Math Department

Euclidean Geometry

Chronicals

Plane-Separation Postulate

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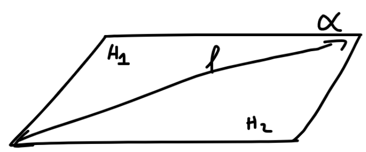
1 Plane-Separation Postulate

So far we have defined two sets of postulates/axioms, the axioms of incidence and the ruler postulate. Now we are going to define a third one, the *Plane-Separation Postualte*. This postulate states the following:

Let α beff a plane and l a linfe contained within α. There exist two subsets of α, H_1 and H_2 such that:

- $\alpha - l = H_1 \cup H_2$.
- H_1 and H_2 are convex sets.
- If $A \in H_1$ and $B \in H_2$, then $\overline{AB} \cap l \neq \emptyset$.



Remember that in general a set $\mathcal{A}$ which contains points is said to be convex if and only if for each pair of points P and Q of $\mathcal{A}$, $\overline{PQ} \subseteq \mathcal{A}$. Also remember that if the first part of the third item of the list is false, the whole proposition is true. This for definition of implication.

Now that we have a defined the Plane-Separation Postulate, let's make and prove some basic questions.

1.1 Can H_1 and H_2 be empty?

The answer in our current context is no. *Proof.* Let α be a plane and l be a line lying in α, then l separates α in two halves H_1 and H_2, this by the plane-separation postulate.

Now we are going to prove that H_1 cannot be empty nor contain a single element. We know that there exist two points $A, B \in l$ and a third point C such that $C \in \alpha$ and is non-collinear to A and B, this by the axioms of incidence. Without losing generality, let that point C lie on H_1. Now, once again by the axioms of incidence, $\overleftrightarrow{CB}$ exists.

Thus, there exists a point D such that $C - D - C$ and $D \in H_1$. If $D \notin H_1$ then either $D \in l$ or $D \in H_2$. Let us observe what happens in both cases.

When $D \in l$. We know that $l \cap \overleftrightarrow{BC} = \{C\}$ this by the theorem of intersection of two lines. Then by definition of intersection, $D \in l \cap \overleftrightarrow{BC}$, thus $D = C$. This is a contradiction because is given that $C - D - C$ and by definition $D \neq C$.

When $D \in H_2$. To be continue...

1.2 Pasch Theorem