CS 198:205 Fall 2016

Homework 4 (Induction)
DUE: **SUNDAY MORNING, October 30h,** 2016 at 5:00am

Prove the following using mathematical induction in an essential way. *Please lay out your proofs systematically, as indicated in lectures:*

1. *Define* predicate $P(n)$, such that "for all $n \geq b, P(n)$" is to be proven. [b and n are integers]
2. BASIS: *Prove* $P(b)$ (where b is the constant above)
3. I.H. *Assume* $P(k)$ for some arbitrary k, $k \geq b$ /* write out the proposition – it will be used in the proof below */
4. *Prove* $P(k+1)$ for the k above. /* Useses I.H. Be sure the proof does not start with the statement P(k+1) – that is an incorrect proof: "Assuming what is to be proved". */

(Steps 3 and 4, together, are called the "Induction step" .)

Before doing this homework be sure you've read the examples in Chapter 5.1. We suggest you try to solve the exercises in the book before you look at the solutions.

Question A. Prove $1^2 + 4^2 + 7^2 + ... + (3n-2)^2 = \frac{1}{2}n(6n^2 - 3n - 1)$ for $n \geq 1$

Question B. Prove $11^n - 6$ is divisible by 5 for $n \geq 1$

Question C. Prove using mathematical induction that every 4×2^n chessboard can be covered by L-shaped tetrominos (4 squares arranged in an L-shape). (Actually, the original problem is about covering a 3×2^n chessboard, though the basis is not n=1. It is more challenging to find the base case, and worth extra points.) /* This proof is not going to be mathematical. Please check example 14 on triominos/page 327 in the book, which I did in class. */

Question D. Prove by **strong induction** that every number n $\geq$ 18 can be written as the sum of 3's and 10's. (Read example 4 in Section 5.2, and my lecture notes first.)

Question E. Find the flaw in the following proof that if P(n) is "$a^n = 1$" then $P(n)$ holds for all $n \geq 0$. *In your answer, give the line number(s) where the proof goes wrong, and state precisely what is the error or what is missing.* (One answers such questions by thinking why induction proofs work in general, and then finding what aspect breaks down.)

TO PROVE: $P(n)$ holds for all $n \geq 0$

1. BASIS: Prove P(0): $a^0 = 1$ (Trivial proof)

2.ASSUME Strong I.H. $a^j = 1$ for $0 \leq j \leq k$ for

NOW PROVE $a^{k+1} = 1$

Proof

3)	LHS	$= a^{k+1}$		
4)		$= a^{2k-(k-1)}$	since $k+1 = 2k-(k-1)$	
5)		$= a^{2k}/a^{k-1}$	since $a^{b-c} = a^b/a^c$ in arithmetic	QED
6)		$= a^k \times a^k/a^{k-1}$	since $a^{2k} = a^{k+k} = a^k \times a^k$	
7)		$= 1 \times 1/1$	by strong IH for a^k and a^{k-1}	
8)	$=$ RHS			