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Section 06
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HW 3

1. Prove the following formally using a Natural Deduction proof that only uses the inference rules on the handout that are on the lecture notes page. (be sure not to use equivalence or Rosen inference rules)
   a. P /\ Q |- P \/ R

      1. P /\ Q Given
      2. P by /\ Elim-1
      3. **P \/ R** **by \/ Intro-2**

   b. P -> ~Q, Q |- ~P

      1. P -> Q Given
      2. [P] Assume
      3. ~Q by Implication Elim 1,2
      4. False by F Intro-3,5
      5. **~P** **by F Elim-4**

   c. P -> (Q -> R) |- (P /\ Q) -> R

      1. P -> (Q -> R) Given
      2. [P] Assume
      3. Q -> R by Implication Elim-1,2
      4. [Q] Assume
      5. R by Implication Elim-3,4
      6. P/\Q by /\ Intro-2,4
      7. P/\Q -> R by Implication Intro-5,6
      8. **(P /\ Q) -> R** **by Repeat 7**

   d. ~Q |- Q-> P

      1. [Q] Assume
      2. ~Q Given
      3. False by F Intro-1,2
      4. Q -> P by F Elim-3
      5. **Q -> P** **by Repeat 4**

e. P \/ (Q /\ R) |- P \/ Q

1.	[P]	Assume
2.	P \/ Q	by \/ Intro-1
3.	[Q /\ R]	Assume
4.	Q	by /\ Elim-3
5.	P \/ Q	by \/ Intro 4
6.	**P \/ Q**	**by \/ Elim-1-5**

2. Give Natural Deduction proofs of the following (the proof are much more challenging)

a. A \/ B, ~B \/ C |- A \/ C

1.	[A]	Assume
2.	A \/ C	by \/ Intro-1
3.	[B]	Assume
4.	~B \/ C	Given
5.	[~B]	Assume
6.	False	by F Intro-3,5
7.	A \/ C	by F Elim-6
8.	[C]	Assume
9.	A \/ C	by \/ Intro-8
10.	A \/ C	by \/ Elim-5,7,8,9
11.	[~B]	Assume
12.	A \/ B	Given
13.	[A]	Assume
14.	A \/ C	by \/ Intro-12
15.	[B]	Assume
16.	False	by F Intro-10,14
17.	A \/ C	by F Elim-15
18.	A \/ C	by \/ Elim-13,14,15,17
19.	[C]	Assume
20.	A \/ C	by \/ Intro-8
21.	**A \/ C**	**by \/ Elim-1,2,3,10,11,18,19,20**

b. A -> B |- ~B -> ~C

1.	A -> B	Given
2.	[A]	Assume
3.	B	by Implication Elim-1,2
4.	[~B]	Assume
5.	False	by F Intro-3,6
6.	~B -> ~C	by F Elim-7
7.	~B -> ~C	by Repeat 6

8. **~B -> ~C by Repeat 7**

3. Section 1.4, Problems on pages 53-56
   a. #10

Let C(x) be the statement that "x has a cat", let D(x) be the statement "x has a dog", and let F(x) be the statement "x has a ferret." Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

a. A student in your class has a cat, a dog, and a ferret

\Exists(x) [C(x) /\ D(x) /\ F(x)]

b. All students in your class have a cat, a dog, or a ferret.

\Forall(x) [(C(x) /\ ~D(x) /\ ~F(x)) \/ (D(x) /\ ~C(x) /\ ~F(x)) \/ (F(x) /\ ~D(x) /\ ~C(x))] ***(if we assume that a student with one of the pets cannot have either of the others)***

c. Some student in your class has a cat and a ferret, but not a dog.

\Exists(x) [C(x) /\ F(x) /\ ~D(x)]

d. No student in your class has a cat, a dog, and a ferret.

\Forall(x) [~(C(x) /\ F(x) /\ D(x))]

e. For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.

\Exists(x) \Exists(y) \Exists(z) [C(x) /\ D(y) /\ F(z)]

b. #60 (omit [d])

Let P(x), Q(x), and R(x) be the statements "x is a clear explanation," "x is satisfactory," and "x is an excuse," respectively. Suppose that the domain for x consists of all English text. Express each of these statements using quantifiers, logical connectives, and P(x), Q(x), and R(x).

a. All clear explanations are satisfactory.

\Forall(x) [P(x) /\ Q(x)]

b. Some excuses are unsatisfactory.

\Exists(x) [R(x) /\ ~Q(x)]

c. Some excuses are not clear explanations.

\Exists(x) [R(x) /\ ~P(x)]

4. Section 1.5, problems on pages 65-67
   a. #10

Let F(x,y) be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a. Everybody can fool Fred.

\Forall(x) F(x,Fred)

b. Evelyn can fool everybody.

\Forall(x) F(Evelyn, x)

c. Everybody can fool somebody.

\Forall(x) \Exists(y) F(x,y)

d. There is no one who can fool everybody.

\Forall(x) \Forall(y) F(~x, y)

e. Everybody can be fooled by somebody.

\Forall(x) \Exists(y) F(y,x)

f. No one can fool both Fred and Jerry.

\Forall(x) [~(F(x,Fred) /\ F(x,Jerry))]

g. Nancy can fool exactly two people.

\Exists(x) \Exists(y) F(Nancy, x /\ y)

h. There is exactly one person whom everybody can fool.

\Exists(x) \Forall(y) F(y,x)

i. No one can fool himself or herself.

\Forall(x) F(~x, x)

j. There is someone who can fool exactly one person besides himself or herself.

\Exists(x) \Exists(y) F(x,y /\ ~x)

b. #28[a,c,e,i,j]

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a. \Forall(x) \Exists(y) [x^2 == y]

True

c. \Exists(x) \Forall(y) [xy == 0]

True

e. \Forall(x) [x <> 0 -> \Exists(y) [xy == 1]]

True

i. \Forall(x) \Exists(y) [x + y = 2 /\ 2x - y == 1]

False; (let x = 2 and show me a plausible y)

j. \Forall(x) \Forall(y) \Exists(z) (z == (x + y) / 2)

True