Handout 14: 3.3, 3.7 Exponential Functions

1. A certain type of tinted automobile glass 1 cm. thick allows only 75% of the light to pass through.
   a. How much light will pass through such glass when its thickness is 2 cm ?
   b. How much light will pass through such glass when its thickness is 3 cm ?
   c. How much light will pass through such glass when its thickness is 10 cm ?
   d. How much light will pass through such glass when its thickness is x cm ?

2. At what interest rate compounded quarterly must $2500 be invested for 2 years so that the accumulated amount is $2800?

3. A common method for financing retirement is to invest a large amount of money at a given rate r compounded annually and then withdraw a fixed amount of money from this investment each year. The maximum amount of withdrawal W possible if an investment P is to last n years is given by the formula

$$(1 + r)^n = \frac{W}{W - Pr}$$

   a. Solve this equation for W.
   b. If $300,000 is invested in an account at a rate of 6% with the intention of making withdrawals for 15 years, how much can be withdrawn each year?

4. Caffeine is a chemical stimulant that is found in coffee and cola. A typical human body eliminates 10% of this compound each hour after ingestion.
   a. Write a function Q that models the amount of caffeine present in the body t hours after the ingestion, given that Q_0 is the amount ingested.
   b. Suppose that Jerome has a double espresso coffee (60 mg of caffeine) at 8 am---just before physics lab. How much caffeine remains in his system when the lab is over at 11: 30 am?

5. For what values of x is $2^x < 3^x$?

6. True or False
   a. $(x^2 - 1)^0 = 1$ for every real number x.
   b. $f(x) = e^x + e^{-x}$ is an even function.
   c. If $f(x) = e^x + e^{-x}$, $(f(x))^2 = 2 + f(2x)$.
   d. The equation $e^{-x} + 1 = 0$ has exactly one solution.
   e. The range of an exponential function is all real numbers.

7. The function $p(t) = 97 \cdot 3^{\frac{t}{10}}$ models the population of a colony of unicyclists, where t is the number of years since 2000.

   a. What was the population of the colony in 2000?

   b. What is the yearly growth rate of the colony?

8. A sample of radioactive material has a half-life of 15 days.

    a. Write a function that gives the percent of the material that is left after t days.

    b. How much of the sample is left after 1 day?

    c. What is the daily decay factor?

    d. What is the daily decay rate?

9. Suppose the amount of cholesterol in your arteries increases by 15% every 5 years, and you currently have 50mg of cholesterol in your arteries.

    a. Find a function that gives the amount of cholesterol in your arteries at any time (in years from now).

    b. How long do you have until the amount of cholesterol reaches 100mg? *You should solve this problem graphically.*

10. What is the hourly decay rate for a substance that decays by 30% every 24 hours?