2019-04-29

Something to Lean on; fun with interactive theorem provers

Alex J. Best
BU Math Retreat 2019

1. Thank the audience for being awake.

Mathematicians make mistakes.

like a living organism mathematics notices and fixes itself eventually (this is harder for a false proof of a true statement!).

Mathematicians make mistakes.

Sometimes they publish these mistakes.

like a living organism mathematics notices and fixes itself eventually (this is harder for a false proof of a true statement!).

Mathematicians make mistakes.

Sometimes they publish these mistakes.

Sometimes nobody notices.

like a living organism mathematics notices and fixes itself eventually (this is harder for a false proof of a true statement!).

The problem

Mathematicians make mistakes.

Sometimes they publish these mistakes.

Sometimes nobody notices.

At least for a while...

like a living organism mathematics notices and fixes itself eventually (this is harder for a false proof of a true statement!).

The problem

Mathematicians make mistakes.

Sometimes they publish these mistakes.

Sometimes nobody notices.

At least for a while...

However this uncertainty takes up time and energy, what if referees only needed to judge the importance, novelity and quality of exposition, not check the arguments.

like a living organism mathematics notices and fixes itself eventually (this is harder for a false proof of a true statement!).

Some examples: Grunwald(-Wang) and K-theory

> *Some days later I was with Artin in his office when Wang appeared. He said he had a counterexample to a lemma which had been used in the proof. An hour or two later, he produced a counterexample to the theorem itself... Of course he [Artin] was astonished, as were all of us students, that a famous theorem with two published proofs, one of which we had all heard in the seminar without our noticing anything, could be wrong.*
> — *Tate*

The problem: 2, the cursed prime. This is often an edge case.

Some examples: Grunwald(-Wang) and K-theory

> *Some days later I was with Artin in his office when Wang appeared. He said he had a counterexample to a lemma which had been used in the proof. An hour or two later, he produced a counterexample to the theorem itself... Of course he [Artin] was astonished, as were all of us students, that a famous theorem with two published proofs, one of which we had all heard in the seminar without our noticing anything, could be wrong.*
>
> — *Tate*

> *The groundbreaking 1986 paper "Algebraic Cycles and Higher K-theory" by Spencer Bloch was soon after publication found by Andrei Suslin to contain a mistake in the proof of Lemma 1.1. The proof could not be fixed.*
>
> — *Voevodsky*

The problem: 2, the cursed prime. This is often an edge case.

The new problem

How do you describe the steps of a proof to a computer with as little pain as possible?

(this is one place mistakes enter) Teach the computer to work off as little as possible.

The new problem

The new problem

How do you describe the steps of a proof to a computer with as little pain as possible? Often mathematicians leave unsaid many steps which are intuitive or easily supplied.

(this is one place mistakes enter) Teach the computer to work off as little as possible.

The new problem

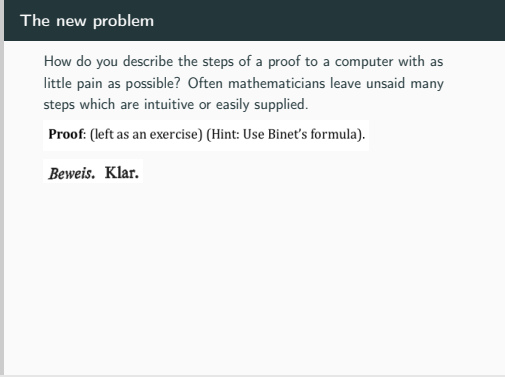
How do you describe the steps of a proof to a computer with as little pain as possible? Often mathematicians leave unsaid many steps which are intuitive or easily supplied.

Proof: (left as an exercise) (Hint: Use Binet's formula).

(this is one place mistakes enter) Teach the computer to work off as little as possible.

Something to Lean on; fun with interactive theorem provers

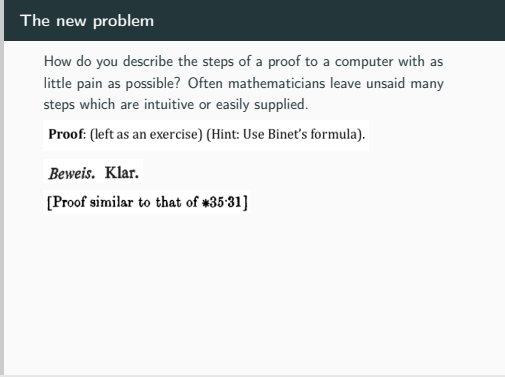
The new problem



(this is one place mistakes enter) Teach the computer to work off as little as possible.

Something to Lean on; fun with interactive theorem provers

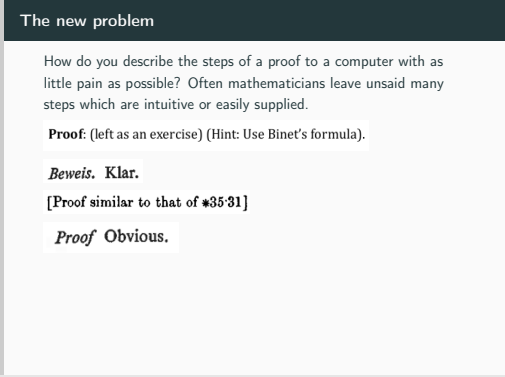
The new problem



(this is one place mistakes enter) Teach the computer to work off as little as possible.

Something to Lean on; fun with interactive theorem provers

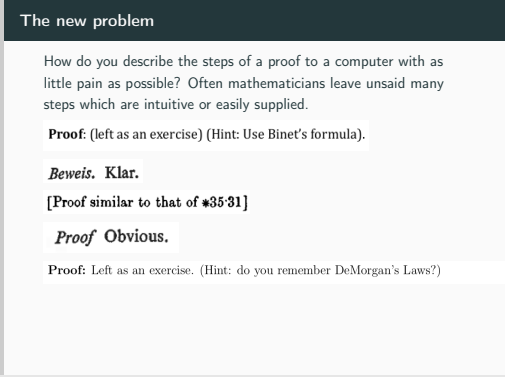
The new problem



(this is one place mistakes enter) Teach the computer to work off as little as possible.

Something to Lean on; fun with interactive theorem provers

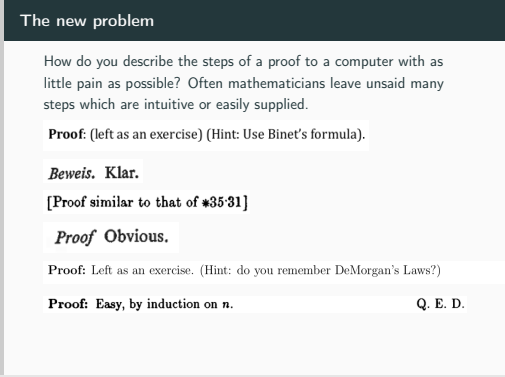
The new problem



(this is one place mistakes enter) Teach the computer to work off as little as possible.

Something to Lean on; fun with interactive theorem provers

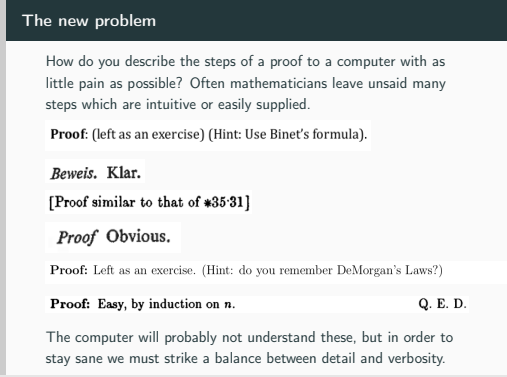
The new problem



(this is one place mistakes enter) Teach the computer to work off as little as possible.

Something to Lean on; fun with interactive theorem provers

The new problem



(this is one place mistakes enter) Teach the computer to work off as little as possible.

Something to Lean on; fun with interactive theorem provers

```
lemma fact_rec (n : ℕ) :
factorial (n + 1) = factorial n * (n+1) :=
begin
-- write out the definition of factorial
unfold factorial,
-- remember {1,...,n+1} = {1,...,n} ∪ {n+1}
rewrite list.range'_concat 1 n,
-- the product of two sequences joined together is
   just the product of the products of each sequence
rewrite list.prod_append,
-- I'm bored already are we done here?
simp,
-- YES!
end
```

Lean will figure out when and how to apply the lemmas.

Something to Lean on; fun with interactive theorem provers

```
lemma fact_rec (n : ℕ) :
factorial (n + 1) = factorial n * (n+1) :=
begin
-- write out the definition of factorial
unfold factorial,
-- remember {1,...,n+1} = {1,...,n} ∪ {n+1}
rewrite list.range'_concat 1 n,
-- the product of two sequences joined together is
    just the product of the products of each sequence
rewrite list.prod_append,
-- I'm bored already are we done here?
simp,
-- YES!
end
```

We can replace all of the above with: `by unfold factorial; simp [list.range'_concat, list.prod_append]`

Lean will figure out when and how to apply the lemmas.