Problem Set #6

1. From Lecture 13b, we have

The likelihood ratio statistic:

Therefore, we have two cases. When , our best estimate for will be and

Otherwise, if , our closest estimate for will be :

We reject when

Making a test size:

According to the notes

Therefore,

Solving,

Which is the same.

2. Similar to the previous question, we have the likelihood function with a subtle difference:

However, we must do an MLE for the variance and the mean. We know that

Therefore, we can set up our ratio:

With

Using the property from Piazza,

To get the sample variance, we need to have :

This takes the form of a t-statistic, so we reject the null hypothesis when

3. We have

Therefore, we want

We differentiate and set equal to zero

Showing this is a minimum:

This is always positive, assuming we don’t have the case where all , since anything squared is positive. Therefore, since the second derivate is positive when the first derivative is zero, this is the minimum.