8.2 *Estimate the theoretical fracture strength of a brittle material if it is known that fracture occurs by the propagation of an elliptically shaped surface crack of length 0.5 mm (0.02 in.) and a tip radius of curvature of 5 × 10^{-3} mm (2 × 10^{-4} in.), when a stress of 1035 MPa (150,000 psi) is applied.*

Solution

In order to estimate the theoretical fracture strength of this material it is necessary to calculate σ_m using Equation 8.1 given that σ_0 = 1035 MPa, a = 0.5 mm, and $\rho_t = 5 \times 10^{-3}$ mm. Thus,

$$\sigma_m = 2\sigma_0 \left(\frac{a}{\rho_t}\right)^{1/2}$$

$$= (2)(1035 \text{ MPa})\left(\frac{0.5 \text{ mm}}{5 \times 10^{-3} \text{ mm}}\right)^{1/2}$$

$$= 2.07 \times 10^4 \text{ MPa} = 20.7 \text{ GPa} \quad (3 \times 10^6 \text{ psi})$$

8.4 *An MgO component must not fail when a tensile stress of 13.5 MPa (1960 psi) is applied. Determine the maximum allowable surface crack length if the surface energy of MgO is 1.0 J/m^2. Data found in Table 12.5 may prove helpful.*

Solution

The maximum allowable surface crack length for MgO may be determined using a rearranged form of Equation 8.3. Taking 225 GPa as the modulus of elasticity (Table 12.5), and realizing that values of σ_c (13.5 MPa) and γ_s (1.0 J/m^2) are given in the problem statement, we solve for a, as follows:

$$a = \frac{2E\gamma_s}{\pi\sigma_c^2}$$

$$= \frac{(2)\left(225 \times 10^9 \text{ N/m}^2\right)(1.0 \text{ N/m})}{(\pi)\left(13.5 \times 10^6 \text{ N/m}^2\right)^2}$$

$$= 7.9 \times 10^{-4} \text{ m} = 0.79 \text{ mm} \ \ (0.031 \text{ in.})$$

8.6 *An aircraft component is fabricated from an aluminum alloy that has a plane strain fracture toughness of 40 MPa* $\sqrt{m}$ *(36.4 ksi* $\sqrt{in.}$ *). It has been determined that fracture results at a stress of 300 MPa (43,500 psi) when the maximum (or critical) internal crack length is 4.0 mm (0.16 in.). For this same component and alloy, will fracture occur at a stress level of 260 MPa (38,000 psi) when the maximum internal crack length is 6.0 mm (0.24 in.)? Why or why not?*

Solution

We are asked to determine if an aircraft component will fracture for a given fracture toughness $(40\ \text{MPa}\sqrt{\text{m}})$, stress level (260 MPa), and maximum internal crack length (6.0 mm), given that fracture occurs for the same component using the same alloy for another stress level and internal crack length. (*Note:* Because the cracks are internal, their lengths are equal to 2a.) It first becomes necessary to solve for the parameter Y, using Equation 8.5, for the conditions under which fracture occurred (i.e., σ = 300 MPa and 2a = 4.0 mm). Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{40\ \text{MPa}\sqrt{\text{m}}}{(300\ \text{MPa})\sqrt{(\pi)\left(\dfrac{4\times 10^{-3}\ \text{m}}{2}\right)}} = 1.68$$

Now we will solve for the product $Y\sigma\sqrt{\pi a}$ for the other set of conditions, so as to ascertain whether or not this value is greater than the K_{Ic} for the alloy. Thus,

$$Y\sigma\sqrt{\pi a} = (1.68)(260\ \ \text{MPa})\sqrt{(\pi)\left(\frac{6\times 10^{-3}\ \text{m}}{2}\right)}$$

$$= 42.4\ \ \text{MPa}\sqrt{\text{m}}\quad (39\ \ \text{ksi}\sqrt{\text{in.}})$$

Therefore, fracture *will* occur since this value $(42.4\ \text{MPa}\sqrt{\text{m}})$ is greater than the K_{Ic} of the material, $40\ \text{MPa}\sqrt{\text{m}}$.

Cyclic Stresses
The S–N *Curve*

8.16 *A fatigue test was conducted in which the mean stress was 70 MPa (10,000 psi), and the stress amplitude was 210 MPa (30,000 psi).*

(a) Compute the maximum and minimum stress levels.

(b) Compute the stress ratio.

(c) Compute the magnitude of the stress range.

Solution

(a) Given the values of σ_m (70 MPa) and σ_a (210 MPa) we are asked to compute σ_{max} and σ_{min}. From Equation 8.14

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 70 \text{ MPa}$$

Or,

$$\sigma_{max} + \sigma_{min} = 140 \text{ MPa} \tag{8.14a}$$

Furthermore, utilization of Equation 8.16 yields

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = 210 \text{ MPa}$$

Or,

$$\sigma_{max} - \sigma_{min} = 420 \text{ MPa} \tag{8.16a}$$

Simultaneously solving Equations 8.14a and 8.16a leads to

$$\sigma_{max} = 280 \text{ MPa} \ \ (40{,}000 \text{ psi})$$

$$\sigma_{min} = -140 \text{ MPa} \ \ (-20{,}000 \text{ psi})$$

(b) Using Equation 8.17 the stress ratio R is determined as follows:

$$R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{-140 \text{ MPa}}{280 \text{ MPa}} = -0.50$$

(c) The magnitude of the stress range σ_r is determined using Equation 8.15 as

$$\sigma_r = \sigma_{max} - \sigma_{min} = 280 \text{ MPa} - (-140 \text{ MPa})$$

$= 420\ \text{MPa}\ \ (60{,}000\ \text{psi})$

8.19 *A cylindrical 2014-T6 aluminum alloy bar is subjected to compression-tension stress cycling along its axis; results of these tests are shown in Figure 8.20. If the bar diameter is 12.0 mm, calculate the maximum allowable load amplitude (in N) to ensure that fatigue failure will not occur at 10^7 cycles. Assume a factor of safety of 3.0, data in Figure 8.20 were taken for reversed axial tension-compression tests, and that S is stress amplitude.*

Solution

From Figure 8.20, the fatigue strength at 10^7 cycles for this aluminum alloy is 170 MPa. For a cylindrical specimen having an original diameter of d_0, the stress may be computed using Equation 6.1, which is equal to

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi\left(\frac{d_0}{2}\right)^2}$$

$$= \frac{4F}{\pi d_0^2}$$

When we divide σ by the factor of safety, the above equation takes the form

$$\frac{\sigma}{N} = \frac{4F}{\pi d_0^2}$$

Solving the above equation for F leads to

$$F = \frac{\sigma \pi d_0^2}{4N}$$

Now taking σ to be the fatigue strength (i.e., 170 MPa = 170×10^6 N/m^2) and incorporating values for d_0 and N provided in the problem statement [i.e., 12.0 mm (12.0×10^{-3} m) and 3.0, respectively], we calculate the maximum load as follows:

$$F = \frac{(170 \times 10^6 \text{ N/m}^2)(\pi)(12.0 \times 10^{-3} \text{ m})^2}{(4)(3.0)}$$

6,400 N

8.20 *A cylindrical rod of diameter 6.7 mm fabricated from a 70Cu-30Zn brass alloy is subjected to rotating-bending load cycling; test results (as S-N behavior) are shown in Figure 8.20. If the maximum and minimum loads are +120 N and –120 N, respectively, determine its fatigue life. Assume that the separation between loadbearing points is 67.5 mm.*

Solution

In order to solve this problem we compute the maximum stress using Equation 8.18, and then determine the fatigue life from the curve in Figure 8.20 for the brass material. In Equation 8.18, F is the maximum applied load, which for this problem is +120 N. Values for L and d_0 are provided in the problem statement—viz. 67.5 mm (67.5×10^{-3} m) and 6.7 mm (6.7×10^{-3} m), respectively. Therefore the maximum stress σ is equal to

$$\sigma = \frac{16FL}{\pi d_0^3}$$

$$= \frac{(16)(120\text{ N})(67.5 \times 10^{-3}\text{ m})}{(\pi)(6.7 \times 10^{-3}\text{ m})^3}$$

$$= 137 \times 10^6\text{ N/m}^2 = 137\text{ MPa}$$

From Figure 8.20 and the curve for brass, the logarithm of the fatigue life (log N_f) at 137 MPa is about 6.5, which means that the fatigue life is equal to

$$N_f = 10^{6.5}\text{ cycles} = 3 \times 10^6\text{ cycles}$$

Stress and Temperature Effects

8.32 *A specimen 975 mm (38.4 in.) long of an S-590 alloy (Figure 8.32) is to be exposed to a tensile stress of 300 MPa (43,500 psi) at 730°C (1350°F). Determine its elongation after 4.0 h. Assume that the total of both instantaneous and primary creep elongations is 2.5 mm (0.10 in.).*

Solution

From the 730°C line in Figure 8.32, the steady state creep rate $\dot{\varepsilon}_s$ is about $1.0 \times 10^{-2}\ h^{-1}$ at 300 MPa. The steady state creep strain, ε_s, therefore, is just the product of $\dot{\varepsilon}_s$ and time as

$$\varepsilon_s = \dot{\varepsilon}_s \times \text{(time)}$$

$$= (1.0 \times 10^{-2}\ \text{h}^{-1})(4.0\ \text{h}) = 0.040$$

Strain and elongation are related as in Equation 6.2—viz.

$$\varepsilon = \frac{\Delta l}{l_0}$$

Now, using the steady-state elongation, l_0, provided in the problem statement (975 mm) and the value of ε_s, determined above (0.040), the steady-state elongation, Δl_s, is determined as follows:

$$\Delta l_s = l_0 \varepsilon_s$$

$$= (975\ \text{mm})(0.040) = 39.0\ \text{mm}\ \ (1.54\ \text{in.})$$

Finally, the total elongation is just the sum of this Δl_s and the total of both instantaneous and primary creep elongations [i.e., 2.5 mm (0.10 in.)]. Therefore, the total elongation is 39.0 mm + 2.5 mm = 41.5 mm (1.64 in.).