① ⓐ $f \circ g = \dfrac{1}{\frac{1}{x+1}} - 1 = \dfrac{1}{x+1} - \dfrac{x+1}{x+1} = \dfrac{-x}{x+1} = \boxed{\dfrac{x+1}{-x}}$

$g \circ f = \dfrac{1}{\frac{1}{x-1}} + 1 = \dfrac{1}{x-1} + \dfrac{x-1}{x-1} = \dfrac{x}{x-1} = \boxed{\dfrac{x-1}{x}}$

ⓑ $f \circ g = \left(\sqrt{4x-5}\right)^2 - 5 = 4x - 5 - 5 = 4x - 10$

$g \circ f = \sqrt{4(x^2-5)} - 5 = \sqrt{4x - 20} - 5 = \sqrt{4x - 25}$

ⓒ $f \circ g = \sqrt{\frac{1}{x^2}} - 5 = \boxed{\sqrt{\dfrac{1 - 5x^2}{x^2}}}$

$g \circ f = \dfrac{1}{\left(\sqrt{x-5}\right)^2} = \boxed{\dfrac{1}{x-5}}$

ⓓ $f \circ g = (x-4)^2 + 1 = x^2 - 4x - 4x + 16 + 1 = \boxed{x^2 - 8x + 17}$

$g \circ f = (x^2 + 1) - 4 = x^2 + 1 - 4 = \boxed{x^2 - 3}$

ⓔ $f \circ g = \left|\left(\sqrt{x-1}\right) + 1\right|$

$g \circ f = \sqrt{|x+1|} - 1$

④ $f \circ g = \dfrac{1}{(x+3) - 2} = \dfrac{1}{x+1}$

$g \circ f = \dfrac{1}{x-2} + 3 = \dfrac{1}{x-2} + \dfrac{3(x-2)}{x-2} = \dfrac{3x - 6 + 1}{x-2} = \boxed{\dfrac{3x-5}{x-2}}$

ⓖ $f \circ g = \dfrac{\left(\sqrt{x+2}\right)}{\left(\sqrt{x+2}\right) - 1} = \dfrac{\sqrt{x+2}\left(\sqrt{x+2} + 1\right)}{\left(\sqrt{x+2} - 1\right)\left(\sqrt{x+2} + 1\right)} = \dfrac{x+2+\sqrt{x+2}}{x+2-1} = \boxed{\dfrac{x+2+\sqrt{x+2}}{x+1}}$

$g \circ f = \sqrt{\dfrac{x}{x-1}} + 2 = \boxed{\sqrt{\dfrac{3x-2}{x-1}}}$

② ⓐ $f(x) = \frac{1}{x}$ $g(x) = x - 11$

ⓑ $f(x) = \sqrt{x}$ $g(x) = (x+2)(x-7)$

ⓒ $f(x) = |x|$ $g(x) = \sin x$

ⓓ $f(x) = \sin x$ $g(x) = |x|$

① ⓐ $f \circ g$ $\{x \mid x \in \mathbb{R}, \text{ and } x \neq 0, -1\}$
   $g \circ f$ $\{x \mid x \in \mathbb{R}, \text{ and } x \neq 0\}$

ⓑ $f \circ g$ $\{x \mid x \in \mathbb{R}\}$
   $g \circ f$ $\{x \mid x \geq \frac{25}{4}\}$

ⓒ $f \circ g$ $\{x \mid x > 0\}$
   $g \circ f$ $\{x \mid x \in \mathbb{R}, \text{ and } x \neq 5\}$

ⓓ $f \circ g$ $\{x \mid x \in \mathbb{R}\}$
   $g \circ f$ $\{x \mid x \in \mathbb{R}\}$

ⓔ $f \circ g$ $\{x \mid x \geq 1\}$
   $g \circ f$ $\{x \mid x \in \mathbb{R}\}$

ⓕ $f \circ g$ $\{x \mid x \in \mathbb{R}, \text{ and } x \neq -1\}$
   $g \circ f$ $\{x \mid x \in \mathbb{R}, \text{ and } x \neq 2\}$

ⓖ $f \circ g$ $\{x \mid x \geq 2, \text{ and } x \neq -1\}$
   $g \circ f$ $\{x \mid -\frac{2}{3} \leq x, \text{ and } x > 1\}$

Handout 12

③ ⓐ $f(g(x)) = (\sqrt{2x-6})^2 + 3 = 2x - 6 + 3 = \boxed{2x - 3}$
$\{x | x \in R\}$

ⓑ $f(g(x)) = 3(\frac{1}{3}(x - 4)) + 4 = x - 4 + 4 = \boxed{x}$
$\{x | x \in R\}$

ⓒ $f(g(x)) = \frac{1}{\sqrt{\frac{1}{5-x}}^2} = \boxed{\sqrt{5-x}}$
$\{x | x \in R, \text{ and } x \neq 5$

ⓓ $f(g(x)) = \frac{1}{\frac{1}{x} - 3} = \frac{1}{\frac{1-3x}{x}} = \frac{x}{1-3x}$
$\{x | x \neq \frac{1}{3}, 0\}$

④ ⓐ $\boxed{f \circ g} = (3x)^2 = 9x^2$

ⓑ $\boxed{g \circ f} = 3(x^2) = 3x^2$

ⓒ $\boxed{h \circ f} = \sqrt{(x^2)} + 1 = |x| + 1$

ⓓ $\boxed{g \circ h} = 3(\sqrt{x} + 1) = 3\sqrt{x} + 3$

ⓔ $\boxed{f \circ h} = (\sqrt{x} + 1)^2 = \sqrt{x}^2 + \sqrt{x} \cdot \sqrt{x} + 1 = x + 2\sqrt{x} + 1$

ⓕ $\boxed{g \circ g} = 3(3x) = 9x$

ⓖ $\boxed{f \circ f} = (x^2)^2 = x^4$

ⓗ $\boxed{h \circ h} = \sqrt{\sqrt{x} + 1} + 1$