Numerical Methods in Scientific Computing - Exercise 1

Problem 2

Both harmonic and harmonic_bunch are stopping once the sum is invariant, the printed result is:

harmonic(): 15.4037
harmonic_bunch(50): 18.6237
harmonic_bunch(100): 19.3149
harmonic_bunch(200): 20.0089
harmonic_bunch(500): 20.9251

Problem 4

a
At first, f1 converges to -0.5 and f2 to 1 which is the correct solution (L'hopital).
Moving closer to 0, f1 becomes 0 and f2 starts to grow again with becoming 0 eventually as well.

Insert number:0.5
f1: -0.489670 f2: 1.042191

Insert number:0.1
f1: -0.499583 f2: 1.001668

Insert number:0.001
f1: -0.500000 f2: 1.000000

Insert number:0.00000001
f1: 0.000000 f2: 1.000000

Insert number:0.000000000000001
f1: 0.000000 f2: 1.054712

B - See image in next

C - The results stay -0.5 and 1 even for a very small x (or just 0):
Insert number:0.000000000000001
f1: -0.500000 f2: 1.000000

Insert number:0
f1: -0.500000 f2: 1.000000

$\boxed{1}$

$\hat{x} \in [x-e_x, x+e_x]$ $\hat{y} \in [y-e_y, y+e_y]$ are machine representation of x, y.

$r_x = \frac{e_x}{x}$, $r_y = \frac{e_y}{y}$ are the relative errors.

we can define $\Delta x \in [-e_x, e_x]$ $\Delta y \in [-e_y, e_y]$ and

$\hat{x} = x + \Delta x$ $\hat{y} = y + \Delta y$ so the mult is

$\hat{x}\hat{y} = (x+\Delta x)(y+\Delta y) = xy + x\Delta y + \Delta x y + \Delta x \Delta y$

$r_{xy} = \frac{xy - \hat{x}\hat{y}}{xy} = \frac{x\Delta y + y\Delta x + \Delta x \Delta y}{xy}$ as $|r_x|, |r_y| << 1$

we can neglect $\frac{\Delta x \Delta y}{xy}$ and get $\frac{\Delta y}{y} + \frac{\Delta x}{x} =$
$= \boxed{|r_x| + |r_y|}$

for the quotient $\frac{\hat{x}}{\hat{y}}$ we get the same result as

if $\hat{z} = \frac{\hat{x}}{\hat{y}} \rightarrow \hat{x} = \hat{z} \cdot \hat{y}$.

$\boxed{3}$ Show that 2's complement is the additive inverse of binary integer.

let α a bit integer, and $\bar{\alpha}$ the bit wise compiement of α.

then $\alpha + \bar{\alpha} = 11\ldots111$ with the length of the number of bits.
now adding $\alpha + \bar{\alpha} + 1 = 0$ as adding 1 to a all 1 bit integer carries it to the most significant bit and beyond.

$\boxed{14}$

B) $f_1(x) = \dfrac{\cos x - 1}{x^2}$ $\quad f_2(x) = \dfrac{e^x - e^{-x}}{2x}$

expanding inner funcs into Taylor series:

- f_1: $\cos x = \sum_0^{k-1} \frac{(-1)^n}{(2n)!} x^{2n} + R(k)$ $\qquad R(k) = \frac{x^k}{k!} f^{(k)}(\xi)$ $\xi \in [0, x]$

then $f_1(x) = \dfrac{\sum_0^{k-1} \frac{(-1)^n}{(2n)!} x^{2n} - 1 + R(k)}{x^2} = \dfrac{\sum_1^{k-1} \frac{(-1)^n}{(2n)!} x^{2n} + 1 - 1 + R(k)}{x^2}$

$= \sum \frac{(-1)^n}{(2n)!} \cdot x^{2n-2} + \frac{R(k)}{x^2}$

the error is $\dfrac{\frac{x^k}{k!} f^{(k)}(\xi)}{x^2} = \frac{x^{k-2}}{k!} f^{(k)}(\xi)$ with $f^{(k)}$

being either cos or sin and $\cos(x) \xrightarrow[x \to 0]{} 1$ we can sy

$\frac{x^{k-2}}{k!} f^{(k)}(\xi) \leq \frac{x^{k-2}}{k!}$

- f_2: $e^x + e^{-x} = \sum_0^{k-1} \frac{2x^{2n-1}}{(2n-1)!} + R(k)$

$f_2(x) = \dfrac{e^x - e^{-x}}{2x} = \dfrac{\sum_0^{k-1} \frac{2x^{2n-1}}{(2n-1)!} + R(k)}{2x} = \sum_1^{k-1} \frac{x^{2n-2}}{(2n-1)!} + \frac{R(k)}{2x}$

now the error is: $R(k) = \dfrac{\frac{x^k}{k!} f^{(k)}(\xi)}{2x} = \frac{1}{2} \frac{x^{k-1}}{k!} f^{(k)}(\xi)$

$f^{(k)} = (e^x + e^{-x})^{(k)} = e^x \pm e^{-x}$ then

$e^x + \frac{1}{e^x} \leq \frac{1}{1-x} + \frac{1}{x+1} = \frac{2}{(1-x)^2}$

$|R(k)| \leq \frac{1}{2} \frac{x^{k-1}}{k!} \cdot \frac{2}{(1-x)^2} = \frac{x^{k-1}}{k!(1-x)^2}$