = (p+1-p)^N = 1$

(b) $P'(m) = P'(n-n')$, $n+n'=N$, $n-n'=m$

$= P'(n*(+1)+(N-n)*(-1)) = P'(2n-N) = P(n)$

(c) this corresponds $P'(0)$, $P'(2n-N=0)$,

when N = odd, $n \Rightarrow$ not integer, ↳ $P'(0)=0$

when N = even, $n \Rightarrow$ integer $\Rightarrow P'(0) = \frac{2n!}{n!n!}(\frac{1}{2})^{2n}$, $(N=2n)$

3

(a) $u \sim P(u)$

$\overline{u^2} = \int_{-\infty}^{+\infty} u^2 P(u)\,du$

$\bar{u}^2 = (\int_{-\infty}^{+\infty} uP(u)\,du)^2$

$\langle(u-\bar u)^2\rangle \ge 0$

$= \int_{-\infty}^{+\infty}(u-\bar u)^2 P(u)\,du$

$= \int_{-\infty}^{+\infty} u^2 - 2u\bar u P(u) + \bar u^2 P(u)\,du$

$= \overline{u^2} - \bar u^2 \ge 0$

(b) $P(+\mu_0) = p$, $P(-\mu_0) = (1-p)$

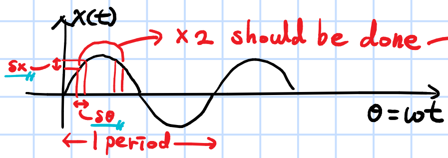
$\bar\mu = \int d\mu\,\mu P(\mu) = +p\mu_0 + (1-p)(-\mu_0) = 2p\mu_0 - \mu_0$

$\overline{\mu^2} = \int d\mu\,\mu^2 P(\mu) = +p(\mu_0)^2 + (1-p)(-\mu_0)^2 = \mu_0^2$

(c) $\sqrt{\langle(\mu-\bar\mu)^2\rangle} = \sqrt{\overline{\mu^2}-\bar\mu^2} = \sqrt{\mu_0^2-\mu_0^2(1-2p)^2} = \mu_0\sqrt{1-(1-4p+4p^2)} = 2\mu_0\sqrt{p-p^2}$

4

$x = A\cos(\omega t + \varphi)$ [totally random ↳ can set as zero.]



× 2 should be done

$P(\theta)d\theta = P(x)dx$

$\Rightarrow \frac{d\theta}{2\pi} = P(x)dx$

$\Rightarrow P(x) = \frac{1}{2\pi}|\frac{d\theta}{dx}| = \frac{1}{2\pi}\frac{1}{\sqrt{A^2-x^2}}$

$\Rightarrow P(x) = \frac{1}{\pi}\,1/\sqrt{A^2-x^2}$

↳ $\int_{-A}^{+A}\frac{1}{\pi}\frac{dx}{\sqrt{A^2-x^2}} = \frac{1}{\pi}\tan^{-1}(\theta)\Big|_0^{+\infty} \times 2$

$= \pi/\pi = 1$