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Explicit computation with Coleman integrals

1. Thank the audience for being awake.

Explicit computation with Coleman integrals

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Coleman integration

Question
Is there p-adic analogue of (path) integration?

Given a p-adic space, ($\approx$ the p-adic solutions to some equations). We can locally write down power series defining a 1-form and try to integrate.

For instance
near a point $\alpha \in \mathbf{G}_m(\mathbf{Q}_p) = \mathbf{Q}_p^\times$:

$$\frac{\mathrm{d}x}{x} = \frac{\mathrm{d}(\alpha + t)}{\alpha + t} = \frac{\mathrm{d}t}{\alpha + t} = \frac{1}{\alpha}\sum\left(\frac{-t}{\alpha}\right)^n \mathrm{d}t$$

Bad topology!

so that

$$\int_\alpha^{\alpha+t}\frac{\mathrm{d}x}{x} = -\sum\frac{1}{n+1}\left(\frac{-t}{\alpha}\right)^{n+1} + C$$

As number theorists it is natural to ask, Minhyong Kim has vastly generalised the above to cases where

$$\mathrm{rank}(\mathrm{Jac}(X))(\mathbf{Q}) \geq \mathrm{genus}(X)$$

This can be made effective, and computable

Theorem (Balakrishnan-Dogra-Muller-Tuitman-Vonk)
The (cursed) modular curve $X_{split}(13)$ (of genus 3 and jacobian rank 3), has 7 rational points: one cusp and 6 points that correspond to CM elliptic curves whose mod-13 Galois representations land in normalizers of split Cartan subgroups.

Their method can also be applied to other interesting curves:

Theorem (WIP B.-Bianchi-Triantafillou-Vonk)
The modular curve $X_0(67)^+$ (of genus 2 and jacobian rank 2), has rational points contained in an explicitly computable set of 7-adic points of cardinality 16.

many authors have tried to make this explicit and useful in examples culminating in