UNIVERSIDAD SERGIO ARBOLEDA

MATHEMATICS

LINEAR ALGEBRA II

Eigenvalues and Eigenvectors

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Chapter 1

Eigenvalues and Eigenvectors

1.1 The vector space of linear combinations

Let $\mathcal{T}$ be the set of all linear transformations from V into W, and let $L(V, W) = \{T | T \in \mathcal{T}\}$, then $L(V, W)$ is a vector space where its operations are defined as:

- ***Vector addition***. Let $T_1, T_2 \in L(V, W)$ then for every $v_1 \in V$

$$(T_1 + T_2)(v_1) = T_1(v_1) + T_2(v_1)$$

- ***Scalar multiplication***. Let $T \in L(V, W)$ and let $\alpha \in F$, then for all $v_1 \in V$

$$(\alpha T)(v_1) = \alpha T(v_1)$$

1.2 Eigenvalues and Eigenvectors

Definition 1.2.1

Definition. Let T be a linear operation on vector space V. A nonzero vector v is called an eigenvector of T if there exists an scalar $\lambda \in F$ such that $T(v) = \lambda v$. The scalar λ is called the eigenvalue corresponding to the eigenvector v.

Theorem 1.2.2

Let T be a linear operator over a finite dimensional vector space V. And let λ be a scalar, then the following are equivalent

1. λ is an eigenvalue of T
2. The operator $(T - \lambda I)$ is singular (it doesn't have inverse)
3. $\det(T - \lambda I) = 0$

Definition 1.2.3

If A is a matrix over a field F, an eigenvalue of A in F is a scalar in F, such that $(A - \lambda I)$ is singular.