Problem Set #1

1. Express , , , and in terms of , , , given :

The mean and median of y will just be the sample mean and median plugged into the equation:

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Plugging in and :

The IQR is only affected by the scale of x but not the translation:

2. Show the interesting optimization interpretations. We can show the first one by differentiating the equation we are trying to minimize and setting it equal to zero.

And we have:

So,

Which is the equation for the mean, so is the value that satisfies the equation:

For the median:

And

Which outputs the sign of the quantity. Therefore, we have

This sum come to zero when half of is to the left of and half of is to the right of , which is the definition of the median . Therefore, is the value that satisfies the equation:

3. What can you say about the distribution of the sample if points fall on the line instead of ?

The line that the points fall on give us insight to the shape and position of the distribution of points. If the points fall on the line , we can tell different properties of the distribution of points:

If , we can tell that the distribution has shorter tails than the standard normal distribution. Therefore, the distribution is more clumped towards the middle than the normal distribution.

If , we can tell that the distribution has longer tails than the standard normal distribution. Therefore, the distribution is more spread out than the normal distribution.

If , we can tell that the median of the distribution is shifted in the negative direction.

If , we can tell that the median of the distribution is shifted in the positive direction.

Problem 4 is attached as code.

Problem 5 is attached as code.

6. Compute the following:

a.

The probability that any given sample value from the sample is equal to N is 1/N since there is a one out of the size of the population chance.

This is the same for any value n.

b.

The probability for each sample value to be N is 1/N, so we just add the probabilities:

c.

The expected value for any sample value would be the mean of the population:

d.

This is equal to one of these events happening and the other one happening without replacement, which we multiply:

e.

This is similar to the previous example where we just multiply the probabilities. We do this for the size of the sample:

7a. The sum of the weights should be one:

We can show this:

7b. Now, we must minimize the standard error:

Following equation 13 from lecture 3:

We split this summation into two possible cases: and are equal or are not equal. We can note that

Therefore, we have

From part a, we know that , so we know . Again, we can look at the cases where and are equal and not equal:

Plugging this into our other equation, we get

With the summation as the only variable.

We will now use the Cauchy-Schwartz inequality, which only works when the variables are linearly dependent. Using this inequality, we get:

Where 1 and are linearly dependent. For this to hold, we have

For some constant a. This shows that all are equal, and we know that the sum is equal to one from part a. Therefore,

The most efficient estimate is the sample mean case stated in the problem.