Does God play Tetris? - Team reference document

Program submission checklist:

- 1. Works on sample inputs given.
- 2. Works on other sensible inputs.
- 3. Works on pathological inputs/corner cases.
- 4. Works in time on the largest possible inputs.
- 5. Works within memory limit (if given) use -Xmx128m for a limit of 128mb for example.
- 6. Compiles! (with warnings on! -Xlint)
- 7. No debug outputs!

Code

Big sample

```
import java.io.*;
import java.util.*;
import java.math.*;
public class samplecode {
  public static void debug(String s) {
    System.out.printf(">>>%s>>>\n", s); //Comment this out to kill n birds with two /
  public static void main(String[] args) throws Exception {
    BufferedReader br = new BufferedReader (new InputStreamReader (System.in));
    String s1 = br.readLine();
    int a = Integer.parseInt(s1.split(" ")[0]);
    String[] arr = s1.split("");
    // Does God play Tetris? used java.util.Collections, it's super effective!
    // A comparator can be defined by
    class MyClassCmp implements Comparator<MyClass> {
      //\ Should\ return\ a\ negative\ integer\ ,\ zero\ ,\ or\ a\ positive\ integer\ as\ the\ first
           argument is less than, equal to, or greater than the second respectively
      public int compare(MyClass a, MyClass b) {
         \textbf{return} \ a.a - b.a; \ \}
        / As far as I can tell this may not be neccessary, but probably best to do anyway
      public boolean equals(MyClass a, MyClass b) {
         return a.a == b.a; }
     // To change an array to a list we can do
    List < String > arrayaslist = Arrays.asList(arr);
    // Or make a general list
    List < MyClass > list = new LinkedList < MyClass > ();
    List < MyClass > list 2 = new Vector < MyClass > ();
    // If we have a comparator already we can do Collections.sort(arrayaslist); // or maybe
    Collections.sort(list, new MyClassCmp());
// If we have a sorted list we can do
    MyClass target = new MyClass(3);
    Collections.binarySearch(list, target, new MyClassCmp());
    SortedSet < MyClass > set = new TreeSet < MyClass > (new MyClassCmp());
     // We can also find maximums
    List < Integer > arr2 = new Array List < Integer > ();
    arr2.add(2);
    arr2.add(5);
    arr2.add(1);
    arr2.add(3);
    int b = Collections.max(arr2);
```

```
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```

```
2
```

Graphs

```
import java.util.*;
// This is wrapped up real nice in a class so it's super duper easy to use
^{\prime\prime}/ and because typing code of any length is O(1)
public class Graph {
    HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String \ , \ LinkedList < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ HashMap < String >> \ adjacents \ = \ new \ Adjacents \ = \ new \ Adjacents \ = \ new \ 
               >>();
     // Add an edge (u, v)
     public void add (String u, String v) {
           if (u.equals(v)) 
               if (!adjacents.containsKey(u)) {
                     \label{eq:linkedList} LinkedList < String > \ newList \ = \ \textbf{new} \ LinkedList < String > () \ ;
                     adjacents.put(u, newList);
                     return:
           if (adjacents.containsKey(u)) {
                if (!adjacents.get(u).contains(v)) {
                     adjacents.get(u).add(v);
          } else {
                \label{eq:linkedList} \mbox{LinkedList} < \mbox{String} > \ \mbox{newList} \ = \ \mbox{new} \ \mbox{LinkedList} < \mbox{String} > () \ ;
                newList.add(v);
                adjacents.put(u, newList);
           if (adjacents.containsKey(v)) {
                if (!adjacents.get(v).contains(u)) {
                     adjacents.get(v).add(u);
          } else {
                LinkedList < String > newList = new LinkedList < String > ();
                newList.add(u);
               adjacents.put(v, newList);
     // Tests whether (u, v) is an edge
     public boolean edge(String u, String v) {
         return (adjacents.containsKey(u) && adjacents.get(u).contains(v));
     // Returns the degree of node u
     public int degree (String u) {
         return adjacents.containsKey(u)? adjacents.get(u).size(): -1;
     public void bfs(String root) {
          LinkedList < String > processed = new LinkedList < String > ();
           String current = root;
          //while (adjacents.get(current)) {
                 // ...
           //}
```

```
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| }
```

Max-Flow

```
import java.util.*;
public class fordfulkerson
     static int fordFulkerson(int[][] cap, int[][] fnet, int n, int s, int t) {
           //ASSUMES: cap[u][v] stores capacity of edge (u,v). cap[u][v] = 0 for no edge.
           //Initialise the flow network so that fret[u][v] = 0 for all u,v
          int flow = 0; //no flow yet
           while (true) {
                //Find an augmenting path using BFS
                int[] prev = new int[n];
                Arrays. fill (prev, -1);
                LinkedList<Integer> queue = new LinkedList<Integer>();
                prev[s] = -2;
                queue.add(s);
                while (!queue.isEmpty() \&\& prev[t] == -1) {
                    int u = queue.poll();
                     for (int v = 0; v < n; v++) {
                          if (prev[v] = -1) { //not seen yet
                                \begin{tabular}{ll} \textbf{if} & (fnet [v][u] > 0 & || & fnet [u][v] < cap[u][v]) \end{tabular} \} \label{eq:cap_u} 
                                    prev[v] = u;
                                    queue.add(v);
                              }
                         }
                    }
                ^{\prime}//See if we couldn't find any path to t (t has no parents)
                if (prev[t] == -1) break;
                // Get the bottleneck capacity;
                \mathbf{int} \hspace{0.1in} \mathtt{bot} \hspace{0.1in} = \hspace{0.1in} \mathtt{Integer.MAX\_VALUE};
                for (int v = t, u = prev[v]; u >= 0; v = u, u = prev[v]) {
                    if (fnet[v][u] > 0) //prefer a backward edge over a forward
                          bot = bot < fnet[v][u] ? bot : fnet[v][u];
                     else //must be a forward edge otherwise
                          bot = bot < cap[u][v] - fnet[u][v]? bot : cap[u][v] - fnet[u][v];
                //update the flow network
                \begin{tabular}{lll} \begin{
                     if (fnet [v][u] > 0) //backward edge -> subtract
                          fnet[v][u] = bot;
                     else //forward edge -> add
                          fnet[u][v] += bot;
                ^{\prime}//Sent 'bot' amount of flow from s to v, so update the flow
               flow += bot;
          return flow;
     }
```

Shortest path

```
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      if (distance[edges.get(i).source] != INF && distance[edges.get(i).destination] >
           distance[edges.get(i).source] + edges.get(i).weight) {
        System.out.println("Negative edge weight cycles detected!");
        return:
    \mathbf{for} (\mathbf{int} i = 0; i < \mathbf{distance.length}; ++i) {
      if (distance[i] == INF)
        System.out.println("There's no path between " + source + " and " + i);
      else
        System.out.println ("The shortest distance between nodes" + source + " and " + i
             + " is " + distance[i]);
   }
 static class Edge {
    int source; // source node
    int destination; // destination node
    int weight; // weight of the edge
public Edge() {}; // default constructor
    public Edge(int s, int d, int w) { source = s; destination = d; weight = w; }
```

Min spanning tree

Gale-Shapley

```
import java.util.*;
public class galeshape {
  int m, n;
  boolean [][] graph;
  boolean seen[];
  \mathbf{int} \ \ \mathsf{matchL}\left[\,\right]; \qquad //What \ \ \mathit{left} \ \ \mathit{vertex} \ \ \mathit{i} \ \ \mathit{is} \ \ \mathit{matched} \ \ \mathit{to} \ \ (\mathit{or} \ -1 \ \ \mathit{if} \ \ \mathit{unmatched})
  int matchR[];
                          //What \ right \ vertex \ j \ is \ matched \ to \ (or -1 \ if \ unmatched)
  int maximumMatching() {
     /\!/Read\ input\ and\ populate\ graph \hbox{\tt [][]}
      //Set\ m\ to\ be\ the\ size\ of\ L,\ n\ to\ be\ the\ size\ of\ R
     Arrays. fill (matchL, -1);
     Arrays. fill (matchR, -1);
     int count = 0;
     \label{eq:formula} \mbox{for } (\mbox{int} \ \ i \ = \ 0\,; \ \ i \ < \mbox{m}; \ \ i + +) \ \{
        Arrays.fill(seen, false);
        if (bpm(i)) count++;
     return count;
  boolean bpm(int u) {
      //try to match with all vertices on right side
      for (int v = 0; v < n; v++)  {
        if (!graph[u][v] || seen[v]) continue;
        seen[v] = true;
        // match\ u\ and\ v\,,\ if\ v\ is\ unassigned\,,\ or\ if\ v\,'s\ match\ on\ the\ left\ side\ can\ be
              reassigned \ to \ another \ right \ vertex
        if (matchR[v] == -1 || bpm(matchR[v])) 

\operatorname{matchL}[u] = v;

           matchR[v] = u;
           return true;
       }
     return false;
```

Number theory

GCD

```
public class gcd {
  static int gcd(int a, int b) {
  int c = 0;
```

```
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    while(a!=0 && b!=0) {
        c = b;
        b = a%b;
        a = c;
    }
    return a+b;
}
static int arrGCD(int[] a) {
    int g = a[0];
    for (int i = 0; i < a.length; i++) {
        g = gcd(a[i],g);
        if (g == 1) break;
    }
    return g;
}</pre>
```

 $lcm(a, b) = ab/\gcd(a, b)$

Sieve of Eratosthenes

```
public class seive {
  public static boolean[] iscompslessthan(int n) {
    boolean[] iscomp = new boolean[n];
    for (int i = 2; i < Math.sqrt((double)n) + 1; i++) {
        if (iscomp[i]) continue;
        for (int j = i*i; j < n; j+=i) {
            iscomp[j] = true;
        }
    }
    return iscomp;
}</pre>
```

Dynamic programming

 ${\bf Discrete~knapsack~problem}$

```
import java.util.*;
public class knapsack
  \textbf{public static } List < Integer > \ zerooneknapsack ( List < Integer > \ values \ , List < Integer > \ weights
       , int W)
     int numItems = values.size();
     int maxWeight = Collections.max(weights);
     int[][] B = new int [numItems+1][maxWeight+1];
     for (int k = 0; k < numItems; k++) {
       for (int w = maxWeight; w >= weights.get(k); w--)
           \  \, \textbf{if} \  \, (\, values \, . \, get \, (\, k\,) \, + \, B\,[\, k\,] \, [\, w\!-\!w\, eig\, h\, t\, s\, . \, get \, (\, k\,) \,] \, > \, B\,[\, k\,] \, [\, w] \, ) \\
            B[k+1][w] = values.get(k) + B[k][w-weights.get(k)];
          else
            B[k+1][w] = B[k][w];
       for (int w = 0; w < w \operatorname{eights.get}(k); w++)
          B[k+1][w] = B[k][w];
     ArrayList < Integer > retval = new ArrayList < Integer > ();
     // The maximum value is noww B[numItems][maxWeight])
     // For items used, code is
      \begin{tabular}{ll} \textbf{for (int} & k = numItems-1, remainingWeight=maxWeight;} & k >= 0; k--) & \{ \end{tabular} 
       if (remainingWeight >= weights.get(k))
          if (B[k] | remainingWeight | == (values.get(k) + B[k] | remainingWeight - weights.get
               (k)]))
            retval.add(k);
             remainingWeight -= weights.get(k);
     return retval;
  }
```

Matching

```
public class kmp {
  static int[] createTable(char[] w) {
     int[] t = new int[w.length];
     int i = 2;
     \quad \textbf{int} \quad j \ = \ 0 \, ;
     t[0] = -1;
     while (i < w.length) {
       if (w[i-1] = w[j]) t[i++] = j+++1;
       else if (j > 0) j = t[j];
       else t[i++] = j = 0;
     return t;
  static int searchKMP(char[] w, char[] s, int[] t) {
     int m = 0;
     int i = 0;
     while ((m + i < s.length) \&\& (i < w.length)) {
       \mathbf{i} \mathbf{f} (\mathbf{s} [\mathbf{m} + \mathbf{i}] == \mathbf{w} [\mathbf{i}]) \quad \mathbf{i} + +;
       else {
         m += i - t[i];
          if (i > 0) i = t[i];
     \mathbf{return} \quad (i == w.length) ? m : -1;
```

Suffix arrays!

Geometry

Centroid of set of point $C = (x_1 + x_2 + \cdots + x_k)/k$. Centroid of figure, triangulate into right triangles X_1, \ldots, X_n and compute $C_x = (\sum C_{ix}A_i)/\sum A_i$, $C_y = (\sum C_{iy}A_i)/\sum A_i$ where the centroid of a right triangle perpendicular to the axis is b/3, h/3.

Simple data structures

```
public class Point implements Comparable<Point> {
  int x; int y;
  public int compareTo(Point p) {return (x-p.x == 0) ? y-p.y : x-p.x;}// left-bottommost
  public float cross(Point p) { return x*p.y - p.x*y; }
}
```

Convex hull, can be used for: furthest points, polygon containment $(P \text{ inside } Q \text{ iff hull}(Q) = \text{hull}(P \cup Q))$,

```
import java.util.*;
public class convexhull
  \textbf{static final double} \ \texttt{eps} = \ \texttt{0.0000000001};
  static int is Anti (Point x0, Point x1, Point x2) {
    double a = (x1.x-x0.x)*(x2.y-x0.y)-(x2.x-x0.x)*(x1.y-x0.y);
     if (a > eps | | -a > eps) return a > 0 ? -1 : 1;
    \mathbf{return} = 0:
  static int isCloser(Point x0, Point x1, Point x2) {
    double d1 = (x0.x - x1.x)*(x0.x - x1.x) + (x0.y - x1.y)*(x0.y - x1.y);

double d2 = (x0.x - x2.x)*(x0.x - x2.x) + (x0.y - x2.y)*(x0.y - x2.y);
    if (d1-d2 > eps | | d2-d1 > eps) return d1 < d2? -1 : 1;
    return 0;
  public static List<Point> hull(List<Point> points) {
     Collections.sort(points);
     final Point p0 = points.get(0);
     points.remove(p0);
     Collections.sort(points, new Comparator < Point > () {
```

```
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                                                                                                             7
         public int compare(Point p1, Point p2) {
         int a = isAnti(p0, p1, p2);
         if (a != 0) return a;
         return isCloser (p0, p1, p2);
    int m = points.size();
for (int i = 1; i < m; i++) { // Remove colinears
       if (isAnti(p0, points.get(i-1), points.get(i)) == 0) {
         points.remove(i-1);
         m--;
       }
    \label{eq:linkedList} \mbox{LinkedList} < \mbox{Point} > \mbox{ hull } = \mbox{new LinkedList} < \mbox{Point} > () \; ;
    if (m < 2) return hull; // All colinear, no hull
    hull push (p0);
     hull.push(points.get(0));
    hull.push(points.get(1));
for (int i = 2; i < m; i++) {
       while (isAnti(hull.get(0),hull.get(1),points.get(i)) <= 0) {
         hull.pop();
       hull.push(points.get(i));
    return hull;
  //Andrew monotone chain is faster still...
```

Closest pair of points

```
import java.util.*;
public class closestpoints {
  public static Point[] closestPair(Point[] arr){
    Point[] ret = {arr[0], arr[1]};
    Arrays.sort(arr);
    return ret;
  }
}
```

Pseudocode

BFS

```
procedure BFS(G, v) is
   create a queue Q
   create a set V
   enqueue v onto Q
   \mathrm{add}\ v\ \mathbf{to}\ \mathrm{V}
   \mathbf{while} \ \mathbf{Q} \ \ \mathbf{is} \ \ \mathbf{not} \ \ \mathbf{empty} \ \ \mathbf{loop}
      t \leftarrow Q.dequeue()
      if t is what we are looking for then
         return t
      end if
       \begin{tabular}{ll} \textbf{for} & all & edges & e & \textbf{in} & G.\, adjacent \, Edges \, (\,t\,) & loop \\ \end{tabular} 
         u \leftarrow G.adjacentVertex(t,e)
         if u is not in V then
            add u to V
            enqueue u onto Q
         end if
      \mathbf{end} \ \mathsf{loop}
   end loop
   return none
end BFS
```

```
procedure DFS(G,v):
  label v as discovered
  for all edges e in G.adjacentEdges(v) do
    if edge e is unexplored then
        w \leftarrow G.adjacentVertex(v,e)
        if vertex w is unexplored then
        label e as a discovered edge
        recursively call DFS(G,w)
        else
        label e as a back edge
        label v as explored
```

Dijkstra

```
function Dijkstra (Graph, source):
                                                                     // Initializations
// Mark distances from
    for each vertex v in Graph:
         dist[v] := infinity;
             source to v as not yet computed
         visited[v] := false;
                                                                    // Mark all nodes as
            unvisited
         previous[v] := undefined;
                                                                    // Previous node in optimal
             path from source
    end for
    \begin{array}{ccc} \text{dist} \left[ \begin{array}{ccc} \text{source} \, \right] & := & 0 \, ; \\ & \text{itself is zero} \end{array}
                                                                    // Distance from source to
                                                                    // Start off with the source
    insert source into Q;
          node
    while Q is not empty:
                                                                     // The main loop
         u := vertex in Q with smallest distance in dist[] and has not been visited;
            // Source node in first case
         remove u from Q;
         \mathtt{visited}\,[\,u\,] \ := \ \mathbf{true}
                                                                    // mark this node as visited
         for each neighbor v of u:
              alt := dist[u] + dist between(u, v);
                                                                    // accumulate shortest dist
                  from source
              if alt < dist[v] && !visited[v]:
                                                                    // keep the shortest dist
                   dist[v] := alt;
                      from src to v
                   previous[v] := u;
                   insert v into Q;
                                                                    // Add unvisited v into the
                       Q to be processed
              end if
         end for
    end while
    return dist;
endfunction
```

Bellman-Ford

```
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predecessor[v] := u

// Step 3: check for negative-weight cycles
for each edge (u, v) with weight w in edges:
    if distance[u] + w < distance[v]:
        error "Graph contains a negative-weight cycle"
```

Ford-Fulkerson

```
Inputs: Graph G with flow capacity c, a source node s, and a sink node t Output: A flow f from s to t which is a maximum f(u,v) \leftarrow 0 for all edges (u,v) While there is a path p from s to t in G_f, such that c_f(u,v) > 0 for all edges (u,v) in p: Find c_f(p) = \min\{c_f(u,v): (u,v) \in p\}
For each edge (u,v) \in p
f(u,v) \leftarrow f(u,v) + c_f(p) \text{ (Send flow along the path)}
f(v,u) \leftarrow f(v,u) - c_f(p) \text{ (The flow might be "returned" later)}
```

Toplogical Sort

```
L ← Empty list that will contain the sorted elements
S ← Set of all nodes with no incoming edges
while S is non-empty do
    remove a node n from S
    insert n into L
    for each node m with an edge e from n to m do
        remove edge e from the graph
        if m has no other incoming edges then
        insert m into S

if graph has edges then
    return error (graph has at least one cycle)
else
    return L (a topologically sorted order)
```

Longest Common Substring

```
function LCSubstr(S[1..m], T[1..n])
     L := array(1..m, 1..n)
     \mathbf{z} := \mathbf{0}
      ret := \{\}
      for i := 1..m
            \quad \textbf{for} \quad j \ := \ 1 \ldots n
                  if S[i] == T[j]
                        \mathbf{i}\mathbf{f} i == 1 or j == 1
                             L[i,j] := 1
                        else
                             L\,[\,\,i\,\,,\,j\,\,]\ :=\ L\,[\,\,i\,\,{-}1\,,j\,\,{-}1]\ +\ 1
                        if L[i,j] > z
                             z := L[i,j]
                             \mathrm{ret} \ := \ \{\widehat{S[i-z+1...i]}\}
                        elif L[i,j] == z
                             ret := ret union \{S[i-z+1...i]\}
                  else L[i,j]=0;
      return ret
```

Point-in-Polygon Test

One simple way of finding whether the point is inside or outside a simple polygon is to test how many times a ray, starting from the point and going ANY fixed direction, intersects the edges of the polygon.

If the point in question is not on the boundary of the polygon, the number of intersections is an even number if the point is outside, and it is odd if inside.

Polygon Stuff

A convex polygon is trivial to triangulate in linear time, by adding diagonals from one vertex to all other vertices. The total number of ways to triangulate a convex n-gon by non-intersecting diagonals is the (n-2)-th Catalan number

Delaunay Triangulation

The most straightforward way of efficiently computing the Delaunay triangulation is to repeatedly add one vertex at a time, retriangulating the affected parts of the graph. When a vertex v is added, we split in three the triangle that contains v, then we apply the flip algorithm. Done naively, this will take O(n) time: we search through all the triangles to find the one that contains v, then we potentially flip away every triangle. Then the overall runtime is $O(n^2)$.

Edit Distance

len s and len t are the number of characters in string s and t respectively

Formulae

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

In general:

$$\sum_{i=1}^{n} i^{m} = \frac{1}{m+1} \left[(n+1)^{m+1} - 1 - \sum_{i=1}^{n} \left((i+1)^{m+1} - i^{m+1} - (m+1)i^{m} \right) \right]$$

$$\sum_{i=1}^{n-1} i^{m} = \frac{1}{m+1} \sum_{k=0}^{m} {m+1 \choose k} B_{k} n^{m+1-k}.$$

Geometric series:

$$\sum_{i=0}^{n} c^{i} = \frac{c^{n+1} - 1}{c - 1}, \quad c \neq 1, \quad \sum_{i=0}^{\infty} c^{i} = \frac{1}{1 - c}, \quad \sum_{i=1}^{\infty} c^{i} = \frac{c}{1 - c}, \quad |c| < 1,$$

$$\sum_{i=0}^{n} i c^{i} = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^{2}}, \quad c \neq 1, \quad \sum_{i=0}^{\infty} i c^{i} = \frac{c}{(1-c)^{2}}, \quad |c| < 1.$$

Harmonic series:

$$H_n = \sum_{i=1}^n \frac{1}{i}, \qquad \sum_{i=1}^n iH_i = \frac{n(n+1)}{2}H_n - \frac{n(n-1)}{4}.$$

$$\sum_{i=1}^n H_i = (n+1)H_n - n, \quad \sum_{i=1}^n \binom{i}{m}H_i = \binom{n+1}{m+1}\left(H_{n+1} - \frac{1}{m+1}\right).$$