The Mathematics Behind the Global Workspace Model

From Dehaene, Kerszberg, & Changeux (1998)

1 Neural Activation Functions

Basic sigmoid activation function for inhibitory units:

$$S^i_{\text{INH}} = \text{sigmoid}\left(\sum_j w_{i,j} S_j\right) \tag{1}$$

Where the sigmoid function is defined as:

$$\text{sigmoid}(x) = \frac{1}{1+e^{-x}} \tag{2}$$

2 Attentional Gating Mechanism

Activation of excitatory units with attentional modulation:

$$S^i_{\text{EXC}} = \text{sigmoid}\left(S^i_{\text{asc}} \cdot F(S^i_{\text{desc}})\right) \tag{3}$$

With ascending (processing) and descending (attentional) inputs:

$$S^i_{\text{asc}} = \sum_j w^{\text{asc}}_{i,j} S_j \quad \text{and} \quad S^i_{\text{desc}} = \sum_j w^{\text{desc}}_{i,j} S_j \tag{4}$$

The attentional modulation function F has properties:

$$F(x) \to 0 \text{ when } x \to -\infty, \quad F(0) = 1, \quad F(x) \to 2 \text{ when } x \to +\infty \tag{5}$$

This creates three processing modes:

$$S^i_{\text{desc}} > 0 : \text{Attentional amplification} \tag{6}$$

$$S^i_{\text{desc}} = 0 : \text{Normal unattended processing} \tag{7}$$

$$S^i_{\text{desc}} < 0 : \text{Attentional suppression} \tag{8}$$

3 Learning Rules

3.1 Reward-Modulated Hebbian Learning

Synaptic weight modification between excitatory units:

$$\Delta w_{\text{post,pre}} = \varepsilon \cdot R \cdot S_{\text{pre}} \cdot (2 \cdot S_{\text{post}} - 1) \tag{9}$$

Where:

- ε is the learning rate
- R is the reward signal ($+1$ for correct, -1 for incorrect)
- S_{pre} is the presynaptic unit activity
- S_{post} is the postsynaptic unit activity

Weights are bounded: $0 \leq w_{\text{post,pre}} \leq 7$

3.2 Vigilance Update Rule

The vigilance signal V modulates workspace activity:

$$\Delta V = \begin{cases} -0.1 \cdot V & \text{if } R > 0 \\ 0.5 \cdot (1 - V) & \text{otherwise} \end{cases} \tag{10}$$

This creates a slowly decreasing vigilance with sharp increases after errors.

3.3 Chemical Hebb Rule

Short-term depression or potentiation of synaptic weights:

$$\Delta w'_{\text{post,pre}} = \begin{cases} -0.5 \cdot w'_{\text{post,pre}} & \text{if } R < 0, S_{\text{pre}} > 0.5, S_{\text{post}} > 0.5 \\ 0.2 \cdot (1 - w'_{\text{post,pre}}) & \text{otherwise} \end{cases} \tag{11}$$

Where w' is a short-term multiplier on the excitatory synaptic weight.

4 Functional Significance

These equations formalize key properties of the global workspace:

1. Selective gating of processor inputs through descending connections
2. Top-down attentional amplification and suppression
3. Learning through reward-dependent plasticity
4. Dynamic stabilization and destabilization of workspace activity
5. Transition between automatic and effortful processing modes