Hi! 😀

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5/28/2018

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Coleman integration

If $C/\mathbf{R}$ is a curve, $P, Q \in C$, $\omega \in \Omega^1_C$ (e.g. $\frac{x\mathrm{d}x}{y}$), we have a path integral

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These can be explicitly computed in many cases! ⌨️

Applications

Rational points: We can sometimes find ω so that

$$\mathrm{Zeroes}\left(\int_{p_0}^{x} \omega\right) \supseteq C(\mathbf{Q})$$ 🕵️‍♀️

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Heights:

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***p*-adic BSD:**

Using the above height pairing one can define a p-adic regulator so that for a modular abelian variety $A/\mathbf{Q}$ conjecturally

$$\mathcal{L}^*(A, 0) = \epsilon_p(A) \frac{|\text{Ш}(A/\mathbf{Q})| \operatorname{Reg}_\gamma(A/\mathbf{Q}) \prod_v c_v}{|A(\mathbf{Q})_{\text{tors}}||A^\vee(\mathbf{Q})_{\text{tors}}|}$$ 😵