Handout 12: 2.10 Composition

1. An automobile manufacturer pays its sales representatives an annual salary plus a bonus of 3% of their sales over \$500,000. Consider the two functions $f(x) = x - 500{,}000$ and $g(x) = 0.03x$. If $x \geq 500{,}000$, which of the following represents the bonus for a sales representative?

a. $f\big(g(x)\big)$

b. $g\big(f(x)\big)$

2. Given $f(x) = x^2$ and $g(x) = 3x + 2$

a. For what value(s) of x does $f\big(g(x)\big)$ attain a minimum value?

b. What is the minimum value of $f\big(g(x)\big)$?

c. For what value(s) of x does $g\big(f(x)\big)$ attain a minimum value?

d. What is the minimum value of $g\big(f(x)\big)$?

3. Write and simplify an expression for $f(g(x))$ and state the domain of the composition.

a. $f(x) = x^2 + 3 \qquad g(x) = \sqrt{2x - 6}$

b. $f(x) = 3x + 4 \qquad g(x) = \frac{1}{3}(x - 4)$

c. $f(x) = \frac{1}{x^2} \qquad g(x) = \sqrt{5 - x}$

d. $f(x) = \frac{2}{x-3} \qquad g(x) = \frac{1}{x}$

4. Let $f(x) = x^2$, $g(x) = 3x$ and $h(x) = \sqrt{x} + 1$. Express each function as a composite of f, g, and/or h.

a. $p(x) = 9x^2$

b. $q(x) = 3x^2$

c. $r(x) = |x| + 1$

d. $a(x) = 3\sqrt{x} + 3$

e. $b(x) = x + 2\sqrt{x} + 1$

f. $c(x) = 9x$

g. $j(x) = x^4$

h. $k(x) = \sqrt{\sqrt{x} + 1} + 1$

5. In the following chart, $f(x)$ has domain $\{x | -3 \leq x \leq 3, x \text{ is an integer}\}$. Fill in the remainder of the chart as completely as possible, crossing out any box that is not defined.

x	-3	-2	-1	0	1	2	3
$f(x)$	7	-1	3	2	0	-5	-6
$3f(x)$							
$f(x-2)$							
$f(2x)$							
$f\big(f(x)\big)$							

6. The functions $p(n)$ and $q(n)$ are defined below.

n	0	1	2	3	4
$p(n)$	1	0	3	4	2
$q(n)$	3	2	0	4	1

Find the following:

a. $p\big(q(2)\big)$

b. $q\big(p(2)\big)$

c. $p\big(p(2)\big)$

d. $q\big(q(2)\big)$

e. Solve $q\big(p(n)\big) = 2$

f. Solve $p\big(q(n)\big) = 2$

7. Given the graphs of $k(x)$ and $j(x)$. Find the following:

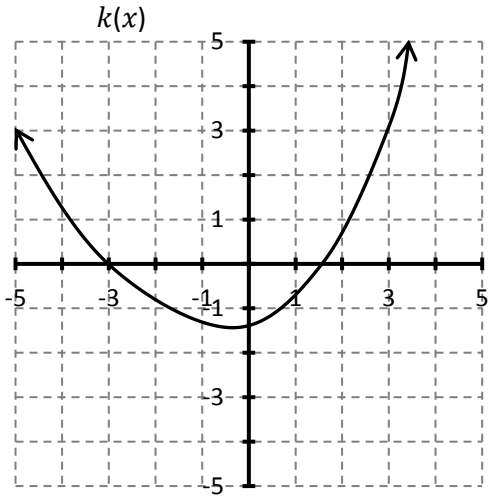
a. $k\big(j(2)\big)$

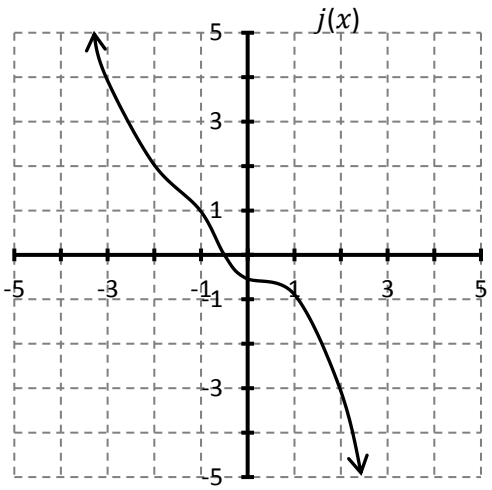
b. $j\big(k(2)\big)$

c. $k\big(k(2)\big)$

d. $j\big(j(2)\big)$

e. Solve $j\big(k(x)\big) = 1$





8. The number of bacteria in a food that has been refrigerated, but is now left on the counter to warm, is given by $N(T) = 20T^2 - 80T + 50, 2 \le T \le 14$, where T is the Celsius temperature of the food. When the food is removed from refrigeration, the temperature is given by $T(x) = 4x + 2, 0 \le x \le 3$, where x is the time in hours.

a. What does the composite function $N\big(T(x)\big)$ represent? *State in words and include units.*

b. According to this model, what is the temperature of the food when it is removed from the refrigerator?

c. Find the number of bacteria in the food if it has been out of the refrigerator for 2 hours.

d. When will the bacteria count reach 2000?

9. Complete the tables given that $h(x) = g\big(f(x)\big)$.

x	$f(x)$
–1	6
0	
1	8
2	
3	7
4	9

x	$g(x)$
5	
6	4
7	1
8	3
9	2
10	0

x	$h(x)$
–1	4
0	–1
1	3
2	0
3	
4	