Alex Jia, Ishan Patel
CS440 ajj64, ip181

Assignment 03

Question 1:

A. P(A, B, C, D, E = true): P(A=true)*P(B=true)*P(C=true)*P(D=true|A,B)*P(E=true|B,C)
= 0.2 * 0.5 * 0.8 * 0.1 * 0.3
= 0.0024

B. P(A, B, C, D, E = false):
P(A=false)*P(B=false)*P(C=false)*[1-P(D=true|A,B)]*[1-P(E=true|B,C)]
= 0.8 * 0.5 * 0.2 * 0.1 * 0.8
= 0.0064

C. P(A=false | B, C, D, E = true):
P(A=false)*P(B,C,D,E=true | A=false) / P(A=false)*P(B,C,D,E=true | A=false) + P(A=true)*P(B,C,D,E = true | A = true)
= 0.8 * [0.5*0.8*0.6*0.3] / 0.8 * [0.5*0.8*0.6*0.3] + 0.2 * [0.5*0.8*0.1*0.3]
= 0.0576 / (0.0576 + 0.0024)
= 0.96

Question 2:

A. Given P(Burglary | JohnsCalls = true, MaryCalls = true) *NOTE: I have followed the book's convention for naming Burglary, JohnCalls, MaryCalls, Alarm, and Earthquake respectively.*

P(B | j, m) = α P(B)$\sum_e$P(e) $\sum_a$ P(a | B, e) P(j | a) P(m | a)
= α P(B)$\sum_e$P(e)$\sum_a$ f_A(A, B, E) x f_J(a) x f_M(a)
= α P(B)$\sum_e$P(e) * $\sum_a$ f_A(a, B, E) x f_J(a) x f_M(a)
= α P(B)$\sum_e$P(e) * [f_A(a, B, E) x f_J(a) x f_M(a) + f_A(a', B, E) x f_J(a') x f_M(a')]
= α P(B)$\sum_e$P(e) * ([0.90 x 0.70 x Matrix_{2x2}(0.95, 0.29, 0.94, 0.001)] + [0.50 x 0.10 x Matrix_{2x2}(0.05, 0.71, 0.06, 0.999)])
= α P(B)$\sum_e$P(e) * [Matrix_{2x2}(0.5985, 0.1827, 0.5922, 0.0063) + Matrix_{2x2}(0.00025, 0.00355, 0.0003, 0.004995)]
= α P(B)$\sum_e$P(e) * [Matrix_{2x2}(0.5985825, 0.183055, 0.59223, 0.0011295)]
= α P(B) * [0.002 x Matrix_{2x1}(0.598525, 0.59223) + 0.998 x Matrix_{2x1}(0.59223, 0.011295)]
= α Matrix_{2x1}(0.00119, 0.999) x Matrix_{2x1}(0.5922459, 0.0014918576)
= α * Matrix_{2x1}(0.00059224, 0.0014919)
= α * Matrix_{2x1}(0.00059224, 0.0014919) / 0.0020841002
= (0.2841718406, 0.715828154)
= (0.284, 0.716)

B. The enumeration algorithm has 7 adds, 18 multiplies, 2 divisions, while the variable elimination algorithm has 7 adds, 16 multiplies, and 2 divisions.

C. Enumeration: O(2*n).
Variable Elimination: O(n).

Question 3:

A. Given $P(X|MB(X)) = \alpha\, P(X|\, U_1, \ldots, U_m)\ \Pi_{Y_i}\ P(Y_i \mid Z_{i1} \ldots)$
$= P(X|\, U_i, \ldots, U_m, Y_i, \ldots, Y_n, Z_{i1}, \ldots, Z_{in})$
$= P(X|\, \text{Parents}(X), Y_i, \ldots, Y_n, Z_{i1}, \ldots, Z_{in})$
$= \alpha\, P(X|\text{Parents}(X), Y_i, \ldots, Y_n, Z_{i1}, \ldots, Z_{in})$
$= P(X|\text{Parents}(X))\, \Pi_{U_i} P(U_i|\text{Parents}(U_i))\, \Pi_{Y_i} P(Y_i|\text{Parents}(Y_i)))\, \Pi_{Z_{i,j}} P(Z_{i,j}|\text{Parents}(Z_{i,j}))$
$= \alpha\, P(X|\text{Parents}(X))\, \Pi_{Y_i} P(Y_i|\text{Parents}(Y_i))$
$= \alpha\, P(X|U_i, \ldots, U_m)\ \Pi_{Y_i} P(Y_i|X, Z_{i1}, \ldots, Z_{in})$
$= \alpha\, P(X|\, U_1, \ldots, U_m)\ \Pi_{Y_i}\ P(Y_i \mid Z_{i1} \ldots, Z_{in})$

B. P(Rain|Sprinkler = True, WetGrass = True)
Possible states:
Rain=True, Cloudy=True, Sprinkler=True, WetGrass=True
Rain=True, Cloudy=False, Sprinkler=True, WetGrass=True
Rain=False, Cloudy=True, Sprinkler=True, WetGrass=True
Rain=False, Cloudy=False, Sprinkler=True, WetGrass=True

C.

	R∩C∩S∩W	R'∩C∩S∩W	R∩C'∩S∩W	R'∩C'∩S∩W
R∩C∩S∩W	0.627	0.0930	0.280	0
R'∩C∩S∩W	0.4074	0.1164	0	0.4762
R∩C'∩S∩W	0.222	0	0.386	0.392
R'∩C'∩S∩W	0	0.024	0.108	0.868

Question 4: *NOTE: I used the notation A' to mean negation(A), or ~A. This was taken from the lecture notes of a concurrent course I am taking, CS206*

A. Net gain = 70% chance for $3,000 buy, $4,000 sell + 30% chance for $4,400 buy, $4,000 sell

= 0.7 * ($4,000 - $3,000) + 0.3*($4,000 - $4,400)
= 0.7 * ($1,000) + 0.3* (-$400)
= **$580 net gain/profit.**

B. We know that Bayes' Theorem is defined as:

P(A|B) = [P(A)*P(B|A)] / [P(A)*P(B|A) + P(A')*P(B|A')]

And we are additionally given information about the mechanic's test:

P(pass(C1)|q^+(C1)) = 0.8, P(pass(C1)|q^-(C1)) = 0.35

For the sake of not having to repeatedly write out names and risk confusing myself:

Let A = event where the vehicle C1 is actually in good condition (i.e q^+)
Let B = event where the mechanic says C1 Passes

P(q^+|Pass) = P(A|B)
= P(A) * P(B|A) / P(A) * P(B|A) + P(A') * P(B|A')
= (0.7) * (0.8) / (0.7) * (0.8) + (0.3) * (0.35)
= **0.8421052**

P(q^+|Fail) = P(A|B')
= P(A) * P(B'|A) / P(A) * P(B'|A) + P(A') * P(B'|A')
= (0.7) * (1-0.8) / (0.7) * (1-0.8) + (0.3) * (1-0.35)
= **0.4179104**

P(q^-|Pass) = P(A'|B)
= P(A') * P(B|A') / P(A') * P(B|A') + P(A) * P(B|A)
= (0.3) * (0.35) / (0.3) * (0.35) + (0.7) * (0.8)
= **0.1578947**

P(q^-|Fail) = P(A'|B')
= P(A') * P(B'|A') / P(A') * P(B'|A') + P(A) * P(B'|A)
= (0.3) * (0.65) / (0.3) * (0.65) + (0.7) * (0.2)
= **0.5820896**

C. Given a pass, the best decision would be to buy C1 (as it is very likely that it is in good condition). Given a fail, the best decision would be to still buy C1 (as the chance of it not being in good condition is only slightly more probable than that of it being in good condition, indicating a nearly equal chance that the car is just fine as it is being bad).

Pass: $E[Utility_{buy}]$ = (4000 - 3000)*0.8421052 + (4000 - 4400) * 0.1578947 = 778.94732
$E[Utility_{sell}]$ = 0

Fail: $E[Utility_{buy}]$ = (4000 - 3000) * 0.4179104 + (4000 - 4400) * 0.5820896 = 185.07456
$E[Utility_{sell}]$ = 0

D. I would not take C1 to the mechanic; while the likelihood of the vehicle being in good condition is already fairly high (70%), the cost of the mechanic in order to raise that percentage around 14% (around 84%) is not worth it, particularly in the event that the test fails. With near equal chances of the mechanic's prediction being false as it is true (given a fail), there is no significant benefit in relying on said additional opinion.

Question 5: Programming Component

A. Given ({Right, N})

$H_{1,1}$	$H_{1,2}$	$T_{1,3}$
$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,1}$	$B_{3,2}$	$H_{3,3}$

Cell 1,1 = 1/1600 Cell 2,1 = 9/800 Cell 3,1 = 9/80
Cell 1,2 = 10/1600 Cell 2,2 = 90/800 Cell 3,2 = 0
Cell 1,3 = 14/1600 Cell 2,3 = 180/800 Cell 3.3 = 1/160

Given ({Right, N} {Right, N})

$H_{1,1}$	$H_{1,2}$	$T_{1,3}$
$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,1}$	$B_{3,2}$	$H_{3,3}$

Cell 1,1 = 0.000025 Cell 2,1 = 0.000250 Cell 3,1 = 0.000350
Cell 1,2 = 0.002500 Cell 2,2 = 0.005625 Cell 3,2 = 0
Cell 1,3 = 0.002500 Cell 2,3 = 0.005625 Cell 3.3 = 0.0000312500

Given ({Right, N} {Right, N} {Down, H})

$H_{1,1}$	$H_{1,2}$	$T_{1,3}$
$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,1}$	$B_{3,2}$	$H_{3,3}$

Cell 1,1 = 0.00000225 Cell 2,1 = 0.00001250 Cell 3,1 = 0.0000175
Cell 1,2 = 0.00022500 Cell 2,2 = 0.00028125 Cell 3,2 = 0
Cell 1,3 = 0.00001250 Cell 2,3 = 0.00028125 Cell 3,3 = 0.000028125

Given ({Right, N} {Right, N} {Down, H} {Down, H})

$H_{1,1}$	$H_{1,2}$	$T_{1,3}$
$N_{2,1}$	$N_{2,2}$	$N_{2,3}$
$N_{3,1}$	$B_{3,2}$	$H_{3,3}$

Cell 1,1 = 0.000000202500 Cell 2,1 = 0.0000006250 Cell 3,1 = 0.0000008750
Cell 1,2 = 0.000020250000 Cell 2,2 = 0.0000140625 Cell 3,2 = 0

Cell 1,3 = 0.000000062500 Cell 2,3 = 0.0000140625 Cell 3,3 = 0.0000253125