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I. For each of the following sets, determine whether 9 is an element of that set (answer with Y or N).

a. {9,9} **Y**
b. {{{9}}, {9}} **N (9 and {9} are considered different elements, same as {{9}})**
c. {9, {9}} **Y**
d. {{{9}}} **N (again, {{9}} and 9 are not the same elements)**

II. For each of the above sets, determine whether {9} is a subset of that set (answer with Y or N).

a. {9,9} **N (but 9 without the "{" and "}" is considered a subset of part a)**
b. {{{9}}, {9}} **Y**
c. {9, {{9}} **Y**
d. {{{9}}} **N ({{9}} and {9} are not the same)**

III. What is the Powerset(Powerset(S)) for S={a,b}

Since a powerset is basically the linear combination of smaller sets that make up a set (in this case that is S, or {a,b}), the powerset of S would be {{}, {a}, {b}, {a,b}, {b,a}}. Since we consider {a,b} and {b,a} to be the same thing, we can ignore the latter ({b,a}). Now Powerset(Powerset(S)) is the same as asking for the Powerset({{}, {a}, {b}, {a,b}}). This is equivalent to { {}, {{}}, {{a}}, {{b}}, {{a,b}}, {{}, {a}}, {{}, {b}}, {{},{a,b}}, {{a}, {a,b}}, {{b}, {a,b}} }.

IV. For each of the following indicate with a (Yes or No) whether it is a power set of some other arbitrary set S, and if yes, provide that S:

a. { {}, {2} } **Yes; S={2}**
b. {} **Yes; S={}**
c. { {}, {2}, { {}, 2 } } **Yes; S={2,{}}**

Section 2.2
The proofs of set equality below should NOT use set identities (which were not covered in lecture)

IV. Let A, B and C be sets.

a. Prove (A - B) - C is a subset of (A - C)

(A - B) will always give you a set of less than or equal size to that of A by itself. Therefore, (A - B) - C would always be less than or equal to (A-C), which is the second part of the proposition. Ex. A={1,2,3}, B={1,2,3,4}, C={}. (A-B) = {}, thus (A-B)-C = {} - {} = {}. Consequently, (A-C)={1,2,3} - {} = {1,2,3}. {} is a \subset of {1,2,3}.

b. A union (B - A) = A union B

(B-A) will always give you a set smaller than or equal to B. So while the proposition above would be false *if we had \intersect symbols* between the sets, a *\union is inclusive*, so A \union B would give you the same elements as that of A \union (B with all elements from A removed (the \union would essentially negate the effect of the set difference.)).

V. Let A, B and C be sets. In which of the following cases can you conclude that A = B

a. A union C = B union C ; Let A={}, B={1}, C={1}. A \union C = {1}, B \union C = {1,1} = {1}
b. A intersect C = B \intersect C This is my answer.
c. Both (a) and (b) above

VI. Define the symmetric difference of sets A and B (written as A [+] B here), to be the set of elements which are in either A or B but not in both.

a. What is {b,g,k,m} [+] {a,g,m}?

{a,b,k} (since {g,m} are common elements in both sets.

b. What can you say about sets A and B that are subsets of universal set U, if A [+] B = U?

If the symmetric difference of sets A and B is equivalent to the universal set U, then that would be saying that either one or both sets A and B must be the empty set {}, because there would be no way to prove that A [+] B = U given that A and B are \subsets of U (see part a for a counterexample to prove this)

c. *Determine if A [+] (B [+] C) = (A [+] B) [+] C either by giving the smallest counter-example, or by giving a proof.

Given arbitrary sets A, B, and C, we need to prove that A [+] (B [+] C) = (A [+] B) [+] C. The two equations are more or less identical, excluding the parentheses. A [+] (B [+] C) is obviously equivalent to (A [+] B) [+] C because of the associative property. In terms of mathematics, the associative property states that in terms of addition, the order of terms used does not affect the result; for arbitrary values x, y, z, x + (y + z) is the same as (x + y) + z; similarly, x + (z + y) is equivalent to the aforementioned expressions. This is also the case for the symmetric difference of two terms. Switching the terms does not affect the solution. Therefore, we can assume that, given the similarities between a symmetric difference and a sum, both sides of the proposition are equal to each other.

BONUS: State and prove a theorem which captures the intuition that the order of elements in a set is not relevant. (First express the intuition that _duplicates_ do not matter by considering the pseudo-math pattern {...,b,...,b,...}, and then state this as a slightly different theorem than the one in the class/notes; now repeat for _order_)

Theory: Given that S = {x, y}; x, y are all \in S for any values of x, y, z.
Proof: Given the arbitrary values of elements within an arbitrary set, we must prove that the order of the elements in said set does not matter.

First of all, we know that x, y are \in S. Therefore, we can assume that S is a subset of any set K \union {x,y} and K \union {y,x}, due to the definition of subsets. If the order of elements of K does not affect its eligibility as a proper subset of S, then the order of elements x, y within S should not matter; in other words, the order of elements in a set is not relevant.