

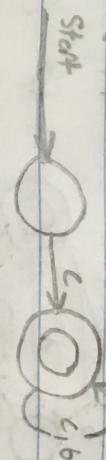
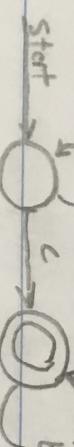
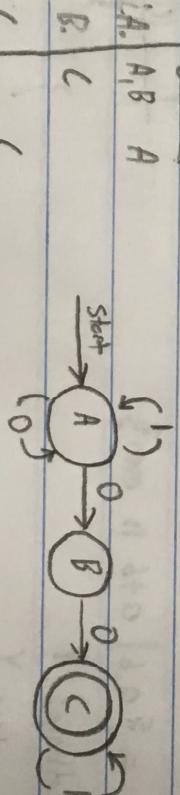
Symmetric: ✓

Transitive: ✓

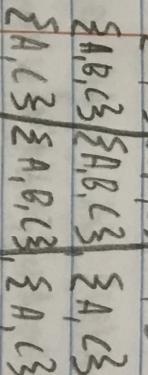
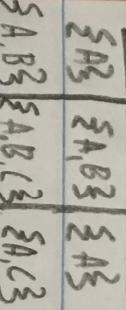
Anti-symmetric: ✓

3. Prove that  $R \circ (S \circ T) = (R \circ S) \circ T$
1.  $(a, b) \in R \circ (S \circ T)$  [def of composition]
  - $\exists c \text{ s.t. } (a, c) \in R \wedge (c, b) \in T$  [def & composition]
2.  $\exists c \text{ s.t. } (a, c) \in R \wedge (d, c) \in S \wedge (d, b) \in T$  [by def.  $\wedge$  our  $\exists$ ]
3.  $\exists d \text{ such that } (\exists c \text{ where } (a, d) \in R \wedge (d, c) \in S \wedge (c, b) \in T)$  [essece.]
4.  $\exists d \text{ such that } (a, d) \in R \wedge (d, b) \in T$  [essece.]
5.  $a, b \in R \circ S \circ T$  [def of essece.]

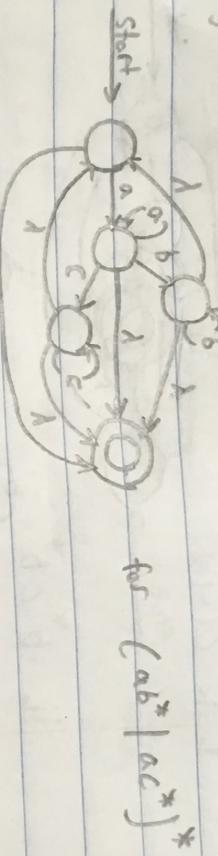
## HW 8

A. i.  $c(c|b)^*$ ii.  $b^*cb^*$ iii.  $b^*c b^* (c|b)^*$ iv.  $b^*c^*$ 2. 0.  $A \mid A, B \quad A$ i.  $c \mid c$ ii.  $(0|1)^*00(1)^*$ 

i. Any number of "1"s and/or any number of "0"s, followed by two "0"s and only number of "1"s.

ii.  $(0|1)^*00(1)^*$ iii.  $0 \mid 1$ 

3. i. Any number of single "a"s followed by an infinite number of "b"s  
and/or any number of single "a"s followed by an infinite number of "c"s.



B.

$$a. \{1, 4, 3, 2, 5, 3, 2, 3, 6, 3, 2, 4, 7\}$$

$$b. \{1, 13, 2, 25, 3, 3, 3, 13, 2, 9, 8, 13\}$$

$$c. \{1, 4, 3, 2, 7, 3, 3, 12, 3, 2, 4, 19, 3\}$$

$$d. \{1, 4, 3, 2, 7, 3, 3, 12, 3, 2, 4, 19, 3\}$$

- e. They are identical in design.

$$2. R_1 = \{a, b \mid a+b \text{ is even}\}$$

Reflexive: Y

Symmetric: Y

Transitive: Y

antisymmetric: N; let  $a=2$ ,  $b=4$ .  $(2,4)$  and  $(4,2)$  both work, but  $a \neq b$ ;  $2 \neq 4$ 

$$R_2 = \{a, b \mid a \times b \text{ is even}\}$$

Reflexive: N; let  $a=3$ ,  $b=3$ ,  $a \times b = 9$ ;  $9 \neq \text{even}$ .

Symmetric: Y

Transitive: Y

antisymmetric: N; let  $a=2$ ,  $b=3$ .  $(2,3)$  and  $(3,2)$  both work, but  $a \neq b$ ;  $2 \neq 3$ 

$$R_3 = \{v, w \mid v \text{ and } w \text{ are strings over letters A to Z, and } v \text{ is } w \text{ reversed}\}$$

Reflexive: N; by definition, if  $w = ab$ ,  $v = ba$ .  $(ba, ba)$  do not relate.