

Advanced Design of Reinforced Concrete

Chapter 1: Introduction, Materials and Properties

1.1 Reinforced Concrete Structures

Reinforced concrete is the union of two materials, plain concrete and steel bars.

Plain concrete - possesses high compressive strength but little tensile strength.

Steel bars - embedded in the concrete which provides the needed tensile strength.

Steel bars are also often used in compression zone of concrete to provide an added compressive strength to the beam.

1.1.1 Reasons why steel and concrete work readily:

1. Bond - prevents slip of the bars relative to concrete.
2. Proper concrete mixture - provides adequate impermeability against water and bar corrosion.
3. Similar rates of thermal expansion:
 1. Concrete - 0.000010 to 0.000013 °C
 2. Steel - 0.000012 °C

Transverse cracks may appear near the steel bars in tension (unless prestressed), such cracks are expected.

1.2 Plain Concrete

Plain concrete is made by mixing cement, fine aggregates, coarse aggregate, water and sometimes, admixtures.

The strength of concrete depends on many factors:

- proportion of the ingredients
- conditions of temperature
- moisture under which it is placed and cured

1.3 Cement

Cement is a material that has adhesive and cohesive properties. These enables mineral fragments to bond and form a solid mass.

1.3.1 Portland cement

- usual hydraulic cement used in reinforced concrete. Concrete used with this attains adequate strength at **14 days** so that forms can be removed and reaches its design strength at **28 days**.

Table 1.3.1 Types of Portland Cement

Type	Uses
I	Ordinary construction, special properties are not desired
II	Ordinary construction, moderate sulfate resistance or moderate heat of hydration is desired
III	When high early strength is desired
IV	When low heat of hydration is desired
V	When high sulfate resistance is desired

1.4 Aggregates

Occupies about 75% of the total volume of concrete.

1.4.1 Normal Weight Concrete

Normal stone aggregates makes the concrete weighs about $2,300\text{kg}/\text{m}^3$. Adding of steel reinforcement, unit weight of concrete for the purpose of calculation is $2,400\text{kg}/\text{m}^3$.

1.4.2 Lightweight Structural Concrete

Lightweight concrete have aggregates made of expanded clays and shales and weighs about 1,120 to $1,840\text{kg}/\text{m}^3$

1.5 Admixtures

1.5.1 Air Entraining Admixtures

Causes air in the form of bubble (about $1\text{mm } \phi$) to form throughout the concrete to increase workability and resistance to deterioration.

1.5.2 Accelerating Admixtures

Accelerate the rate of early age strength development.

1.5.3 Water-reducing and Set-controlling Admixtures

Improve the fresh and hardening of concrete. This reduces the water cement ratio while decreasing the use of cement and increasing strength.

1.5.4 Admixture for Flowing Concrete

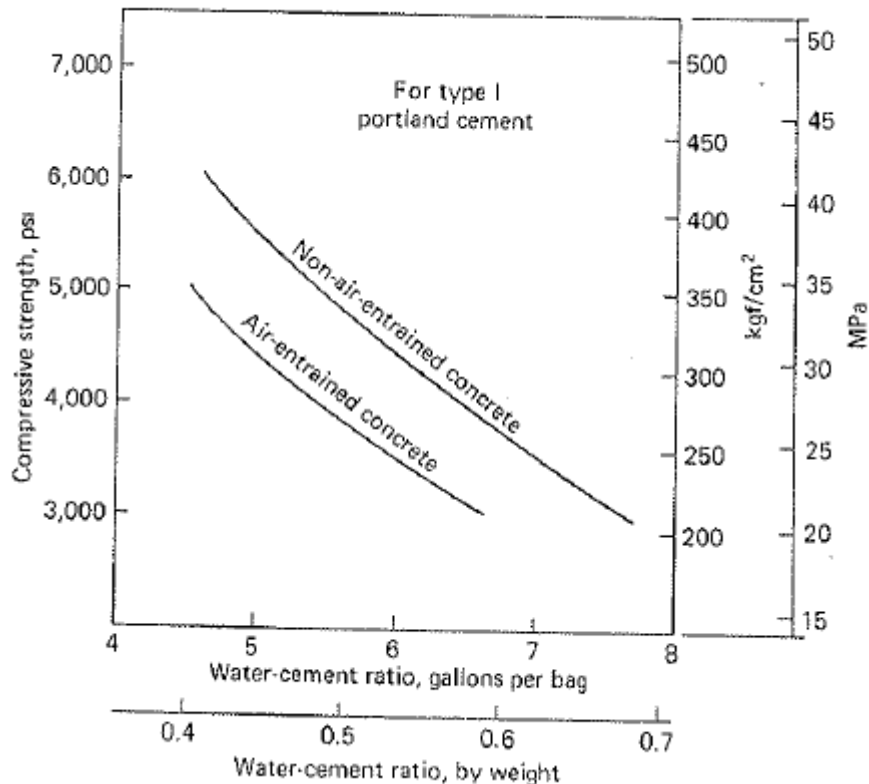
Used in concrete with high volume placement such as slabs, mats and pavements.

1.6 Compressive Strength

The strength of concrete is dependent on the mixture of cement, aggregates, water and sometimes admixtures.

Reference Concrete Strength - uniaxial compressive strength measured from cylinder compression tests.

The most important variable in determining concrete strength is the water-cement ratio. The lower the ratio, the higher the compressive strength.



Nearly all reinforced concrete behavior is related to the 28-days compressive strength, f'_c . Note that this still depends on the size and shape of the test specimen. In normal weight concrete, in average, the 6 x 12in cylinder is 80% of the 150-mm cube strength and 83% of the 200-mm cube strength.

1.7 Tensile Strength

Tensile strength property of concrete greatly affects the size and extent of cracks in structures.

Test in this property is usually done using split-cylinder test by placing the cylinder on its side then applying the load. The split-cylinder test result has been found to be proportional to $\sqrt{f'_c}$:

Modulus of rupture, $f_{ct} = 0.5\sqrt{f'_c}$ to $0.6\sqrt{f'_c}$ (for normal weight concrete) in MPa

Note that test result for modulus of rupture and the one calculated from $f_r = \frac{Mc}{I}$ are different. The computed gives higher values due to compressive strength distribution is not linear when tensile failure is imminent.

1.8 Modulus of Elasticity

Modulus of elasticity of concrete varies with strength, unlike that of steel. For normal weight concrete, ACI-8.5.1 suggests

$$E_c = 57,000\sqrt{f'_c} \quad (1)$$

where E_c is in psi, and roughly

$$E_c = 4,700\sqrt{f'_c} \quad (2)$$

where E_c is in MPa (ACI 318-05M).

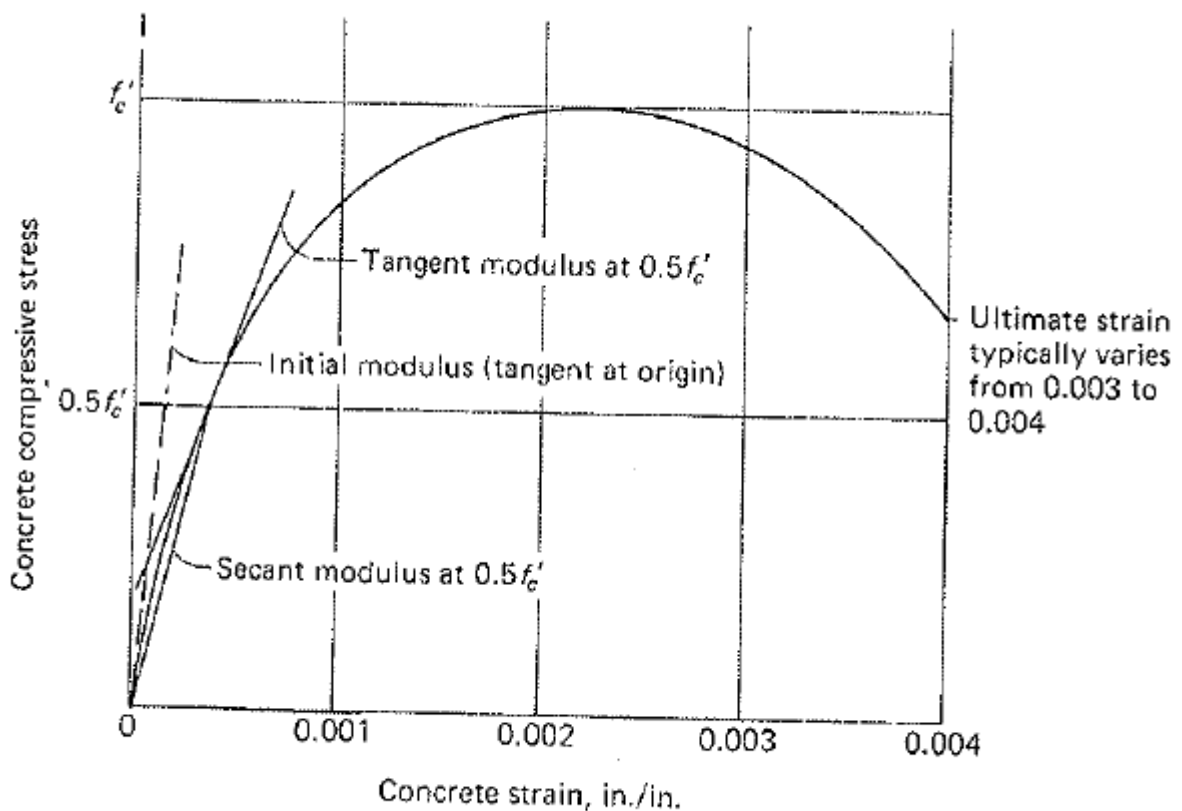


Figure 1.9.1 Stress-strain curve for concrete in compression.

1.9 Creep and Shrinkage

Properties of concrete that are hard to predict and is time-dependent. These becomes hard to predict because of inaccuracies and lots of unknown variables.

1.9.1 Creep

Creep is the property by which a material continues to deform with time under sustained loading. Factors affecting creep are composition of cement, admixture and sizes of aggregates, water-cement ratio, closing of internal voids and flow of water out of cement during drying.

1.9.2 Shrinkage

Volume change of concrete during hardening and curing. This may be controlled by continuously wetting or watering the structure during curing period.

Chapter 2: Design Methods and Requirements

2.1 Design Code

Varies depending on country. These codes must be adopted by a governing body to be legal.

2.2 Working Stress Design Method

A structure is designed so that the resulting stress in an element do not exceed a specified allowable stress. This may be expressed by the following expression:

$$f \leq f_{allow} \quad (3)$$

where:

f = an elastic stress calculated, for example by, $f = \frac{M_c}{I}$

f_{allow} = a predefined allowable stress given by a design code.

2.2.3 Obstacles on WSD Method

1. No simple way to account for different degrees of uncertainty of various kinds of loads.
2. Creep and shrinkage are not easily accounted for by calculation of elastic stress.
3. Concrete stress is not proportional to strain up to its crushing point.

2.3 Strength Design Method

Also called *Ultimate Strength Design (USD)* method or *Load Resistance Factor Design (LRFD)*.

The service loads are increased by factors to obtain load at which failure is considered imminent. The resulting loads are called *factored loads*.

The resulting stress in the structure is then proportioned such that strength is reached when factored loads are acting and can be expressed by:

$$strength\ provided \geq strength\ required \quad (4)$$

where:

strength provided = computed in accordance to provisions of a building/structural code.

strngth required = computed using structural analysis.

Chapter 3: Strength of Rectangular Sections in Bending

3.1 Flexural Behavior and Strength of Rectangular Sections

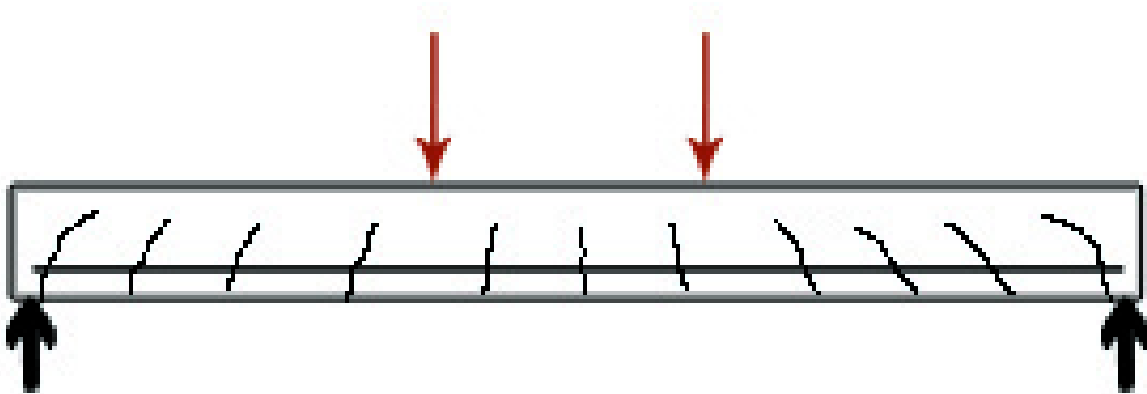


Figure 3.1: Simply supported beam with concentrated loads

On a section in Figure 3.1, flexure-induced stresses will cause compression at the top and tension at the bottom. Concrete, which is strong in compression but weak in tension resists the stresses in compression zone at the top while steel is placed at the bottom of the beam to resist tension stress.

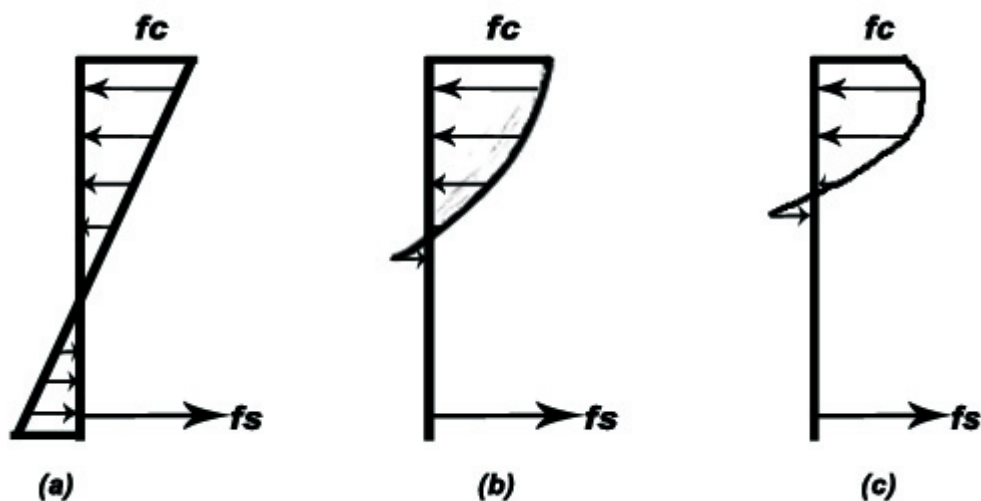


Figure 3.2: Stress distribution at 3 stages

Figure 3.2 above shows stress distributions of a singly reinforced concrete beam in three (3) different stages of its behavior.

1. Stage 1 (Figure 3.2 - a) - shows linear stress distribution in compression down to tension zone. This is also known as *elastic-uncrack stage*.

2. Stage 2 (Figure 3.2 - b) - shows a nonlinear stress distribution and that neutral axis goes higher towards compression zone. This is known as *elastic-crack stage* where concrete in tension area has already reached its cracking capacity.
3. Stage 3 (Figure 3.2 - c) - neutral axis has moved higher as cracks at the bottom grows. This is known as the *inelastic stage*.

Basis of Nominal Strength

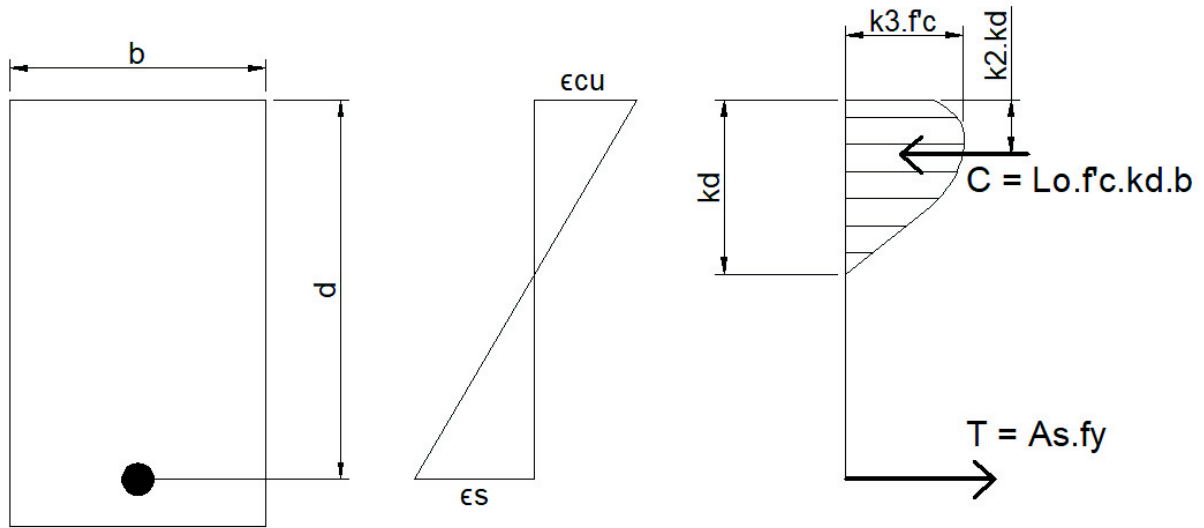


Figure 3.3: Beam Nominal Stress Diagram

On the diagram shown above,

Symbol	Value
ϵ_{cu}	crushing strain of concrete
ϵ_s	strain at steel tension
d	effective depth of beam
b	width of the beam section
kd	the distance from neutral axis to extreme compression fiber
Lo	the product of constants $k1$ and $k3$
As	Area of steel reinforcement
fy	yield strength of steel
$f'c$	compressive strength of concrete
$k2$	shape factor for the stress diagram and dictates the location of compression force resultant based on kd

Basis:

1. It has been assumed that plane section remains plane after bending up to failure of the beam. This assumption is for homogeneous elastic beams but has also been verified using tests to be good even for cracked, reinforced concrete beams at nominal strength.
2. On average (not true on a local cracked section), it is found that the bond of steel and concrete works well.

The nominal strength of the beam is reached when the strain at the extreme compression fiber reached ϵ_{cu} . And this is when the beam fails.

$$M_n = A_s \cdot f_y \cdot (d - k_2 \cdot kd) \quad (5)$$

$$< \text{Empty Math Block} > \quad (6)$$

3.2 Whitney Rectangular Stress Distribution

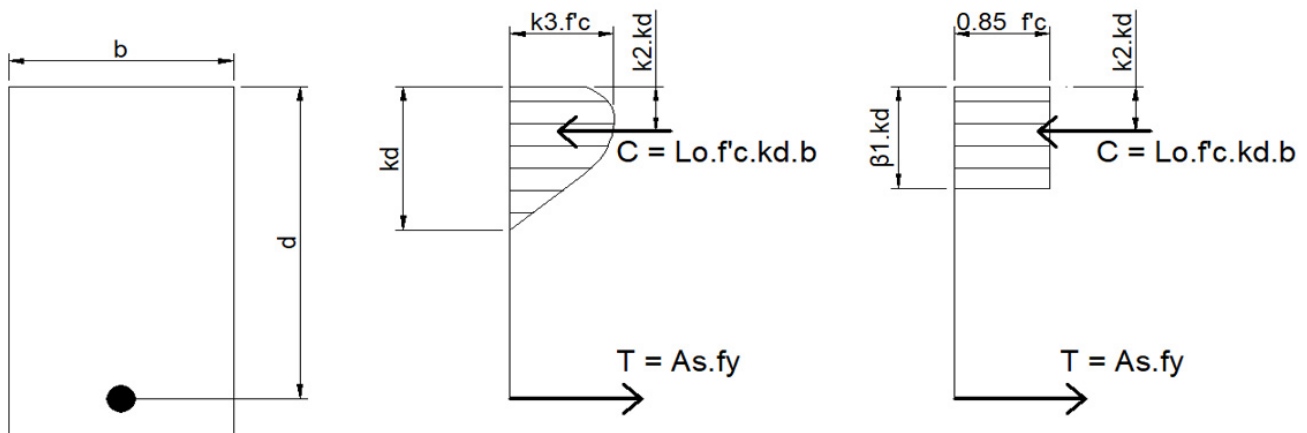


Figure 3.4: Whitney Equivalent Stress Block

3.3 Design of Section in Bending

3.3.1 Maximum Steel Ratios (ρ_{max}):

1. Singly Reinforced Beams

$$\rho_{max} = 0.75 \cdot \rho_{bal} \quad (7)$$

This is just a traditional requirement to ensure that steel yields first.

Note: A more direct way of controlling ductility is to provide maximum value of neutral axis, kd . ACI-B.10.3.3 shows that for singly reinforced beams,

$$kd_{max} = 0.75 \cdot kd_{bal} \quad (8)$$

2. Doubly Reinforced Beams

For ductility requirements, it is much more appropriate to follow and limit kd to $0.75 \cdot kd$.

3.4 Analysis of a Reinforced Concrete Beams at Various Stages

3.4.1 Uncrack - Elastic Stage

Stage just before the concrete cracks at tension fiber.

In this stage, as also shown in Figure 3.3 (a), stress distribution is linear. Finding kd at this stage is as simple as calculation of centroid of the transformed section.

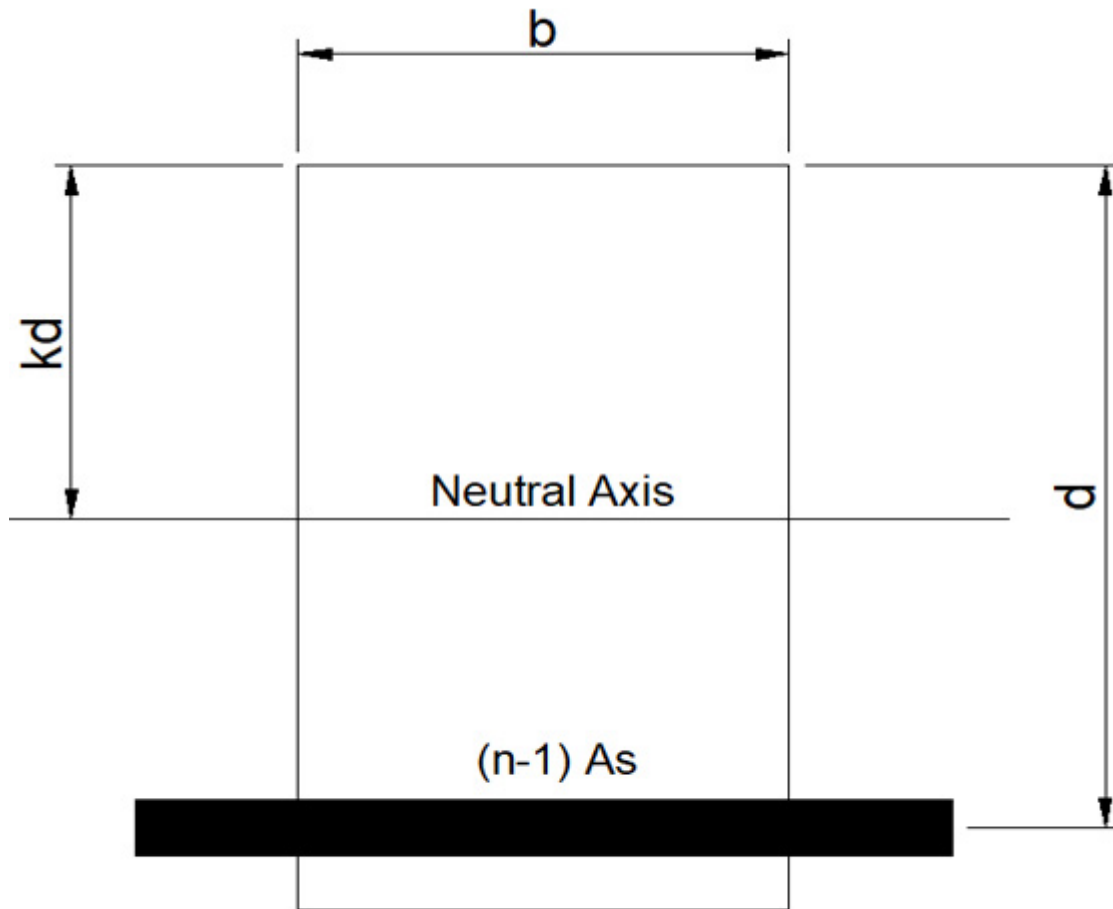


Figure 3.5: Transformed uncrack section

1. Calculate the total area of the transformed uncracked concrete by

$$Ac = b \cdot h + (\eta - 1) \cdot As \quad (9)$$

where:

A_c — total area of transformed uncracked concrete

h — total height of the beam section

η — modular ratio ($\frac{E_s}{E_c}$)

E_s — modulus of elasticity of steel, $200 \times 10^3 \text{ MPa}$

E_c — secant modulus of concrete taken up to $f_c = 0.5 f'_c$, $E_c = 4,700 \sqrt{f'_c}$

A_s —area of steel

2. Now calculate the total moments of areas to the top of section

$$Ma = b \cdot h \cdot \frac{h}{2} + (\eta - 1) \cdot A_s \cdot d \quad (10)$$

where:

Ma — moments of areas of concrete and transformed steel

3. We can now calculate kd by summation of moments:

$$A_c \cdot kd = Ma \quad (11)$$

$$kd = \frac{Ma}{A_c} \quad (12)$$

4. Solving for the stress at cracking, modulus of rupture,

$$f_{ct} = 0.6 \sqrt{f'_c} \quad (13)$$

(Section 1.7)

5. There are 2 ways to solve for the cracking moment now that kd is solved. Either by using the formula

$$M_{cr} = \frac{f_{ct} \cdot I_c}{h - kd} \quad (14)$$

which was derived from

$$f = \frac{M \cdot y}{I} \quad (15)$$

or by using equilibrium and take moment to centroid of compression block using the figure below

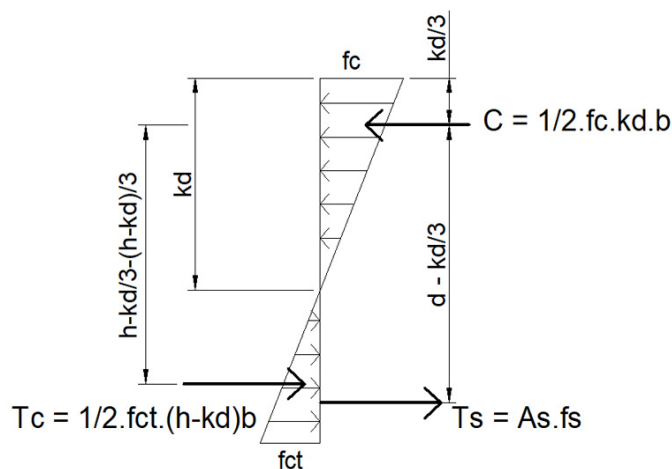


Figure 3.6 Uncracked Beam Stress Diagram

By taking moment to the compression block,

$$M_{cr} = Tc \cdot \left(h - \frac{kd}{3} - \frac{h - kd}{3}\right) + Ts \cdot \left(d - \frac{kd}{3}\right) \quad (16)$$

we should obtain values very close or the same as from eq. (14).

6. After this is the crack stage, where the stress block is no longer linear Figure 3.2 (b), but the behavior is still elastic. That is when strain at concrete ϵ_c is less than the elastic limit ϵ_o .

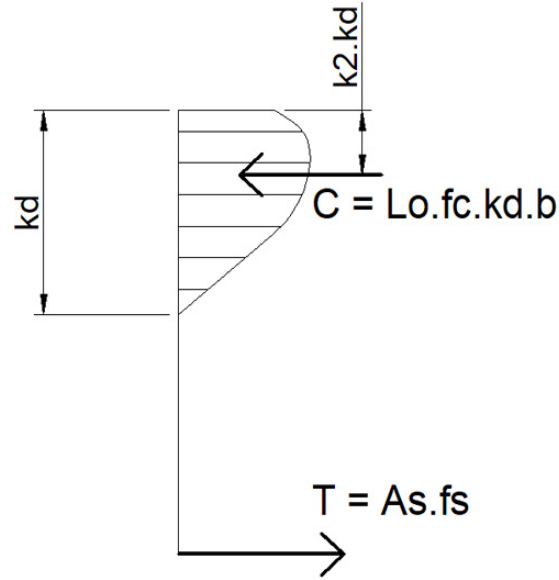


Figure 3.7 Stress Diagram for Crack Elastic Stage

So at any value of ϵ_c at this range, we can use the following formulas in calculating the moment.

$$k_2 = \frac{1}{4} \cdot \frac{(4 - \lambda_o)}{(3 - \lambda_o)} \quad (17)$$

$$\lambda_o = \frac{\epsilon_c}{\epsilon_o} \quad (18)$$

$$L_o = \frac{0.85}{3} \cdot \lambda_o \cdot (3 - \lambda_o) \quad (19)$$

$$fc = 0.85 \cdot f'_c \cdot (2\lambda_o - \lambda_o^2) \quad (20)$$

$$L_o \cdot fc \cdot kd \cdot b = As \cdot fs \quad (21)$$

At this stage, kd is still unknown. We solve for kd assuming steel yields, then we solve for fs to check.

$$fs = E_s \cdot \epsilon_c \cdot \frac{(d - kd)}{kd} \quad (22)$$

If steel does not yield, we know that our assumption is not correct and the value of kd we solve as well is not. We re-calculate for kd by using equilibrium equation replacing fs in the equation in terms of kd , so that we solve it using quadratic equation.

7. (To be continued...)

Chapter 4: Shear Strength and Shear Reinforcement

Shear failure or *Diagonal Tension* on beams is difficult to predict accurately.

Shear failure may be more catastrophic than flexure failure. A concrete beam without proper design of shear reinforcement could fail and collapse suddenly without any warning. Unlike in flexure, an under reinforced beam will show signs before failure like cracking, large deflections, and so they can be sometimes be applied with corrective measures.

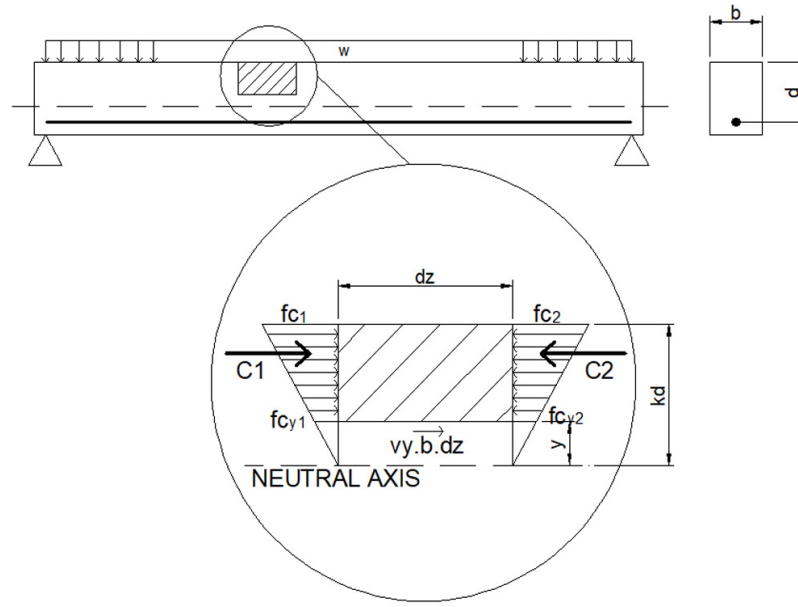


Figure 4.1 Shear Stress Derivation

4.1 Shear Formula Derivation

To initially derive the formula for horizontal shear stress ν , we base it in the figure above. (Note that these assumptions are based on the uncrack section due to failure, hence the triangular stress distribution)

First would be summation of horizontal forces $\sum F = 0$:

$$C_1 + \nu_y \cdot b \cdot dz = C_2 \quad (23)$$

Taking the trapezoidal part of stress blocks,

$$C_1 = \frac{f_{c1} + f_{cy1}}{2} \cdot (kd - y) \quad (24)$$

similarly,

$$C_2 = \frac{f_{c2} + f_{cy2}}{2} \cdot (kd - y) \quad (25)$$

by similar triangles,

$$\frac{f_{cy1}}{y} = \frac{f_{c1}}{kd} \quad (26)$$

$$f_{cy1} = \frac{f_{c1} \cdot y}{kd} \quad (27)$$

equation (24) now becomes

$$C_1 = \frac{f_{c1} + \frac{f_{c1} \cdot y}{kd} \cdot b \cdot (kd - y)}{2} \quad (28)$$

$$C_1 = \frac{1}{2} \cdot f_{c1} \cdot kd \cdot b \cdot [1 - (\frac{y}{kd})^2] \quad (29)$$

By taking moment of the compression solid to the steel tension reinforcement, we got an internal moment M_1 or M_2 depending on the direction. Let's say we consider M_1 first.

$$M_1 = C_c \cdot d \quad (30)$$

where:

$$C_c = \frac{1}{2} \cdot f_{c1} \cdot kd \cdot b$$

Rewriting equation (30),

$$\frac{M_1}{d} = \frac{1}{2} \cdot f_{c1} \cdot kd \cdot b \quad (31)$$

Inserting equation (31) to (29):

$$C_1 = \frac{M_1}{d} \cdot [1 - (\frac{y}{kd})^2] \quad (32)$$

similarly,

$$C_2 = \frac{M_2}{d} \cdot [1 - (\frac{y}{kd})^2] \quad (33)$$

Now going back to equation (23):

$$\nu_y \cdot b \cdot d_z = C_2 - C_1 \quad (34)$$

substituting equation (32) and (33) into (34):

$$\nu_y = \frac{M_2 - M_1}{b \cdot d_z \cdot d} \cdot [1 - (\frac{y}{kd})^2] \quad (35)$$

Now if we take the resultant of $M_2 - M_1$ and divide it to the width of the element, we have the shear force V .

$$\nu_y = \frac{V}{b \cdot d} \cdot [1 - (\frac{y}{kd})^2] \quad (36)$$

Now we have the value for the horizontal shear and remember that the vertical shear stress must be equal to this as

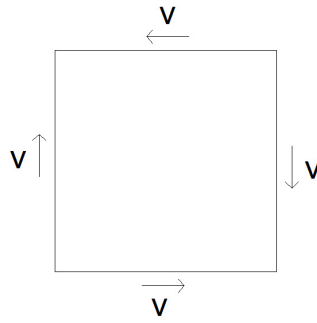


Figure 4.2 Beam Element with Pure Shear

See that if we get the shear stress at neutral axis, at $y = 0$, we get

$$\nu = \frac{V}{b \cdot d} \quad (37)$$

Large bending moments reduces the shear force at which diagonal cracks form to roughly half of the value is the moment is zero or nearly zero.

It is evident then that shear at which diagonal cracks are developed depends on ratio of shear force to bending moment.

4.2 Behavior of Diagonally Cracked Beams

1. The crack, once formed, spreads, traversing the entire beam from tension to compression face splitting it in two.

Because of this, it's a good practice to provide minimum shear reinforcement even if it's not required by calculations as they prevents cracks to grow.

2. Sometimes, diagonal cracks formed and spread up to compression zone then stops and not make it to the compression face. In this case, there is no sudden collapse by this failure.