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Section 06
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HW 4

Section 1.5

A. Prove using the rules of propositional and predicate equivalence from Rosen that the following equivalences hold. Be sure to do a precise formal proof, with lines justified by equivalence rule names.
   1. Forall x. P(x) \/ Exists x. ~P(x) == True
      1. P(x) Universial Instantiation
      2. ~P(x) Existential Instantiation
      3. True Inverse 1,2 (P \/ ~P == True)

   2. Exists x. P(x) == ~ Forall x. ~P(x)
      1. \Exists(x) P(x) Given
      2. \Forall(x) P(x) Quantifier de Morgan 1
      3. ~~\Forall(x) P(x) 2-negation 2
      4. ~\Forall(x) ~P(x) Quantifier de Morgan

B. Express the _negation_ of each of the following formulas so that all negation symbols appear only immediately before predicates:
   1. Exist x. Exist y. P(x,y) \/ Forall x. Forall z. Q(x,z)
      ~Existx. ~Exist y. P(x,y) \/ ~Forall x. ~ Forall z. Q(x,z)
   2. Exist x. Exist y. (Q(x,y) <-> Q(y,x))
      ~Exist x. ~Exist y. (Q(x,y))

Section 1.6

B. For each of the following claims
- Either prove they are true using rules of inference of the predicate calculus: you may use either Rosen or NatDedn _propositional_ reasoning rules, and the Rosen quantifiers rules in Table 6, augmented with my restrictions on constant symbols (see handout on inference rules).
- Or provide a counter-example, by defining appropriate predicate atoms (e.g., (likes(bob,eve),..) in my updated lecture notes); for full credit, use the smallest possible universe for the quantifiers.
  [Hint: think first about what the statements say _intuitively_, maybe using something familiar instead of predicates P and Q.]

a. Exists y.(P(y) /\ Q(y)) |- Exists x.P(x) /\ Exists y.Q(y)

1. \Exists y.(P(y) /\ Q(y)) Given
2. P(x) /\ Q(x) Existential Intialization
3. Q(x) /\ P(x) Commutative
4. P(x) Simplification
5. \Exists(x). P(x) Existential Generalization
6. \Exists (x). P(x) /\ \Exists (y). Q(y) And Introduction

b. Exists y.(P(y) \/ Q(y)) |- Exists x.P(x) /\ Exists x.Q(x)
   1. P(y) \/ Q(y) Existential Intitialization
   2.   [~P(y)] Assume
   3.   Q(y) Disjunctive syllogism
   4. \Exists(x) Q(x) Existential Generalization
   5. \Exists (x) Q(x) /\ \Exists(x) P(x) And Introduction

c. Forall x.P(x) /\ Forall y.Q(y) |- Forall z.(P(z) \/ Q(z))

Let x = all humans, P(x) = “x can inhale”, Q(x) = “x can exhale”, y = all children, z = all babies.

It is the case that all humans can inhale, and all children can exhale, but it is not the case that all babies can either exhale or inhale.