EXPLICIT COMPUTATION WITH COLEMAN INTEGRALS AND APPLICATIONS

ALEX J. BEST (*)

Abstract.

Coleman integration is a p-adic integration theory that provides a way integrate 1-forms on curves over p-adic fields [5]. The applications of this theory to arithmetic questions are varied, but include definitions of p-adic polylogarithms, explicit determination of rational points and torsion points on curves, and the computation of regulator maps in K-theory.

We will discuss the problem of computing these integrals in practice for special classes of interesting curves, such as hyperelliptic or superelliptic curves. Via Monsky-Washnitzer cohomology this problem is related to that of computing zeta functions of curves over finite fields [1]. One can try to extend methods for computing the zeta function to also compute Coleman integrals. Several algorithms can be developed for Coleman integration in this way [3, 4] with advantages and disadvantages of each. We will also mention work in progress to use such computations as part of quadratic Chabauty to determine all rational points on interesting modular curves (as pioneered in [2]), in cases that were as far as we know previously undetermined.

References

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Date: May 2019.

2010 *Mathematics Subject Classification.* Primary 11G20; Secondary 11Y16, 14F30.

Key words and phrases. Coleman integration, superelliptic curves, Kedlaya's algorithm.

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Boston University
Email address: alexbest@bu.edu