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Intraday price bubbles and their influencing factors: a small- and micro-cap analysis

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INTRADAY PRICE BUBBLES AND THEIR INFLUENCING FACTORS: A SMALL- AND MICRO-CAP ANALYSIS

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DEDICATION

I dedicate this dissertation to my family. First and foremost, I recognize my wife, Mallory, for her unyielding support and inspiration throughout this academic journey, which would not have been possible without her. To my parents and sisters, thank you for believing in me and offering your love and support through all my pursuits.

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ABSTRACT

This dissertation explores episodes of intraday stock price bubbles in small- and micro-cap stocks, a type of price bubble that is understudied in traditional financial literature. Stock price bubbles, marked by significant deviations of market prices from fundamental value, often cause undue harm to market participants and hinder the efficient allocation of capital. Despite their long history and recurring nature, the anatomy of price bubbles is still highly debated – their existence, causes, and impacts. Drawing on market efficiency, microstructure, and behavioral finance theories, the research applies a number of traditional and more recently developed statistical tests to detect intraday bubbles and explores factors influencing the size of these events. Utilizing a two-paper dissertation format, the study analyzes 342 bubble events occurring in Russell Microcap Index constituent stocks between 2018 and 2022. This period, marked by significant market events such as COVID-19 and the meme stock trading frenzy, provides a unique and dynamic context for exploring these phenomena. The first empirical paper utilizes minute-by-minute trade data to test for the presence of bubble dynamics through trends, patterns, exuberance, and nonstationary segments of the price series. The second empirical paper employs regression analysis to assess the relationship between multiple predictor variables and the magnitude of the intraday bubble while controlling for company fundamentals, news, market returns, volatility, and industry. Findings suggest that the complex nature of intraday trading generates price bubbles in our sample of small- and micro-cap stocks. This research not only underscores the need for more sophisticated bubble detection methods that account for time-scale dependencies but also provides actionable insights for market participants and regulators to better monitor and mitigate the risks associated with intraday bubbles.

Keywords: stock price bubble, intraday, market efficiency, behavioral finance, bubble tests

CHAPTER 1: INTRODUCTION

Overview

Price bubbles have been a recurring phenomenon and persistent problem in economic markets for centuries. Most discussions about these bubbles begin with references to tulips and the South Sea Company before leading to conversations around the technology, housing, and cryptocurrency industries. However, this study focuses on a unique type of bubble that has been less recognized and analyzed: intraday stock price bubbles in individual small- and micro-cap stocks.

Regardless of the circumstances around it, the term price bubble typically evokes a similar concept: a phenomenon where the market price of an asset far exceeds its intrinsic value. This seemingly simple conceptual definition belies the technical complexity of understanding precisely when, why, where, how often, and how long bubbles occur. The phenomenon is not new, as MacKay (1841/2020) observed nearly two centuries ago: "Money, again, has often been a cause of the delusion of the multitudes. Sober nations have all at once become desperate gamblers, and risked almost their existence upon the turn of a piece of paper" (p. 3). Despite their many occurrences and an almost equal number of studies conducted by the academic elite, the precise understanding of a stock bubble's anatomy remains elusive for academics and practitioners alike. While this study does not aim to end ongoing debates around price bubbles, it provides novel insight into a specific type (i.e., stock price bubbles in small- and micro-cap stocks) that form intraday and are often fleeting.

Despite extensive research on long-term bubbles, the specific characteristics and dynamics of intraday price bubbles largely remained underexplored until the recent meme stock trading frenzy led by Gamestop and its Roaring Kitty protagonist. Unfortunately, numerous cases of unusual, bubble-like, intraday price activity both before and after GameStop have failed to receive the media attention and scrutiny they deserve. The lack of focus on anomalous price inefficiencies at shorter horizons (intraday or several weeks) on small- and mid-cap equities in favor of studies of market efficiency at long horizons or on large-cap indices poses a greater threat to market stability than ever before. This is particularly true as individual investors and their behavioral peculiarities influence a market structure traditionally dominated by institutional investors and their algorithms. One goal of this study is to draw focus on the prevalence of these intraday bubble events, the potential risks they present to all market participants, and whether individual investors are the main drivers of these large price movements.

The study's time period (2018-2022) was purposefully selected given the significant market events that characterized this five-year period: the rise of commission-free trading in 2019, the onset of COVID-19 lockdowns in March 2020, and the aforementioned meme stock frenzy in early 2021. Collectively, these events led to an influx of new, inexperienced retail investors determined to manage their investments on mobile apps without the guidance of licensed financial advisors. Frino et al. (2019) show that individual investors tend to employ gambling-like behavior by overinvesting in lottery stocks, which they describe as low-priced, high-volatility, and high-skew stocks with a low probability of high returns. Although individual investors are often discussed as main players in such events, it is possible that institutional investors are also contributing to the amplification of price bubbles if they are as confident in predicting the price bubble dynamics as individual traders.

This study aims to examine a sample of suspected intraday price bubbles in small and micro-cap stocks, documenting their prevalence, price efficiency characteristics, and key factors influencing their magnitudes. By focusing on the period from 2018-2022, this research captures the impact of significant market events and the unusual influx of retail investors on bubble dynamics. Through this lens, the study seeks to encourage market participants and regulators to address these localized (short-duration) inefficiencies by offering evidence of useful detection methods and the factors driving intraday bubbles.

This dissertation follows a two-paper format. The first paper focuses on the application and evaluation of various bubble detection methods to identify intraday deviations from market efficiency and bubbles in small- and micro-cap stocks. The second paper is an empirical study investigating the factors influencing the magnitude of these intraday events. Together, these papers address a gap in the literature and provide interesting empirical findings on the phenomenon of intraday stock price bubbles. I also discuss likely implications for market participants and regulators that future researchers can focus on.

Problem Addressed

Stock price bubbles, regardless of their cause or duration, can create numerous problems for all investors (both retail and professional), regulators, and organizations supporting financial market structure (e.g., stock exchanges, clearing agencies, and market makers). Incorrect asset pricing can misallocate resources, lead to crises, threaten market stability, distort economic growth, and exacerbate wealth inequality.

Minsky (2008) discusses how the economy is cyclical, partly driven by overconfidence, resulting in asset bubbles and resource misallocation. Malkiel (2010) highlights how the dot com boom of the late 1990s resulted in the misallocation of financing to companies without credible long-term viability. Shiller (2015) discusses many historical bubble events and notes that the misallocation of resources has negative consequences on suppliers of capital when a bubble bursts and hinders economic growth as capital is diverted from productive assets.

Due in part to the housing and real estate bubble, the Great Recession exemplifies the potential magnitude of crises that can result from bubbles and lead to capital diversions. Blinder and Zandi (2010) describe the government response as an alphabet soup of policies designed to support a collapsing economy through unprecedented capital infusions. According to Harbert (2019), between the Troubled Asset Relief Program (TARP) and other government interventions, the Great Recession required a bailout of over $500 billion.

As technology advances, retail trading becomes more prominent, and market structure grows in complexity, the short-term stock price bubbles highlighted in this study threaten market stability and raise concern about the financial well-being of individual investors lacking the financial expertise and wealth to endure rapid price declines when a bubble bursts. The Deloitte Center for Financial Services (2021) reports that over 10 million Americans opened retail brokerage accounts in 2020, leading to roughly six million individuals accessing mobile trading apps in January 2021 alone. These new investors, from diverse backgrounds with varying levels of investing experience, often utilize advanced strategies such as day trading, leverage, and options, which carry risks they may not fully understand. Further, as documented in empirical and experimental studies, an influx of inexperienced traders into markets may significantly drive price bubbles (Dufwenberg et al., 2005; Efremidze et al., 2017).

Some supporters of the efficient market hypothesis (EMH), discussed in more detail later, argue that market participants may invest based on the assumption that at any given time, the market price of a stock reflects its fundamental value. However, investing with such price efficiency blinders can result in a substantial financial loss when a stock price does not represent its true value. The recent, well-publicized, explosive stock price events of GameStop, AMC, Bed Bath and Beyond, and others have reignited discussions on price bubbles, their underlying characteristics, and their implications for market efficiency. While the meme stock bubbles were not limited to small and micro-cap companies, the phenomenon may have indirectly led to more news coverage of speculative interest in these smaller companies as seen through GameStop, AMC, and Bed Bath and Beyond – all small or micro-cap companies at the time of their bubble events. Thus, despite the longstanding discussions of the general efficiency of the U.S. equity market, there remains an opportunity to refine our understanding of how and why stock price bubbles occur by focusing on smaller market capitalization stocks. The amount of capital directed to some of the recent intraday bubble events is not insignificant. For example, Melvin Capital was losing more than $1 billion a day during the peak of the GameStop bubble event (Chung, 2022). While the literature has discussed the economic and business implications of longer-duration asset bubbles, more research is needed to understand economic consequences and appropriate policies to deal with intraday asset bubbles. I offer my view on the potential implications but do not examine or document them. It should be a subject of future research.

Research Questions

This dissertation explores the phenomenon of intraday stock price bubbles through the following primary research questions:

1. How well do existing bubble tests identify the formation of intraday price bubbles in small- and micro-cap stocks?
2. How did the prevalence and characteristics of intraday bubbles evolve across significant market events between 2018 and 2022?
3. To what extent do liquidity, short interest, retail trader participation, opening stock price, and market capitalization influence the size of intraday bubbles?

The answers to these questions are highly relevant to the current trading landscape and the consideration of future rules and regulations supporting fair and orderly market trading. The study follows a two-paper format, with each paper structured as an empirical study examining archival stock data in small- and micro-cap stocks from 2018-2022.

Significance of the Proposed Research

This study explores a modern phenomenon of short-term, intraday stock price bubbles in individual small- and micro-cap stocks, contrasting the longer-term market- and sector-wide bubbles that most extant literature examines. The study analyzes various methods for detecting bubble-like price behavior and factors that may influence the size of these intraday bubble events.

The risky strategies some retail investors use, such as day trading or even a simple buy-and-hold approach, can become problematic if the buy order occurs at a bubble's peak. Market participants should be aware of the current market and firm-specific conditions influencing market prices before deciding to buy or sell a specific security. However, if the generally accepted belief is that market prices are accurate and fair, investors may forgo thorough research due diligence, assuming that orders will be executed at reasonable prices. Unfortunately, a poorly timed trade at runaway prices can result in significant losses in a short period.

It may be that, historically, short-lived individual price bubbles in small market capitalization stocks have been ignored by investors, regulators, and academics due to the limited economic impact of the price moves. However, the investing landscape has evolved recently as commission-free trading, COVID-19, and social media-motivated investment strategies have inspired new investors to bet on individual stocks. For instance, Cepton Inc. and Inspirato Inc.

(two micro-cap companies before their respective price bubbles) combined to turn over nearly $825 million in shares in a single day in 2022.

Furthermore, some studies (e.g., Mackintosh, 2019) show that retail investors tend to prefer micro-cap stocks, which typically have wider spreads and more price volatility. Van der Beck and Jaunin (2021) suggest that retail traders' preference for these stocks may stem from a belief that they are better able to influence the prices of these companies. However, inexperienced retail investors are not alone in their preference for small stocks; some successful professional investors, such as Warren Buffett, have continuously expressed their admiration for investing in small companies, given the higher likelihood of inefficiencies and stocks being undervalued. At the 1995 Berkshire Hathaway annual meeting, Buffett noted the higher likelihood of inefficiencies and stocks being undervalued in this segment, stating, "I do think, if you're working with very small amounts of money, that there almost always are some significant inefficiencies someplace — to find things" (Buffett, 1995, 1:12:00). Therefore, as more and more retail investors open brokerage accounts and begin investing in these small stocks, the economic importance of micro-cap mispricings is likely to grow. While Markowitz's (1952) modern portfolio theory addresses this risk through broad diversification via market portfolios, many studies to be discussed later recognize bubbles in broad-based, diversified indices as well.

This study will provide a new perspective on intraday pricing and volatility in small- and micro-cap stocks for market participants and regulators to consider. It will offer some insights into the extent of bubble risk in this segment of the investable universe, which can be helpful for pricing risk in these securities and assessing the effectiveness of existing regulatory surveillance. Despite the previous empirical evidence of mid- and large-cap stock susceptibility to bubbles, the study will focus on small- and micro-cap stocks, given the growing retail investor base in equity markets and their interest in this speculative segment.

While it may be unrealistic for regulators to expect rules, oversight, and enforcement actions to prevent all stock price bubbles, there may be an opportunity to refine existing or develop new rules to improve market efficiency. Specifically, self-regulatory organizations (SROs), such as stock exchanges and FINRA, should consider the evolving investing landscape to ensure that exchange listing and surveillance rules are structured to maintain fair and orderly markets.

Finally, this study contributes to the literature stream on stock price bubbles, which lacks depth on shorter duration (intraday) price bubbles that form and often crash more rapidly than previously studied historical bubble events.

Ethical Considerations

This dissertation, including both empirical studies, does not involve the participation of human subjects, nor was any data obtained from individual participants. Therefore, this research is exempt from review by the Graduate and Professional Schools Institutional Review Board (IRB) at Pepperdine University. The exemption form is included in Appendix A.

CHAPTER 2: OVERVIEW OF THE RESEARCH AREA AND APPROACH

Introduction

Chapter 1 introduced the intraday stock price bubble phenomenon, highlighting the risks they pose to financial market participants and the need for more thorough research and awareness around their existence and structure. The study first identifies a sample set of intraday price bubbles between 2018 and 2022 by detecting outlier intraday price changes, both upside and downside. Subsequently, the research analyzes how well highly-cited bubble detection tools capture patterns in the intraday price series of these events that are consistent with bubble dynamics. The second study employs regression analysis to examine the relationship between multiple variables and the size of the intraday bubble.

This research is grounded in well-established economic and financial theories, encompassing topics such as the EMH, capital asset pricing model, behavioral finance, and market microstructure. This chapter aims to provide an overview of the foundational literature, establishing the theoretical framework that underpins both studies. It outlines the methodological approach, detailing how the study will address the research questions, thus setting the stage for the in-depth analyses presented in the remaining chapters.

Foundational Literature Review

The research for this two-paper study on short-natured bubble formations and their influencing factors is related to several strands of literature, collectively known as modern finance theory. This initial literature review introduces the foundational ideas upon which many bubble research studies build. It covers the EMH, along with a brief discussion of the capital asset pricing model (CAPM) and the concept of noise in stock prices. While bubble theory, behavioral finance, and market microstructure are introduced here, more extensive discussions of these topics will be presented within the literature reviews for studies one and two.

Efficient Market Hypothesis

Efficiency in the context of stock prices and the EMH, according to Fama (1970), often cited as the most prominent thought leader and proponent of efficient market theory, means that equity prices reflect all available information at any given moment and that future prices are not possible to predict based on past prices and information. Malkiel (2003) operationalizes the definition by suggesting that efficiency means that markets do not allow above-average returns without accepting above-average risks.

Under an EMH model, prices can fluctuate randomly and in unpredictable directions in short-term random walks. Therefore, supporters of the theory argue that attempting to make consistent profits by stock picking is a dismal task. Fama (1970) and Malkiel (1989) note that stock prices should fully and correctly reflect all relevant information. The terms fully and correctly have competing views by scholars regarding how these terms are measured. Notwithstanding these ongoing debates, there is an implication that accuracy is a qualifier for efficient markets. Therefore, if a bubble is considered a deviation from fundamental value, or inaccurate pricing, it may imply that inefficient markets can exist, at least temporarily.

Before Fama's (1970) work and the widespread acceptance of the EMH, other individuals laid the groundwork through various theorems and empirical studies. Bachelier's (1900) theoretical work established the concept of random walks in stock prices. He used probability theory and a Brownian motion stochastic process to model stock price movements, demonstrating that speculation was a fair game with an expected outcome value of zero. Samuelson (1965) developed a theorem to show that a random walk is consistent with competitive markets where price discrepancies are quickly arbitraged away. There were also early critics of random walks and market efficiency. Mandelbrot (1963, 1966) used Pareto stable distributions and fractal geometry to show that extreme events are much more likely than suggested by normal distributions. In the strictest interpretation of efficient markets, bubble-like price activity should not occur, or at least be statistically so rare that it can be discounted as an impossibility.

Fama's (1970) seminal work on the EMH provides a foundational description of an efficient market where prices accurately reflect all available information and serve as reliable signals for resource allocation. A crucial caveat to his work is the assumption that prices fully reflect all available information at any given time, thereby defining market efficiency. Fama tests this theory empirically under the three well-known forms of the EMH: the weak, semi-strong, and strong.

According to Fama (1970), under the weak-form of the EMH, utilizing only price and volume data, random walk tests show that future prices are unpredictable, which misaligns with bubbles. Support for the weak form of EMH rejects technical analysis as an effective strategy for obtaining excess returns. The semi-strong form is tested using martingale fair game models, asserting that all publicly available information is reflected in a stock’s price. Support for the semi-strong form rejects both fundamental analysis and technical analysis as tools for obtaining excess returns, but it is susceptible to insider trading as a means for excess returns. Finally, the strong form of the EMH supports the belief that public and non-public information is reflected in a stock’s price. Fama discusses the strong-form model for academic purposes but rejects it as a plausible scenario in real-world settings.

Both before and following his comprehensive review of theoretical and empirical work on the EMH, Fama conducted his own empirical studies and statistical tests on market efficiency. Fama (1965) tested the autocorrelation of large-cap U.S. companies over 30 years and showed that past prices do not predict future prices. Fama (1988, 1992, 1998) continued to refine and expand on the EMH as markets evolved, and critics questioned the model's validity. While Eugene Fama is undoubtedly a thought leader in market efficiency, many other academics have also published well-known work supporting the EMH model. However, much of the empirical work on the EMH focuses on the semi-strong form models, given the limited support for the strong-form of the hypothesis due to insider trading, as noted by Seyhun (1986, 2000).

The basic premise of the semi-strong version of the EMH is that stock prices quickly adjust to reflect all publicly available information. Many studies have found support for this form of the EMH through empirical research, such as examining price adjustments to stock splits (Fama et al., 1969) and the incorporation of earnings and dividend announcements (Hillmer & Yu, 1979; Jennings & Starks, 1985) into stock prices. Fama (1998) argues that positive and negative abnormal returns are equally likely and due to chance.

Despite the widespread support for the EMH, there are also price anomalies, which it fails to explain fully. Some studies rejecting the EMH have also been used to indicate the possibility of a bubble. A brief discussion of EMH anomalies is presented here, while a more detailed analysis of these studies and their statistical tests related to price bubbles will be covered in the literature discussion for the bubble detection paper.

Many market anomalies, such as historical bubble events, the January effect, small-firm effects, and the P/E effect, among others, have been recognized by both supporters and critics of the EMH (Malkiel, 1989, 2003; Russel & Torbey, 2002). Rozeff and Kinney (1976) were among the first to show that stocks appeared to significantly outperform in January compared to other months. Some studies (e.g., Bhabra et al., 1999; Haugen & Jorion, 1996) have demonstrated that a January seasonality (and a new November effect) anomaly was still occurring, which they attribute to tax-loss selling strategies. Yet, some studies (Patel, 2016; Perez, 2018) have rejected the January effect, suggesting it has dissipated over time. Another highly cited anomaly that questions the EMH is the small firm (or size) effect. Banz (1981) showed that small NYSE-listed stocks had statistically significantly higher returns than large NYSE-listed stocks over a 40-year period. He suggests this anomaly results from the misspecification of the CAPM, which will be discussed briefly later in this section. Similar to the January effect, Schwert (2003) suggests that the magnitude of the size effect anomaly has declined significantly in more recent sample periods. This discussion of EMH anomalies covers just a few examples, as it is intended only to introduce evidence of inefficient trading that suggests markets can sometimes be less than efficient.

The EMH has a rich history dating back over 100 years, which has led to significant debate regarding its accuracy – a debate that continues today. One notion of market efficiency, which justifies the research of both its proponents and critics, is presented by Marqueering et al. (2006). They suggest that EMH price anomalies wax and wane over time depending on a multitude of factors that impact price formation, such as technological innovation, emerging rules and regulations, and the ever-changing behavioral motivations and sentiment of individual investors. As such, the recent evolutions in financial markets previously discussed may have led us to a moment where micro-cap price anomalies are waxing.

While a comprehensive review of the EMH is beyond the scope of this study, it is introduced as a foundational literature stream for the two-paper study on price bubbles, given its importance to modern finance theory. One of the difficulties in addressing the validity of the EMH, which appeared in many of the studies just discussed, is the joint hypothesis problem. For example, the results of a study rejecting market efficiency should not be confounded by other hypotheses, such as the existence of bubbles in markets. Nevertheless, a grounding in efficient markets theory is essential to understand the bubble literature in the first study and the microstructure and behavioral finance literature in the second study.

Capital Asset Pricing Model (CAPM)

A discussion of the EMH without at least introducing pricing models, such as the CAPM and arbitrage pricing theory, would be an incomplete introduction to principles of modern finance theories and their influence on asset bubble studies. The CAPM is an influential pricing model used by market participants to calculate the expected return on an investment with consideration of risk. The principles of the CAPM are rooted in Markowitz's (1952) modern portfolio theory (MPT), which emphasizes the optimal portfolio as one that maximizes expected return for a given level of risk. Shortly after Markowitz developed MPT, the CAPM was introduced by Sharpe (1964), Lintner (1965), and Mossin (1966).

Both the CAPM and MPT rely on market efficiency in terms of pricing to ensure the accuracy of their models. If price anomalies occur, including price bubbles, the risk-return trade-off of a stock can become disconnected from its equilibrium level resulting in a beta that may be much higher than it otherwise would be absent a price anomaly. The issue compounds if the CAPM is applied across an entire portfolio with multiple individual asset mispricings.

Much like with the EMH, many critics conducted studies to highlight the shortcomings of the CAPM. Friend and Blume (1970) empirically show that the CAPM implies an inverse relationship between risk and return on a random portfolio of between 25 and 100 stocks. This conclusion suggests a counterintuitive logic to the assumptions of the risk-return relationship. Ross (1973) notes that there is an overly simplistic assumption of homogeneity among investor beliefs assumed in the CAPM and proposes an alternative, the arbitrage pricing theory (APT). The APT is a factor model suggesting arbitrage opportunities will force prices to their equilibrium levels, but even arbitrage has limitations (Shleifer & Vishny, 1997).

Following APT, additional factor models, such as the three-factor model (Fama & French, 1993) and the five-factor model (Fama & French, 2015), were introduced as alternatives to CAPM and APT. This discussion on the CAPM and its extensions is only a cursory review intended to provide a glimpse into the difficulty in assessing the efficient and fair market value of financial assets, such as stocks. This difficulty is addressed again when discussing the problem of identifying a consensus on what does and does not constitute a bubble. While this discussion only scratches the surface of asset pricing models, the brief overview provides relevant context to the remaining literature discussions.

Noise

Noise is a final foundational theory necessary to introduce before addressing the two studies in more detail. An important distinction is that noise has multiple connotations in financial markets – noise traders and noise in stock prices. For now, the focus is on noise in stock prices through an introduction to the concept and clarification of how noise-like movement in stock prices differs from bubble-like movement.

While the concept of a noise factor was introduced around the time the EMH was emerging (Grossman, 1967; Grossman & Stiglitz, 1980), it was expanded more formally by Black (1986). In his seminal work on the topic, Black (1986) stated, “Noise makes financial markets possible, but it also makes them imperfect” (p. 530). He also noted that noise is not necessarily bad for markets as it acts as an element that motivates market participants to provide liquidity. Without noise and price movements around the actual fundamental value, there would be no incentive to trade other than to shift asset allocations. Based on Black's (1986) description, noise can be seen as the daily cost of doing business in the stock market.

To better understand noise in stock prices, it is important to consider its sources and impacts. The microstructure literature will discuss how quote spreads, liquidity, and other factors influence the amount of noise in stock prices. This discussion is not intended to be comprehensive, but rather, it is included here to show that it is generally accepted that stock prices can fluctuate around fundamental values in the near term. The intraday bubble trends discussed in each study seek to examine price fluctuations that exceed the price movements often considered noise.

Summary

The literature discussion sets the groundwork for the literature streams to be covered in each of the two studies. To understand why bubbles should not occur and the problems they create, it is vital to understand the origin and long-studied ideas behind market efficiency and pricing models. These streams of literature are mature, dating back over 100 years. Therefore, even in a survey on market efficiency, it would be challenging to introduce every study that has influenced the way academics and practitioners think about market pricing. Nevertheless, the papers highlighted cover some of the most highly respected works on the topics.

Description of the Research Agenda

The dissertation adopts a two-paper format, each based on a unique empirical study. While each study is independent, both are based on the same intraday bubble events.

The first study utilizes various bubble detection tests established in the literature to identify intraday price patterns consistent with bubble dynamics. While detecting quickly forming bubbles over the sample period is the primary focus of the first study, additional descriptive information related to the bubbles and their distribution over time will be captured.

The second study assesses factors influencing the magnitude of the intraday bubble events. The study also analyzes how the characteristics of the bubbles evolved during significant socioeconomic events during the study period. Both studies focus on short-lived bubbles, that is, bubbles that form and crash or decline in price rapidly. Long-lasting bubbles that are sustained for months or years and impact entire sectors of the market are not this dissertation's focus.

The two-paper format is appropriate to address the research questions presented previously for several reasons. First, while the studies are related, they required two distinct analyses using unique statistical tests that contribute separately to apparent gaps in the research literature on stock price bubbles. Second, the literature domain informing these studies is mature and robust. Breaking the dissertation into two papers supported a more digestible and focused approach to the analysis. Third, the two-paper format facilitates a dual path to publication and the opportunity to collaborate on various topics within each study. Finally, the two-paper design provided greater flexibility and adaptability during the data collection and analysis process.

Justification of the Research Agenda and Approach

It is appropriate to use the two-paper format to address the research questions presented previously for several reasons. First, while the studies are related, they require two distinct analyses, contributing separately to apparent gaps in the research literature on stock price bubbles and each requiring an extensive review and application of empirical statistical tools. Second, the literature domain informing these studies is mature and robust. Breaking the dissertation into two papers supports a more digestible and focused approach to the analysis. Third, the two-paper format facilitates a more straightforward path to publication and the opportunity to collaborate on various topics within each study. Finally, the two-paper design provides for greater focus for the analysis of the issues.

While the studies are technically sequential, with study one providing a sample for study two, aspects of each can be conducted in tandem. Specifically, the data collection process in study two is not contingent on the completion of study one, given that the population for study two is known. Therefore, the data can be collected and cleaned while study one bubble tests are conducted. The benefit is that there will be a narrow gap between data analysis in study one and study two, resulting in additional time for synthesizing and interpreting the findings and iterating the dissertation. Together, the research findings from the studies will benefit academics studying stock price bubbles and market participants encountering bubbles in practice.

Bubble Sample Selection and Preliminary Analysis

Sample Selection Criteria

Both empirical studies are quantitative and focused on small- and micro-cap stocks listed on a national securities exchange between 2018 and 2022. Limiting the sample to five years allowed me to accomplish empirical analysis of the events within reasonable time limits of the research project, but still provided enough observations to examine common features among them. I, of course, cannot exclude the possibility that the results may be sample-specific, limiting generalizability until other studies examine samples from different time periods.

Both the Nasdaq stock market and Finra, the self-regulatory entity that oversees U.S. broker-dealers, use the categorizations in Table 1 to define market capitalizations (FINRA, 2018):

Table 1

Market Capitalization Thresholds

Company Size	Capitalization
Large-Cap	>$10 billion
Mid-Cap	$2 billion - $10 billion
Small-Cap	$250 million - $2 billion
Micro-Cap	<$250 million

The observation set was selected from stocks included in the Russell Microcap Index, which consists of the smallest 1,000 securities in the Russell 2000 index plus the following 1,000 smallest eligible securities by market cap. The index reconstitutes annually, in July, and had on average approximately 1500 constituents across the five-year period of this study. This segment of companies is appropriate for a study on intraday and short-term bubbles given the generally accepted view that smaller market capitalization stocks are more volatile than large-cap stocks. Multiple empirical studies (Banz, 1981; Fama & French, 1992 Reinganum, 1981) also confirm this discrepancy in volatility, implying there may be more price inefficiencies in small stocks. Additionally, the recent increase in retail trader participation has led to an increased interest in small, speculative stocks. A review of limit-up, limit-down (LULD) volatility trading halts between 2019 and 2022 suggests these smaller companies accounted for over 85% of the LULD halts during this period (Table 2). Together, these points support the selection of a small- and micro-cap index for this study, given that they may be more susceptible to the brief price anomalies subject to this study.

Table 2

Limit-Up, Limit-Down Trading Halts by Market Cap (2019-2022)

Market Cap	Count
Less than $250m	10,996
$250m - $2b	4,629
$2b - $10b	1,542
>$10b	865

The index constituents were obtained directly from FTSE Russell for each year of the study period. The identifier (i.e., ticker) for each constituent was used to obtain daily aggregate data from the Compustat-Capital IQ database utilizing the Wharton Research Data Services (WRDS) platform. The data points collected from the Compustat-Capital IQ database are summarized in Table 3.

Table 3

Compustat – Capital IQ Data Points and Codes

Data point	Code	Data point	Code	Data point	Code
Company Name	CONM	Daily Trading Volume	CSHTRD	Security Status	SECSTAT
Ticker Symbol	TIC	Current EPS	EPS	Foreign Incorporation Status	FIC
CUSIP	CUSIP	Daily Opening Price	PRCOD	Issue Type	TPCI
Issuer CIK Number	CIK	Daily Closing Price	PRCCD	Deletion Date	DLDTE
Primary Listing Exchange Code	EXCHG	Daily High Price	PRCHD	Deletion Reason	DLRSN
Shares Outstanding	CSHOC	Daily Low Price	PRCLD		

The initial review of the constituents resulted in approximately 2,500 unique tickers and 1.7 million daily price observations. To filter this number to a manageable observation set of bubble events, three measures of bubble size were considered:

- OpenHigh: the percentage change from the daily opening price to the intraday high.
- PrCloseHigh: the percentage change from the previous day closing price to the current day high.
- LowHigh: the percentage change from the intraday low to the intraday high.

However, PrCloseHigh and LowHigh were eliminated as options for this study despite their potential value as bubble measures for alternative studies. PrCloseHigh was removed as a measure partly because it includes a crossover between days, increasing the likelihood of material news announcements (e.g., earnings information, mergers, corporate developments, etc.), significantly influencing the following day price increase. Similarly, the LowHigh measure potentially introduces negative intraday bubbles, where a stock opens and subsequently experiences a rapid decline before recovering later in the day. Notably, studies on negative bubbles, especially in context of retail investors, have gained traction in bubble literature in recent years (Chen et al., 2022; Van Eyden et al, 2023). However, negative bubble may necessitate a different set of variables, or literature, than positive bubbles when attempting to understand their dynamic. Therefore, these alternate measures of bubbles were excluded, despite their merit for inclusion in other studies within the price bubble domain.

To reduce the observation set, OpenHigh was analyzed across percentile thresholds to obtain extreme upside intraday price movements. The threshold of approximately 28% was based on utilizing the 99.8th percentile of all 1.7 million observations, leaving approximately 3,400 observations. The downside threshold of approximately 27% was based on the 90th percentile changes from the intraday high price and closing price on the observation date, resulting in 342 bubble observations. Therefore, the final sample for both studies consisted of 342 intraday bubble events where the subject stock traded up at least 28% intraday and down 27% or more from the high price, representing both the upside and downside period of a bubble. The upside and downside thresholds represent extreme movements of more than three standard deviations from the mean of all 1.7 million observations.

Preliminary Analysis

A preliminary analysis covering Q3 2018 and Q3 2021 was conducted to assess the viability of the study and ensure access to reliable data. These periods were chosen to capture potential changes in trading patterns before and after the onset of the COVID-19 pandemic and meme stock trading. The data collection methods generally followed the methods used in the formal studies, although some aspects of the preliminary analysis were conducted prior to the finalization of the research plan and bubble threshold parameters. As a proxy for intraday bubbles, daily price data was sorted in descending order by percentage differences between the high and opening price with a secondary filter to remove closing prices that were not at least 10% less than the high price. Descriptive statistics compared the top 20 stocks that appeared to represent a bubble against the population of stocks not included in the bubble sample. Also, price graphs, autocorrelations, and Augmented Dickey-Fuller tests were run for a few stocks exhibiting bubble-like behavior.

The preliminary findings resulted in promising trends supporting the proposed study. Descriptive statistics comparing the full sample and bubble observations in Q3 2018 and Q3 2021 are included in Appendix B. First, there was a significant difference in the average daily mean volume between micro-cap companies in Q3 2018 (264,000) and Q3 2021 (668,000), which supports the idea that trading in these stocks is becoming more prevalent. Further, the average daily volume for the bubble stocks is approximately three times greater than the full sample in 2018 and nearly six times greater in 2021. This finding may suggest that the intraday bubbles in this study are more than fleeting order imbalances and may reflect a madness of crowds, a phenomenon where collective behavior leads to extreme market movements (Mackay, 1841/2020).

During the preliminary review, it was noted that on July 8, 2021, COHN opened at $20.30 before trading up to $49.95 (an increase of approximately 146%) and closing at $27.05 with no apparent news to justify the intraday volatility. Even more interesting, COHN traded over 18 million shares on this date while only having 1.3 million shares outstanding and even less in the public float.

Other findings show that the average price and market capitalization of bubble companies were lower in 2018 and 2021 than in the whole sample. The outstanding shares for 2021 bubble companies were lower than in the whole sample, but the average was nearly the same for the two samples in 2018. This discrepancy may reflect the recent trend of retail investors looking to replicate a GameStop short squeeze on other companies with limited shares available to borrow.

Additionally, access to reliable intraday data was essential to this study, specifically the first study. The vendor selected for intraday trading data was Polygon.io, which offered access using a JSON API subscription. Given the importance of this data, test files were obtained and cross-checked against Refinitiv pricing data to confirm accuracy prior to the start of the study.

While this preliminary review examined stylized facts through descriptive statistics, it offered interesting takeaways that were considered when the study was formalized.

CHAPTER 3: DETECTING INTRADAY BUBBLES: A MULTI-TEST APPROACH

Introduction

Price bubbles can significantly impact the stability of financial markets and the well-being of their participants, from retail investors and professional traders to stock exchanges, regulators, and the myriad of organizations crucial to financial market infrastructure. While stock price bubbles have been well-studied, albeit with substantial ongoing debates, shorter-term and small-cap price bubbles are a less explored area. The established literature (Griffin et al., 2011; Malkiel, 2010; Shiller, 2002) on stock price bubbles often focuses on long-term phenomena across sectors, industries, or even entire markets. In contrast, this study examines a different category of bubbles: those that form and burst rapidly, often within a single trading day. This study seeks to explore intraday stock price bubbles in individual small- and micro-cap stocks between 2018 and 2022.

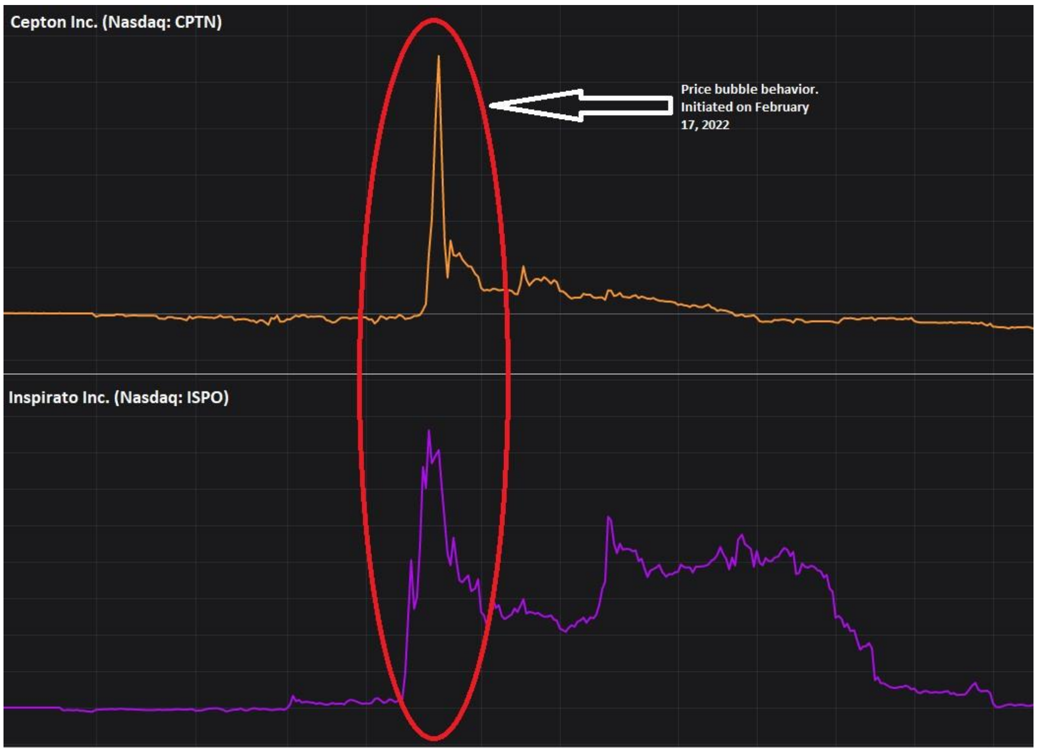
To illustrate the nature of these phenomena, consider the following striking example: On February 17, 2022, two Nasdaq-listed stocks, Cepton Inc. and Inspirato Inc., experienced highly unusual trading activity, exemplifying what this paper terms intraday price bubbles. Cepton's stock surged by as much as 746% from the previous day's closing price, while Inspirato's stock increased by as much as 772%. Specifically:

- Cepton opened at $9.47 and traded to $80.16 before closing at $42.00 on a volume of 445,000 shares, resulting in a turnover of $18.35 million.
- Inspirato opened at $12.38 and traded up to $108 before closing at $92.65 on a volume of 12.67 million shares, resulting in a turnover of $804 million.

These volumes far exceeded their 30-day averages. Cepton's typical daily volume was approximately 239,000 shares per day, while Inspirato's was approximately 181,000. Notably, as of June 2024, both stocks are trading well below $5.00. Figure 1 illustrates the extreme price movement of these two stocks over a one-month period.

Figure 1

Cepton and Inspirato One-Month Price Chart Around February 17, 2022: Bubble Event



The extreme price jumps observed in Cepton and Inspirato raise intriguing questions about market dynamics. Despite a thorough examination of news sources and SEC filings surrounding February 17, no company announcements or rumors sufficiently explain the dramatic price movements. While Cepton rang the opening bell at Nasdaq on February 17, such an event hardly justifies a 700% price increase. Paradoxically, Inspirato announced noncompliance with Nasdaq listing standards on February 18, news that typically exerts downward pressure on stock prices. Yet, Inspirato’s stock maintained a higher price for longer than Cepton's, even after the initial bubble burst.

Bloomberg News reported that both stocks were subject to speculation on social media message boards, suggesting that retail investors may have been behind the stock price surge (Lipschultz, 2022). If true, it raises various questions: Why were these stocks targeted? Did their low public float make them attractive targets? Were the price rallies attempts at short squeezes? If so, what factors made these attempts successful in these specific stocks? What dictates the bursting of the bubble? Why is it difficult to replicate this strategy for other stocks with seemingly similar characteristics? While this study does not explicitly address all of these questions, it does provide insight tools used for identifying some of these bubbles.

Unfortunately, the aforementioned price events of Cepton and Inspirato, and many others like them, often fail to garner significant attention. These micro-cap bubble-like events may avert substantial criticism and analysis because of their fleeting nature and limited market-wide impact. Even in the wake of the highly publicized GameStop incident, discussions failed to expand beyond the well-known meme stock companies highlighted by financial media. Regulators, legislators, self-regulatory organizations (SROs), and academics primarily focus on broader market stability and price efficiency. While these organizations and market participants may have their own unique perspectives on market stability and their role in helping maintain it, the general form of the definition follows the European Central Bank's assertion that financial market stability is the condition where the financial system, its intermediaries and market infrastructures, can withstand shocks without a subsequent unraveling of the system. Clearly, the recent short-term price bubbles in small stocks have not been detrimental to our overall financial market stability, but they are far from inconsequential to certain groups of investors and could also be related to broader market conditions or market-wide bubbles.

In fact, these short-term volatile price swings are arguably more relevant and economically impactful today than ever before. According to data published by the Bank of New York Mellon (as cited in Versace et al., 2022), individual investors' share of U.S. equity trading volume increased from just over 10% in 2011 to almost 25% in Q1 2021, with subsequent reports suggesting fluctuations between 17% and 24% of total equity volume. Further, they note that over 10 million Americans opened brokerage accounts in 2020. This increased retail participation in the stock market is not a trivial development, given that:

- Individual investors tend to be overconfident, which leads to more trading turnover than professional traders (Barber & Odean, 2000).
- Retail investors tend to employ gambling-like behavior, overinvesting in "lottery" stocks, which are described as low-priced, high-volatility stocks with positively skewed returns (Frino et al., 2019). This description matches many of the micro-cap stocks that are the subject of this study.
- Retail investors use momentum and contrarian strategies to respond quickly to overnight returns (Pagano et al., 2021).

These suggestions could mean that individual investors may be both followers and (at least partial) creators of short-term bubbles in micro-cap stocks. Consequently, the economic importance of these quickly forming bubbles is growing, even if they do not pose an immediate threat to market-wide financial stability.

While the second empirical study in this dissertation will focus more acutely on retail investor variables, the bubble detection study finds that bubbles in our sample occur more frequently and more intensely after COVID-19 lockdowns, which aligns with the increased participation among individual investors during this time period. The explosive bubble and long-term memory tests were the most successful at identifying a larger proportion of intraday bubbles in our sample.

The remainder of the discussion of the bubble detection paper will be structured as follows: (1) a brief review of the bubble theory literature and an introduction to the bubble tests, (2) an overview of the research design, (3) a review of the data collection methodology, (4) a discussion of the results, and (5) a review of the limitations, contributions, and future research considerations.

Bubble Definitions

Historical and Contemporary Perspectives

Discussions on financial bubbles often begin with Mackay's (1841/2020) account of the Dutch tulip mania of the 1600s and the Ponzi-like South Sea Company in the 1700s, which Garber (1990) referred to as the famous first bubbles. While these events occurred centuries ago, many of the underlying causes Mackay (1841/2020) described as popular delusions and the madness of crowds continue to influence bubble formation today. The challenge lies in detecting these bubble events ex-ante, which is often considered impossible, and agreeing on appropriate ex-post detection methods. This review of bubble literature will first explore commonly accepted bubble definitions and frameworks before highlighting important theoretical papers and empirical studies. Academics and economists use varying definitions and measurement techniques to detect bubbles, but contemporary definitions include:

- Stiglitz (1990): A bubble exists when an asset's price is high today because investors believe it will be higher tomorrow, even when fundamental factors do not justify its current value.

- Chicago Fed (2012): A bubble exists when the market price of an asset exceeds its fundamentally determined price by a significant amount for a prolonged period.
- Scherbina (2013): A bubble is simply a deviation of market prices from an asset's fundamental value.

These definitions, while seemingly straightforward, contain inherent ambiguities that complicate the process of testing for and identifying bubbles, both ex-post and ex-ante. For instance, terms like exceeds, fundamental factors, and prolonged period in the Chicago Fed's definition are open to interpretation. Scherbina's (2013) definition, while more concise, still relies on the challenging task of assessing fundamental value across various pricing and EMH specifications. Distinct from the Chicago Fed in two ways, Scherbina's (2013) description does not imply bubbles have to be positive or make assertions as to whether the price deviation must be prolonged. However, Scherbina (2013) notes that positive bubbles are more likely than negative ones due to the non-trivial costs associated with arbitrage as the stock price overvaluation increases. The conceptual idea of a bubble seems straightforward. However, the technical ambiguities associated with bubble definitions lead to many inconsistencies and discrepancies among researchers in studies assessing market bubbles, before they occur or after.

Applicability of Operationalized Definitions

Recognizing the limitation of conceptual definitions, some researchers have attempted to operationalize the concept of a bubble. Siegel (2003) reviews and analyzes bubble definitions presented in the existing literature, acknowledging the generally accepted bubble concept of asset prices deviating from fundamental values for extended periods, but he also recognizes the limitation of such a definition. Without operationalizing the definition, Siegel (2003) argues that one could project an endless future stream of cash flows (or unnecessarily truncate the future cash flow projections) to justify (or discredit) current market prices. Therefore, Siegel (2003) developed a measurement that tests for a bubble over a period long enough for the present value of future cash flows to equal at least one-half of the current price. If the realized return of this present value is more than two standard deviations from the expected return, then a bubble is confirmed.

While Siegel's (2003) definition has merit and adds a quantitative dimension to bubble identification, it may not be suitable for all contexts, particularly for this study. First, many micro-cap companies do not have positive cash flows (and some may never have during their time as a public company) to distribute to shareholders, so Siegel's (2003) model will be challenging to implement. Second, the study utilized a broad index to generalize bubbles across the entire stock market, whereas the proposed research focuses on individual stocks. Finally, Siegel (2003) analyzed a bubble period with a run-up in price that occurred over long periods (months or years), which is different from the type of bubble formation of interest in this study. Siegel's (2003) operational definition is highlighted not because it has any superior relevance to other operationalized bubble definitions but because the limitations in fitting his model into this study are common across many studies and quantitative measures in the literature domain.

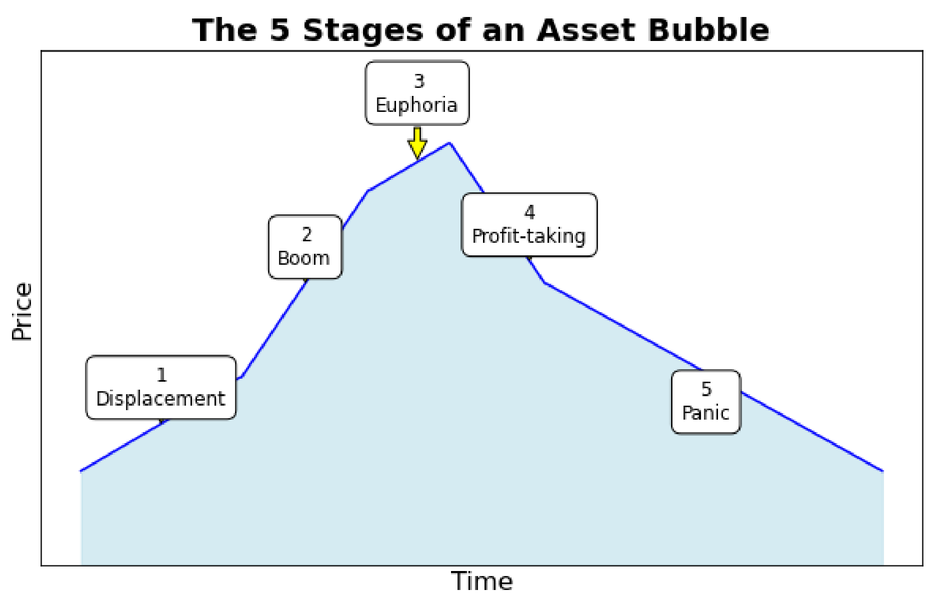
Bubble Dynamics: The Minsky Model

In addition to defining a bubble, describing elements of their occurrence (from formation to deconstruction) is helpful in providing insight into the individual behavioral aspect of buying into or selling out of a bubble. While not developed with asset bubbles specifically in mind, Minsky and Kaufman (1986) studied financial instability and identified five stages of an economic credit cycle. Despite its original application, there was a renewed interest (Auerbach et al., 2010; Cassidy, 2008; Rosser et al., 2012; Yellen, 2009) in applying Minsky and Kaufman's (1986) five-stage model to price bubbles following the housing and mortgage crisis in 2008.

The five stages of the model are (1) Displacement, (2) Boom, (3) Euphoria, (4) Profit-Taking, and (5) Panic, as shown in Figure 2. In the context of stock price bubbles, displacement is the initial divergence of market prices from fundamentals, where boom and euphoria are in the exponential growth phase. Profit-taking is the initial sign of the bubble bursting, followed by panic when prices rapidly decline.

Figure 2

Minsky's Five Stages of a Bubble



A unique feature of the stock price bubbles in this study, which differs from the illustration in Figure 2 and Minsky's original contention, is that the transition from displacement to boom happens quickly rather than gradually if an initial displacement phase even occurs. A second potential difference between the bubbles in this study and Minsky's model is the manifestation of the euphoria, profit-taking, and panic stages. The model assumes the bubbles are behavioral, but as will be discussed in study two, market microstructure frictions may interplay with human sentiment during rapidly forming bubbles. The compressed timeframe of intraday bubbles may require reconsideration of how these stages form and are influenced by high-frequency or other intraday trading dynamics.

The apparent takeaway from the literature suggests that price bubbles are conceptually clear but operationally obtuse and possibly subjective. Tyc (2012) posits that the morphology of bubbles is dependent upon the factors leading to their occurrence, and a descriptive framing of them provides little value as it does not get to the core of the issue. In this context, the following section will transition away from elucidating a singular bubble definition and move to discussions on theoretical contributions and empirical bubble detection methods.

Theoretical Studies

In this discussion, I will overview that there are opposing views of what bubbles are, including whether they are consistent with rational investor behavior. The early theoretical landscape on stock price bubbles explored whether a bubble's formation and subsequent crash are the result of feedback loops and consistent with rationality, aptly referred to as rational bubbles. Blanchard (1982) finds theoretically and empirically that deviations from an equilibrium pricing model's (e.g., CAPM) fundamental value are not inconsistent with rationality and that bubbles may be deterministic. He notes that crowd psychology, more commonly referred to as herd behavior today, and other factors extraneous to fundamentals influence prices. Tirole (1982, 1985) argues that with infinite horizons over generations of investors, bubbles are not inconsistent with optimal behavior and market equilibrium.

The counterintuitive suggestion that bubbles may be rational rests on investors' heterogeneous expectations of future prices, which incentivizes trade and creates feedback loops that encourage bubbles. Milgrom and Stokey (1982) oppose this idea without directly referencing bubbles by arguing in support of a no-trade theorem, even when individual investors possess private information. However, their argument is highly idealized as it relies on the assumption that before receiving confidential information, the market for a stock is in a state of ex-ante Pareto optimal allocations. Therefore, investors attempting to trade based on private information will be met by counterparties unwilling to trade as they will know the only incentive to do so will be based on confidential information, which means it will be a losing trade. This argument favoring common knowledge and no-trade scenarios opposes the idea of bubbles generated from heterogeneous investors and feedback loops. Allen et al. (1993) argue that common knowledge is different from beliefs (about future prices), and while most investors may recognize a stock is trading above its fundamental value based on knowledge, they still trade based on the belief that they will be able to sell it at a higher price to someone else.

While early debates on price bubbles centered on the ideas of rational bubbles, heterogeneous expectations, and common knowledge, these discussions remain relevant today. Scherbina (2013) identifies a new generation of rational bubbles characterized by herding, limited liability, and perverse incentives. This modern perspective on bubble formation incorporates new behavioral elements while maintaining a rational framework.

A group of recent theoretical studies focus on the greater rational fool theory (Barlevy, 2015; Harrison & Kreps, 1978; Liu & Conlon, 2018; Scheinkman & Xiong, 2003). The theory suggests that investors are willing to pay successively higher prices with the expectation that they will be able to sell the asset at an even higher price to someone else. Scheinkman and Xiong (2003) introduce a theoretical model for bubble formulation under the greater rational fool theory. Their study notes that bubbles go hand in hand with high trading volume and price volatility, which coincides with the effects of the meme stock social media trading strategies.

Many media members, regulators, and legislators linked the retail trading behavior around GameStop and other meme stocks to this theory. However, even in such a circumstance (rational or not) bubbles pose harm to the fool left holding the hot potato when the price crashes.

Kindleberger (1991) asserts that bubbles are represented by a sharp rise of prices in a continuous process (motivated by speculation rather than fundamentals) followed by a similarly rapid decline in price. He claims a fundamental problem in the theoretical literature around bubbles is that there is a one mind, one purpose assumption of rationality among investors when in actuality, heterogeneous beliefs, information, and time horizons dominate homogeneous rational decision-making. Garber (1990) argues that relegating speculative events as inexplicable bubbles without considering and exhausting all financial and nonfinancial market fundamentals is an imperfection of many theoretical bubble studies. Garber (1990) interprets fundamentals broadly, including non-financial fundamentals, and argues that economists often are too quick to declare a bubble without adequately investigating the related motivations of investors, suggesting that even the most recognized bubbles are rationally explainable.

Unlike Garber (1990), Malkiel (2010) suggests that rationality and bubbles can co-exist. He notes that bubbles often begin with exogenous behavioral factors that lead to rational conclusions about future expected profits. Shiller (2000) argues that behavioral responses to these exogenous factors, such as news media and emotional framing, lead to feedback loops that encourage bubble formation and cascading effects. Cass and Shell (1983) describe this behavior as sunspots (extrinsic random variables) and self-fulfilling prophecies. Malkiel (1990) argues that the EMH does not hold in these bubble cases due to limits and risks associated with the level of arbitrage needed to counteract the bubble completely. He points to the study by De Long et al. (1990) as an example of why arbitrage is not a perfect counter to bubble formation. De Long et al. (1990) developed a theoretical model that suggests arbitrageurs may maximize utility by riding the wave of a bubble rather than attempting to burst it.

The competing debates and theoretical models of bubbles and their interaction with rationality are copious. If nothing else, there appears to be a consensus that markets experience price anomalies from time to time, but a problem is the lack of consensus on measuring the price anomalies and whether they constitute a bubble (rational, speculative, or otherwise). A repeated and unanswered debate converges around whether the irrational inclinations of individual investor decisions fully consider the unknown risk factors inherent in bubbles. The theoretical papers discussed in this section are only a modest introduction to the vast body of bubble literature, but many are considered cornerstone discussions on the topic and maintain relevance to the behavioral literature that informs a portion of the variable selection in the second paper of this study. The following section highlights the bubble detection methodologies employed in this study and some of the empirical papers that utilized the measures previously. Some of the tests were originally applied to market efficiency studies, which were extended to studies of bubbles. I will provide an overview of each test and its applicability to this study.

Bubble Detection Tests

The following discussion will focus on empirical studies that utilized statistical tests to either directly or indirectly assess the presence of bubbles in a price series. There are numerous statistical techniques, each with strengths and shortcomings, available to assess whether a bubble may be present in stock prices. The tests utilized for this study are the more robust and widely tested methods in the literature. The outcome of the below studies is included for context but not to suggest an implication for the potential result of these tests as they will be applied novelly to individual intraday bubbles in this study.

Autocorrelation Tests

Autocorrelation, often referred to as serial correlation, tests are fundamental statistical tools used in time series analysis, initially utilized in many early market efficiency studies. These tests assess whether stock prices are correlated with their lagged values, potentially indicating non-random price changes – a possible sign of a price bubble.

In a key study, Fama (1965) uses serial correlation tests to show no strong autocorrelation among Dow Jones Industrial Average stocks over four-, nine-, and 16-day periods. Similarly, Fama and French (1988) found low levels of autocorrelation in the short-run but significant negative autocorrelations over 3–5-year periods due to a mean-reverting factor.

Compared to earlier studies, Campbell et al. (2001) find that firm-level volatility is increasing compared to market volatility, and the correlation between individual stocks is declining. A central tenet of this dissertation is the focus on bubbles in individual stocks that may not occur in unison with larger market indices or even peer companies. Therefore, the Campbell et al. (2001) study findings around increased volatility at the individual stock level and decreasing correlation in stocks moving together support a need for more attention to individual stock price bubbles, this study's focus. However, Campbell et al. (1993) suggest that serial correlation is a function of volume, as autocorrelation is low on high-volume days and high on low-volume days. They attribute this pattern to market makers hedging against non-informed traders' demand. These findings will be compared to my results in later sections. The intraday bubbles in this study are expected to reflect significant autocorrelations, given the focus on events where prices rise and fall rapidly, with an expected correlation with their previous value.

Runs Test

The Wald-Wolfowitz runs test, developed by mathematicians Abraham Wald and Jacob Wolfowitz, is a non-parametric test for randomness in a two-valued data sequence. Although Wald and Wolfowitz (1940) did not develop this test for bubble detection or even specifically for stock price time series analysis, its application in these areas has proven to add value to the literature. The test assesses the sequence of runs in a series against the expected number of runs in a stochastic, random process, making it an appropriate addition to the bubble detection tests under investigation in this study.

In one of his seminal works, Fama (1965) showed that a group of stocks did not have signs of significant runs, supporting random walks in stock prices. Others (e.g., Chiat et al., 1983; Dickinson & Muragu, 1994; Urrutia, 1994) used similar runs tests on international stock markets with results suggesting the support of the EMH, even if stock prices were not always perfect random walks.

Blanchard and Watson (1982) applied a runs test to the price of gold, showing that bubbles were not supported in their test period. They used the test to determine if each element in a sequence is independent and also analyzed the series for kurtosis, or fat tails, which may suggest the presence of a bubble. The results of their runs test did not support the existence of a bubble in gold prices during the late 1970s, but the tail test did find a significantly high kurtosis.

While some have utilized a runs test to suggest bubbles may not be present in a series in favor of random walks, many others have used the test to find just the opposite. Stanley (1997) found that nonparametric runs tests find significant autocorrelation in price premiums in experimental markets. Harman and Peterson (2003) find that speculative bubbles are present through evidence of positive abnormal runs in New York Stock Exchange, American Stock Exchange, and Nasdaq-listed stock indices.

The Wald-Wolfowitz runs test is a valuable tool for evaluating the randomness of a data sequence. The flexibility of the test allows for its application to financial market time series despite its original application to binary sequences. Studies utilizing the runs test have led to conflicting conclusions regarding whether stock prices follow random walks or follow patterns consistent with bubbles. The inclusion of this test in the study provides additional contextual insight into the intraday sequence of the observation set.

Variance Ratio Test

The variance ratio test, developed by Lo and MacKinlay (1988), is a statistical tool designed to determine whether stock prices follow random walks., Lo and Mackinlay (1988) found that stock prices do not follow random walks in weekly and monthly holding periods, especially for small market capitalization stocks. While not explicitly designed to detect bubbles, the test's ability to identify non-random price movements makes it a valuable tool in assessing potential bubble-like behavior in this study. It is important to note that their application of this test was not conducted on intraday data or on individual stocks, which are the focus of this study. However, their work laid the foundation for numerous subsequent studies applying the variance ratio test in financial contexts. For instance, Liu and He (1991) and Chang (2004) utilized the test to detect random walks in exchange rates, broadening its utility beyond equity markets.

In the realm of stock market efficiency, Ryoo and Smith (2002) assess the efficiency of Korean stock prices by imposing successively larger artificial price limits to manipulate market efficiency on daily data. Their findings revealed that the market's behavior increasingly approximated a random walk as these price limits were lifted. This study, while informative, still focused on daily data rather than the intraday movements central to the current research. Huang (1995) conducted a similar variance ratio analysis in Asian stock markets but determined that prices do not follow random walks at various holding periods. This divergence in findings underscores the test's sensitivity to different market conditions at different time frames, highlighting a potential limitation. Further, as it is applied in this study to intraday data based on minute aggregate data rather than tick-level data, patterns or trends at more granular tick levels may be lost. Finally, the test is sensitive to the lag, or window selection for comparing variances across two sub-sample periods. The difference in outcome of the variance test when using varying lag length will be highlighted in the findings section.

Unit Root Tests

Unit root tests offer another crucial statistical tool for assessing potential deviations from efficiency and, with certain extensions, the presence of bubbles in time-series data through stationarity tests. The unit root tests utilized in this study are the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests. Each of these tests is an autoregressive model that produces test statistics to compare against critical values for assessing the presence of unit roots or stationarity in a time series. In the ADF and PP tests, the null hypothesis is that the series has a unit root and is nonstationary. In contrast, the KPSS test is the opposite as it is considered a stationarity test with a null hypothesis that the time series is stationary. A critical difference between the ADF and PP tests is how the serial correlation is managed – the ADF test uses lags while the PP test applies a non-parametric correction to the t-statistic. These three tests are complementary and often analyzed concurrently as a robustness tool for assessing unit roots and stationarity in time series.

The Dickey-Fuller test, established by Dickey and Fuller (1979), is the most common, or at least most well-known, test used to determine the presence of unit roots. However, there are many variations, some of which were designed for the detection of explosive price behavior, such as may be expected in a bubble environment. As one of the first widely accepted unit root methods, the original Dickey-Fuller test uses an autoregressive model of order one with a unit root as the null hypothesis. While the test has been used widely, Said and Dickey (1984) note a primary limitation: it does not allow for large, more complex time series with more than one lag.

The augmented Dickey-Fuller (ADF) addresses the singular lag issue by including multiple lags, or differencing terms, in the equation. The appropriate lag length is often determined using either the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC). Additionally, the model allows for a constant or trend to be applied to the intercept in the model. The ADF test has been used widely in the financial market literature on market efficiency and bubble events. For so-called explosive bubbles, Diba and Grossman (1985) suggest that if price levels are nonstationary at levels but stationary in first differences, that would mean an explosive bubble does not exist in that particular data series.

Choudry (2010) uses a multitude of tests, including the ADF test, to assess whether price shocks in stock indices are permanent rather than temporary. This is consistent with the dynamics of a unit root and, arguably, market efficiency. Craine (1993) uses the ADF test to show whether the price-dividend ratio of stocks contains a rational bubble. While others have utilized the test in similar studies, there are limitations to its power and effectiveness, as noted by Dods and Giles (1995), who recognize the test’s sensitivity to lag selection. Researchers utilizing the test could exhibit selection bias by manually setting the number of lags and manipulating the results. This study utilizes AIC for lag selection, which attempts to determine the best-fit model.

While widely used, some of the limitations of the ADF test led to the rapid development of extensions and modifications. The Phillips and Perron (1988) test is an adjusted version of the Dickey-Fuller test that uses a nonparametric method to address serial correlation and heteroscedasticity in error terms of a time series. This test is appropriate given the likelihood of serial correlation in stock price data, especially in explosive intraday price movements. It can be used alongside other unit root stationarity tests for robustness, as Rosini and Shenai (2020) suggested. According to Afriyie et al. (2020), the PP test performs better than the ADF tests with large samples. However, it suffers from weakness due to the potential of structural breaks in a time series, thus suppressing the detection of nonstationarity. Therefore, while the PP test may potentially benefit from a larger stock price time series sample, the robustness of the results is limited by the higher likelihood of multiple structural breaks or bubble-like occurrences as the sample size grows.

The KPSS test was introduced as an alternative to the Dickey-Fuller and PP tests by moving the presence of a unit root from the null hypothesis to the alternative. Kwiatkowski et al. (1990) note the significant number of empirical studies on stock data that fail to reject the presence of a unit root using traditional tests and express concern that these tests may result in excessive type II errors. As an alternative, they proposed making the null hypothesis reflect that a time series is trend stationary, which should be used in conjunction with other unit root tests for robustness and a more nuanced interpretation of time series data. Like the ADF and PP tests, the KPSS test has been utilized in numerous studies to assess the presence of unit roots, evaluate market efficiency, or detect behavior consistent with bubbles. Notably, Bohl (2003) used the KPSS test in conjunction with other tests to conclude that the US stock market exhibited multiple periods of collapsing bubbles. However, they noted that the results of the KPSS test were not significantly conclusive.

It is essential to emphasize a point related to historical studies utilizing the ADF, PP, and KPSS tests versus this study on intraday bubbles. Often, these tests were applied to either indices representing the overall market or to a set of stocks over a long period of time, during which stock and market fundamentals have time to evolve. Given sufficient time, a broad spectrum of variables can influence assessments of fair prices at any moment in time. Researchers attempting to specify these models over lengthy periods to address market efficiency struggle to apply consistent assumptions of market fundamentals (e.g., interest rates) across studies to determine fair prices of assets. The unique structure of this study eliminates some of these concerns, given that on an intraday basis, there is not likely to be a regime-switching expectation of interest rates or dividend levels in individual stocks and, even if there were, it would be difficult to control for these changes. Consequently, intraday data is less susceptible to long-term structural changes, making it more suitable for studying the immediate formation and burst of bubbles.

Supremum ADF (SADF) and Generalized Supremum ADF (GSADF) Tests

While traditional unit root tests provide valuable insights, more specialized tests have been developed to specifically detect explosive behavior in asset prices. Explosive behavior refers to rapid, unsustainable price increases that are characteristic of bubbles. Extending the ADF test with a forward recursive regression technique, Phillips et al. (2011) developed the PWY, or SADF, test for exuberance (explosive behavior) in stock prices. Using the recursive regression technique allows for iterative sub-sample regressions until the entire sample is included, identifying the most explosive sub-sample period. The method can determine not only the presence of bubbles but also the timing of the origination and collapse of a bubble. Their original test used the monthly returns of the Nasdaq composite price index between 1973 and 2005 and confirmed explosive behavior in stock prices but notably not in the corresponding dividends. Price explosions without corresponding increases in aggregate dividends may suggest deviations from intrinsic values, potentially signaling bubbles.

Despite the success of the Phillips et al. (2011) test, it is constrained by similar problems from early specification tests for rational bubbles, including those tests conducted by Diba and Grossman (1988b), which failed to allow for the real-world scenario of periodically collapsing bubbles. To address this limitation, Phillips et al. (2015) utilized a generalized supremum augmented Dickey-Fuller test that provides flexibility in the recursive regression windows rather than the fixed widths of SADF. The Phillips et al. (2015) test is commonly known as the PSY, or GSADF, test. Their empirical test covered S&P 500 data from 1871 to 2010 and claimed to have greater discriminatory power in bubble detection than other tests, including the SADF test. Additionally, the GSADF test can detect multiple bubble periods within a single time series, which could occur in both intraday and longer-term time series. Both tests have been utilized in various studies aimed at detecting explosive behavior in asset prices, with varying degrees of success (e.g., Harvey et al., 2016; Long et al., 2016; Monschang & Wilfling, 2021).

Notwithstanding the power and wide-ranging usage of the SADF and GSADF tests, they are not without their limitations. Harvey et al. (2016) note that they can lead to spurious results and type I errors due to the critical values dependence on the time-varying volatility of the underlying data. Phillips et al. (2015) highlight the benefit of the flexible sub-sample window period but acknowledge that the selection of window length can potentially bias the outcomes.

The SADF and GSADF tests are included in this study due to their unique structural design to explicitly detect explosive behavior rather than just nonstationarity, as the ADF, PP, and KPSS tests are designed. However, their sensitivity to window size selection and potential for type I errors encourages careful application and interpretation in parallel with the other tests discussed. These tests, when used alongside traditional unit root tests, broaden our toolkit for understanding and detecting intraday stock price bubbles.

Hurst Exponent

The final test included in this study to complement the others is the Hurst exponent, which measures long-term memory and self-similarity. Hurst (1951) originally developed it in the 1950s as a tool in hydrology, based on fractal geometry. Nevertheless, its applicability to time series data has resulted in its application to various fields of study, including asset bubble tests. The Hurst exponent ranges in value from 0 to 1, with a value greater than .5 suggesting a trend (closer to 1, the stronger the trend) and values less than .5 suggesting mean reversion. A Hurst exponent at or near .5 would represent a random walk. Therefore, values greater or less than .5 could indicate market inefficiencies and bubble behavior.

Studies using the Hurst exponent to analyze stock price data have produced conflicting results. Ramirez et al. (2008) and Couillard and Davidson (2005) use the Hurst exponent model to show that stock prices do not reflect long-term trends. However, Martinez et al. (2017) suggest that the Hurst exponent can detect early-stage bubbles in the presence of herding in individual stocks. It is worth considering this bubble test given the significant amount of herding that was speculated among retail investors during the sample period. While a promising tool for the detection of trends, the Hurst Exponent has a few limitations that should be considered from the perspective of bubble detection. First, like many of these tests that have been introduced, it is designed to detect trends, or long-term memory, not explosive behavior. Also, Lo (1991) notes that the Hurst exponent is biased and sensitive to high-frequency autocorrelation that is often present in financial data, which could also be expected in this study.

Concluding Thoughts on Bubble Tests and the Hypothesis

The suite of statistical tests proposed for this study provides a diverse suite of statistical tools and tests to assess the presence of intraday bubbles in individual small- and micro-cap stocks. Each test contributes unique insights while complementing the others, allowing for a robust analysis of rapid intraday price movements. The primary hypothesis in this study is that within the selected sample of intraday events, there are price bubbles. The analysis will consider how well the collection of tests uncovers inefficiencies via runs, trends, patterns, or explosive behavior as evidence suggesting the presence of price inefficiencies or price bubble dynamics.

Autocorrelation and runs tests are designed to reveal patterns and assess randomness, while the variance ratio test considers the highly-supported random walk view of stock prices. The variety of unit root tests (i.e., ADF, PP, and KPSS) add depth by examining the time series for unit roots and nonstationarity through various specifications that help identify stochastic trends. These foundational tests, while not specifically designed for bubble detection, offer valuable insights into the underlying characteristics of the price movements.

Building upon this foundation, our tests incorporate the more specialized SADF and GSADF tests, which are specifically designed to detect explosive behavior characteristics of bubbles. These tests focus on detecting rapid, unsustainable increases in stock prices that may occur in irregular segments of an intraday time series. Both tests support the timestamping of the explosive periods, while the GSADF test has the flexibility to detect multiple episodes of explosive behavior in a single time series. Complementing these tests, the Hurst exponent is useful for data that exhibit complex systems characteristics (including financial data) and detects long-term memory of a time series, especially against the backdrop of herding behavior among retail traders.

By leveraging the strength of each method, the analysis can temper their individual limitations, thereby enhancing the reliability and robustness of the findings. The multifaceted approach may help provide a more nuanced understanding of intraday bubble dynamics, offering better insight to market participants and future researchers.

Research Approach and Design

The objective of this study is to assess how well statistical time series tests produce results that align with the price dynamics of bubbles. Fundamentally, this study is quantitative in nature, which aligns with the mature theory framework outlined by Edmondson and McManus (2007). Within a mature domain, they suggest a researcher may "…test a theory in a new setting, identify or clarify the boundaries of a theory, examine a mediating mechanism, or provide new support for or against previous work" (Edmondson & McManus, 2007, p. 1159). This statement aligns well with the purpose and design of both empirical studies conducted in this dissertation.

Financial market literature is robust, and the statistical methods utilized have been applied to a multitude of time series settings. However, there are still many unanswered questions about market efficiency and bubbles, providing an opportunity for researchers to test the bounds of existing theory. Also, the frequency of research focused on intraday bubbles is much less common, providing an opportunity for new insights. The investing landscape evolved tremendously between 2018 and 2022 due to both the exogenous shock of COVID-19 and the endogenous shock of democratized finance through commission-free mobile app-based investing platforms.

Given these recent market developments and the need for more research in the area of intraday bubbles in individual stocks, this study aims to utilize established statistical tests in a novel setting. By seeking to uncover patterns and dynamics that may not even be apparent in daily pricing aggregates, the study addresses a risk that is relevant in the current trading dynamic, given the apparent shift toward more day trading activity among less sophisticated investors in recent years.

Data Collection Methods and Instruments

Study Population and Sampling

The observation set of 342 was selected from constituents of the Russell Microcap Index where upside price movements fell in the 99.8^{th} percentile and downside price movements were in the 90^{th} percentile from this high price on an intraday basis between 2018 and 2022. The intraday minute data was obtained via Python through a subscription to Polygon.io API data feeds. While intraday tick-by-tick level data is available via Polygon feeds, the one-minute aggregates were chosen given that the average volume across all observations on the bubble event dates was over 37 million. Even by utilizing a conservative estimate of 500 shares per trade, accessing tick-level data would result in over 25 million data points across the 342 observations. Therefore, time and data management constraints required the selection of a shorter time interval for the intraday ticks, with minute aggregate bars selected as the interval of choice. While some limitations of the one-minute interval selection are discussed later, it was determined to provide sufficient data points to capture the intraday trends consistent with price bubbles.

The Polygon minute aggregates feed provides various data points for each bar, including timestamp, open, high, low, close, volume, vwap, and number of transactions. There are 390 minutes the primary trading session, which implies a maximum of 133,380 data points.

However, not every stock in the data set has a trade executed every minute of the trading day, which is seen in that the total number of minute observations is approximately 97,500 with an average of approximately 425 trades per minute bar.

Methods and Instruments

This study employs a variety of statistical tests to detect deviations from EMH, which broadly could be qualified as behavior consistent with price bubble dynamics. Additionally, we include tests designed specifically to detect explosive price patterns consistent with bubble dynamics. The list of methods includes autocorrelation, runs tests, variance ratio analysis, unit root tests, the explosive SADF and GSADF tests, and the Hurst exponent. The analysis focuses on minute-by-minute aggregate data for small- and micro-cap stocks, capturing trading dynamics within intraday periods. Each statistical test is applied to the intraday high price at each minute interval to identify patterns of nonrandomness, trends, and explosive behavior. The tests are conducted using software tools such as Python and R, leveraging specialized packages for time series analysis. The results are then compared across different market periods, including pre-COVID, post-COVID lockdowns, and the meme-stock trading frenzy during the years of 2021 and 2022 to assess the impact of these events on bubble dynamics.

Data Analysis and Results

Autocorrelation Analysis

The discussion on bubble detection methods begins with the autocorrelation function (ACF), which is utilized to understand the temporal dependencies of a series. The objective of the test is to assess the persistence of trends in intraday price behavior through observation of high prices and lagged autocorrelations. High prices are used for the series for each stock at one-minute aggregates during the primary trading session, 9:30 a.m. - 4:00 p.m., on the observation date. The high price was selected as opposed to other price data in the minute aggregate bars (i.e., open, low, or close) to ensure the ACF captured the maximum intraday price appreciation, or bubble peak, for the observation set. The test was run using Python with the lag length set to the minimum of 40 or the length of the series minus one if less than 40 observations. In addition to visual inspection of plots and autocorrelation summaries, the Ljung and Box (1970) Box-Ljung test was used to improve the robustness of the autocorrelation assessment.

A review of the ACF output suggests that past high prices have a significant influence on current high prices at one-minute intervals at up to 40 lags on the observation date across 341 of the 342 observations. The one observation without significant autocorrelation, IRIX-8/12/2019, was unique in that it has only 24 price observations compared to an average of 285 (maximum of 390) for all observations in the data set. The summary statistics in Table 4 highlight the high level of autocorrelation present in the intraday data. For example, at lag 10 the autocorrelation at the 50th percentile is .7122 and at lag_20 it is nearly .5, which represents some level of trend in the series, likely counterbalanced by a degree of noise.

Table 4

Autocorrelation Table Summary Statistics

	Count	Mean	Std	Min	25th	50th	75th	Max
lag_0	342	1.00	0.00	1.00	1.00	1.00	1.00	1.00
lag_1	342	0.95	0.07	0.42	0.95	0.97	0.98	0.99
lag_2	342	0.90	0.12	-0.05	0.90	0.94	0.97	0.99
lag_3	342	0.86	0.16	-0.11	0.84	0.91	0.95	0.98
lag_4	342	0.82	0.19	-0.18	0.78	0.88	0.93	0.97
lag_5	342	0.78	0.21	-0.31	0.73	0.85	0.91	0.97
lag_6	342	0.75	0.22	-0.41	0.68	0.83	0.89	0.96
lag_7	342	0.72	0.23	-0.33	0.64	0.79	0.87	0.95
lag_8	342	0.69	0.24	-0.46	0.59	0.77	0.86	0.94
lag_9	342	0.66	0.25	-0.50	0.56	0.74	0.84	0.94
lag_10	342	0.64	0.26	-0.52	0.52	0.71	0.82	0.93
lag_15	342	0.53	0.27	-0.40	0.37	0.61	0.74	0.90
lag_20	342	0.44	0.28	-0.30	0.26	0.50	0.66	0.86
lag_25	340	0.37	0.28	-0.43	0.19	0.43	0.59	0.81
lag_30	338	0.31	0.28	-0.45	0.12	0.36	0.52	0.77
lag_35	336	0.26	0.26	-0.54	0.08	0.29	0.47	0.73
lag_40	335	0.21	0.25	-0.58	0.02	0.23	0.41	0.69

Note: Std = standard deviation. 25th, 50th, and 75th represent percentiles.

In addition to summary statistics, visual inspection of ACF plots can help identify levels of autocorrelation in a series. Figure 3 depicts the plots for the five observations with the largest Box-Ljung statistics, all of which have p-values of effectively 0.00. For each observation, even at lag_40, the autocorrelation is still above .50. Figure 4 depicts the plots for the five observations with the smallest Box-Ljung statistics. While the autocorrelations for these observations quickly move into the confidence interval within the first few lags, the significant autocorrelation at the early lags appears to have a larger impact on the Box-Ljung statistic, resulting in the rejection of the null hypothesis (i.e., no autocorrelation) in favor of the alternative.

Figure 3

Autocorrelation Plots of the Five Observations with the Largest Box-Ljung Statistic

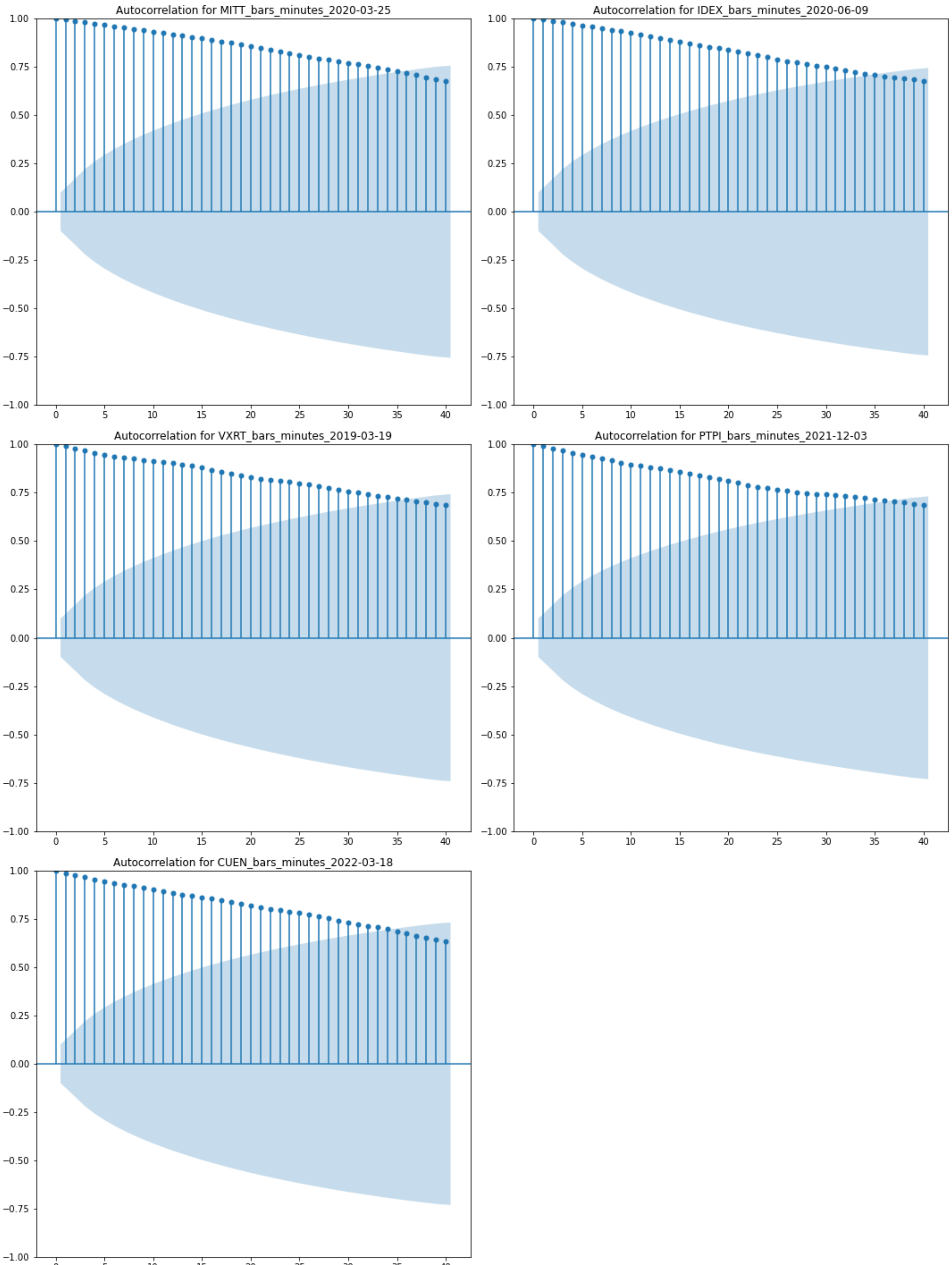
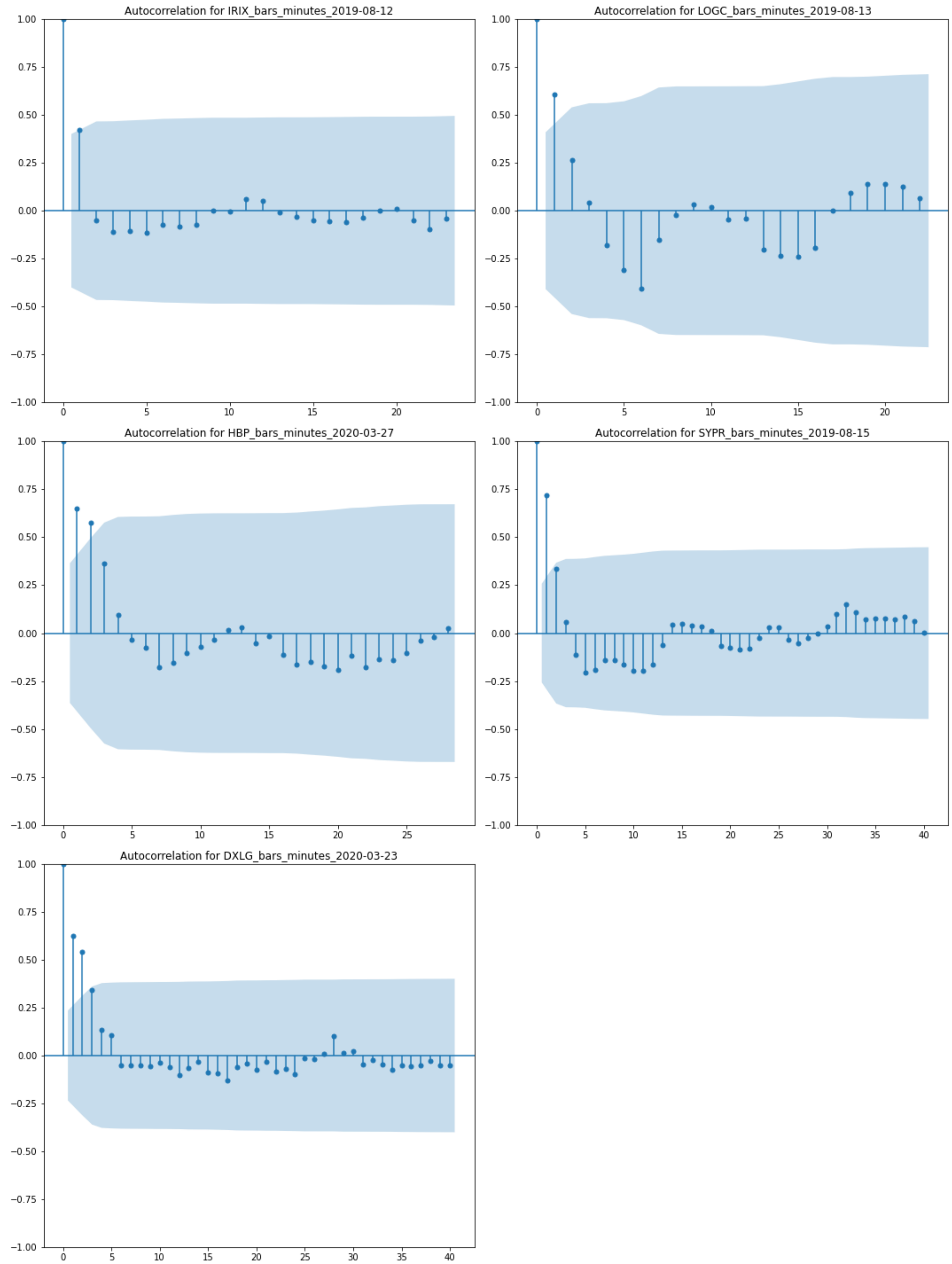


Figure 4

Autocorrelation Plots of the Five Observations with the Smallest Box-Ljung Statistic



Like the majority of tests to be discussed, the ACF is not designed to detect the presence of bubbles in a time series. However, it can, and does in this observation set, detect trends in series through price dependence, a pattern that is expected in the case of price bubbles. Therefore, it can be argued that the results of the ACF tests support the possibility of intraday bubbles in the observations set.

Runs Test

The Wald-Wolfowitz runs test was applied to the intraday data to assess the randomness of the sequence of high prices at each minute aggregate bar. The test, while not specifically designed to detect bubbles, has been utilized (Chiat et al., 1983; Dickinson & Muragu, 1994; Fama, 1965; Harman & Peterson, 2003; Stanley, 1997; Urrutia, 1994) to assess random walks in stock prices. Therefore, the test was applied to the intraday data set to check for the absence of randomness in favor of trends or patterns that occur in bubble dynamics.

The runs test was conducted using Python by leveraging the Wald-Wolfowitz function to calculate z-stats and p-values for each intraday series. In this test, the null hypothesis is that the sequence is random, where large z-stats and small p-values support trends. Table 5 summarizes the results of the runs test across the five-year period of the observations with subsets across the significant market events during this period. There were 83 instances (24.27%) where the null hypothesis was rejected at a significance level of .05 or better, with the majority of these occurring after the COVID-19 lockdowns in March 2020. The percentage of non-random price sequences during this period exceeds that of the pre-COVID lockdowns.

Table 5

Runs Test Summary Across Market Events

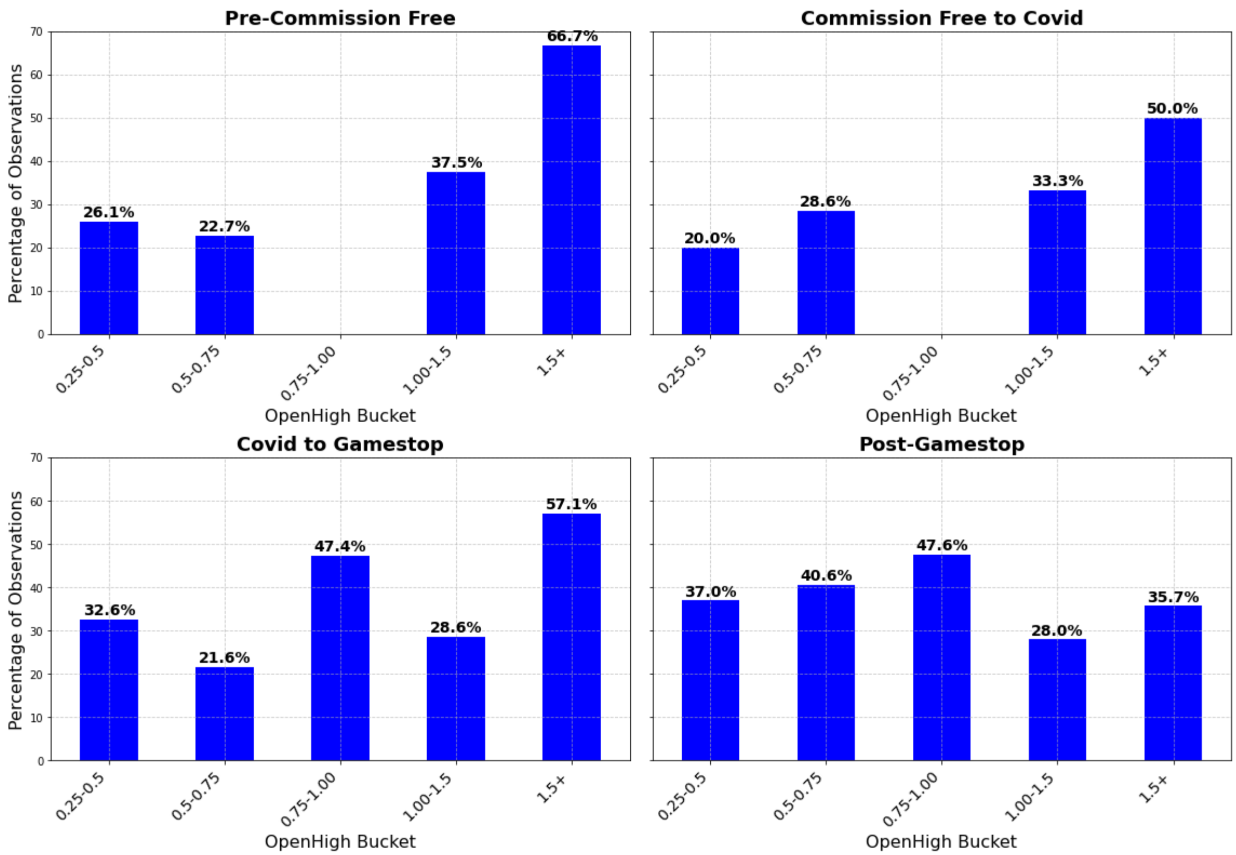
Category	Count	Percentage
Before Commission Free Trading	60	17.54%
p <= 0.01	4	6.67%
0.01 < p <= 0.05	4	6.67%
0.05 < p <= 0.10	8	13.33%
p > 0.10	44	73.33%
Between Commission-Free Trading and COVID	24	7.02%
p <= 0.01	1	4.17%
0.01 < p <= 0.05	3	12.50%
0.05 < p <= 0.10	2	8.33%
p > 0.10	18	75.00%
Between COVID Lockdowns and Gamestop Frenzy	120	35.09%
p <= 0.01	18	15.00%
0.01 < p <= 0.05	15	12.50%
0.05 < p <= 0.10	8	6.67%
p > 0.10	79	65.83%
After Gamestop Frenzy	138	40.35%
p <= 0.01	22	15.94%
0.01 < p <= 0.05	16	11.59%
0.05 < p <= 0.10	14	10.14%
p > 0.10	86	62.32%

Note. The percentages of each sub-period are based on the total 342 observations

These results are supported by the visualization in Figure 5, which shows that the greatest proportion of significant events are detected post-March 2020 COVID-19 lockdowns. Additionally, larger bubbles appear to have a higher percentage of nonrandom price sequences prior to COVID-19, whereas there is a more even distribution of detectable nonrandom patterns across all bubble sizes in the bottom two bar charts. While the runs test confirmed some instances of nonrandom behavior, in nearly 75% of the observations, I was unable to reject the null hypothesis of random walks. The results align with the literature, which is replete with varying results from utilizing the runs test to assess randomness in stock market prices and returns. A recurring limitation is the use of minute aggregates, which may hide some of the patterns or trends that occur at more granular tick-level data. Further, the application of this test to intraday price sequences in small- and micro-cap companies is a novel exploration, which may result in market microstructure order dynamics that influence price patterns. Similarly, oscillating market motivations on an intraday basis at small tick intervals may obscure patterns or trends that develop over the course of a day. The runs test has limitations that should be considered in any study. First, the dichotomous nature (e.g., up or down) does not provide context on the size of the price change. Second, it may not capture more complex dependencies or patterns in a time series beyond detecting runs. Finally, the test does not test directly for random walks. Rather, the test's null hypothesis is randomness in the specific sequences being evaluated, which means that even if we are unable to reject the null, it does not necessarily imply random walks. Therefore, despite confirming instances of nonrandom behavior, the results should be interpreted in conjunction with other tests in this study to better understand its power in intraday bubble detection.

Figure 5

The Proportion of Runs Test Observation Significant at the .10 Level Across Bubble Size and Time



Note. The x-axis represents bubble size ranges, while the y-axis represents the proportion of observations found to have runs within each range of bubble magnitude.

Variance Ratio Analysis

The variance ratio test is designed to detect random walks in a series by calculating the variance ratio for single period returns against multi-period returns in a single time series. A variance ratio significantly greater than one implies the series does not follow random walks and may reflect momentum or trends. In contrast, a variance ratio significantly less than one implies potential mean reversion in the series. Therefore, a variance ratio not significantly different from one suggests the series reflects a random walk.

The variance ratio test was conducted using Python by leveraging the Lo-Mackinlay function to calculate the ratio and p-value for each observation, again using the high price from the one-minute aggregates. The test was run across multiple lag periods to detect variance ratios at varying intervals. Table 6 summarizes the test outcomes at varying intervals and significance levels of .05 and .01. The count of total significant observations is greater at shorter lag intervals, with 69 and 28 at significance levels of .05 and .01, respectively. The decay in the count of significant observations is greater at the .05 level than at .01. Based on the 69 observations with significance at the .05 level, the results indicate more significant variance in shorter lags. However, 273 observations (nearly 80%) reflect random walks. These results should be interpreted within the context of the structure of this study on intraday bubbles. One line of intuition may be that longer lags reveal trends that exert more influence than noise at shorter intervals. However, the nature of market microstructure, a complex and influential factor, may impact short-term trends more heavily than at longer lags. For example, large orders may be fragmented into many smaller orders that place the same directional strain on prices. These events, which are conceptually aligned with bubbly behavior, suggest that short-term deviations from random walks may manifest as bubble mania waxes and wanes in short intervals. Finally, the test was conducted at minute aggregates, so a tick-by-tick analysis of trades may result in very different attributes.

Table 6

Variance Ratio Count of Significant Observations

Lag	**P-value <.05**	**P-value <.01**
2	69	28
5	56	25
10	34	20
20	28	14

Further analysis of the variance ratio test results across significant market events highlights that the test does a slightly better job of detecting deviations from random walks after COVID-19 lockdowns than it does before this time period. As highlighted in Table 7, before COVID-19, 17.86% of observations showed deviations from random walks at the .05 significance level, whereas 20.9% deviated from random walks following COVID-19 lockdowns. Additionally, the bar graph in Figure 6 depicts the distribution of significant observations at the .10 significance level over time, scaled by the size of the intraday bubble. Noticeably, the proportion of detectable events is spread evenly across all bubble sizes post-COVID, whereas they are more detectable at larger bubbles pre-COVID. This may represent that the bubbles reflect stronger trends post-COVID.

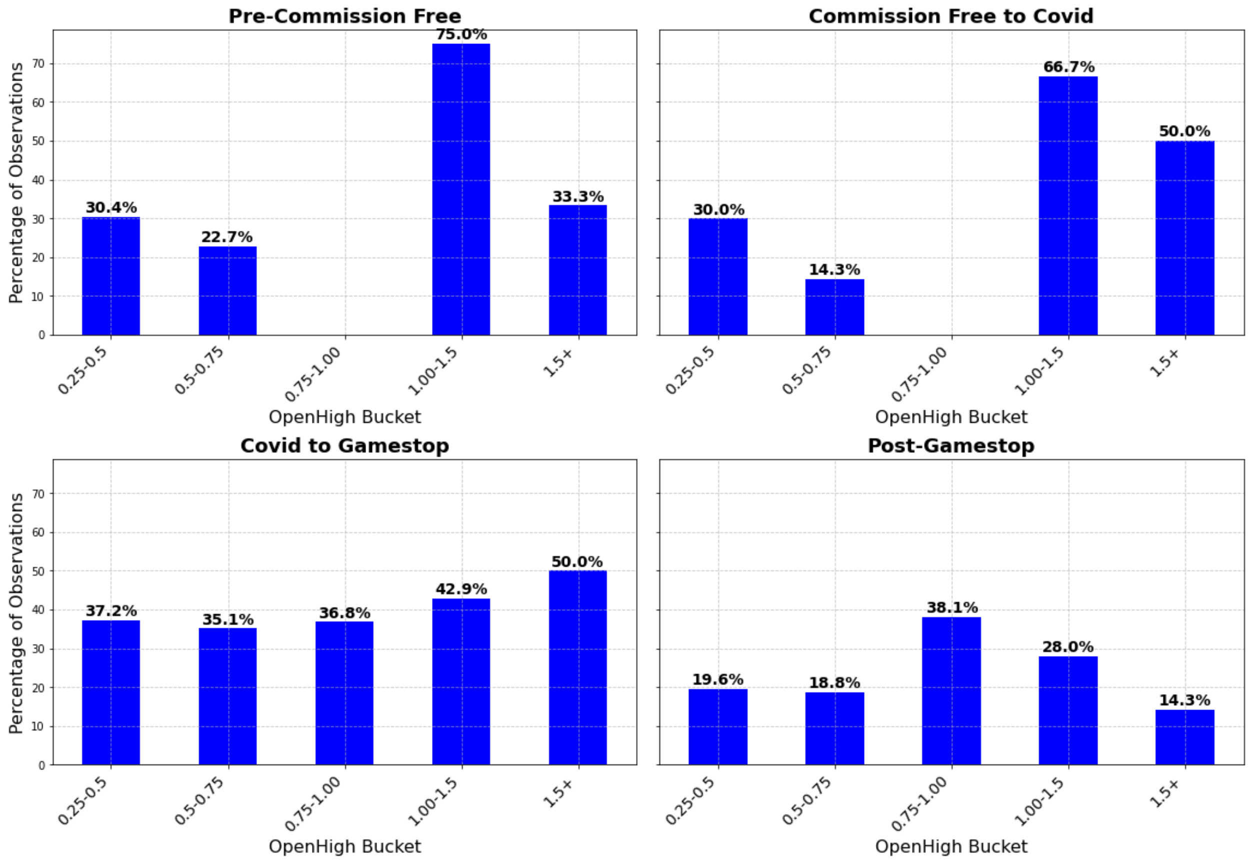
Table 7

Variance Ratio Summary Distribution Over Market Events at Lag 2

Category	Count	Percentage
Before Commission-Free Trading	60	17.54%
p <= 0.05	12	20.00%
0.05 < p <= 0.10	7	11.67%
p > 0.10	41	68.33%
Between Commission Free Trading and COVID	24	7.02%
p <= 0.05	3	12.50%
0.05 < p <= 0.10	4	16.67%
p > 0.10	17	70.83%
Between COVID Lockdowns and Gamestop Frenzy	120	35.09%
p <= 0.05	30	25.00%
0.05 < p <= 0.10	16	13.33%
p > 0.10	74	61.67%
After Gamestop Frenzy	138	40.35%
p <= 0.05	24	17.39%
0.05 < p <= 0.10	8	5.80%
p > 0.10	106	76.81%

Figure 6

Scatterplot of Variance Ratio Significance Across Bubble Size and Time



Note. The x-axis represents bubble size ranges, while the y-axis represents the proportion of observations found to have variance ratio trends within each range of bubble magnitude

The variance ratio test was conducted to detect the presence of random walks in the series of high prices from one-minute aggregates. Leveraging the Lo-Mackinlay function, the analysis compared single-period returns to multi-period returns across multiple lag intervals to identify significant variance ratios. The results revealed that the number of significant observations was higher at shorter lags, suggesting that shorter intervals are more likely to show deviations from random walks, potentially due to short-term momentum or mean reversion effects. However, the majority of observations did not deviate significantly from random walks, indicating that either the test is not effective in identifying most violations of efficiency in these extreme price events, a large portion of these events have random movements, or the minute-by-minute data loses some runs that occur within a minute period.

The higher incidence of significant variance ratios may be associated with fragmented large orders or short-term bubbly dynamics. Additionally, the comparison of pre- and post-COVID-19 lockdown periods revealed a slight increase in the detection of deviations from random walks in the latter period. This suggests that the market dynamics during and after the lockdowns may have introduced more pronounced short-term trends. These results should be interpreted cautiously, given the likely scenario of varying results at tick intervals of less than a minute. As with any statistical test, we should consider possible shortcomings in its applicability. In short, the Lo-Mackinlay test is sensitive, as shown in the results discussed above, with lag lengths leading to issues in interpreting these results. Also, Lo and Mackinlay (1988) suggest their test statistic is heteroscedastic robust, but they also recognize that there are some concerns around varying variances.

Unit Root Tests

The ADF, PP, and KPSS tests were conducted using Python to assess whether the intraday minute aggregate data exhibit characters consistent with unit roots or stationary series. Each test was run four times (i.e., levels with a constant, levels with a trend, first differences with a constant, and first differences with a trend) using the AIC criterion for lag selection. Given the complementary and similar nature of these tests, the results are discussed and compared together.

As would be expected, utilizing a trend rather than constant results in fewer instances of a unit root as a portion of the trend that a bubble may exhibit is accounted for in the model. Table 8 summarizes the outcomes of all 12 models across the three tests, highlighting that most observations become stationary upon first differencing. Notably, the KPSS test results in the largest count of nonstationary events when first differencing, while the PP test resulted in no nonstationary series at first differences. The PP test, unlike the ADF test, is robust to heteroscedasticity and autocorrelation, which may explain why nonstationarity is removed for all observations at first differences for this test. There were no observations that resulted in determinations of unit roots across all 12 specifications of the three tests, or even across the 10 tests when excluding the PP test at first differences given it had zero cases of unit roots. There were only five observations where none of the tests resulted in unit root determinations, while there were 26 observations that resulted in unit root determinations in seven or more tests, which means all of the observations confirm unit roots in at least one of the first differences tests.

Table 8

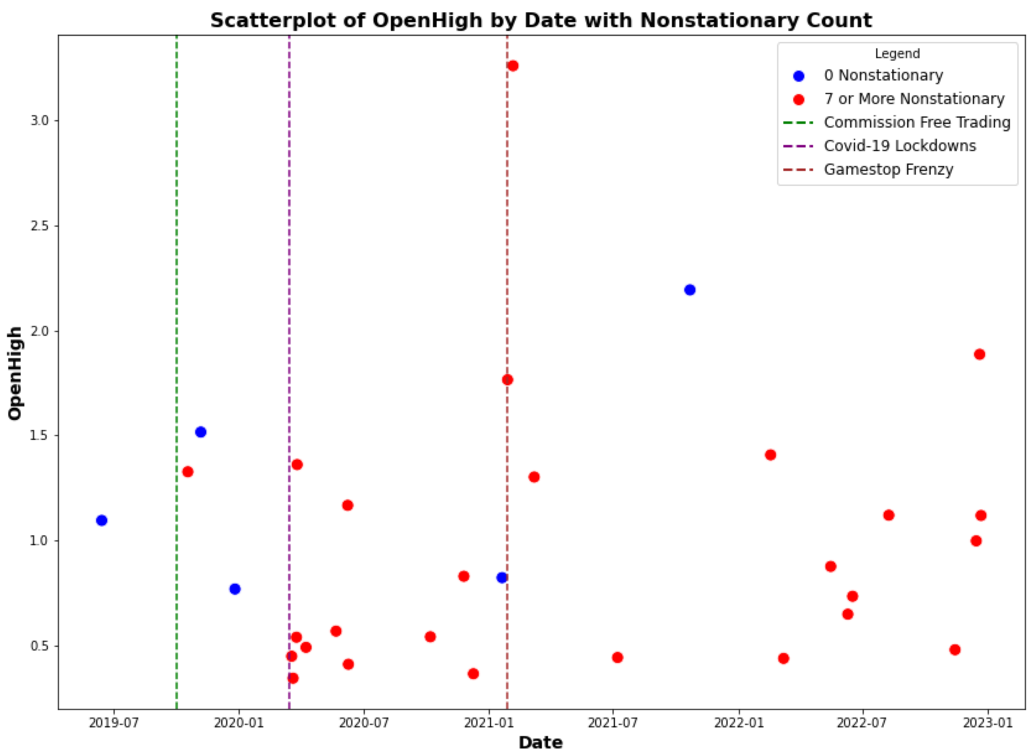
Nonstationary/Unit Root Breakdown by Test

Test	Nonstationary/Unit Root	Proportion of Total
ADF - Levels, Constant	271	79.24%
ADF - Levels, Trend	228	66.67%
ADF - 1st Differences, Constant	7	2.05%
ADF - 1st Differences, Trend	10	2.92%
PP - Levels, Constant	284	83.04%
PP - Levels, Trend	219	64.04%
PP - 1st Differences, Constant	0	0.00%
PP - 1st Differences, Trend	0	0.00%
KPSS - Levels, Constant	291	85.09%
KPSS - Levels, Trend	276	80.70%
KPSS - 1st Differences, Constant	23	6.73%
KPSS - 1st Differences, Trend	21	6.14%

As shown in Figure 7, 25 of 26 observations, which support the presence of unit roots in seven or more tests, occurred after COVID-19 lockdowns in March 2020. In the five observations with no support of unit roots in the tests, three occurred prior to COVID-19 and one occurred prior to the GameStop event. The spread of bubble size, as shown by OpenHigh on the y-axis, does not appear to show significant trends. There are larger bubbles at the instances where unit roots are present, but there are few observations with zero cases of unit roots to compare against, limiting any practical takeaway.

Figure 7

Distribution of Observations of Zero Unit Roots Confirmed vs. More than Seven Confirmed



The combined use of the ADF, PP, and KPSS tests provides a detailed view of the stationarity and trend characteristics of the high prices for the minute-by-minute aggregates. At levels, prices appear to be mostly nonstationary, suggesting the random walk hypothesis when a unit root is confirmed or possibly cases of trends and persistence in nonstationary data absent a unit root. By first differencing the price series, if it is a random walk, it should become stationary, as supported by the results. If, after first differencing, the series continues as nonstationary, then the presence of an explosive bubble is supported.

SADF and GSADF Tests

The Phillips et al. (2011) SADF test, along with the Phillips et al. (2015) GSADF test, will be discussed in tandem given their similarity in design, structure, and output. Both tests were conducted using the exuberance (exuber) package in the R program. Unlike the other unit root tests, the specification is designed to test for explosive behavior at levels, so there was not a need to apply these tests at first differences.

Table 9 shows the breakdown of explosive events across both tests. A point worth noting is that all the incidents of explosive behavior detected by the SADF test are also captured by the GSADF test, which serves as a robust confirmation of expected outcomes. The GSADF test has additional power to capture explosive events missed by the SADF test due to its ability to recognize multiple periods of exuberance. Nearly half of the observations in the data set have at least one period of exuberance at the .05 level of significance, while over a third of the observations experience explosive behavior at the .01 level of significance.

Table 9

Count of Explosive Behavior in SADF and GSADF Tests at .05 and .01 Significance Levels

SADF (PWY, 2011) at .05 Significance		
	Count	Proportion
Explosive	102	29.82%
Not Explosive	240	70.18%
Total	342	

SADF (PWY, 2011) at .01 Significance		
	Count	Proportion
Explosive	84	24.56%
Not Explosive	258	75.44%
Total	342	

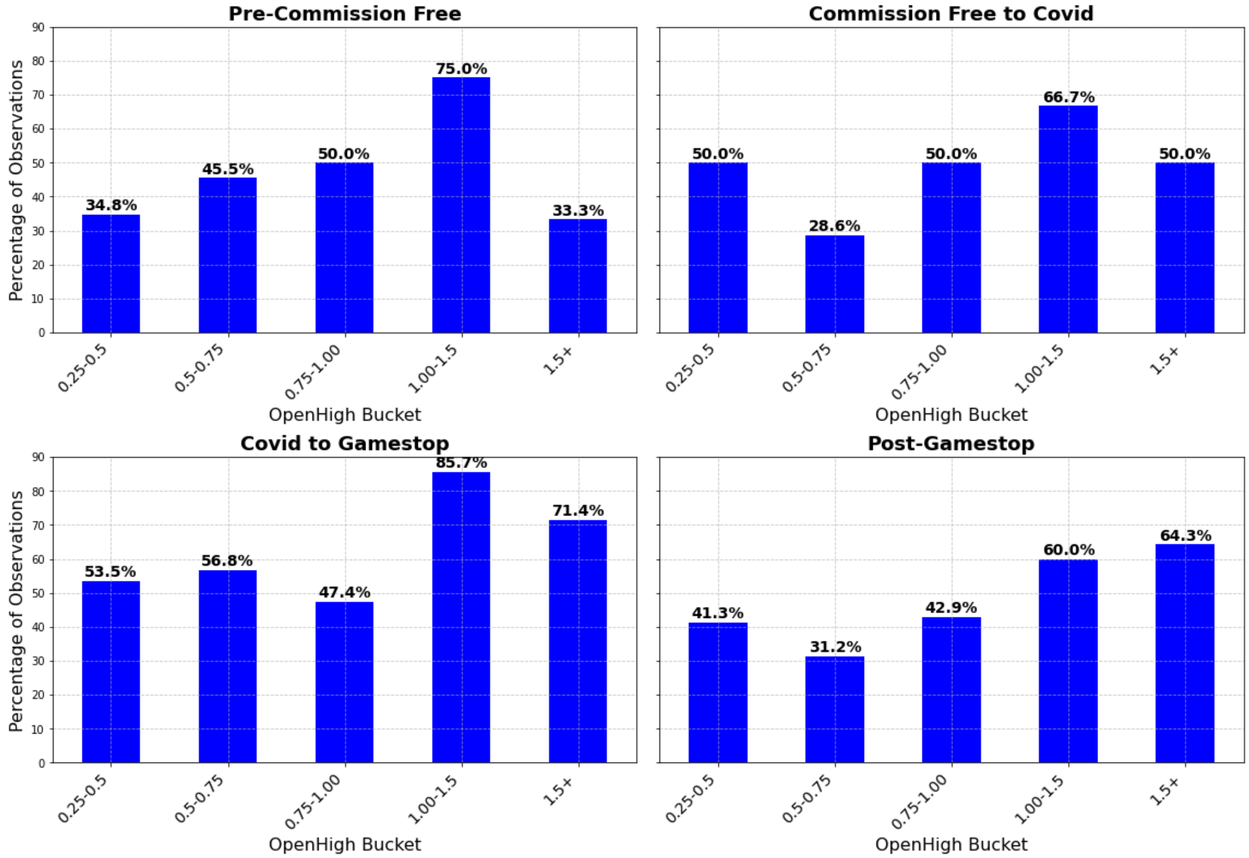
GSADF (PSY, 2015) at .05 Significance		
	Count	Proportion
Explosive	169	49.42%
Not Explosive	173	50.58%
Total	342	

GSADF (PSY, 2015) at .01 Significance		
	Count	Proportion
Explosive	129	37.72%
Not Explosive	213	62.28%
Total	342	

Figure 8 displays how the proportion of detectable explosive events is spread across different bubble-size buckets over time. Similar to the Runs and Variance Ratio tests, explosive behavior is detected at higher proportions when bubbles are larger pre-COVID. However, the GSADF test still captures explosive behavior in approximately 30-50% of the observations in the three smaller bubble-size buckets. This exceeds the nonrandomness and trends detectable at these bubble sizes using the Runs and Variance ratio tests. Post-COVID, the GSADF test detects a higher proportion of explosive behavior across all bubble sizes. Again, this supports the idea that these events were more intense post-COVID than they were before. While not displayed in Figure 8, approximately 85% of the events determined not to be explosive had an OpenHigh of less than 100%, with a mean of 71% and median of 59%. For the explosive events, 70% had a bubble size of less than 100%, with a mean of 90% and a median of 64%. These outcomes align with the logical expectation that larger bubbles are more likely to test positive for explosive behavior in the SADF and GSADF tests. Likewise, the results appear to align with the trends in the distribution of nonstationary events in the tests discussed previously.

Figure 8

Proportion of Explosive Events at .05 Significance Level



Hurst Exponent

The Hurst exponent test was conducted in Python using the rescaled range (R/S) version of the Hurst exponent value, which is more reliable in small samples and volatile data, making it appropriate for this study. Unlike the other tests explored in this study, the Hurst exponent does not produce test statistics and critical values. Rather, a value of .5 represents a random walk, a value greater than .5 represents a trending series, and a value below .5 represents a mean reverting series. In some studies bootstrapping is used to calculate confidence intervals, but that was not explored for purposes of this study given the Hurst exponent was one of many tests used. Notably, the Hurst exponent package requires a minimum of 100 observations to reliably calculate the Hurst exponent value, so 32 observations were dropped from the data set. Table 10 shows that over 90% of the observations appear to be trending when the cutoff was set at .45 < H < .55 for random walks. This total of 282 trending events is very similar to the number of nonstationary observations at levels when conducting the ADF, PP, and KPSS tests.

Table 10

Hurst Exponent Implications

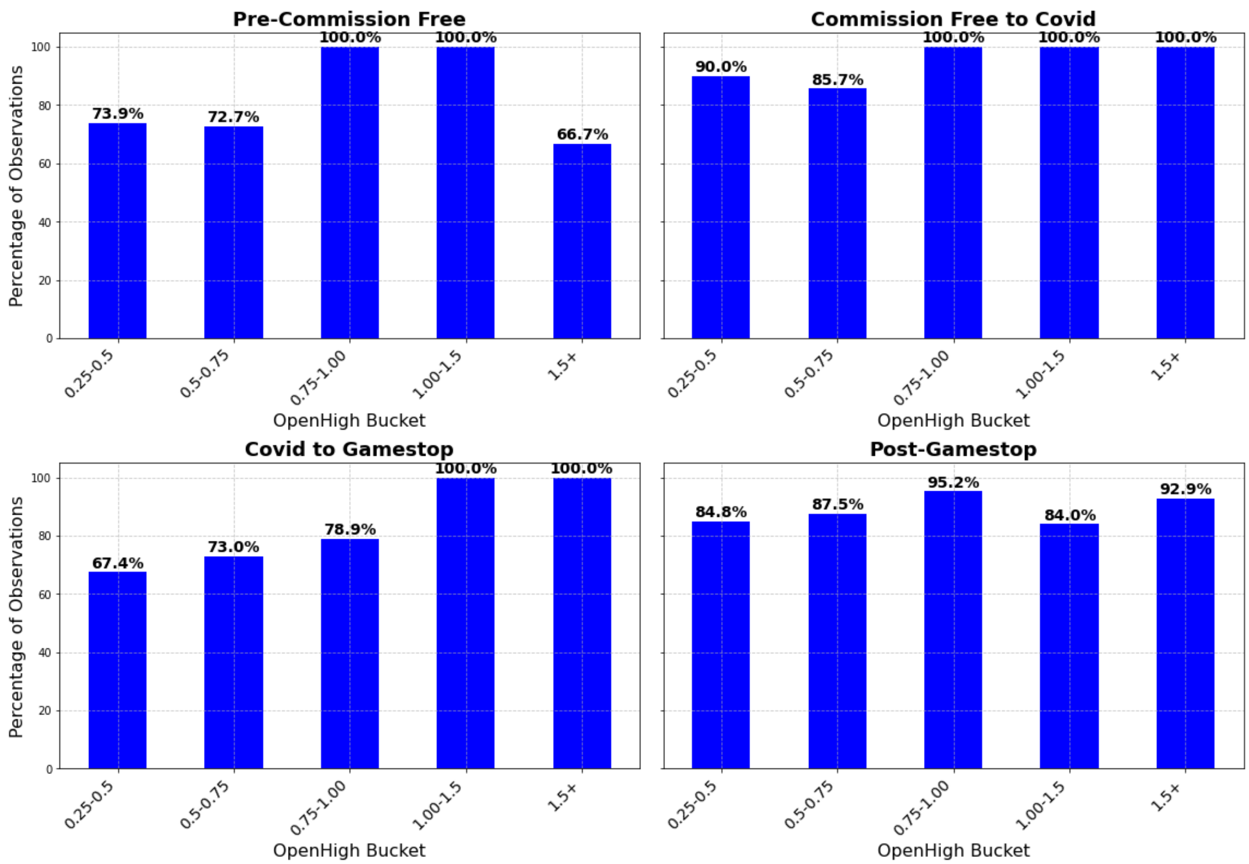
Hurst Memory	Count	Proportion
Mean Reverting	3	0.97%
Random Walk	25	8.06%
Trending	282	90.97%
Total	310	100.00%

Note. 32 observations are lost due to a minimum threshold of 100 minute bars for the Hurst Exponent test.

The bar chart in Figure 9 shows that trending behavior was detected in nearly 100% of observations with a bubble size greater than 100%, excluding the 150% bucket in the pre-commission-free trading window. While there is some variation across time and bubble size, the Hurst Exponent is robust to these factors as it consistently captures trending behavior in more than 60% of the observations across all categories. There were 25 observations suggested to be random walks by the Hurst exponent, of which only two had a bubble size of more than 100%. There were only three observations showing mean reversion, all of which had modest bubbles – significantly less than 100%. This suggests that the size of the bubble has some correlation with the outcome of the Hurst Exponent test. However, the distribution over the 5-year period does not appear to have a significant impact on the occurrence of mean reversion or trends as these events are spread evenly over the 5-year period. The Hurst exponent results provide additional context on the memory, or trend, of high prices in the minute-by-minute aggregates data, with results suggesting trending behavior in 90% of the eligible observations (some events were removed due to an insufficient number of observations).

Figure 9

Proportion of Hurst Exponent Trending Observations



Summary of Results

In sum, the study applied 17 bubble tests across the 342 observations, including four versions of each type of unit root test (i.e., ADF, PP, and KPSS). While the results of the bubble tests applied in this study may not offer a direct comparison or definitively establish that bubbles are or are not present in the time series, collectively, they do provide valuable insight into the intraday price patterns of a unique set of intraday bubble events over an important five-year period in US financial markets. Recognizing the individual nuances of each test, one way to compare across is to compare based on the test's implication regarding whether the outcome is consistent with time series behavior that is consistent with bubble dynamics.

Appendix C summarizes the results of all 342 observations across 16 of the bubble tests performed in the study. Table 11 explains the abbreviations used in the summary table. Of the 342 bubble events, each had at least one test result suggesting the time series shares characteristics with bubble dynamics. However, when excluding the unit root tests at levels, 24 events failed to show signs of nonstationary, trends, or explosive behaviors. There were 293 observations that showed to be stationary at first differences across all three unit root tests. However, 270 of 293 were suggested to have either a trend, patterns, runs, or explosive behavior in one or more of the other tests. Table 12 shows the count of observations where the test suggests there may be bubble like behavior in the price series. Not surprisingly, the unit root tests at levels show signs of nonstationarity in 65-85% of the observations, which is aligned with established literature on stock prices (Diba & Grossman, 1985). The GSADF and Hurst exponent tests all do a decent job at detecting the bubble-like pattern in the intraday prices of our sample as they detect the behavior at approximately 50% and 83% of the events.

Table 11

Summary Table Key

Legend	Test	Implication of ✓
ADF - L,C	Augmented Dickey-Fuller test at levels with constant intercept	Unit Root/Nonstationary
ADF - L,T	Augmented Dickey-Fuller test at levels with trend intercept	Unit Root/Nonstationary
PP - L,C	Phillips-Perron test at levels with constant intercept	Unit Root/Nonstationary
PP - L,T	Phillips-Perron test at levels with trend intercept	Unit Root/Nonstationary
KPSS - L,C	Kwiatkowski-Phillips-Schmidt-Shin at levels with constant intercept	Unit Root/Nonstationary
KPSS - L,T	Kwiatkowski-Phillips-Schmidt-Shin at levels with trend intercept	Unit Root/Nonstationary
ADF - D,C	Augmented Dickey-Fuller test at 1st differences with constant intercept	Unit Root/Nonstationary
ADF - D,T	Augmented Dickey-Fuller test at 1st differences with trend intercept	Unit Root/Nonstationary
PP - D,C	Phillips-Perron test at levels 1st differences with constant intercept	Unit Root/Nonstationary
PP - D,T	Phillips-Perron test at levels 1st differences with trend intercept	Unit Root/Nonstationary
KPSS - D,C	Kwiatkowski-Phillips-Schmidt-Shin at levels with constant intercept	Unit Root/Nonstationary
KPSS - D,T	Kwiatkowski-Phillips-Schmidt-Shin at levels with trend intercept	Unit Root/Nonstationary
Runs Trend	Wald-Wolfowitz Runs Test	Run or mean reverting
VR - T or R (5 lags)	Lo-Mackinlay Variance Ratio test	Trends/Patterns
Explosive (GSADF)	Generalized Sup Augmented Dickey-Fuller test	Explosive behavior
Hurst Trend or Reverting	Hurst Exponent	Long-term memory/pattern

Table 12

Number of Observations that Each Test Supports Bubble-Like Price Behavior

Test	Observations with bubble dynamics	Proportion of Total
ADF - L,C	271	79.24%
ADF - L,T	228	66.67%
PP - L,C	284	83.04%
PP - L,T	219	64.04%
KPSS - L,C	291	85.09%
KPSS - L,T	276	80.70%
ADF - D,C	7	2.05%
ADF - D,T	10	2.92%
PP - D,C	0	0.00%
PP - D,T	0	0.00%
KPSS - D,C	23	6.73%
KPSS - D,T	21	6.14%
Runs Trend	83	24.27%
VR - T or R (5 lags)	81	16.37%
Explosive (GSADF)	169	49.42%
Hurst Trend or Reverting	285	83.33%
Average	140.50	40.63%
Average excluding first differences	218.70	63.22%

The top 10 bubble events based on size of the bubble as well as the top 10 based on volume of trading activity are shown in Table 13. The Hurst exponent recognizes long-term memory in all 20 of these extreme events, while seven of 10 are recognized as explosive in the GSADF test. The results of these tests for these price and volume outliers support the results from the larger data set, suggesting that the GSADF test and Hurst exponent are superior in capturing potential bubble events.

Table 13

Results for the Top 10 Bubbles, Based on Size and Volume

Ticdate	Criteria	ADF - L,C	ADF - L,T	PP - L,C	PP - L,T	KPSS - L,C	KPSS - L,T	ADF - D,C	ADF - D,T	PP - D,C	PP - D,T	KPSS - D,C	KPSS - D,T	Runs Trend	VR - T or R (5 lags)	Explosive (GSADF)	Hurst Trend or Reverting
AAME-2/5/2021	size	✓	✓	✓	✓	✓	✓					✓					✓
BNGO-10/16/2019	size	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓
ECOR-3/30/2020	size	✓	✓	✓	✓	✓	✓									✓	✓
ICD-4/9/2020	size	✓	✓	✓	✓	✓	✓							✓	✓	✓	✓
KELYB-8/11/2020	size		✓	✓	✓		✓									✓	✓
KELYB-8/21/2020	size		✓	✓		✓	✓										✓
LTRPB-8/1/2022	size	✓	✓	✓	✓	✓	✓									✓	✓
NUZE-11/11/2021	size						✓					✓	✓				✓
UONEK-6/16/2020	size	✓	✓	✓	✓	✓	✓									✓	✓
UONE-6/16/2020	size	✓	✓	✓	✓	✓	✓							✓		✓	✓
EXPR-1/27/2021	volume	✓	✓	✓	✓	✓	✓							✓		✓	✓
HYMC-3/11/2022	volume	✓	✓	✓	✓	✓	✓								✓		✓
KODK-7/29/2020	volume		✓		✓	✓	✓							✓			✓
OLB-11/2/2021	volume	✓	✓	✓	✓	✓	✓										✓
OPGN-3/8/2021	volume	✓	✓	✓	✓	✓	✓										✓
PHUN-10/22/2021	volume													✓		✓	✓
SPRO-9/22/2022	volume	✓	✓	✓	✓	✓	✓									✓	✓
TYME-2/3/2021	volume					✓											✓
VCNX-2/19/2021	volume	✓	✓	✓		✓	✓										✓
XELA-7/8/2021	volume	✓	✓	✓	✓	✓	✓					✓			✓	✓	✓

Discussion

Applicability of Bubble Tests

The study is designed to evaluate how well traditional bubble detection tests detect intraday bubbles in individual stocks. Unfortunately, academic research has not yet discovered a singular test that can objectively answer the bubble/no bubble quandary – this study included.

However, by employing a variety of tests, including autocorrelation, runs tests, variance ratio analysis, unit root tests, SADF and GSADF explosive tests, and the Hurst exponent, the study has identified patterns of nonrandomness, trends, and explosive behavior indicative of bubble dynamics based on intraday minute aggregate data in small- and micro-cap stocks. Notably, the presence of significant autocorrelation and nonstationarity in high prices, along with the frequent occurrence of explosive events as detected by the GSADF test and Hurst exponent, underscores the potential value of these tests. Further, the impact of market events, such as the COVID-19 lockdowns and the meme-stock trading frenzy, further highlights the sensitivity of these stocks to external shocks. However, unraveling the thread of influence on these intraday bubbles is in itself a difficult task.

Despite the merit and promise of some bubble tests, it is important to acknowledge their limitations. While they provide valuable insights, they are not perfect and may not capture all bubble events. The mixed results, particularly from the runs test and variance ratio analysis, illustrate the challenges in identifying bubble behavior. Nevertheless, the novel and collective application of these tests to intraday data offers a granular understanding of bubble formation and dissipation, supporting the hypothesis of bubble dynamics in many of the observations.

Limitations

Each bubble test evaluated in this study has been utilized in highly cited studies, which supports the validity of their application to financial market time series, such as stock prices. Nevertheless, there were a few notable limitations regarding the research design and the applicability of these tests in this study. First, as noted, the tests were conducted utilizing minute aggregate data obtained from Polygon.io, which may have resulted in some patterns being lost between minutes. Further, if a bubble peak occurs very early or late in the trading day, there will be a limited number of minute observations available. In our data set, the average and median number of observations prior to the high price were 72 and 41, respectively, suggesting that the majority of the high prices occurred earlier in the trading day. While this limits the number of observations in some cases, it is partially mitigated by the fact that these tests do not discriminate between upside and downside patterns or trends. Given that there is a sizable downside movement in these events, the tests should have substantial observations to capture the bubble dynamics on the downside price movement.

Trading today, especially in extreme events, is rapid and hundreds of trades across thousands of shares can occur in less than a minute. Second, the tests are sensitive to specification bias, such as lag selection, variance ratio length, and minimum observation choice for the Hurst exponent. Third, the study is limited by a lack of cross-validation by testing observations that do not meet the bubble criteria utilized for selection of the observation set. Finally, there may be limited generalizability of these findings given that bubbles of different sizes may exhibit unique characteristics.

Future Research Considerations

These results offer a robust framework for future research and practical implications for traders and regulators aiming to navigate and mitigate the risks associated with intraday bubbles. The findings emphasize the need for further refinement of detection methods and the incorporation of more granular data to enhance the accuracy and reliability of bubble identification in financial markets. The SADF and GSADF tests time stamps the explosive periods, providing an opportunity to analyze specific events during this time, such as company news, release of economic reports, and order flow/depth information. This proposed granular analysis at the moment(s) of explosive patterns can provide additional insight into the factors triggering and influencing these bubble events.

While this study was explicitly focused on intraday bubble formation and crash, there is clearly an opportunity to expand the application of these tests over a multi-day window and on bubbles at less extreme outlier cutoffs. If nothing else, this study highlights that looking for bubbles in daily aggregate over multiple days, weeks, months, or years can conceal intraday bubbles that deserve more acute analysis. Finally, the influx of retail investors, with their propensity for lottery stocks and zero-dated options, has changed the intraday trading dynamic and heightened the risk of these bubble events to all market participants. In the 342 observations in this study, the average total dollar value traded on the day of these events was over $35 million, highlighting that there is significant money at risk in these events, even among small- and micro-cap stocks.

In conclusion, this study provides valuable insights into the detection of intraday bubbles using traditional bubble tests. While no singular test can definitively identify bubbles, the collective application of multiple tests offers a multi-perspective view of bubble dynamics in small- and micro-cap stocks. Acknowledging the limitations, such as data aggregation and sensitivity to specification bias, this research underscores the need for further refinement and more granular data to improve bubble detection. The findings have significant implications for future research, traders, and regulators, highlighting the evolving landscape of retail investing and the importance of timely detection and analysis of intraday bubbles.

CHAPTER 4: EXPLORING INTRADAY BUBBLE SIZE THROUGH REGRESSION

Introduction

Background and Motivation

In the dynamic world of financial markets, where fortunes can shift within minutes, the phenomenon of intraday bubbles, specifically in small- and micro-cap stocks, represents a fascinating yet understudied frontier of price discovery and market behavior. Despite extensive research on the factors influencing price discovery and bubbles in broad market contexts, the unique characteristics of intraday bubbles in individual small stocks remain largely unexplored. Much of the bubble literature explores and seeks to explain longer-duration bubbles and deviations from intrinsic value resulting from, various processes, including limits to arbitrage, information asymmetry, self-fulfilling expectations, herd behavior, or other behavioral biases. Further, many, but not all, of these studies conduct a market-wide or index-based analysis of market bubbles.

This study aims to investigate to what extent specific characteristics of individual stocks contribute to larger magnitude intraday bubble formations, providing insight into the market microstructure characteristics and behavioral motivations of investors that may influence bubble dynamics. Despite the long history of price bubbles, this study addresses a critical recurring issue underscored by the recent influx of individual investors and whether it amplified intraday bubbles. Alternatively, both institutional and individual traders may play fairly equal roles in intraday bubble formations, a possibility that we could address empirically.

Although financial markets have evolved significantly from the early studies on market efficiency and the opposing studies on behavioral finance-driven price anomalies, many discussions are still relevant today. However, the market structure and trading dynamics have evolved such that the common themes of market liquidity, herd behavior, loss aversion, and gamification dynamics, among others, have new connotations and may have a more significant impact on price formation than in years past.

While this study does not directly analyze meme stock trading, the hypothesized impact of this development on intraday price bubbles requires a brief discussion. There are various theories on why recent meme stocks experienced significant price increases over the past two years, one of which is the rise of retail investors and their tendency toward herd mentality through social media-promoted investment theses. Hsieh et al. (2020) find that the Google search volume index serves as free access to information for uninformed investors, which leads to herding amongst the individuals who rely on the same source of information. Further, Hu et al. (2020) find that positive sentiment regarding a stock on Reddit and other social media platforms has a positive relationship with intraday stock returns. Also, Dixon (2022) shows that over 36% of Reddit users are 18-29, which captures college-aged individuals and recent graduates. Wu (2022) finds that young investors often exhibit irrational behavior, while Nagel (2009) notes that inexperienced investors play a role in bubble formation.

Beyond the behavioral impact of individual investors, the microstructure environment that supports intraday dynamics may not have fully adjusted to the shocks of COVID-19 and meme stock volatility, leading to increased individual stock volatility during the sample period. Professional traders using shorting strategies in hard-to-borrow stocks were forced to cover short positions at substantial losses. Likewise, brokerage firms were exposed to significant overnight risks from customer accounts with significant margin positions.

Finally, regulators faced unprecedented intraday volatility in some stocks that resulted in a considerable number of market-wide circuit breakers and individual LULD halts. These circuit breakers and LULD halts were designed to dampen volatility during extreme price movements, but they had minimal, if any, success in limiting intraday volatility during this tumultuous period in financial markets. These issues are documented thoroughly (Chiu & Yahya, 2022; Kelleher & Cisewski, 2021; SEC et al., 2021), so they are only mentioned here briefly to acknowledge that this study is motivated by individual trading behavior as well as the market microstructure that bridges investor demand and price discovery.

Research Question

The second study of this two-paper dissertation examined the influence of multiple variables (independent and control) on small- and micro-cap stock price bubble characteristics and their prevalence before and following significant financial market events. The analysis builds upon study one by analyzing the intraday bubbles identified during this dissertation's initial data analysis using the previously described bubble detection tests. Given the recent changes in the intraday investor population and profile, understanding both who trades and how they are motivated are crucial elements for regulators to consider when addressing market inefficiencies in future rule proposals and market structure updates. As such, we seek to answer the following question: To what extent do liquidity, short interest, retail trader participation, stock price level, and market capitalization influence the size of intraday bubbles?

Literature Review

As with the bubble detection study, multiple streams of literature provide a necessary theoretical basis for the bubble size regression analysis. The literature discussed in this section will focus on two broad themes: behavioral finance and market microstructure. Both domains are mature and contain seminal work by many respected financial thought leaders. Within each general theme, the review will cover essential topics that inform the independent (as well as control) variables selected for the regressions.

Behavioral Finance

Barberis and Thaler (2002) describe behavioral finance as a field that studies how some financial phenomena can be explained using models that assume not all market participants are entirely rational. Despite the academic literature on rational bubbles, many academics and most practitioners consider stock price bubbles to be irrational market events driven by the behavioral peculiarities of individual investors. While behavioral finance is a broad field covering an array of topics, this literature discussion will focus on the most pertinent topics relevant to this study.

The first significant work on behavioral finance dates back to the early- and mid-1900s, when Selden (1912), Keynes (1936), and Festinger (1957) focused on individual psychological triggers that affect investment decisions. Selden (1912) studied an individual's mental attitude and perspective to explain short-term price fluctuations. Keynes (1936) argued that individuals' 'animal spirits' are an immutable factor that causes market prices to diverge from fundamental values. While Festinger (1957) did not have a specific focus on capital markets, his work on cognitive dissonance, the discomfort associated with conflicting beliefs or behaviors, has been referenced by many supporters of behavioral finance in the years since he first introduced the concept. However, it was in the late 1970s and throughout the 1980s and 1990s when behavioral finance emerged as a primary field of study focused on explaining the market anomalies that the EMH seemingly failed to explain (Kahneman & Tversky, 1979, 1981; Thaler, 1980; Shefrin & Statman, 1985). The EMH contrarian academics often demonstrated behavioral finance theory through experiments and the development of concepts such as cognitive dissonance, prospect theory, bounded rationality, herd behavior, and the fear of missing out (FoMo).

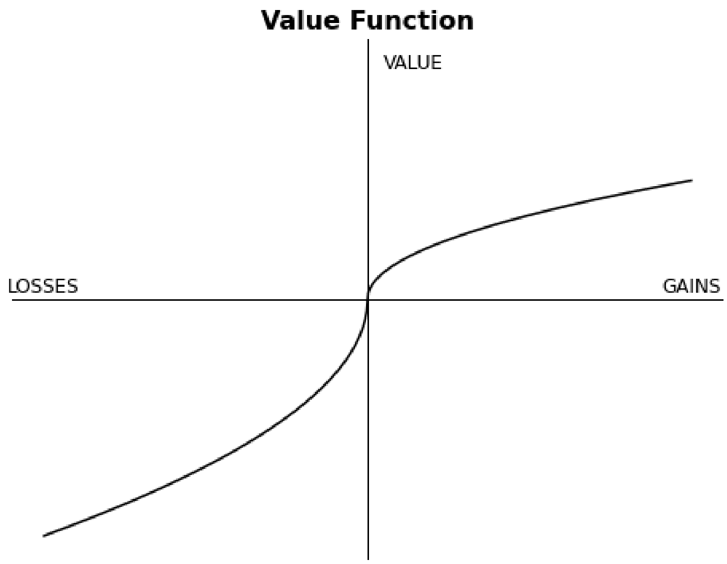
Cognitive dissonance was introduced by Festinger (1957), who described it as the discomfort associated with conflicting beliefs or behaviors. Although Festinger (1957) did not have a specific focus on capital markets, it has been referenced in many academic studies on the behavioral aspect of financial decision-making. During the run-up of stock prices towards a bubble, investors may experience cognitive dissonance in their beliefs that a stock price will continue to rise despite contradictory evidence of its fundamental value. Olsen (2008) argues that investors resist changes to investment decisions when news or results contradict their original expectations, leading to overconfidence as they seek information that confirms their beliefs.

Related to cognitive dissonance, Shefrin and Statman (1985) introduced the idea that investors sell winning investments too soon and hold losing investments too long, a disposition effect. Odean (1998) analyzed trading records from a discount brokerage and found that the disposition effect can have a material impact on stock prices and economic significance, particularly for retail investors. This finding is relevant today and for this study, given the recent increase of retail participation in the stock market and the likelihood that an investor will continue to hold a stock purchased during a bubble even after it bursts. Further, if there is a significant retail presence in a stock with a low public float, the lack of sufficient sellers could lead to liquidity issues and a prolonged bubble period – inviting new purchasers (herding) into overpriced stocks.

Prospect theory, the idea that individuals make decisions in the presence of risk through subjective evaluations of gains and losses instead of based on rational probabilities, can help provide insight into the cognitive errors made by investors leading to or influencing stock price bubbles. In their seminal paper, Kahneman and Tversky (1979) show that individuals have a propensity to overweight small probabilities and are risk averse when faced with a risk decision resulting in gains and risk-seeking when faced with a risk decision resulting in losses. They explain this behavior based on an individual's tendency to use reference points to assess value, illustrated in a value function curve, shown in Figure 10.

Figure 10

Gains and Loses Value Function



Similarly, Thaler (1980) argued that traditional economic theory is normative in that it explains how consumers should make decisions, suggesting it fails to account for how consumers do make decisions. Thaler (1980) builds on prospect theory's ideas by providing examples of bounded rationality in decision-making. Bounded rationality is the idea that individuals tend to make good enough, rather than optimal, decisions (Simon, 1955). Further, Thaler (1980) introduces the endowment effect to describe the tendency of individuals to underweight opportunity costs (foregone gains) and overweight out-of-pocket costs (losses). This idea is referred to as the endowment effect given an individual's tendency to place a higher value on an object endowed than they would on the same object in the open market if they did not own it. This endowment effect may result in uninformed investors holding stocks well after a bubble begins to burst, resulting in significant financial loss. Thaler argues that it is unreasonable for economists to expect average consumers to make programmatic rational decisions, as assumed in many pricing models. The idea of individuals' bounded rationality is linked to an alternative to the efficient market hypothesis: the adaptive market hypothesis (AMH). Lo (2004) introduced the AMH conceptual model to explain the EMH shortcomings by linking traditional economic models with individuals' use of heuristics and satisficing to adapt to changing environments.

These concepts and individuals' tendency to overweight low probability outcomes are highly relevant as this study's subject companies are often highly speculative with above average volatility, so-called lottery stocks. Several recent studies have shown that retail and young (retail and professional) investors are attracted to these lottery stocks, which aligns with the principles of prospect theory (Bali et al., 2021; Du et al., 2022; Greenwood & Nagel, 2009; Zhu et al., 2021). As discussed widely in popular media, an unprecedented number of individuals have opened brokerage accounts and managed their own investments since 2020 (World Economic Forum, 2022). According to a Motley Fool survey (Caporal, 2021), young people and inexperienced investors account for many of these new brokerage accounts. Therefore, as new investors pile into lottery stocks, which may have limited liquidity, they can generate momentum rapidly, pushing up stock prices and creating a herd-like effect, potentially leading to a bubble.

Herd behavior is another important concept in behavioral finance that can provide valuable insight into stock price bubbles. According to Mackay (1841/2020), "Men, it has been well said, think in herds; it will be seen that they go mad in herds, while they only recover their senses slowly, and one by one" (p. 3). Avery and Zemsky (1998) show that informationally complex market structures with an unknown mix of informed and noisy traders, herd behavior is likely and short-term price bubbles can occur. The rise in retail investors (often noise traders) and the increased use of options and social media to influence investment decisions has arguably increased the multidimensionality of stock prices.

Studies on the meme stock trading frenzy of 2021, headlined by GameStop, support increased herding in stocks during this period, which may have been partially influenced by a FoMo among new investors. Dim (2021) suggests that social media investment analysts (SMA's) can occasionally provide alpha based on their beliefs about future stock prices, and by sharing information freely on sites such as Reddit and Seeking Alpha, herding can occur. However, Bradley et al. (2021) shows that while there was an increase in usage of investment advice from social media post-GameStop, the informativeness of the social media advice declined versus pre-GameStop. The emerging trend of social media-based investment advice may lead large groups of new and uninformed traders into speculative and highly volatile stocks.

Some academics point to a fear of missing out, or FoMo, as a behavioral characteristic that can psychologically influence human decisions. Przybylski et al. (2013) define FoMo as an individual's apprehension that others may be having rewarding experiences from which one is absent. While it is not domain-specific, FoMo's impact on investment decisions has been studied and is increasingly a consideration of behavioral finance. Recently, conceptual and survey-based studies (Argan et al., 2022; Clor et al., 2020; Gupta & Shrivastava, 2021) have shown that FoMo has a direct positive effect on loss aversion and herd behavior in investment decisions.

Given the recent increase in retail participation in equity markets, this discussion highlights why studies on behavioral finance are just as critical today as ever. These studies and the concepts mentioned previously, among others, help provide a better understanding of trading motivations, strategies, and decisions that collectively impact prices and potentially contribute to price bubbles.

Market Microstructure

Understanding the behavioral tendencies of investors can shed some light on the characteristics of short-natured stock bubbles, but it is equally as critical to consider how these behavioral motivations manifest into market prices – broadly described as market microstructure. There are various definitions, which usually describe market microstructure as the study of how financial markets operate, the structure of organized trading, and how assets are priced within the existing trading rules and market mechanisms (Brummer & March, 2013; Kissell, 2013; Vega & Miller, 2009). Madhavan (2002) separates market microstructure into four subtopics: (1) price formation and price discovery, (2) market structure and market design, (3) information transparency, and (4) corporate finance asset pricing. Madhavan (2002) argues that these four subtopics explain how market prices reflect investors' latent demands.

One critical aspect of understanding how prices fluctuate throughout the day and can potentially deviate from fundamental values is recognition of the various types of traders, why they trade, and their price impact. Harris (2015) notes that traders consist of agency and proprietary traders, where agency traders arrange trades for others and proprietary traders choose when, what, and how much they want to trade. The motivation of proprietary traders can vary – trades may be liquidity-related, speculative, arbitrage-based, manipulative, or simply a mechanism to move cash from the present to the future. While there are many order types, most generally can be categorized as either a market order, which demands liquidity, or a limit order, which supplies liquidity. Many market orders, or a single large market order, may 'walk the book' in stocks with minimal limit order depth, causing rapid price increases (or declines). While tracing buy and sell orders and order types to their origin is difficult, intuition suggests that retail investors may rely more on market orders than limit orders. This may be partly due to their simplicity, and this intuition is supported by the competitive nature of purchasing order flow. The many nuances and debates on payment for order flow are outside the scope of this proposal, but it is introduced here to acknowledge that there is significant demand for retail market orders, which can lead to quick profit opportunities for wholesalers and market making firms.

The preceding discussion on order flow and order types introduces the importance of liquidity in maintaining fair and orderly markets. In a market microstructure context, liquidity refers to the ease of trading without significantly influencing prices up or down. Madhavan (2000) notes that stock liquidity and how quickly prices adjust when new orders enter a market are impacted by various factors, including market size, order depth, and volume. If stocks have a relatively small number of shares in the public float, which excludes insiders and significant shareholders, large orders in either direction can quickly remove liquidity from the order book. These dynamics create feedback loops when there is considerable short interest in a stock, as there will be very few shares to borrow in the event of a margin call, leading to open market purchases and continued upward pressure on the price of a stock. The topic of liquidity leads to another critical market microstructure consideration within this study – limits to arbitrage.

Schleifer and Vishny (1997) discuss falsehoods in many pricing models that assume arbitrageurs will keep prices from becoming overly disconnected from fundamental values. An important reason they note as a limit to using arbitrage to main efficient markets is that arbitrage traders are concentrated to a minority number of sophisticated traders. Also, they acknowledge that arbitrage trading is a risky strategy that does not guarantee a positive payoff. Noise traders, duration of price anomalies, costs to borrow shares, capital requirements, and regulatory constraints can all limit the effectiveness of arbitrage. In a similar paper, De Long et al. (1990) developed a theoretical model that portrays noise traders as unsophisticated and unpredictable, which results in the distortion of prices. In contrast to the principles of the EMH, the study suggests that the risk-averse nature of arbitrageurs prevents those traders from taking a large enough position against noise traders to stabilize prices and can potentially allow noise traders to earn excess returns. Both papers are relevant to this study and highlight how aspects of market microstructure can limit arbitrage opportunities and price corrections, at least in the near term.

While not an exhaustive list, other notable papers discussing market microstructure concepts related to this study include options trading activity, high-frequency trading, and regulatory frameworks. Easley and O'Hara (1998) conducted an empirical study using intraday options data to conclude that informed traders are more likely to trade in options when information asymmetry is present in markets. However, the recent rise of the retail investor (often labeled as noise and uninformed traders) has led some studies (Aramonte & Avalos, 2021; Li et al., 2021) to suggest that they prefer options linked to market sentiment due to their lottery-like attributes.

O'Hara (2015) focuses on high-frequency trading (HFT) and the strategies HFT firms employ to profit from the price formation mechanics within a market design. O'Hara notes that in a world of HFT, post-trade details (e.g., volume, price, venue, trade time) are not as valuable data as the order information itself, pre-trade. Many HFT strategies are executed via algorithms or computer code, which are more concerned with profit opportunity than fundamental price equilibrium.

As is common in financial regulation, new rules and updated market (micro)structures are reactionary to destabilizing market events, such as the GameStop short-squeeze. By examining these microstructure elements in the context of small- and micro-cap bubbles, this study aims to identify potential triggers or amplifiers of intraday bubble formation. In doing so, one desired outcome of is to draw attention to these microstructure triggers and encourage more proactive surveillance of their interplay with intraday bubbles.

Research Design and Approach

Recognizing the occurrence of stock price bubbles as an event distinct from general market noise or volatility is an important, albeit challenging, endeavor that was covered in the first study. This second study of the dissertation employs regression analysis to evaluate the impact of liquidity, short interest, retail participation, price level, and company size on the magnitude of intraday bubble formation. As such, this study aims to enrich the literature on stock price bubbles with new insights into the characteristics and factors driving intraday bubble formation in individual small- and micro-cap stocks. Additionally, a deeper understanding of the influence of the proposed variables will provide market participants, especially regulators, with valuable data to enhance rules and regulations that support price discovery and market efficiency within the framework of fair and orderly markets.

This study seeks to provide new insights into the single-stock price gyrations that are often disregarded as one-off anomalies that do not threaten market-wide stability. As has been discussed, the changing landscape of U.S. equity markets makes any argument that downplays the economic risk of idiosyncratic microcap stock price bubbles increasingly difficult to defend. Instead, we argue that these seemingly one-off price bubbles are more prevalent and economically more significant than most market participants believe. The purpose was not to understand every determinable factor influencing short-natured stock price bubbles or price inefficiency in every publicly traded stock. Instead, I looked to analyze intraday bubble formations in microcap stocks through regression analysis over a purposefully selected 5-year period from 2018-2022 through the influence of five factors: liquidity, short interest, stock price level, retail trader sentiment, and market capitalization.

Like study one, this study intends to fill a gap by analyzing a unique type of stock price bubble in the types of companies that have been mostly ignored. The preliminary data analysis conducted for this dissertation supports the need for closer surveillance of microcap stocks. As noted, the analysis showed that the average daily trading volume of constituents in the Russell Microcap Index was almost three times greater in Q3 2021 than in Q3 2018, having grown from approximately 260,000 to 680,000 shares with a max volume nearly five times larger (approximately 120 million in Q3 2018 and 510 million Q3 2021). As the trading volume in these stocks grows, the risk to market participants, big and small, along with general market stability, will also increase. I hope this study highlights that intraday bubbles and microcap price volatility do not begin and end with GameStop and the handful of other meme stocks.

Hypothesis Development

The hypotheses for this study relate to behavioral finance concepts and the interplay of market microstructure mechanics, together influencing the shape and structure of intraday bubbles in small- and micro-cap stocks. The hypotheses are tested using regression analysis for the specific bubbles selected based on the 98.8th upside percentile and 90th percentile downside criteria. The DV, bubble size, is measured based on the OpenHigh variable.

The IVs include liquidity measures (e.g., float, average trading volume), short interest, retail sentiment, price level, and market capitalization. A better understanding of these variables' impact on intraday bubbles in small- and micro-cap stocks can help market participants manage risk and help regulators develop new or modified guardrails to enhance market efficiency.

Although not formally hypothesized in this section, this study will also analyze an additional factor of interest in the dissertation: assessing the prevalence and trend of these price anomalies across important market events. The period selected provides three distinct sub-sample periods for comparison: October 2019 through March 2020 (the move to commission-free trading across most major brokerage platforms), March 2020 through January 2021 (the start of the COVID-19 pandemic lockdowns), and January 2021 through December 2022 (the GameStop short-squeeze trading frenzy). One additional sub-sample (i.e., January 2018 through September 2019) sets a baseline period before the three significant events.

This analysis aims to provide a better understanding of how these significant market events impacted price efficiency and intraday trading patterns in small- and micro-cap companies. By examining the effects of the independent variables, the study seeks to provide regulators with new insight into potential manipulative investment strategies during this period. Moreover, it may help lead to new or amended rules, either at the SRO or Federal level, which support the ongoing objective of maintaining fair and orderly markets. The following hypotheses were developed to systematically investigate these relationships and their implications for intraday bubble size.

H1: Liquidity is negatively associated with the size of intraday price bubbles

This hypothesis assesses the relationship between the DV, bubble size or magnitude, and the IV, liquidity. While there are numerous measures of liquidity to consider, as suggested by the microstructure literature, this hypothesis is measured using public float and average volume as a percentage of total shares outstanding (TSO). The expectation is that when an intraday price bubble occurs, lower liquidity (indicated by a smaller public float or lower average volume as a percentage of TSO) will induce more upward pressure on intraday bubble formation. Public float refers to the shares of a stock that are freely tradeable and not subject to lock-ups or held by insiders or major holders (10% or more). Although not proposed as a predictor variable for this hypothesis, the average effective spread is added as a control in some of the regression model specifications discussed in the results section. The expectation is that a lower availability of tradeable shares will increase volatility and upward price pressure, especially in conjunction with a large short interest in a stock.

H2: Short interest as a percentage of public float is positively associated with the size of intraday price bubbles

The selection of this hypothesis stems from anecdotal observations of social media message boards following LULD circuit breaker trading halts in individual stocks following the initial GameStop short squeeze. In alignment with Tuckett and Taffler (2008), the belief is that GameStop-led retail traders often search for the next phantastic object, which they describe as an imagined mental representation of something that fulfills an immediate desire. It was widely reported that GameStop and AMC both had large short positions in their stocks when their stock prices took off in 2021, which some characterized as a short squeeze. Traditionally, absent a short squeeze, highly shorted stocks may underperform markets given the view that their stocks are overvalued. Therefore, the relationship between short interest and the size of price bubbles may conflict before and after the GameStop event.

H3: Retail trader sentiment is positively associated with the size of intraday price bubbles

This hypothesis examines the impact of retail trader participation on intraday bubble formation. Media speculation and research suggest that retail traders were a primary driving force behind the meme stock price anomalies in 2021. The expectation is that a higher proportion of volume, specifically buy interest from retail traders, will positively influence the size of a price bubble. Retail trader sentiment will be measured in two ways:

- Retail Sentiment (retail_sentiment): Retail Sentiment (retail_sentiment): The percentage change between the 30-day average ratio of retail buy volume to total volume and the 5-day average ratio of retail buy volume to total volume for the five days prior to the observation date and the 30 days prior to these five days. This measurement compares the 5-day average ratio with the 30-day average ratio prior to those five days, with an increase for the 30-day to 5-day average suggesting stronger retail buy sentiment.
- 5-Day Sentiment (5day_sentiment): The 5-day average ratio of retail volume to total volume.
- Retail volume to total volume (1-day lag): Retail volume proportion to total volume on day prior to observation date.

Retail investor participation data is sourced from the WRDS intraday indicators dataset, utilizing the Boehmer et al. (2021) methodology, which identifies retail order flow based on sub-penny price improvement in marketable orders, consistent with the mechanics of payment for order flow that often occurs with retail order flow. Boehmer et al. (2021) also found that positive marketable retail order flow is associated with higher stock returns in the subsequent week, supporting this hypothesis.

H4: Opening stock price is negatively associated with the size of intraday price bubbles

This hypothesis tests the impact of stock price on the characteristics of bubble formation, with an expectation of a negative relationship. This reasoning is based on the growth in retail investors and their interest in low-priced, highly volatile stocks, which they often treat as lottery-like gambles. Studies by Frino et al. (2019) and Bali et al. (2021) suggest that this behavior can significantly impact retail investor investment decisions. The stock price will be measured based on the opening price on the observation date.

H5: Market capitalization is negatively associated with the size of intraday price bubbles

This hypothesis examines the relationship between company size, as measured by market capitalization, and the size of intraday price bubbles, with an expectation of a negative association. The primary focus is on small- and micro-cap stocks, based on anecdotal practitioner observations of unusual intraday price fluctuations in these companies. This intuition is supported by LULD data presented earlier in this paper, which shows that small stocks experience significantly more volatility trading pauses than their larger counterparts.

Data Collection Methods and Instruments

As with the first study, the regression analysis utilizes the observation set selected from the Russell Microcap Index, based on the bubble criteria previously noted. These observations represent events where the intraday high price was at a minimum 28% greater than the opening price and the closing price was at least 27% less than the intraday high between 2018 and 2022. Unlike the previous study, study two utilizes data points beyond intraday pricing information.

The additional data points are based on literature in the field of market efficiency and stock price bubbles, along with recently developed theories on the behavior of markets following the COVID-19 and meme stock events that were associated with increased retail investor participation in financial market trading. The data points were obtained from various market data providers, including WRDS, Polygon.io, Refinitiv, Capital IQ, Federal Reserve Economic Data (FRED), and FINRA. Table 14 highlights the variables used in the regression along with the source of the data.

Table 14

Data Descriptions for Regression Variables

Dependent Variable	**Definition**	**Source**
OpenHigh	Percentage change from opening price and intraday high, in decimal	Manual calculation using Compustat via WRDS
Log Transformed	Log transformed open to high precentage	Manual calculation
Box-Cox Transformed	Box Cox transformed open to high percentage	Manual calculation
Predictor Variables	**Definition**	**Source**
float_refinitiv	Ratio of float shares to total shares outstanding	Refinitiv Excel Plug-In
log_float_refinitiv	Log transformed float percentage	Manual calculation
low_float_refinitiv_25th	Low float threshold variable at less than 25th percentile	Manual calculation from float_refinitiv variable
volume_tso_ratio	Ratio of retail volume (buy or sell) to total volume on the observation date	Manual calculation using WRDS intraday indicators
short_float_capiq	Ratio of shore shares to public float shares	S&P Capital IQ Excel Plug-In
log_short_float_capiq	Log transformed short interest percentage	Manual calculation
high_short_capiq_75th	High short threshold variable at greater than 75th percentile	Manual calculation from short_float_capiq variable
retail_sentiment	Percentage change from the 30-day retail buy volume to to volume ratio and 5 day buy volume to total volume (5 days prior to observation date and 30 days prior to the 5 days)	Manaul calculation, retail data obtained from WRDS
5day_sentiment	5-day average of retail buy volume to total volume prior to observation date	Manaul calculation, retail data obtained from WRDS
retail_percent_volume	Retail volume (buy and sell) ratio of total volum on the day prior to the observation date	Manaul calculation, retail data obtained from WRDS
prcod	Opening price	Obtained from Compustat - Security Daily files
low_price_dummy	Threshold variable representing low price at less than 25th percentile	Manual calculation
market_cap	Company market capitalization	Manually calculated using opening price and TSO
log_market_cap	log transformed market capitalization	Manual calculation
Control Variables	**Definition**	**Source**
sec_filing	Dummy variable representing SEC news filing +/-5 days from observation date	SEC Edgar API
pre_covid	Dummy variable representing observations occuring prior to Covid lockdowns on March 2020	Manual calculation
pre_gamestop	Dummy variable representing observations occuring prior to Meme trading in January 2021	Manual calculation
russell_microcap_return	Difference in observation date return of Russell Microcap Index and S&P 500	Manual calculation
vix	CBOE volatility index	CBOE
60_day_return	Momentum variable based on 60 day return excluding 3 days prior to observation date	Manual calculation
1_year_return	Momentum variable based on 1 year return excluding 30 days prior to observation date	Manual calculation
eps_growth	1 year growth rate in EPS	Capital IQ
cpi_rate_monthly	Monthly CPI rate	FRED
10_year_tbill		FRED
tic_volatility	Standard deviation of daily retursn for 60 days prior to observation date, excluding the 3 days prior	Manual calculation
nyse_dummy	Dummy variable representing observations that are listed on an NYSE exchange	WRDS - Computstat/Capital IQ
energy_dummy	Energy sector dummy	WRDS - Computstat/Capital IQ
financials_dummy	Financials sector dummy	WRDS - Computstat/Capital IQ
industrials_dummy	Industrials sector dummy	WRDS - Computstat/Capital IQ
information_technology_dummy	IT sector dummy	WRDS - Computstat/Capital IQ
materials_dummy	Materials sector dummy	WRDS - Computstat/Capital IQ
real_estate_dummy	Real Estate sector dummy	WRDS - Computstat/Capital IQ
utilities_dummy	Utilities dummy	WRDS - Computstat/Capital IQ
communication_services_dummy	Communication Services dummy	WRDS - Computstat/Capital IQ
consumer_discretionary_dummy	Consumer Discretionary dummy	WRDS - Computstat/Capital IQ
consumer_staples_dummy	Consumer Staples dummy	WRDS - Computstat/Capital IQ

Python, SPSS, and R were used to produce the majority of the statistical data analysis, regressions, and plot visualizations included in the results section. Linear multivariate regression and quantile regression, along with diagnostic tests to assess the assumptions underlying linear regression, are employed to test the hypotheses.

Data Analysis and Results

Exploratory Data Analysis

Basic exploratory analysis of the response variable and primary IVs was conducted prior to beginning regression testing. The DV is OpenHigh, which represents the percentage change between the opening price of the day and the high price of the day. As such, OpenHigh acts as a measure of the size of the intraday bubble. Other measures were considered, including the difference between the prior day close price and the intraday high, as well as the difference between the low price of the day and the high price of the day. While these measures have merit and may overlap with the bubbles from the OpenHigh measure, they also can introduce bubble types that are not the focus of this study, significantly expanding the scope.

Measuring a bubble from the previous day close and the current day high introduces a greater opportunity for market-moving news to explain the price movements, given that most material news announcements are issued outside of normal market hours. Additionally, pre- and post-market trading sessions are notoriously more volatile and less liquid than the primary trading session. Therefore, bubbles that occur during this period may not deflate quickly due to low liquidity until the primary trading session opens. Similarly, measuring a bubble as the difference between the low and high price of the day introduces the possibility of negative bubbles, where the open price is the high price of the day. Negative bubbles raise questions similar to positive bubbles, but these events warrant their own analysis independent of the positive bubbles we seek to explore in this study.

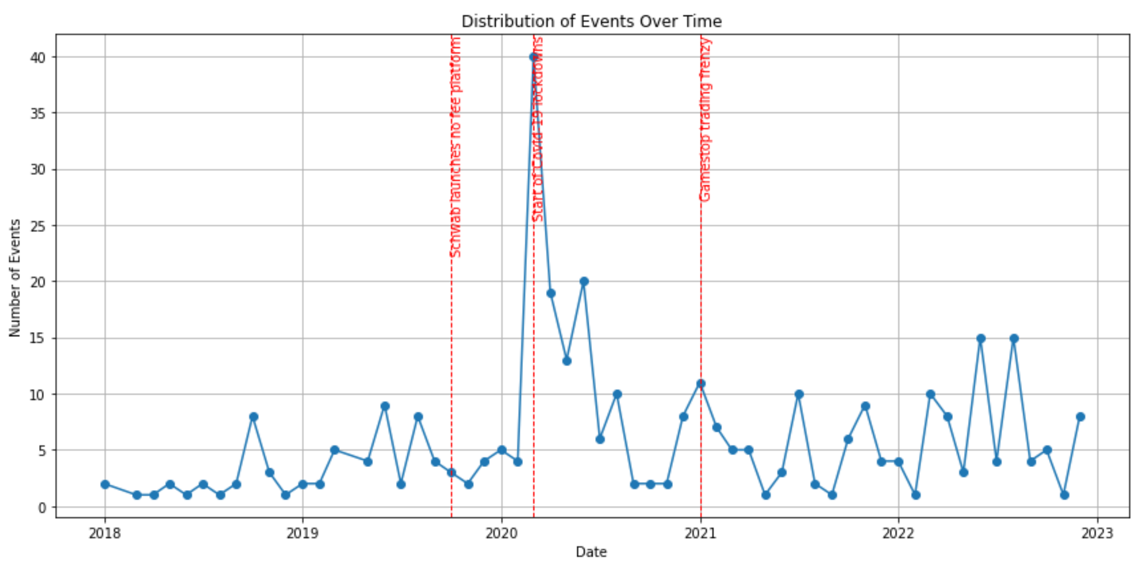
This study specifically focuses on events where a stock opens, experiences a large price increase, and subsequently undergoes a significant price decrease within a single trading day. To identify these events, the following filtering methodology was applied to the data set:

- Selected observations at the 99.8th percentile of price movements between the open and high prices.
- From this subset, filtered for observations that fell in the 90th percentile of downside movement between the high price and close price

This approach resulted in a total of 342 observations spread between January 2018 and December 2022. The filtering methodology identifies extreme events exhibiting both upside and downside price fluctuations in a single day, which this study describes as intraday bubbles. Large price rises that do not reverse within an intraday timeframe were excluded as they may have unique underlying characteristics, thus requiring separate investigation. As shown in Figure 11, there is a noticeable increase in the number of extreme events following the adoption of no fee trading platforms in late 2019. This trend is especially evident following the onset of COVID-19 lockdowns in early 2020 and the Gamestop/Meme stock trading frenzy in early 2021.

Figure 11

Distribution of Intraday Bubble Observations Over Time with Event Overlay

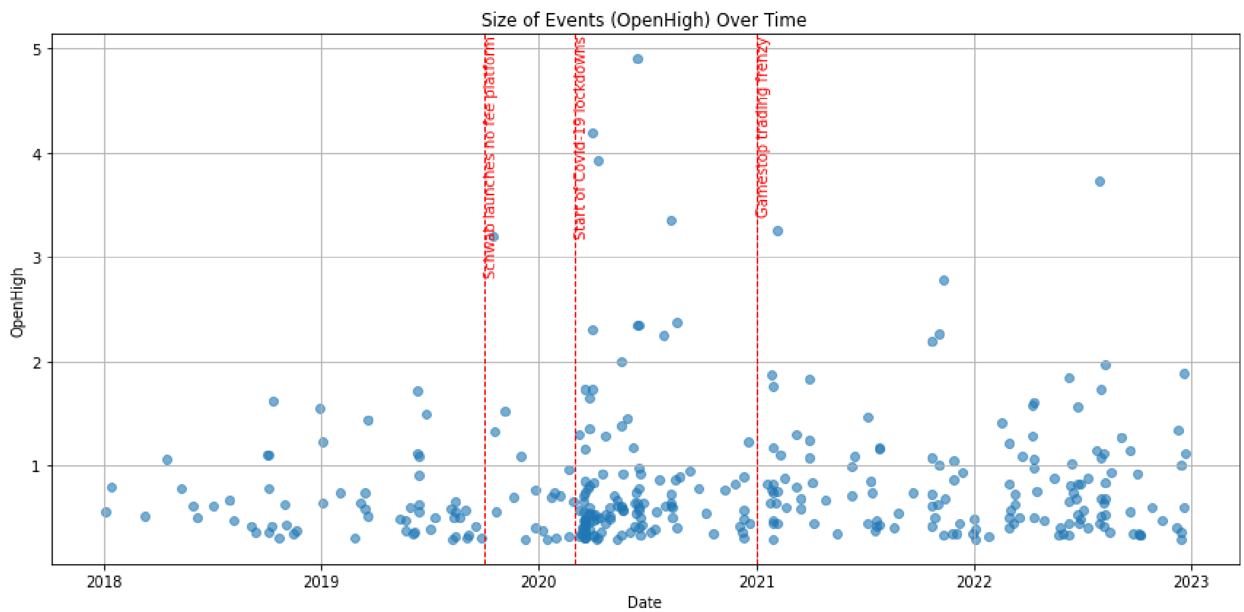


Note. This figure represents time on the x-axis and the number of bubble events on the y-axis. The figure is overlayed with three vertical red lines representing major market events: the widespread adoption of commission-free trading, the start of the COVID-19 lockdowns, and the GameStop short squeeze.

Figure 12 demonstrates a trend of more extreme intraday events following the COVID-19 lockdowns and the GameStop event. Specifically, I observe a higher frequency of events with OpenHigh values exceeding 200%, particularly in the periods following these market disruptions. These initial observations suggest that market structure changes (commission-free trading, new market participants, increased squeeze attempts) and significant events (COVID-19 and meme stocks, among others) may have influenced the frequency and magnitude of intraday bubbles. As the literature on bubbles suggests, trader behavior and bubble size are sensitive to environmental factors, such as disruptions in trading, which occurred frequently through trading halts during this period (Smith et al., 1988). Proxies for these factors are included as dummy variables in the regression analysis and discussed further in the results section.

Figure 12

Size of Intraday Bubble Observations Over Time with Event Overlay



Note. The figure represents time on the x-axis and the size of bubble events on the y-axis. The figure is overlayed with three vertical red lines representing major market events: the widespread adoption of commission-free trading, the start of the COVID-19 lockdowns, and the GameStop short squeeze.

Descriptive Statistics, Correlations, and Plots of DV and primary IVs

Prior to conducting regression analysis, descriptive statistics, correlations, and basic plot visualizations were examined to gain insights into relationships and patterns among the DV and IVs. A summary of the descriptive statistics is included in Table 15.

Table 15

Descriptive Statistics of Dependent and Independent Variables

Variable	Count	Mean	Std	Min	25%	50%	75%	Max
OpenHigh	342	80.55%	62.59%	28.84%	43.28%	61.04%	92.16%	490.84%
float_refinitiv	342	66.44%	25.87%	4.28%	51.41%	71.16%	87.21%	99.97%
volume_tso_ratio	327	6.16%	24.48%	0.01%	0.23%	0.62%	1.90%	283.15%
short_float_capiq	342	5.93%	15.85%	0.00%	0.42%	2.21%	6.03%	171.08%
retail_sentiment	322	15.00%	53.41%	-100.00%	-15.92%	8.95%	31.66%	389.71%
5day_sentiment	322	12.86%	5.81%	0.00%	8.76%	12.46%	16.69%	31.31%
prcod	342	$6.78	$19.27	$0.13	$0.81	$2.19	$4.95	$265.00
market_cap	342	$185,429,965	$1,091,220,482	$2,499,560	$20,618,021	$45,286,295	$95,486,820	$18,482,955,000

Note. The variables in this table represent the predictor variables used in the regression analysis: bubble size, float share percentage of TSO, 30-day proportion of trading volume to TSO, shares sold short as a percentage of float shares, prior 30-day change in retail buy volume to total volume, 5-day average retail buy volume to total volume, opening price, and market capitalization

The average size of the bubble is 80.55%, with a standard deviation of 62.55%, indicating considerable variability in bubble magnitude. This aligns with the episodic nature of intraday price movements in individual stocks. The largest bubble observation is over 490%, whereas the smallest bubble was 28.8%, which corresponds to the 99.8th percentile criteria used for the sample selection.

The float metric suggests that, on average, around two-thirds of a company's TSO are freely tradeable, ranging from as little as 5% to nearly 100%, but it is notable that the 25th percentile float value is 51%, suggesting that in this sample of events low float may not play a large role on average. The prior 30-day average volume as a percentage of TSO is only 6%, suggesting low average trading relative to total shares. This contrasts with the observation date, where the average volume as a percentage of TSO is 191%. Notably, the maximum of 283% suggests the presence of significant outliers.

The retail sentiment metric has a mean of 15%, with fairly significant variability suggested by the standard deviation of 53%. This implies that the average percent change in retail buy volume to total volume ratio in the five days pre-bubble event compared to 30 days pre-bubble is 15%, consistent with greater retail participation in a stock leading up to the bubble, but there were also substantial declines in retail participation, as the 25th percentile of this variables is -15.92%. The five-day sentiment measure indicates that retail buy volume is approximately 13% of the total volume preceding the bubble event. Thus, we have two different versions of retail participation, with prior 30-day change and prior 5-day average level, and both show substantial variability.

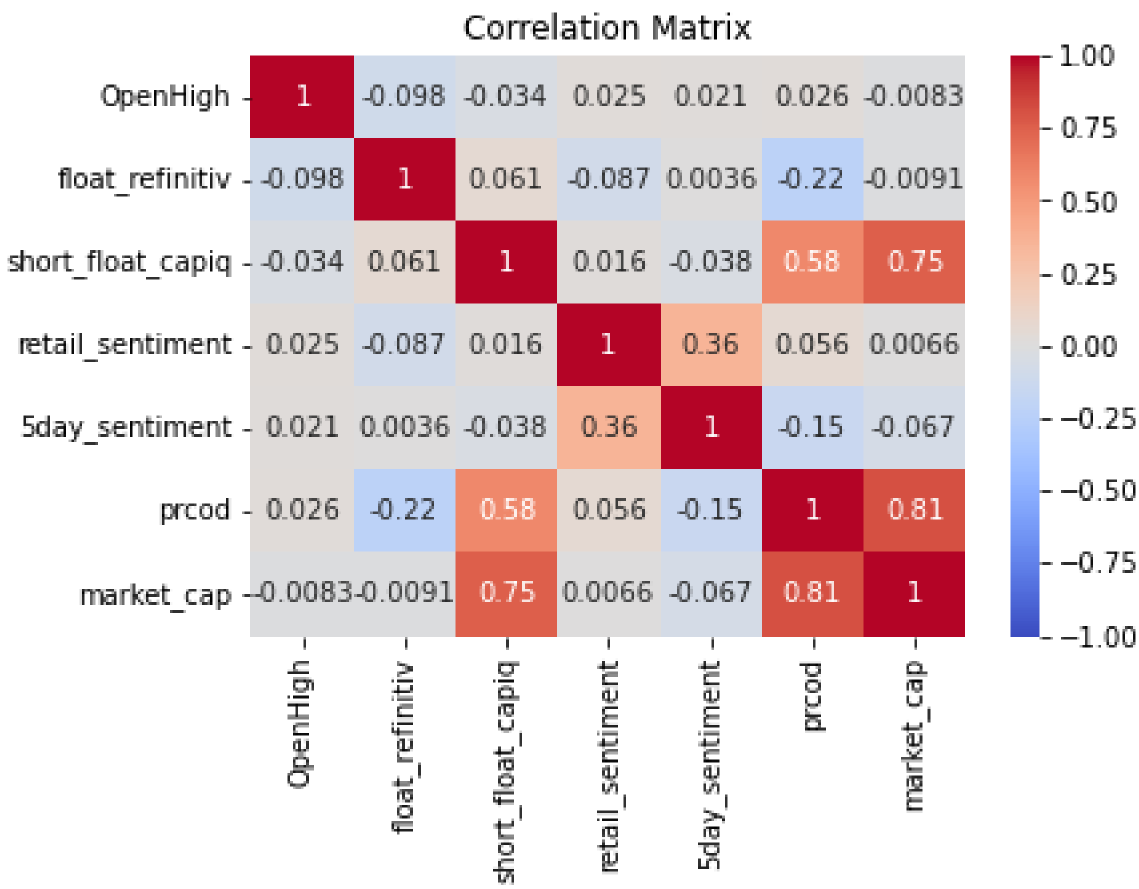
The average opening stock price is $6.78, but the large standard deviation is indicative of extreme outliers. Specifically, three observations had opening prices close to or greater than $100. Excluding these three outliers reduces the average to approximately $5, with a substantial reduction in standard deviation from $19.27 to under $10. Thus, suggesting the transformation of the variable to reflect its potential nonlinear influence, either into a dummy variable or logarithmic measure. Likewise, the mean and median of market cap are influenced by two outlier observations, both of which are GameStop.

The descriptive statistics reveal several key considerations for the regression analysis. The potential nonlinearity of the relationship between the DV (OpenHigh) and the IVs suggests the possible need for transformations to apply linear regression analysis and to meet the normality assumption of regression residuals. Multicollinearity, particularly between similar measures like retail sentiment and 5-day sentiment, could distort the regression coefficients and will be checked using variance inflation factors (VIFs). The significant nonlinear variability in market capitalization and intraday stock returns could lead to possible heteroscedasticity, which could invalidate standard errors and necessitate diagnostic tests and the potential use of robust standard errors. Finally, the presence of outliers in variables such as short interest and market capitalization suggests the need for robust regression techniques or transformations to mitigate their nonlinear influence.

Analysis of the correlation matrix (see Figure 13), which includes the primary variables included in the subsequent regressions, revealed several key insights. The DV (OpenHigh) shows weak correlations with all IVs, suggesting that bubble size may be influenced by nonlinear relationships or by factors not captured in the predictor variables. Market capitalization and opening price level are highly correlated (0.81). The short interest variable moderately correlates with both opening price (0.58) and market cap (0.75). This may indicate that larger, higher-priced companies are more likely to be overvalued, hence a larger short interest. The two retail participation variables correlate moderately with each other but weakly with the other variables. Given their moderate correlation, they are considered together and independently in the regression specifications. These observations underscore the need to consider non-linear relationships, interaction effects, and other critical factors that are not captured by the initial versions of the predictor variables.

Figure 13

Correlation Matrix of Primary Regression Variables



Histograms and scatterplots were reviewed to understand the distribution of the observations for each variable as well as the IVs' relationship with the DVs. The histograms and scatterplots are included in Figure 14 and Figure 15, respectively. As the descriptive statistics suggested, the histograms reveal some nonnormality with significant outliers in most variables, other than both retail sentiment measures, which appear normally distributed. The scatterplots do not indicate strong linear relationships between the IVs and the DV. Given the non-normal distributions and the lack of clear linear relationships, log and exponential transformations of the variables will be addressed in the regression results section when discussing residual diagnostics.

Figure 14

Histograms for Primary Regression Variables

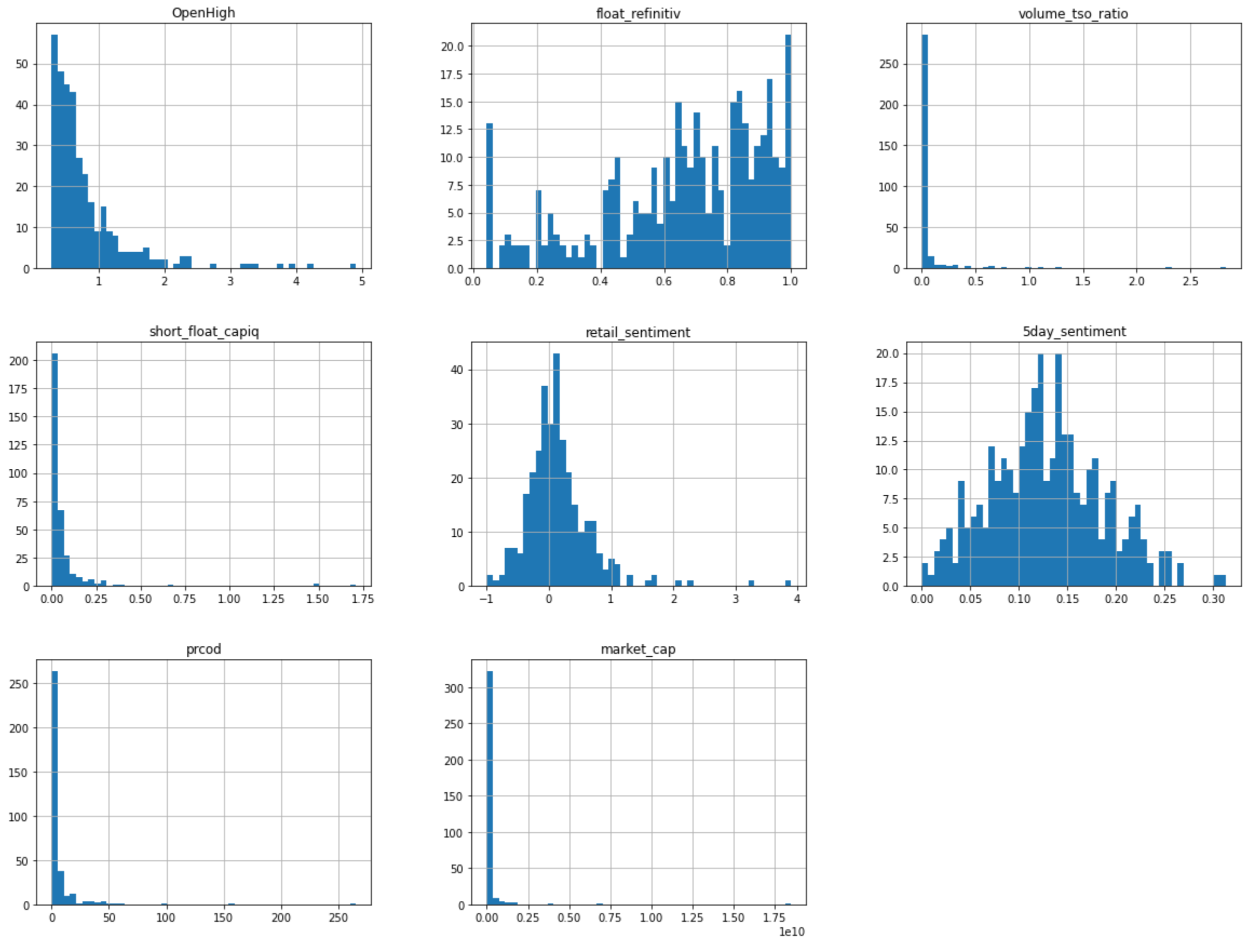
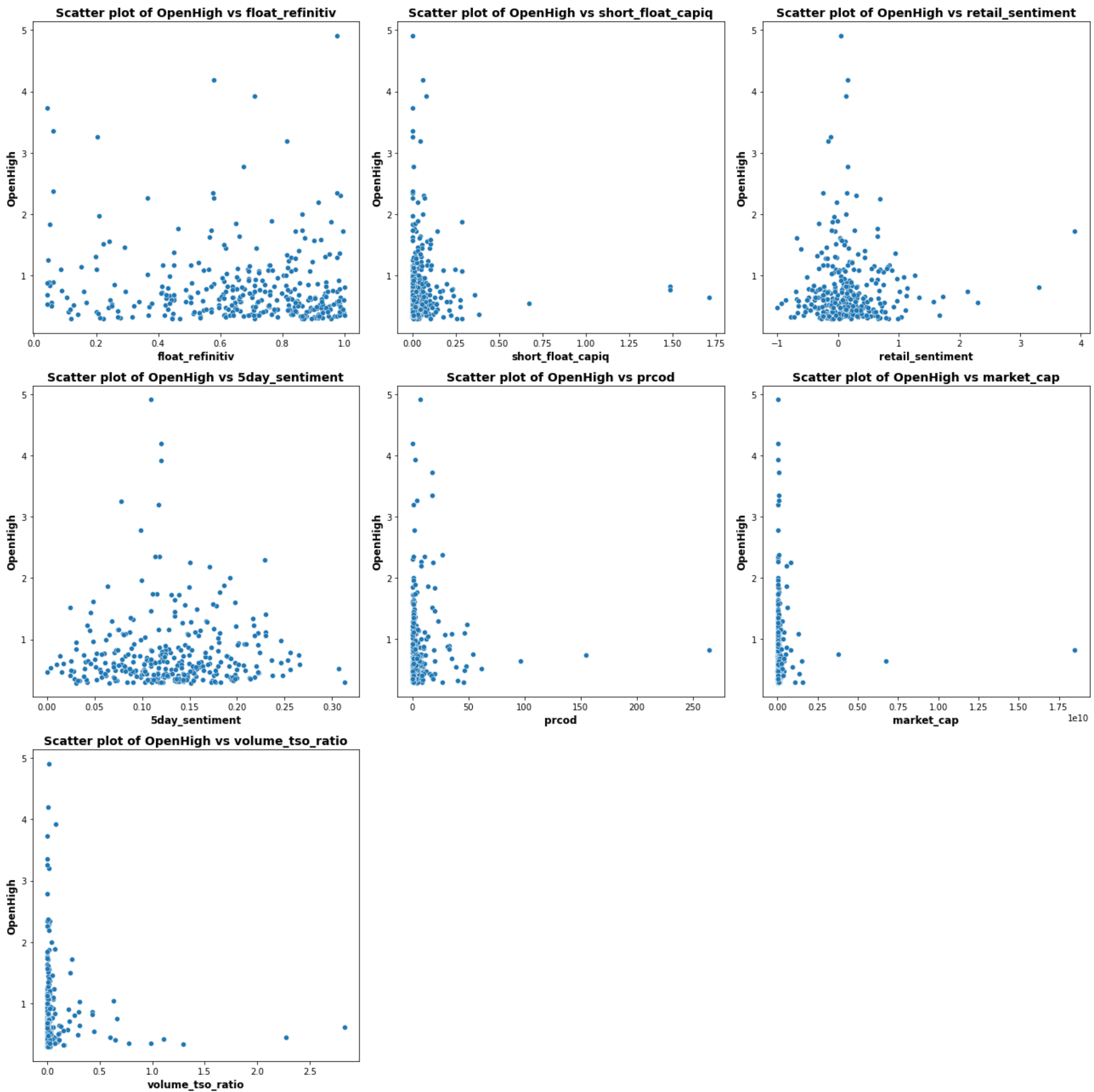


Figure 15

Scatterplots for Primary Regression Variables



The exploratory data analysis yielded crucial insights into the relationships between the predictor variables and response variable, helping inform the subsequent regression analysis. Key findings include:

- Descriptive statistics revealed considerable variability in key variables, particularly in the bubble size DV, with a mean of 80.55% and a standard deviation of 62.55%. Additionally, most variables have non-normal distributions with outliers.
- The correlation matrix identified moderate correlations between market cap, price level, and short interest. While this suggested potential multicollinearity, the transformation of price from a continuous to a dummy variable minimizes the concern. OpenHigh showed weak correlations with all IVs, possibly indicating non-linear relationships or missing variables.
- Histograms and scatterplots illustrated the non-normality and the lack of strong linear relationships between the IVs and the DV, underscoring the need to closely assess residual diagnostics in the regression analysis.

The exploratory findings helped inform the regression analysis and discussion, guiding the choice in variable transformation, model specification, and diagnostic testing. It reinforces the complex nature of intraday bubble events and the need for a nuanced analytical approach.

Regression Results

The hypotheses and variables utilized in the regression analyses were based on a review of the literature covering bubbles, market efficiency, behavioral finance, and market microstructure. Table 16 summarizes the predictor variables utilized as the foundation of the regression analyses prior to including control variables, transformations, and interactions.

Table 16

Predictor Variable Definitions

Coded Variable Name	Descriptive Name	Description	Measurement Scale
float_refinitiv	Float	Publicly tradeable shares	Percentage of total shares outstanding
volume_tso_ratio	Volume	30-day average volume as a	Percentage of TSO, excluding observation date and two prior days
short_float_capiq	Short Interest	Shorted shares	Percentage of public float
retail_sentiment	Retail Sentiment	Change in retail trading volume	Difference in retail buy volume as a percentage of total volume between 35 days and 5 days prior to observation date
5day_sentiment	5-day Sentiment	Recent retail buying activity	Average retail buy volume to total volume in 5 days prior to observation date
market_cap	Market Cap	Company market capitalization	Based on opening price on event date
prcod	Price Level	Stock Price	Opening price on event date

The predictor variables were selected to test the hypotheses regarding the size of the intraday bubbles in our dataset of small- and micro-cap stocks. Each variable corresponds to an aspect of the literature and anecdotal practitioner observations of intraday trading activity. Float and volume serve as proxies for liquidity, which are expected to be negatively associated with bubble size (H1). Short interest is hypothesized to have a positive relationship with bubble size due to potential short squeezes and retail enthusiasm for highly shorted stocks following GameStop (H2). Likewise, the retail measures are expected to have a positive relationship with the size of the intraday bubbles (H3). Market capitalization and opening price are hypothesized to have a negative relationship with bubble size (H4 and H5). The variables will collectively provide insight into these intraday events before and after significant market events, such as industry-wide commission-free trading, COVID-19, and meme stock fervor.

Multivariate Regression Specification Results

Given the complex nature and temporal dynamics of intraday trading in individual stocks, the discussion begins directly with multivariate regression rather than assessment of bivariate regressions beyond the discussion in the exploratory analysis. Over 20 different specifications were tested and progressively refined to address issues of residual normality, fit, and theoretical considerations. This section will discuss key specifications, illustrating model fit and violations of regression assumptions.

Specification 1 consists of the normalized DV and IVs. The model fit is poor, and the IV coefficients are not significant. Further, as expected, there is moderate nonnormality in the distribution of the residuals when using the normalized variables in the model, suggesting the need to transform the variables to improve the normality of residuals. Table 17 shows the model fit and ANOVA coefficients.

Table 17

Regression Output for Specification 1

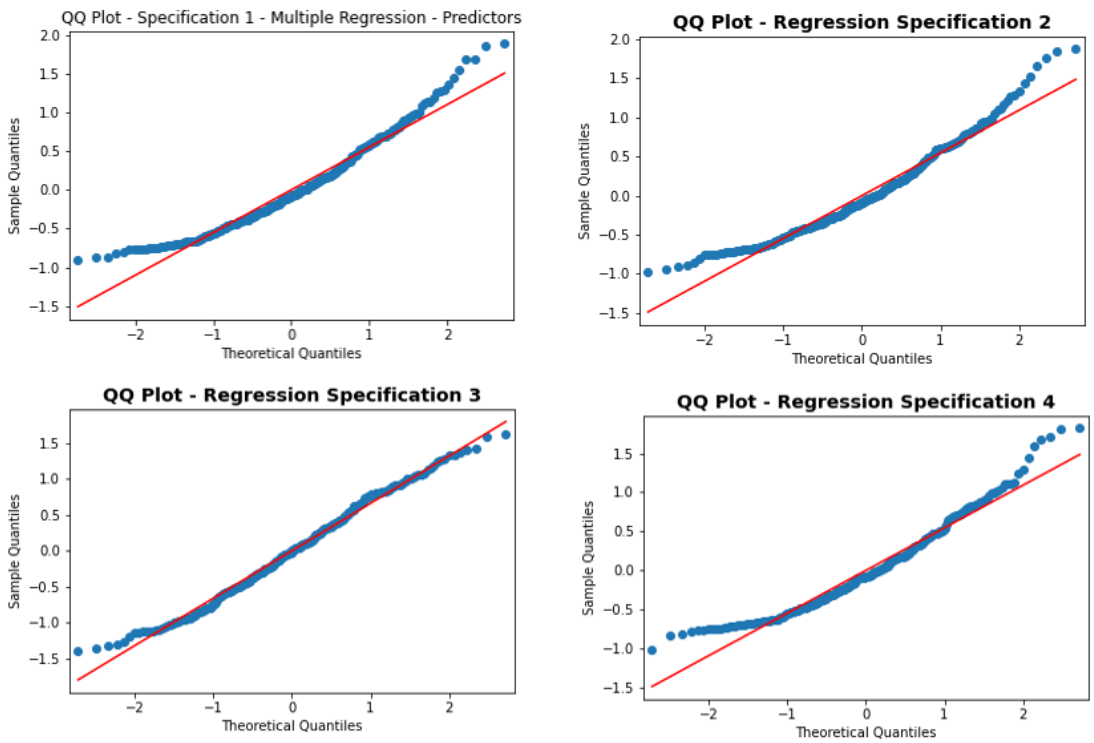
Specification	***1***
Dependent Variable	**OpenHigh**
Constant	0.682
Float Percentage of TSO	0.09
Volume to TSO Ratio, 30-day average	-0.163
Short Interest Percentage of Float	-0.049
Retail Buy Sentiment, 30-day change	0.013
Retail Buy Sentiment, 5-day average	0.3
Opening Price	0.001
Market Capitalization	0
Number of Observations	308
R^2	0.0067
Adjusted R^2	-0.0165
F-Statistic	0.290 (0.958)

Note: Values for the variables represent coefficients using standard errors. The coefficients are not significant but would otherwise be marked by *** p<0.01, ** p<0.05, * p<0.1.

Specification 2 uses the log-transformed version of OpenHigh and specification 3 uses a Box-Cox transformation. Additionally, in both specifications 2 and 3, the nonnormally distributed IVs (float, volume, short interest, price level, and market cap) were log-transformed. The DV in the fourth specification is log-transformed OpenHigh, and the float, short, and price level continuous variables were replaced with threshold dummy variables. The low float threshold is set to below the 25th percentile (51%), the high short interest is set to greater than the 75th percentile (6%), and the low price is set to below the 25th percentile ($5). The QQ-Plots in Figure 16 show that the residuals were normally distributed, with some deviations in the tails when using the log transformation, while the Box-Cox transformation indicates a good fit.

Figure 16

QQ Plots for Specifications 1-4



Heteroscedasticity is not a concern in these initial models, as supported by visual inspection of the residual vs. fitted plots and results of the Breusch-Pagan test, as seen in Table 18. Likewise, multicollinearity is not an issue in these models as the VIFs in Table 19 are all below five, which is commonly cited as the threshold for potential concern.

Table 18

Breusch-Pagan and Lagrange Multiplier Tests for Heteroscedasticity, Specifications 1-4

	LM Statistic	LM P-value	F-Statistic	F-Test p-value
Specification 1	1.110	0.993	0.155	0.993
Specification 2	4.337	0.740	0.612	0.746
Specification 3	2.154	0.951	0.302	0.953
Specification 4	8.006	0.332	1.144	0.336

Table 19

VIFs of Specifications 1-4

Specification 1	
const	16.992
float_refinitiv	1.047
volume_tso_ratio	1.060
short_float_capiq	2.456
retail_sentiment	1.201
5day_sentiment	1.234
prcod	4.192
market_cap	5.491

Specification 2	
const	501.577
log_float_refinitiv	1.253
log_volume_tso_ratio	1.636
log_short_float_capiq	1.589
retail_sentiment	1.302
5day_sentiment	1.673
log_prcod	2.631
log_market_cap	2.494

Specification 3	
const	501.577
log_float_refinitiv	1.253
log_volume_tso_ratio	1.636
log_short_float_capiq	1.589
retail_sentiment	1.302
5day_sentiment	1.673
log_prcod	2.631
log_market_cap	2.494

Specification 4	
const	380.552
low_float_refinitiv_25th	1.141
log_volume_tso_ratio	1.339
high_short_capiq_75th	1.240
retail_sentiment	1.276
5day_sentiment	1.449
low_price_5	1.552
log_market_cap	1.702

The model fit is poor in all four models, where the adjusted R-squared is negative in specification 1 and less than .01 in specifications 2, 3, and 4. The regression output for specifications 2, 3, and 4 is displayed in Table 20. Low price and log market cap are significant in all three specifications at either the .05 or .1 level, with both being negatively related to bubble size, as represented by the negative coefficient.

Table 20

Regression Output for Specifications 2-4

Specification	***2***	***3***
Dependent Variable	**Log_OpenHigh**	**BoxCox_OpenHigh**
Constant	0.964	0.820
Log Float	0.088	0.117
Log 30-day Volume to TSO Ratio	-0.029	-0.038
Log Short	0.044	0.051
Retail Buy Sentiment, 30-day change	0.027	0.029
Retail Buy Sentiment, 5-day average	0.699	0.948
Log Price Level	0.089**	0.106**
Log Market Cap	-0.085**	-0.090*
Number of Observations	308	308
R^2	0.024	0.0230
Adjusted R^2	0.002	0.0000
F-Statistic	1.067 (0.385)	1.004 (.429)

Note: All specifications reflect standard errors as heteroskedasticity is not a concern. Coefficient significance levels are denoted as *** p<0.01, ** p<0.05, * p<0.1.

Specification	***4***
Dependent Variable	**Log_OpenHigh**
Constant	0.615
Low Float (<=25th percentile)	-0.128
Log 30-day Volume to TSO Ratio	-0.026
High Short (>=75th percentile)	0.126
Retail Buy Sentiment, 30-day change	0.037
Retail Buy Sentiment, 5-day average	0.316
Low Price (<=$5)	-0.181*
Log Market Cap	-0.061*
Number of Observations	308
R^2	0.028
Adjusted R^2	0.006
F-Statistic	1.242 (.280)

Note: All specifications reflect standard errors as heteroskedasticity is not a concern. Unstandardized Beta significance levels are denoted as *** p<0.01, ** p<0.05, * p<0.1.

While these three initial specifications are not significant and the model fit is poor, patterns begin to emerge:

- The log-transformed price level (Specifications 2 and 3) and the low-price threshold dummy (Specification 4) are significant at the 5% or 10% level. However, the sign of the coefficients suggests that higher-priced stocks are associated with larger intraday bubbles, which does not support H4.
- Log market cap is consistently significant across all three models. Further, the coefficient suggests that smaller companies are prone to larger bubbles, consistent with H5.
- Contrary to expectations, liquidity, short interest, and retail sentiment do not show significant relationships with bubble size

Specifications with Controls and Interactions

Building on these initial models, several control variables are introduced to improve model fit and account for broader market conditions. While the controls were added to both the specifications with float, short, and price as continuous variables as well as dummies, the model fit is better with the threshold variables. Therefore, the remainder of the regression outputs displayed below will discuss the results utilizing the threshold variables.

Given this study's interest in bubble events across major market events during the 5-year period, a pre-COVID-19 dummy variable is added to the model. In alignment with literature on short-term overreaction to news events (Howe, 1986; Mahani & Poteshman, 2008), a dummy variable to represent news events based on SEC filings made +/- five days from the bubble observation date is included. These news events consist of all SEC filing types, which may or may not contain market-moving (material) news and may be either positive or negative in nature.

Additional specifications were considered on narrower definitions of materiality, but materiality can be a subjective determination. Therefore, the analysis included a generalized dummy variable of all SEC filings, but future research should consider the impact of specific filing types.

Two additional controls are incorporated to account for market-wide factors. First, the Chicago Board Options Exchange Volatility Index (VIX), often referred to as the market's fear gauge, is added to represent expected market volatility over the next 30 days, serving as a barometer for the market sentiment (Whaley, 2000). While a higher VIX could be associated with larger bubbles, given increased volatility, it may also indicate that investors are less willing to make risky bets on intraday bubble events (Bollen & Whaley, 2004). Second, pulling from Fama and French's (1996) three-factor model, a measure of market return across the universe of small- and micro-cap companies, the Russell Microcap Index return on the observation date is included to help isolate stock-specific behavior from broader market moves.

In addition to including control variables, additional variations of the previous specifications 2, 3, and 4 were modeled. First, each of the float and short interest variables was included independently of the other to determine if they share some portion of the variance in the dependent variable. However, the model's fit and coefficients' significance were not improved when these were included individually. Also, given the correlation between the two retail sentiment measures (.35) and the potential for shared variance, the 5-day sentiment measure was dropped from the model in favor of the 30-day percentage change in daily average retail buy volume, which resulted in a better model fit than 5-day sentiment and their inclusion together.

Both specifications 5 and 6 have a notably improved model, albeit still modest, with an adjusted R-squared of approximately .05, an increase from less than .01 in the first three specifications. Further, the 30-day average volume to TSO, low price, company size, pre-covid, VIX, and market return variables are all significant. The volume liquidity measure and market cap are negatively associated with bubble size, which aligns with H1 and H5. However, the low-price variable suggests lower prices result in smaller bubbles – this does not support H4, which assumed lower-priced stocks would be more susceptible to large intraday bubbles. VIX is also negatively related to bubble size, meaning that more expected volatility results in smaller bubbles. The market return index variable is positively related to bubble size, suggesting larger market returns lead to larger bubbles in this data set. The other variables are not significant, as seen in Table 21. The patterns observed in the residual analysis of specifications 5 and 6 are consistent across all subsequent models discussed in this paper. Specifically:

- Normality: Specifications using the log-transformed OpenHigh as the dependent variable consistently show slight non-normality in residuals (See Figure 17 for QQ-Plots). Despite this difference, the outputs of the two models are very similar, suggesting that the log-transformed dependent variable is sufficiently normally distributed for this study.
- Heteroscedasticity: Log-transformation specifications consistently exhibit heteroscedasticity (Box-Cox transformation does not) as seen in visual inspection of residuals vs. fitted plots and review of the Breusch-Pagan test. Therefore, robust standard errors are used for log-transformed specifications, while standard errors are used for Box-Cox transformations unless explicitly stated otherwise.
- Multicollinearity: VIF analysis indicates that multicollinearity is not a concern in specifications 5 and 6 nor in any subsequent specification discussed.

Table 21

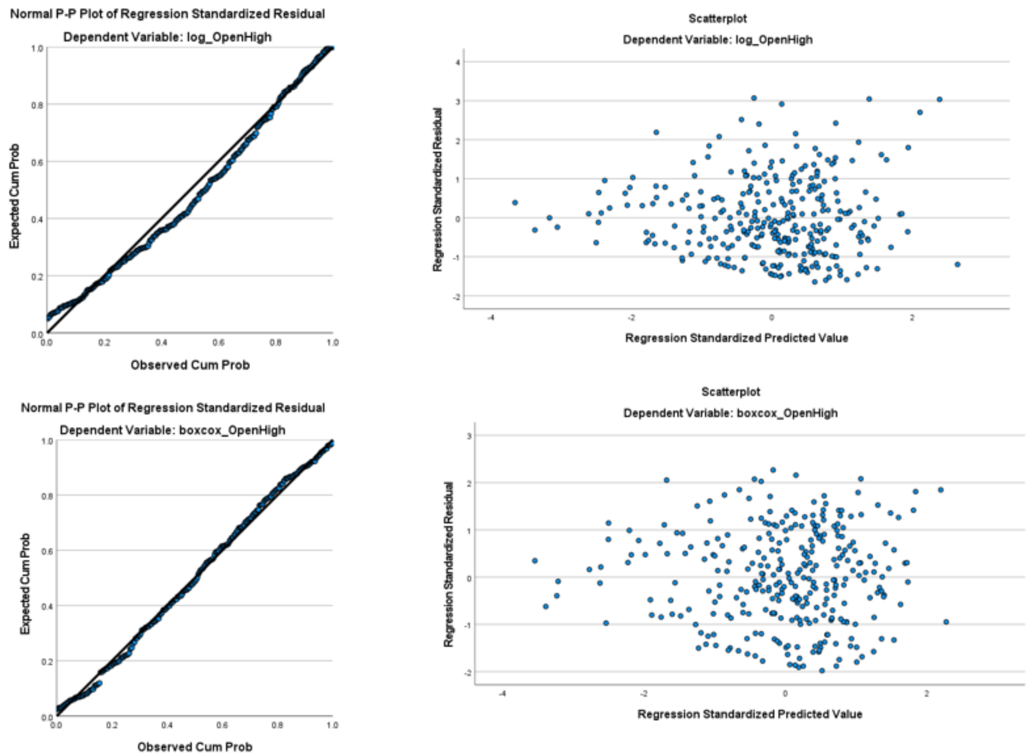
Regression Output for Specifications 5 and 6

Specification	***5***	***6***
Dependent Variable	**Log_OpenHigh**	**BoxCox-OpenHigh**
Constant	1.247*	1.186*
Low Float (<=25th percentile)	-0.117	-0.146
30-day Volume to TSO Ratio	-0.280**	-0.323**
High Short (>=75th percentile)	0.109	0.110
Retail Buy Sentiment, 30-day change	0.060	0.077
Low Price (<=$5)	-0.181*	-0.192*
Log Market Cap	-0.084**	-0.086**
Pre-COVID Dummy	-0.186**	-0.231**
News Filing Dummy	0.133	0.158
VIX	-0.004*	-0.006**
Russell Microcap Index Return	2.314**	2.470**
Number of Observations	307	307
R^2	0.086	0.081
Adjusted R^2	0.055	0.050
F-Statistic	2.766 (.003)	2.606 (.004)

Note: Specification 5 reflects robust standard errors, and Specification 6 reflects standard errors. Coefficient significance levels are denoted as *** p<0.01, ** p<0.05, * p<0.1.

Figure 17

QQ-Plots and Residuals vs. Fitted Plots for Log and Box-Cox Transformed Dependent Variables



Over the next five specifications, additional control variables are added progressively, and a new retail sentiment measure is utilized. Given the limited significance of the retail buy sentiment variable, in either the 30- or 5-day form, it is replaced by a one-day lag measure of retail volume (buy or sell) to total volume. This measure of retail participation is also obtained from the WRDS intraday indicators database, based on the Boehmer et al. (2021) methodology, as were the other retail measures. This expanded measure of retail sentiment is included to see if the buy and sell volume together explains more of the variance in bubble size than measures of the retail buy sentiment alone.

The four new control variables are a dummy variable for NYSE and NYSE American listed stocks, average effective spread the five days prior to the bubble observation date, a momentum measure in the form of the 60-day return, and the one-year growth in earnings per share (EPS). There is significant competition between NYSE and Nasdaq market centers for order flow and stock listings, with each exchange arguing to have a superior market model. Isolating this key difference in listing exchange among the data set may help provide additional insight into bubble size. The average effective spread is yet another proxy for liquidity, as tighter average spreads may indicate more depth or liquidity in a given stock, tying into H1 and the other liquidity measures. Consistent with Carhart (1997), the 60-day return is included as a momentum variable to see if it impacts bubble size. The EPS growth rate is added to the model to include a company-level fundamental variable into consideration as a potential predictor of intraday bubble size, which is used by Rosenberg et al. (1985) in a study on market inefficiency. Beyond these four controls, additional variables, such as variance of daily returns, interest rates, and industry sectors, are added, but the models become successively worse as the adjusted R-squared and significance of the coefficients begin to decline (See Appendix D for additional regression outputs).

In Table 21, the model fit improves by approximately 1.5% when progressing from specification 6 to specification 9 as the new retail measure, the NYSE dummy variable, and the average expected spread are added to the regression. However, when the 60-day momentum and EPS growth rate variables are added in specification 10 and 11, respectively, the model fit levels out before declining with additional variables or interactions. While the overall model fit is improving with the new retail measure, it still remains insignificant even at the .10 level. Additionally, there is little change in the significance of the variables that have been included in previous models, except for the news filing variable, which becomes significant at the .05 level, whereas it was not significant in previous models. Notably, the constant is significant up to specification 9, at which point it fails to gain significance in subsequent specifications. This suggests that the model does not predict a meaningful baseline level of the bubble size dependent variable when independent variables are zero. Conceptually, this supports the expectation of randomness (at least in the very short-run) in stock prices, absent new information,

While not apparent in Table 22, the low float and NYSE variables are nearly significant as they both have a p-value of .11. However, the negative coefficient of the float variable does not align with H1, as the expectation was that low float stocks would lead to larger bubbles. Interestingly, retail sentiment, as measured in this study, remains insignificant across all specifications, suggesting that retail participation may not be a key driver of intraday bubble size. While not displayed below, the variables in these specifications were applied identically to the Box-Cox transformed OpenHigh variable, resulting in similar results with nothing additional to add to this portion of the discussion. All specifications considered, when managing the trade-off between complexity and explanatory power, specification 9 emerges as the best fit model given the adjusted R-squared of .069 along with the theoretical and practical relevance of the variables, with seven of 11 showing significance. As additional variables are added in specifications 10 and 11, as well as those included in specifications in Appendix D, the model fit begins to deteriorate.

Table 22

Regression Output for Specifications 7-11

Specification	***7***	***8***	**9**	**10**	**11**
Dependent Variable - Log_OpenHigh					
Constant	1.154*	1.279*	1.085	1.063	1.043
Low Float (<=25th percentile)	-0.108	-0.116	-0.133	-0.137	-0.128
30-day Volume to TSO Ratio	-0.297**	-0.297*	-0.281**	-0.250*	-0.226**
High Short (>=75th percentile)	0.109	0.104	0.115	0.12	0.120
Retail Volume to Total Volume, 1-Day Lag	0.303	0.291	0.272	0.26	0.287
Low Price (<=$5)	-0.202*	-0.211**	-0.211**	-0.222**	-0.240**
Log Market Cap	-0.083**	-0.090**	-0.080**	-0.079*	-0.077*
Pre Covid Dummy	-0.178**	-0.178**	-0.179**	-0.178**	-0.171**
News Filing Dummy	0.171**	0.175**	0.167**	0.167**	0.172**
VIX	-0.004*	-0.004*	-0.004*	-0.004*	-0.004*
Russell Microcap Index Return	2.127**	2.086**	2.080**	2.071**	2.146**
NYSE Dummy		0.131	0.142	0.142	0.135
Average Effective Spread (5-day lag)			1.602	1.656	1.585
60-Day Momentum				-0.011	-0.014
1-year EPS Growth					-0.015
Number of Observations	301	301	301	300	288
R^2	0.092	0.099	0.106	0.107	0.110
Adjusted R^2	0.061	0.065	0.069	0.067	0.064
F-Statistic	2.947 (.002)	2.8952 (<.001)	2.854 (<.001)	2.642 (.0017)	2.404 (.0035)

Note : All specifications use log-transformed dependent variables. Coefficient significance levels are denoted as *** p<0.01, ** p<0.05, * p<0.1. Significance levels reflect robust standard errors as heteroskedasticity is present. The reduction in the number of observations is a result of missing data related to the intraday retail trading, spread, and EPS variables.

Before turning to a more detailed discussion of the implications of the various specifications on the five hypotheses, the dynamics of bubble formation across different magnitudes were conducted via quantile regression analysis on specification 9 at the 25th, 50th, and 75th percentiles (Table 23). This approach resulted in several insights:

- The pseudo R-squared increases from .045 at the 25th percentile to .0718 at the 75th percentile, suggesting that the specification may better explain larger bubbles occurring at or above the 75th percentile.
- The effective spread becomes significant at the 75th percentile, indicating this factor may be particularly important for larger bubbles. Also, the historical volume to TSO ratio is only significant at the 25th percentile, suggesting that more liquidity may play less of a role in minimizing a bubble as it becomes larger, which may indicate a herd-like effect of buyers dominating the impact of sellers in the market.

- While consistent with the OLS specifications in some ways, such as the significance of low prices, other variables, such as pre-COVID and news filing, diverge from the OLS findings.
- The quantile regression reveals nuances in bubble formation that are not captured by OLS regression alone, suggesting that different mechanisms may be at play for bubbles of different magnitudes.

Table 23

Quantile Regression Output, Specification 12-14

Specification	***12***	***13***	**14**
Dependent Variable Quantile	**25th Percentile**	**50th Percentile**	**75th Percentile**
Constant	-0.253	0.550	2.982**
Low Float (<=25th percentile)	-0.098	-0.139	-0.149
30-day Volume to TSO Ratio	-0.300**	-0.133	-0.338
High Short (>=75th percentile)	0.053	0.053	0.200
Retail Volume to Total Volume, 1-Day Lag	0.242	0.249	0.174
Low Price (<=$5)	-0.209**	-0.160	-0.342**
Log Market Cap	-0.021	-0.056	-0.161**
Pre Covid Dummy	-0.113	-0.148	-0.203
News Filing Dummy	0.103	0.163	0.170
VIX	-0.004*	-0.006**	-0.005
Russell Microcap Index Return	1.153	0.496	2.566
NYSE Dummy	0.138	0.266**	0.148
Average Effective Spread (lagged 5-day average)	0.520	1.730	3.323**
Number of Observations	301	301	301
Pseudo R-Squared	0.045	0.0585	0.072

Note : Quantile regression captures relationships between the independent variables and the dependent variable at different levels of bubble size. Coefficient significance levels are denoted as *** p<0.01, ** p<0.05, * p<0.1.

The various specifications discussed in this section highlight the complexity of modeling intraday bubble activity and highlight the level-dependent influence of various factors on bubble size. This analysis provides valuable insights for the subsequent discussion of the hypotheses.

Results and Limitations

To this point, the hypotheses have been mentioned only briefly in discussions of the various specifications. A more comprehensive analysis is warranted to fully understand the implications of the findings. Each of the five hypotheses is examined in the context of the results from specifications 5 through 9, along with the quantile regression, which represents the best model fits. This approach allows for a systematic examination of the evolution of the findings as controls were introduced, assessing the consistency of support (or lack thereof) for each hypothesis. The hypotheses are restated in Table 24, and the regression results for the relevant specifications are consolidated in Table 25.

Table 24

Hypothesis Summary Table

Hypothesis	Description
H1	Liquidity is negatively associated with the size of intraday price bubbles
H2	Short interest as a percentage of public float is positively associated with the size of intraday price bubbles
H3	Retail trader sentiment is positively associated with the size intraday price bubbles
H4	Stock price is negatively associated with the size of intraday price bubbles
H5	Market capitalization is negatively associated with the size of intraday price bubbles

Table 25

Regression Outputs for Hypothesis Discussion

Variable	5	6	7	8	9
Dependent Variable	Log_OpenHigh	BoxCox_OpenHigh	Log_OpenHigh	Log_OpenHigh	Log_OpenHigh
Constant	1.247*	1.186*	1.154*	1.279*	1.085
Low Float (<=25th percentile)	-0.117	-0.146	-0.108	-0.116	-0.133
30-day Volume to TSO Ratio	-0.280**	-0.323**	-0.297**	-0.297*	-0.281**
High Short (>=75th percentile)	0.109	0.11	0.109	0.104	0.115
Retail Sentiment/Participation	0.06	0.077	0.303	0.291	0.272
Low Price (<=$5)	-0.181*	-0.19*	-0.202*	-0.211**	-0.211**
Log Market Cap	-0.084**	-0.086**	-0.083**	-0.090**	-0.080**
Pre Covid Dummy	-0.186**	-0.231**	-0.179**	-0.178**	-0.178**
News Filing Dummy	0.133	0.158	0.171**	0.175**	0.167**
VIX	-0.004*	-0.006**	-0.004*	-0.004*	-0.004*
Russell Microcap Index Return	2.314**	2.470**	2.127**	2.086**	2.080**
NYSE Dummy				0.131	0.142
Average Effective Spread (5-day lag)					1.602
Number of Observations	307	307	301	301	301
R^2	0.086	0.081	0.092	0.099	0.106
Adjusted R^2	0.055	0.050	0.061	0.065	0.069
F-Statistic	2.766 (.003)	2.606 (.004)	2.947 (.002)	2.895 (<.001)	2.854 (<.001)

Note : All log-transformed specifications utilize robust standard errors, where as the Box-Cox transformation relies on standard errors. Unstandardized Beta significance levels are denoted as *** p<0.01, ** p<0.05, * p<0.1.

H1: Liquidity is negatively associated with the size of intraday price bubbles

The three measures that act as a proxy for liquidity in the regression specifications are public float, average daily volume as a percentage of shares outstanding, and the average effective spread. However, the spread variable was not initially hypothesized and was only included in specification 9. These variables are not correlated and uniquely measure liquidity, suggesting they could capture unique aspects of the variance in bubble size. Findings include:

- The low float and 30-day volume measure coefficients are negative across all five specifications. The coefficient for spread is positive in specification 9, but the implication is the same as the other two measures, as wider spreads are associated with less liquidity. The coefficients are summarized in Table 26.
- The coefficient for low float is negative, suggesting smaller bubbles, contrary to the hypothesis. However, this measure is not statistically significant across the specifications.

- The signs of the coefficient for average volume and spread support H1, but only the volume measure is statistically significant. The significance level and size of the volume coefficient remain consistent across the specifications.
- The strong and consistent results for volume to TSO ratio imply that establishing consistent trading volume can potentially limit the size of bubbles. This finding could encourage listing exchanges to modify or enhance trading volume listing rules. For example, Section 802.01 of NYSE's continued listing standards requires listed equities to average just 100,000 shares traded per month.

Table 26

Coefficient Impact of Liquidity on Bubble Size, based on Specification 9

Variable	Coefficient	Implication to Bubble Size
Low Float	-0.133	If the observation is a low float stock, the log_OpenHigh decreases by .133, and the actual bubble size is decreased by 12.44%
Volume/TSO Ratio	0.281**	For every 1-unit increase (100%), the log_OpenHigh decreases by .281, and the actual bubble size is decreased by 24.50%. Scaling to a .1-unit increase (10%), the decrease in bubble size is 2.76%.
Spread	1.602	For every 1-unit increase (100%), the log_OpenHIgh increases by 1.602, and the actual bubble size increases by 396.10%. Scaling to a .1-unit increase (10%), the increase in bubble size is 17.37%

H2: Short interest as a percentage of public float is positively associated with the size of intraday price bubbles

Short interest was included as a hypothesis partly due to the retail investor's fascination with GameStop and its unusually large short interest (over 100% of its float). Additionally, highly shorted stocks may be susceptible to a short squeeze, like with GameStop, leading to bubble-like spikes in stock prices. Retail investors created the Meme stock movement following GameStop, where they target other stocks that they attempt to influence. Findings include:

- The high short threshold variable is not significant across the specifications despite approaching the .10 significance level in some specifications.
- The magnitude is stable, and the coefficient is positive across all models as control variables are added.
- While not significant, the specifications suggest that stocks in the 75th percentile of short interest are associated with larger bubbles, consistent with the hypothesis.
- The stable level of this measure, despite the inclusion of various controls and alternative measures of liquidity and retail participation, suggests that there may be untested nonlinear or interaction effects that can be investigated in future studies.
- If the observation is a high short-interest stock, the log_OpenHigh increases by .1151, and the actual bubble size is increased by 12.19%. However, the lack of statistical significance across all specifications for short interests limits the ability to draw strong conclusions.

H3: Retail trader sentiment is positively associated with the size of intraday price bubbles

The so-called rise of retail investors in recent years has been well publicized in the media and recently published research. However, dissecting order flow to precisely track a trade's origination with a retail or institutional investor is a difficult task. This study leans heavily on the Boehmer et al. (2021) methodology and the WRDS intraday indicators data to obtain retail information. Findings include:

- The retail investor variables exhibit consistency in the direction of the coefficient as each measure (30-day change in retail buy sentiment, 5-day average buy sentiment, and 1-day lag retail buy and sell volume to total volume) is positive in all specifications.

- Despite the consistently positive coefficients, there are no statistically significant relationships with bubble size across the models. However, in any follow-on research, consideration should be given to a retail sentiment threshold variable, which may better capture non-linear effects.
- There is a significant increase in the magnitude of the coefficients when transitioning from the buy sentiment to buy and sell interest measure in specifications 7, 8, and 9.
  - In specification 9, every one-unit increase (100%) in retail volume to total volume results in an increase in log_OpenHigh of .2718. In terms of bubble size, this equates to an increase of 31.23%. Scaling to a .1-unit increase (10%) results in a 2.75% increase in the bubble size.
  - In specification 6, using the retail buy sentiment measure, a one-unit increase is a .0599 increase in log_OpenHigh. In terms of bubble size, this equates to only a .6% increase in bubble size for a .1-unit increase.
- The lack of significance in the retail variables could be a result of more complex relationships with bubble size, the choices of measurement period and lags, or the complexity of market microstructure and difficulty of pinpointing the source of some orders.

H4: Stock price is negatively associated with the size of intraday price bubbles

As discussed earlier in the paper, price level is hypothesized to have a positive relationship with bubble size. This hypothesis ties back to the recent increase in retail investors and their affinity for low-price, lottery-like stocks. A low-price threshold of approximately $5 (aligned with the 25th percentile) was used for this variable. Findings for this hypothesis include:

- The low-price coefficient is consistently significant and negative across all specifications, which does not support the hypothesis as it posited that lower-priced stocks would be associated with larger bubbles.
- The significance improves from .1 to .05 as the model becomes more complex.
- If the observation is a low-priced stock, log_OpenHigh decreases by 0.2107 in specification 9. In terms of bubble size, this equates to a decrease of approximately 19%.

The contradictory findings relative to this hypothesis suggest that the initial understanding or assumptions about low-priced stocks need to be reconsidered, at least for this specific data set. Also, there may be regulatory limitations preventing some investors (retail or institutional) from participating in very low-priced stocks, limiting the intraday momentum effects of a bubble. Also, lower-priced stocks have tighter limit-up and limit-down circuit breaker parameters, leading to more volatility trading pause interruptions, which are partially intended to slow down extreme price movements.

H5: Market capitalization is negatively associated with the size of intraday price bubbles

Both empirical studies of this dissertation are partially motivated by practitioner experience, suggesting that small companies experience unusual intraday price behavior consistent with bubbles more often than larger companies. Therefore, including a hypothesis regarding company size was well suited for the theme of this study. Key findings include:

- There is consistency in the significance (.05 level), magnitude, and direction of the company size variable across all specifications.
- The direction of the coefficient suggests that as market capitalization increases, the bubble size decreases, which supports this hypothesis.

- For a one-unit increase in log market cap, the log OpenHigh decreases by .0804. In terms of market cap and bubble size, a 10% increase in market cap results in a .78% decrease in bubble size.
- The association of larger companies with better price efficiency may be related to more analyst coverage, more institutional ownership, and increased liquidity.

The specifications strongly support this hypothesis, which also supports the general perspective of this study and intraday activity in small stocks. However, the observation set for this study was selected from a microcap index, so an argument can be made that all of the companies in this study have a small market cap. This encourages future research on stocks from a large-cap index to assess consistency in this relationship.

Discussion

Conclusion

This study set out to unravel the complex factors influencing the magnitude of intraday bubbles in a sample of small- and micro-cap stocks between 2018 and 2022. It addresses a significant gap in the existing literature, which is mostly focused on longer-term bubbles in broader market or industry indices. By focusing on intraday bubbles in individual small- and micro-cap stocks, the research offers a novel perspective on an understudied but increasingly important market segment.

Set against a backdrop of rapidly evolving market dynamics (i.e., shifting patterns of who and how market participants engage in equity trading along with the increasingly intricate complexity of intraday market microstructure), the study was challenged with addressing market realities in a small sample of events, while striving for applicability across companies and bubbles of varying size. The findings reveal a nuanced interplay of factors influencing the size of intraday bubbles, underscoring the complexity of investor behavior and market dynamics that challenge traditional models of pricing and efficient markets.

The hypotheses were formulated through a dual lens: firmly grounded in various strands of academic research on financial market dynamics while also reflecting contemporary market events and trends faced by practitioners. The integration of seminal theoretical contributions with real-world observations provides a sound framework for a study on intraday bubbles in individual small- and micro-cap stocks. The results of the study were mixed in their support of the hypotheses and are summarized in Table 27.

Table 27

Summary of Hypothesis Findings

Hypothesis	Findings
Liquidity (H1)	Partially supported with historical volume showing a significant negative relationship with bubble size. Float and spread measures were not statistically significant; float was not directionally supported
Short Interest (H2)	Directionally supported but not statistically significant as a continuous variable or as a high short threshold variable
Retail Sentiment (H3)	Directionally supported but not statistically significant when as either buy sentiment, buy participation, or total patricipation. Large deviation in coefficient size when viewing total participation versus buy only
Stock Price (H4)	Not supported; lower-priced stocks are associated with smaller bubbles in the sample selection used in this study
Market Capitalization (H5)	Strongly supported; larger market capitalization companies are associated with smaller intraday bubbles within the range of company sizes in this data set for this study

Despite not all hypotheses being supported, either due to shortcomings of existing theories to explain such events or empirical misspecifications, this study provides valuable insights into the complex nature of intraday price bubbles in small- and micro-cap stocks. As trading technologies continue to evolve, retail participation grows, and an interest in small stocks continues to grow, it is vital that market participants, regulators, and academics continue to broaden our understanding of these intraday bubble dynamics as it is crucial for maintaining fair and orderly markets.

Limitations

The nature of stock price bubbles, limitations of existing theories, intraday trading dynamics, data access, and study design present some limitations to consider when assessing the potential implications of the regression results. The real-time nature of intraday trading involves innumerable temporal dynamics, making it difficult to capture all relevant factors. The missing variable issue, an inevitable concern for most studies on intraday pricing, is present in this study.

Financial market data's abundance acts as a double-edged sword. While there is a vast body of information available to enlighten our understanding of intraday trading, studying a large swath of companies across multiple years on an intraday basis can quickly lead to millions of data points. At such levels of data, there is an inevitable trade-off regarding time, cost, complexity, and generalizability.

This study focused on extreme outliers regarding intraday price movements, which may have muted or distorted some of the relationships between the independent variables and the dependent variable, given there may be little variance at such extremes.

While these limitations are important to acknowledge, the study nonetheless manages to uncover important considerations for practitioners as well as for future research.

Future Research Considerations

The temporal dynamics of intraday trading provide a challenging yet vast opportunity for understanding the interplay of company fundamentals, market-wide factors, market microstructure, and investor sentiments with individual stock price patterns. This study adds to the research in this area by exploring intraday bubbles. However, future researchers should consider varying levels of bubble size, expanding the generalizability of the regression findings. Additionally, future studies should consider assessing how these variables relate to intraday events where no bubble occurred, providing a comparative analysis. Also, despite its upward trend, small- and micro-cap stocks still account for a small portion of all equity trading. As such, researchers should consider intraday bubble dynamics in mid- and large-cap stocks.

CHAPTER 5: CONCLUSIONS AND IMPLICATIONS

This chapter synthesizes the findings from the two empirical studies conducted in this dissertation exploring intraday price bubbles and their implications for research, theory, and business practice. The studies address a contemporary issue of intraday price inefficiency in small- and micro-cap stocks, which has been an area of focus among regulators and market participants in light of COVID-19 volatility and meme stock trading. The paper concludes by addressing the limitations of the research and proposing directions for future studies.

Historical Context and Persistent Challenges

Price bubbles have been a societal phenomenon for centuries, long before stock exchanges, long before high-speed trading, long before complex market microstructures, and certainly long before meme stock trading strategies. Yet, economists, academics, regulators, and market participants still debate Greenspan's (1996) question to central bankers: how do we know when irrational exuberance has unduly escalated asset values? Irrational exuberance is a polarizing phrase that divides many when dissecting the presence, cause, and characteristics of stock price bubbles. Regardless of one's definition, categorization, or causation of a bubble, the acknowledgment of its presence, at least temporarily, is hardly debatable.

Study Focus and Contribution

This dissertation set out to address a gap in the academic literature by providing insight into an understudied segment of price bubbles, those that form and crash within a single day of trading: intraday bubbles. Further, the study focuses on bubbles that are idiosyncratic to a single stock rather than through the lens of broader market indices. Finally, the study focuses on small- and micro-cap stocks, which offer less price stability than their large-cap counterparts and may attract less sophisticated retail investors who often have less risk tolerance and liquidity than professional traders.

Research Questions

The research was designed to address three questions:

- How well do existing bubble tests capture the formation of intraday price bubbles in small- and micro-cap stocks?
- How did the prevalence and characteristics of intraday bubbles evolve across significant market events between 2018 and 2022?
- To what extent do liquidity, short interest, retail trader participation, opening stock price, and market capitalization influence the size of intraday bubbles?

Key Findings

Despite their limitations, both empirical studies resulted in important insights into intraday bubbles in small- and micro-cap stocks. Importantly, the studies draw awareness to the scale of these individual intraday bubbles in terms of trading volume on the bubble observation date (average of 37 million shares traded) and the dollar volume of shares traded on the observation date (average of more than $356 million in value traded per day, combined buy or sell volume). These numbers suggest that these events are not merely block trades or temporary order imbalances that impact a small number of market participants. Rather, these numbers suggest that there is a significant amount of money at risk to both retail and institutional investors when these events occur.

The first study highlighted that many of these existing bubble tests (e.g., unit root tests, runs tests, explosive price tests, and the Hurst Exponent), at best do a fair job at identifying bubble events. Collectively, they capture bubble-like price dynamics in approximately two-thirds of the events studied despite the clear bubble path of their intraday prices, suggesting that more work needs to be done to develop tools to detect bubble events in real-time.

The second study provides valuable insights into some of the factors influencing the size of intraday price bubbles in small- and micro-cap stocks. There appears to be a strong relationship suggesting that historical trading volume can help limit the size of intraday bubbles, magnifying the need for stock exchanges to consider the implementation of new listing standards pertaining to average daily trading volume or develop tools to monitor low-volume stocks more closely. Similarly, lower stock prices are associated with smaller bubbles, which contradicts our original hypothesis but provides an opportunity to consider alternative explanations for the relationship, such as the LULD parameters for lower-priced stocks versus higher-priced stocks.

We find there is, on average, a 15% increase in retail buy volume to total volume from the 30 days prior to the five days prior to the bubble observation date, suggesting a potential signal. Notably, we do not find dominance by retail investors during these intraday events, as the average retail volume to total volume on the observation date is 22% across all 342 observations. However, expanding the observation set will improve the robustness and generalizability of these findings, which must be considered alongside the limitations of the Boehmer et al. (2021) methodology for the detection of retail orders.

Collectively, these two studies present a novel lens for analyzing the uniqueness of intraday bubbles in individual stocks. The studies do not answer all questions pertaining to price bubbles, and they may, in fact, invite new questions, but they address a gap in the literature and leave academics and practitioners with much to consider.

Contributions to Theory

Detecting Intraday Bubbles: A Multi-Test Approach

Our findings suggest that there are limits to the ability of statistical tests commonly used in traditional bubble theory research to detect intraday bubbles in single stocks. Of 342 events studied, 270 exhibited patterns or trends consistent with bubble dynamics. While 270 is over three-quarters of the observations, the results were inconsistent across the various tests utilized. Researchers should continue to develop extensions of existing tests or turn to new measures when determining the presence of bubbles. Further, there may be time-scale dependency of bubble phenomena, meaning the characteristics of bubbles may vary over different periods of time and differing sizes.

The presence of detectable intraday bubbles challenges some aspects of the efficient market hypothesis, particularly in its weak- and semi-strong forms. While even some of the EMH's staunchest supporters acknowledge the presence of occasional bubbles, they often simply write off these events as chance results and temporary overreactions that cause deviations from market efficiency (Fama, 1998; Malkiel, 2003, 2010). Certainly, markets could be efficient at times, but discounting intraday bubbles as short-term deviations in otherwise efficient markets does not provide due consideration to the long-term problems that these bubbles may present to market participants. As such, market efficiency theory should be considered, studied, and analyzed across all horizons, given that different market participants engage in financial markets within varying horizons.

Finally, by focusing on the identification of intraday bubbles, the study supports the extension of behavioral finance theories to assess the psychology of market participants, whether retail or institutional, operating in short, day trading time scales.

Exploring Intraday Bubble Size Through Regression

By identifying the key factors that influence the size of intraday bubbles, this study contributes to the understanding of bubble dynamics and the factors influencing their size. The significance of the relationship between trading volume and smaller bubbles contributes to theories on market liquidity and price stability. Additionally, the surprisingly positive relationship between price level and bubble size can inform theoretical studies on price levels and the potential mitigating role lower prices have on bubble size.

More than contributing to existing theory, the study emphasizes the need for new theoretical frameworks that can explain the unique characteristics of intraday price bubbles. These phenomena are not as rare as one might assume, and they pose significant risks to market participants when they appear.

Contributions to Business Practice

This study provides important considerations for financial market practitioners, including traders, market makers, brokerage firms, stock exchanges, clearing agencies, and regulators. As we were able to confirm inefficiencies in a large proportion of extreme intraday events, the first study emphasizes the need for the financial industry to develop more robust bubble detection tools. The second study shows that the size of these intraday bubble events should not be written off as simply a downstream effect of retail meme trading, as the various measures of retail sentiment did not have significance in their explanatory power for bubble size.

By putting a spotlight on these intraday volatility events, regulators should consider further exploration of the variables with strong relationships impacting the size of the bubble in an effort to reduce the size or likelihood of these events. To this end, regulators (including stock exchanges in their duty as an SRO) could consider enhancements or modifications of rules and surveillances, such as enhanced liquidity requirements in the form of increased minimum volume thresholds for listed stocks. Likewise, developing a real-time GSADF test, which time stamps bubbles, could support faster response to intraday bubbles.

For individual investors, this research highlights the importance of due diligence when making investing decisions. This study highlights that there are episodes of market inefficiencies in intraday data of small- and micro-cap stocks, which may exist for minutes or hours. Thus, the nature of high-speed trading and a trend toward day trading strategies can put the originator of a poorly timed trade at substantial financial risk.

With a better awareness of the frequency and impact of intraday bubbles through the recent GameStop and other meme stocks events, brokerage firms may find this study useful as they consider enhanced warnings and alerts around trading in stocks when patterns are consistent with bubbles. While this is a challenging suggestion, given it could indirectly stifle trading in stocks with significant moves not associated with bubbles, it underscores the need for more advanced, real-time bubble detection tools. Additionally, brokerage firms should enhance investor education tools to equip new, unsophisticated investors with better awareness of when and how market inefficiencies can arise, along with basic tutorials on fundamental research.

The ever-increasing prevalence of high-speed trading and complex market microstructure suggest that practitioners, especially brokerages and regulators, aim to limit these intraday bubble events. However, it is acknowledged that implementation and execution of enhanced surveillance around these events is no easy task. Therefore, academia and practice should collaborate in interdisciplinary studies to develop a framework for bubble detection and prevention.

Limitations

This research, encompassing both the bubble detection study and the regression analysis, faces several notable limitations. A primary concern is the granularity of the data utilized, specifically in the bubble detection study. Due to constraints related to time, data processing capacity, and data availability, minute aggregate data was used for the intraday bubble detection tests. Given that some tests failed to capture trends or patterns in the price series, a more granular analysis may uncover additional bubble-like patterns in the observation set.

The focus on extreme outliers may have distorted or concealed relationships between some variables and the size of the intraday bubbles. In such extreme events, there may be minimal variance, limiting the ability of the regressions to detect significant relationships. Widening the bubble parameters and including more observations may reveal novel relationships to consider when evaluating intraday bubbles.

The temporal dynamics of the intraday trading landscape pose a challenge in capturing all relevant factors influencing bubbles. This challenge is magnified when the bubble peak occurs early or late in the trading day, limiting the number of observations on either side of the bubble. Given the sensitivity of bubble test specifications to assumptions regarding lag or variance length as well as minimum observation thresholds, the tests may provide inconsistent results, as shown in the results of the first study.

As with many financial market studies, there are challenges in addressing the missing variable problem, given the volume of financial data available. There is a risk of overfitting a model and increasing complexity at the expense of parsimony. This study attempted to limit complexity by focusing on a small subsection of intraday bubble events, but this also creates an issue by limiting the generalizability of the results across a larger swath of the market.

Despite the limitations highlighted here and earlier in the paper, this research provides valuable insights into intraday bubble dynamics and a solid foundation for future studies assessing the prevalence and characteristics of intraday bubbles in individual stocks.

Future Research Considerations

This study provides a foundation for future research exploring intraday stock price bubbles and their influencing factors. The first paper highlights the need for more advanced bubble detection tools or modifications of existing tests. While the GSADF and Hurst Exponent tests capture explosive trends or patterns in approximately 50% and 83% of the observations, respectively, more than 50 of the 342 bubble observations remained undetected. Future research could focus on developing or refining bubble detection methods to increase their sensitivity and accuracy. Additionally, investigating smaller, or tick-by-tick, trade increments may uncover additional time series trends indicative of bubble behavior. The SADF and GSADF tests' ability to timestamp explosive price behavior provides a starting point for a deeper analysis of bubble events via an event study. Such studies could analyze order flow, market depth, and trade origination to understand the microstructure dynamics driving explosive price movements.

To address these challenges, interdisciplinary collaborations with data scientists and machine learning experts could yield more sophisticated bubble detection algorithms. Advanced machine learning techniques may help identify complex patterns and predictive signals of bubble formation that traditional methods might miss. Also, studies examining potential regulatory responses or market structure changes could provide valuable insights for policymakers aiming to mitigate the negative impacts of these rapid price movements.

The second paper sets the stage for future researchers to consider time-scale dependencies when evaluating stock price bubbles. The significance, in size and reach, of the intraday bubbles presented in this study encourages consideration of price bubbles beyond traditional long-term, market-wide contexts. Future research should explore the catalysts for changes in retail sentiment and their impact on bubble dynamics, as well as characteristics of institutional involvement in such events.

Further, there is an opportunity to expand the selection of bubble events at less extreme cases to improve the generalizability of regression findings. While our focus has been on intraday bubbles in small- and micro-cap stocks, similar research on mid- and large-cap stocks is warranted, broadening our understanding of intraday bubbles across various market segments. By pursuing these research directions, scholars can develop a more comprehensive and nuanced understanding of intraday bubble formation.

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APPENDIX A: IRB APPROVAL LETTER

PEPPERDINE IRB NON-HUMAN SUBJECTS NOTIFICATION FORM FOR RESEARCH THAT DOES NOT INVOLVE HUMAN SUBJECTS

Pepperdine University's Institutional Review Board (IRB) is required to review and approval all research that meets the definition of human subjects research. The code of federal regulations provides the following definitions:

- **For the purposes of the IRB, research is defined as a systematic investigation designed to develop or contribute to generalizable knowledge.**
- **Human subject means a living individual about whom an investigator (whether professional or graduate student) conducting research obtains**

  (1) Data through intervention or interaction with the individual, or
  (2) Identifiable private information.

IMPORTANT INSTRUCTIONS: If your research does ***not*** involve the participation of human subjects ***and*** you are ***not*** using/collecting any data that has been obtained from individual participants, then your research is **not** subject to IRB review and approval.

EXCEPTION: If you are not certain whether your proposed activity meets the definition of non-human subjects research, or if your study requires a formal written determination (e.g., as requested by sponsors, funding agencies, and/or journals), please complete this form and submit it along with either **1)** a one page abstract (outlining the study's research design and methodology), or **2)** a draft of your research project (does not need to be finalized) by email to andrea.quintero@pepperdine.edu and copy gpsirb@pepperdine.edu.

We may reach out with clarification questions as needed; otherwise, if your research's non-human subjects status is confirmed, the Pepperdine IRB office will issue a confirmation of non-human subjects verification.

Investigator Name: Benjamin Sawyer
Status (Check One): ☐ Faculty ☒ Graduate Student ☐ Undergraduate Student
Faculty Chair (if applicable): Levan Efremidze
Proposal Research Title: Intraday Price Bubbles and Their Influencing Factors: The Analysis of Small and Microcap Stocks

I verify that this proposed research does not involve the use of human subjects, either directly or indirectly.

Benjamin Sawyer — 7/17/2023
Principal Investigator(s)/Student Signature — Date

[signature] — 8/18/2021
Faculty Chairperson Signature — Date

Revised 05/16/2023

APPENDIX B: PRELIMINARY DATA ANALYSIS

Descriptive statistics comparing Q3 2018 and Q3 2021

Table 1. Descriptive Statistics

PANEL A. Q3 2018, Full Sample

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Shares Outstanding	95,452	207,000	1,091,680,000	32,341,438	42,112,977	9.257	187.342
Market Capitalization	95,452	$ 122,578	$ 3,119,205,250	$ 350,128,743	$ 317,042,922	1.700	5.109
Volume	95,452	-	127,146,400	264,072	1,165,712	44.509	3,346.889
Opening Price	93,581	$ 0.02	$ 990.00	$ 18.54	$ 32.09	18.113	503.487
Closing Price	95,452	$ 0.02	$ 995.00	$ 18.66	$ 31.97	17.814	493.173

PANEL B. Q3 2021, Full Sample

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Shares Outstanding	111568	207,000	479,030,000	38,327,544	45,818,723	4.032	24.662
Market Capitalization	111568	$ 5,200,000	$ 4,803,711,640	$ 386,096,774	$ 372,928,349	2.189	8.927
Volume	111568	-	510,576,700	668,469	5,487,676	39.969	2233.263
Opening Price	111568	$ 0.04	$ 889.00	$ 17.91	$ 29.70	13.968	332.784
Closing Price	111568	$ 0.13	$ 901.00	$ 17.89	$ 29.71	13.990	333.912

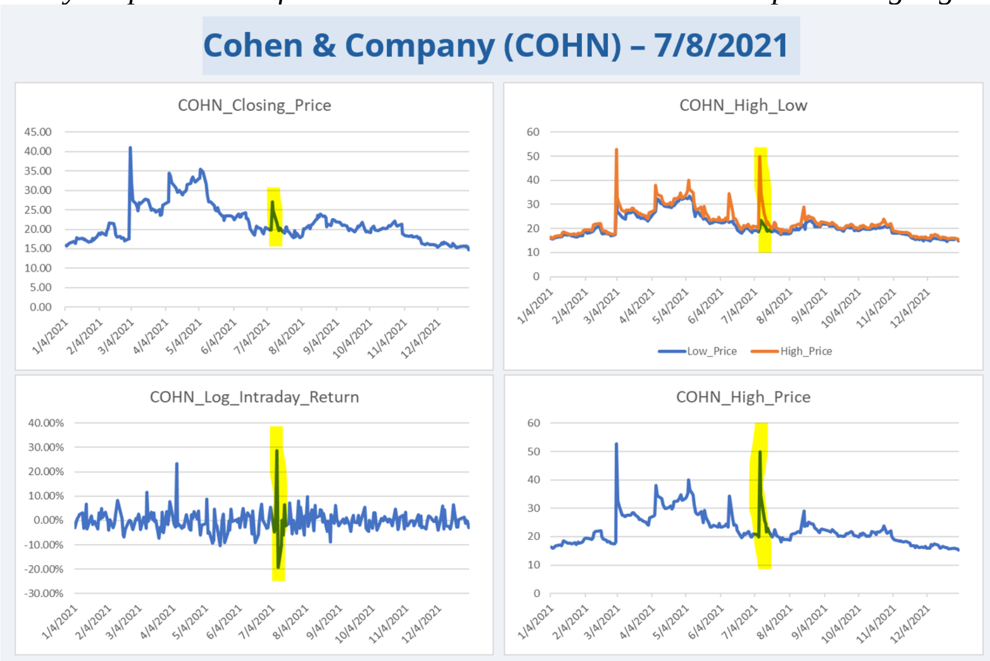
PANEL C. Q3 2018, Top 20

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Shares Outstandingg	4769	584,000	186,348,000	36,701,509	38,587,316	2.461	5.952
Market Capitalization	4769	$ 464,947	$ 993,395,020	$ 120,211,515	$ 159,484,766	2.245	5.154
Volume	4650	106	84,297,590	910,158	3,497,072	11.303	175.644
Opening Price	4769	0.02	98.00	5.33	11.35	4.879	25.851
Closing Price	4769	0.02	92.06	5.32	11.35	4.866	25.648

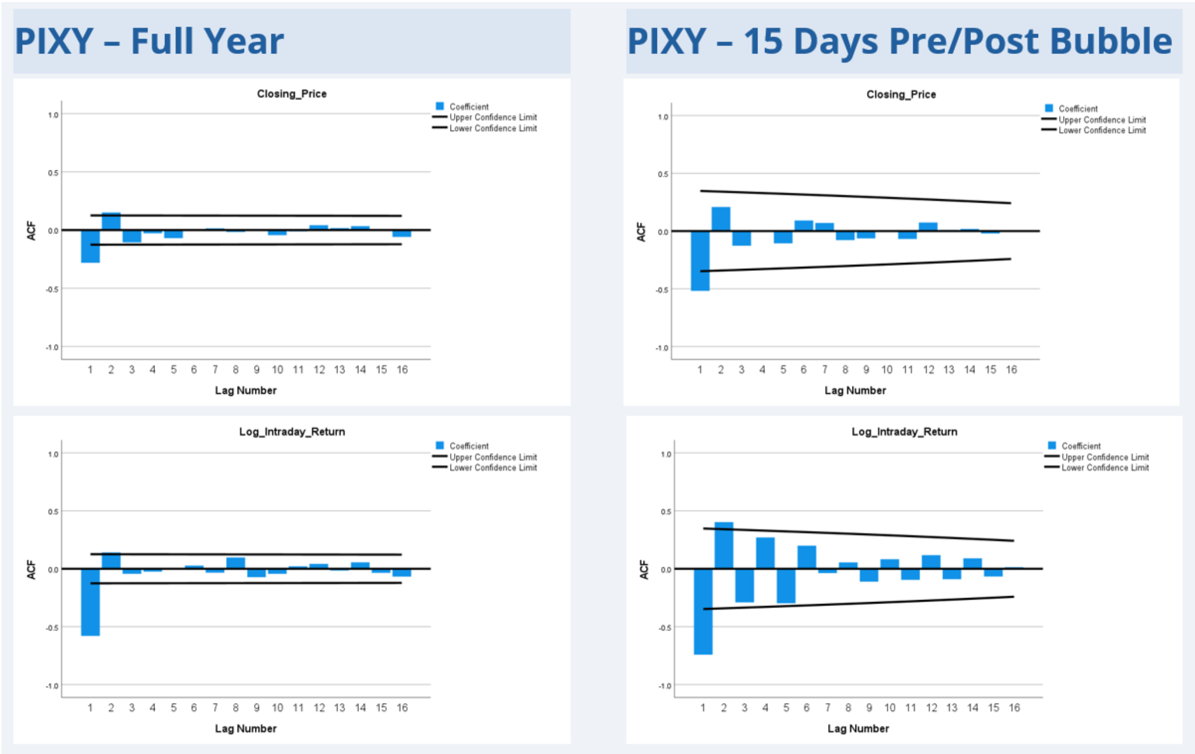
PANEL D. Q3 2021, Top 20

	N	Minimum	Maximum	Mean	Std. Deviation	Skewness	Kurtosis
Shares_Outstanding	4507	551,000	259,432,000	26,299,487	30,649,161	3.306	14.660
Market_Capitalization	4507	$ 17,693,560	$ 1,417,581,000	$ 163,958,688	$ 188,218,625	2.553	7.953
Volume	4507	-	510,576,700	4,155,409	24,451,187	11.953	173.919
Opening_Price	4497	0.42	198.50	17.04	32.14	3.649	12.904
Closing_Price	4507	0.46	199.51	17.05	32.18	3.653	12.944

Full-year price trend for COHN with the theoretical bubble period highlighted in yellow



Autocorrelation and Augmented Dickey Fuller test for PIXY



Time Series Tests for Variable: Closing_Price (PIXY – Full Year)

	Values
Test(3)	Augmented Dickey-Fuller
Alternative Hypothesis(3)	Stationary
P-Value(3)	0.68796
Note(3)	None
Truncation Lag(3)	6

Computations done by R package tseries

Time Series Tests for Variable: Closing_Price (PIXY – 15 Days Pre/Post Bubble)

	Values
Test(3)	Augmented Dickey-Fuller
Alternative Hypothesis(3)	Stationary
P-Value(3)	0.5681
Note(3)	None
Truncation Lag(3)	3

Computations done by R package tseries

APPENDIX C: FULL DATA SET BUBBLE TEST SUMMARY

Ticker and Date	ADF-L,C	ADF-L,T	PP-L,C	PP-L,T	ADF-D,C	ADF-D,T	PP-D,C	PP-D,T	KPSS-L,C	KPSS-L,T	KPSS-D,C	KPSS-D,T	Runs Trend	VR - T or R (5 lags)	Explosive (GSADF)	Hurst Trend or Reverting
AAME-6/18/2020		✓	✓	✓												✓
AAME-2/5/2021	✓	✓	✓	✓					✓	✓	✓					✓
ABSI-8/11/2022	✓	✓	✓	✓					✓	✓			✓		✓	✓
ACOR-9/25/2020	✓	✓	✓	✓					✓	✓			✓		✓	✓
ACOR-10/20/2020				✓					✓	✓			✓			✓
AGE-5/29/2020									✓					✓	✓	✓
AGFS-5/18/2020	✓	✓	✓						✓	✓					✓	✓
AGRX-10/8/2018	✓	✓	✓	✓					✓	✓					✓	✓
AGRX-4/12/2022		✓	✓	✓					✓	✓			✓	✓	✓	✓
AKUS-6/29/2022	✓		✓						✓	✓		✓			✓	✓
ALDX-3/13/2020		✓							✓	✓			✓		✓	✓
AMPE-6/14/2019															✓	✓
AMST-1/29/2021	✓	✓	✓	✓						✓					✓	✓
AMST-6/10/2021	✓	✓	✓	✓					✓	✓					✓	✓
AMTX-7/3/2018	✓	✓	✓	✓						✓					✓	✓
ANIX-12/10/2020	✓	✓	✓						✓	✓				✓		✓
ARC-3/20/2020	✓	✓								✓	✓			✓	✓	✓
AREC-2/21/2020		✓	✓	✓					✓	✓						✓
ARL-7/15/2021	✓		✓						✓	✓		✓				✓
ARMP-1/28/2020									✓	✓					✓	
ARMP-1/28/2021	✓	✓	✓	✓		✓			✓	✓			✓	✓	✓	✓
ASRT-5/11/2020			✓	✓					✓	✓			✓			✓
ATHX-4/11/2022	✓	✓	✓	✓					✓	✓			✓		✓	✓
ATHX-4/29/2022										✓			✓		✓	✓
ATLC-7/31/2018	✓	✓	✓	✓					✓	✓						✓
ATNF-10/7/2022		✓	✓	✓					✓	✓				✓		✓
ATNF-12/16/2022	✓	✓	✓	✓						✓					✓	✓
ATNF-12/20/2022	✓	✓	✓	✓					✓	✓	✓				✓	✓
ATXI-10/11/2018	✓	✓	✓	✓					✓	✓					✓	✓
ATXI-3/1/2022	✓		✓						✓	✓			✓			
AUTO-3/25/2020	✓	✓	✓	✓					✓	✓				✓	✓	✓
AUTO-4/17/2020	✓	✓	✓	✓						✓					✓	✓
AVXL-2/4/2021	✓	✓	✓	✓					✓	✓						✓
AWH-5/16/2022	✓	✓	✓	✓		✓			✓	✓	✓	✓			✓	✓
AYTU-3/10/2020	✓		✓	✓					✓	✓			✓		✓	✓

AYTU-12/10/2020	✓	✓							✓	✓			✓			✓
BIOL-5/11/2018	✓		✓	✓					✓							✓
BIOL-3/18/2020	✓	✓			✓	✓			✓	✓		✓				
BLPH-6/24/2022	✓	✓		✓	✓										✓	
BNGO-6/14/2019			✓	✓					✓							
BNGO-8/30/2019	✓	✓	✓	✓					✓	✓					✓	✓
BNGO-10/16/2019	✓	✓	✓	✓					✓	✓			✓	✓	✓	✓
BNGO-10/18/2019	✓	✓	✓	✓					✓	✓	✓				✓	✓
BNGO-3/20/2020	✓	✓	✓	✓					✓	✓			✓		✓	✓
BNTC-6/24/2022		✓		✓					✓	✓			✓		✓	✓
BNTC-6/28/2022	✓	✓	✓	✓					✓	✓			✓			✓
BOXL-11/21/2018	✓	✓	✓	✓					✓	✓			✓		✓	✓
BOXL-3/14/2019			✓	✓					✓					✓		✓
BOXL-10/22/2019	✓		✓						✓	✓						✓
BOXL-3/13/2020	✓	✓	✓	✓					✓	✓						✓
BOXL-5/19/2020	✓	✓	✓	✓					✓	✓					✓	✓
BOXL-5/20/2020	✓	✓	✓	✓					✓	✓					✓	✓
BYFC-6/19/2020	✓	✓	✓						✓	✓						✓
CCRN-3/19/2020	✓	✓	✓	✓					✓	✓				✓	✓	✓
CEMI-7/22/2021	✓	✓	✓	✓						✓					✓	✓
CETX-5/23/2019	✓	✓	✓	✓						✓						✓
CETX-6/5/2019	✓	✓		✓						✓					✓	
CETX-6/13/2019	✓	✓							✓	✓					✓	✓
CETX-6/27/2019										✓					✓	✓
CETX-8/20/2019	✓	✓	✓	✓					✓	✓						
CFRX-6/6/2019	✓		✓						✓	✓			✓			
CGTX-8/4/2022	✓	✓	✓	✓					✓	✓				✓		
CLIR-5/13/2019	✓	✓	✓	✓					✓	✓						✓
CLNN-12/16/2022	✓	✓	✓	✓					✓			✓			✓	✓
CLRO-10/31/2018	✓	✓							✓	✓						✓
CLRO-11/16/2018		✓	✓	✓					✓	✓						✓
CLRO-3/24/2020	✓		✓						✓	✓				✓		✓
CLSD-9/4/2019	✓		✓						✓	✓			✓			✓
CMPI-12/21/2020									✓							✓
CMT-5/11/2020									✓	✓				✓		✓
CNSP-3/1/2022	✓	✓	✓	✓					✓	✓					✓	✓
CNST-11/6/2019																✓
CNST-12/9/2019	✓		✓	✓					✓	✓					✓	✓

COCP-3/9/2020	✓	✓	✓	✓					✓	✓					✓	✓
COGT-6/10/2022	✓	✓	✓	✓					✓	✓	✓					✓
COHN-7/8/2021	✓	✓	✓	✓					✓	✓						✓
CPHC-8/13/2020	✓								✓							✓
CRIS-6/10/2020	✓	✓	✓	✓					✓	✓						✓
CRVS-7/7/2020	✓	✓	✓						✓	✓						
CSLT-5/21/2020	✓	✓	✓	✓					✓	✓					✓	✓
CUEN-3/18/2022	✓	✓	✓	✓					✓	✓			✓			✓
CVM-1/27/2021	✓	✓	✓	✓					✓	✓			✓	✓	✓	✓
CVM-1/28/2021	✓	✓	✓	✓					✓	✓				✓	✓	✓
CVM-4/18/2022	✓	✓	✓	✓					✓	✓					✓	✓
CYCN-6/10/2022	✓	✓	✓	✓					✓	✓			✓		✓	✓
CYN-5/27/2022	✓	✓	✓	✓					✓	✓						✓
CYTO-11/12/2021	✓	✓	✓						✓	✓						
CYTO-3/11/2022	✓	✓	✓	✓					✓	✓						✓
DATS-10/22/2021			✓						✓				✓		✓	✓
DDE-1/12/2018									✓						✓	✓
DIT-9/21/2021	✓	✓	✓	✓						✓						✓
DMS-8/11/2022	✓	✓	✓	✓					✓	✓			✓		✓	✓
DRIO-9/18/2019	✓	✓	✓						✓				✓			✓
DRMA-6/10/2022	✓	✓	✓	✓						✓						
DRMA-6/30/2022	✓		✓	✓					✓	✓				✓		✓
DSS-3/3/2022	✓	✓	✓	✓					✓	✓					✓	✓
DXLG-3/23/2020	✓												✓			
DXLG-6/4/2020	✓	✓	✓	✓					✓	✓			✓		✓	✓
DYAI-12/9/2020	✓	✓	✓	✓	✓	✓			✓	✓		✓		✓	✓	✓
DYAI-7/27/2021	✓	✓	✓						✓	✓		✓				✓
ECOR-3/30/2020	✓	✓	✓	✓					✓	✓					✓	✓
EFTR-8/9/2022	✓	✓	✓	✓					✓	✓	✓		✓		✓	✓
ELOX-12/17/2020	✓	✓	✓	✓						✓				✓	✓	✓
ENVB-3/1/2022	✓	✓	✓						✓	✓						✓
EOLS-5/22/2020	✓	✓	✓	✓					✓	✓		✓			✓	✓
EVC-6/19/2020	✓	✓	✓	✓					✓	✓			✓	✓	✓	✓
EXPR-1/27/2021	✓	✓	✓	✓					✓	✓			✓		✓	✓
FATBB-11/3/2021	✓	✓	✓	✓					✓	✓						✓
FATBB-11/4/2021	✓	✓							✓							✓
FAT-3/19/2020	✓		✓						✓	✓					✓	
FAT-12/11/2020		✓	✓	✓					✓	✓						✓
FCEL-6/12/2019	✓	✓	✓	✓					✓	✓						

FRBA-3/16/2020			✓						✓	✓					✓	
FTEK-3/16/2020	✓		✓	✓												
FTK-2/17/2022	✓	✓	✓	✓	✓	✓				✓						
FTNW-2/26/2019	✓		✓	✓					✓	✓						✓
FUV-11/20/2019			✓						✓	✓						✓
GANX-4/12/2022	✓	✓	✓	✓					✓							✓
GDEN-3/16/2020	✓								✓	✓				✓	✓	✓
GFN-10/7/2020	✓	✓	✓	✓					✓	✓		✓				✓
GLSI-3/8/2021	✓	✓	✓	✓		✓			✓	✓					✓	✓
GLYC-6/12/2020	✓	✓	✓	✓					✓	✓				✓	✓	✓
GME-1/25/2021	✓	✓	✓						✓	✓					✓	
GME-1/28/2021	✓	✓	✓	✓					✓	✓			✓	✓	✓	✓
GME-2/5/2021	✓		✓						✓	✓			✓			✓
GNMK-2/28/2020	✓		✓	✓					✓	✓						✓
GNMX-1/2/2019	✓	✓	✓	✓						✓					✓	✓
GNMX-8/7/2019	✓	✓	✓	✓					✓	✓						✓
GRTS-1/20/2021																✓
GTXI-10/5/2018			✓	✓					✓	✓					✓	✓
GTXI-3/7/2019	✓	✓	✓	✓					✓	✓				✓	✓	✓
HALL-3/18/2020			✓	✓					✓	✓						✓
HALL-10/7/2022	✓		✓						✓						✓	✓
HARP-12/12/2022	✓	✓	✓	✓					✓	✓						✓
HBP-3/27/2020	✓		✓													
HBP-4/30/2020									✓							
HCWB-10/28/2021	✓	✓	✓	✓					✓	✓			✓	✓		
HILS-6/13/2022									✓	✓			✓			
HSDT-5/24/2022	✓		✓	✓					✓	✓						
HSGX-9/5/2018	✓		✓						✓						✓	✓
HSGX-10/4/2018	✓	✓	✓	✓					✓	✓			✓		✓	✓
HSGX-1/3/2019	✓	✓	✓	✓					✓	✓					✓	✓
HSGX-3/19/2019	✓	✓	✓						✓							✓
HYMC-3/11/2022	✓	✓	✓	✓					✓	✓				✓		✓
HZN-3/16/2020	✓	✓	✓	✓					✓						✓	✓
ICCC-1/29/2021	✓	✓	✓	✓					✓							
ICD-3/20/2020	✓	✓	✓	✓					✓	✓	✓				✓	
ICD-4/2/2020	✓	✓	✓						✓	✓						✓
ICD-4/9/2020	✓	✓	✓	✓					✓	✓			✓	✓	✓	✓
ICVX-12/15/2022	✓	✓	✓	✓					✓	✓	✓			✓	✓	✓
IDEX-3/20/2020	✓		✓						✓	✓			✓		✓	✓
IDEX-6/9/2020	✓	✓	✓	✓					✓	✓		✓	✓	✓	✓	✓

IMPL-8/19/2021		✓	✓	✓					✓	✓						✓
IMUX-3/25/2020	✓	✓	✓	✓					✓	✓					✓	
INKT-8/17/2022		✓	✓						✓	✓						✓
INMB-7/14/2020		✓	✓	✓											✓	
INTZ-7/27/2021	✓	✓	✓	✓					✓	✓				✓	✓	✓
INUV-4/13/2020	✓	✓	✓	✓					✓	✓					✓	✓
IRIX-8/9/2019	✓				✓	✓			✓			✓				
IRIX-8/12/2019						✓										
ISO-12/22/2022	✓	✓	✓	✓					✓	✓	✓				✓	✓
ISPC-11/26/2021	✓	✓	✓	✓					✓	✓						
ISPC-11/29/2021		✓	✓	✓						✓					✓	✓
ISPC-12/30/2021	✓	✓	✓	✓					✓	✓					✓	✓
IZEA-10/22/2021									✓							✓
JSPR-9/26/2022	✓		✓						✓	✓			✓		✓	✓
KELYB-8/11/2020		✓	✓	✓						✓					✓	✓
KELYB-8/21/2020		✓	✓						✓	✓						✓
KELYB-8/24/2020	✓	✓							✓	✓			✓			✓
KELYB-4/1/2021	✓	✓	✓	✓						✓					✓	✓
KELYB-4/5/2021									✓	✓						✓
KODK-7/29/2020		✓		✓					✓	✓			✓			✓
KODK-8/18/2020	✓	✓	✓	✓					✓	✓			✓		✓	✓
KOD-12/2/2019	✓	✓							✓	✓						✓
KOPN-4/21/2020	✓	✓	✓	✓					✓	✓					✓	✓
KTRA-4/11/2022	✓	✓	✓	✓					✓	✓				✓	✓	✓
LBPH-7/13/2021	✓		✓						✓	✓			✓			✓
LEE-1/29/2020	✓	✓								✓				✓		✓
LFLY-7/27/2022	✓	✓	✓	✓					✓	✓			✓		✓	✓
LIFE-12/11/2020	✓	✓	✓	✓					✓	✓					✓	✓
LINK-2/1/2019	✓	✓	✓	✓					✓	✓					✓	✓
LIXT-4/12/2022	✓		✓						✓	✓						✓
LOGC-8/13/2019	✓	✓	✓	✓	✓	✓										
LQDA-12/26/2019																✓
LTRPA-8/1/2022	✓	✓	✓	✓					✓	✓					✓	✓
LTRPB-7/16/2020	✓		✓	✓												
LTRPB-8/12/2020	✓	✓	✓						✓							
LTRPB-2/18/2021	✓		✓						✓							✓
LTRPB-3/31/2021	✓	✓	✓	✓					✓	✓					✓	
LTRPB-4/1/2021	✓	✓	✓						✓	✓						✓

LTRPB-7/13/2022									✓							✓
LTRPB-8/1/2022	✓	✓	✓	✓					✓	✓					✓	✓
LTRPB-8/2/2022	✓		✓						✓							✓
LTRPB-8/4/2022	✓	✓	✓	✓												
LTRY-8/3/2022	✓	✓	✓	✓					✓	✓						✓
LTRY-8/19/2022	✓	✓	✓	✓					✓	✓						✓
LTRY-10/3/2022	✓	✓	✓	✓						✓					✓	✓
LTRY-10/5/2022	✓	✓	✓	✓					✓	✓					✓	✓
LWAY-3/19/2020			✓	✓												
MBII-3/16/2020	✓		✓						✓	✓					✓	✓
MCRB-8/10/2020			✓	✓					✓	✓					✓	✓
MEDS-6/10/2021	✓	✓	✓	✓					✓	✓					✓	✓
MGYR-8/11/2020	✓		✓	✓						✓					✓	
MIRO-3/23/2022	✓	✓	✓	✓					✓						✓	✓
MITT-3/25/2020	✓	✓	✓	✓					✓	✓		✓			✓	✓
MITT-3/26/2020	✓	✓	✓	✓					✓	✓	✓			✓	✓	✓
MVIS-4/1/2020	✓		✓	✓					✓	✓					✓	✓
MVIS-4/2/2020			✓	✓					✓	✓						✓
NBSE-11/14/2022	✓	✓	✓	✓					✓	✓	✓				✓	✓
NBY-9/3/2019	✓		✓						✓							✓
NCSM-4/3/2020	✓	✓							✓	✓			✓	✓	✓	✓
NES-6/17/2020	✓		✓						✓				✓			✓
NES-6/18/2020	✓								✓						✓	
NES-12/13/2021	✓	✓	✓						✓	✓	✓		✓			✓
NES-1/3/2022	✓		✓						✓	✓					✓	✓
NINE-5/12/2020	✓	✓	✓	✓					✓	✓				✓	✓	✓
NINE-6/8/2020	✓	✓	✓	✓					✓	✓	✓		✓	✓	✓	✓
NRBO-6/14/2021	✓	✓	✓	✓					✓	✓			✓		✓	✓
NRBO-7/20/2021	✓	✓	✓	✓					✓	✓			✓		✓	
NTRP-1/22/2020	✓	✓	✓	✓					✓	✓			✓			✓
NUZE-11/11/2021										✓	✓	✓				✓
NWHM-4/8/2020	✓	✓	✓	✓					✓	✓	✓	✓	✓		✓	✓
NXTP-6/9/2022	✓	✓	✓	✓					✓	✓					✓	✓
OBLN-5/23/2019	✓	✓	✓						✓	✓						
OCN-3/20/2020	✓	✓	✓	✓					✓	✓					✓	
OLB-11/2/2021	✓	✓	✓	✓					✓	✓						✓
ONDK-3/25/2020									✓					✓		✓
OPGN-3/8/2021	✓	✓	✓	✓					✓	✓						✓
ORMP-7/12/2022	✓		✓						✓	✓	✓			✓		✓

PHUN-2/20/2020	✓	✓	✓	✓					✓	✓					✓	✓
PHUN-3/18/2020	✓	✓	✓	✓						✓				✓	✓	✓
PHUN-4/30/2020	✓	✓	✓	✓					✓	✓			✓		✓	✓
PHUN-5/20/2020	✓								✓	✓			✓			✓
PHUN-10/22/2021													✓		✓	✓
PIXY-3/9/2018	✓	✓	✓	✓						✓	✓					✓
PIXY-6/14/2022				✓						✓			✓		✓	✓
PNBK-6/17/2020	✓	✓	✓	✓										✓	✓	
PNRG-2/9/2021	✓	✓	✓	✓					✓	✓					✓	
POLA-3/8/2022	✓	✓	✓	✓					✓	✓		✓	✓			✓
PPSI-12/9/2021	✓	✓	✓	✓					✓	✓					✓	✓
PRPH-6/18/2020	✓		✓						✓	✓						
PRPL-8/14/2019	✓	✓	✓						✓	✓					✓	✓
PRPO-4/6/2020			✓	✓					✓	✓						✓
PRPO-5/21/2020	✓	✓	✓	✓					✓	✓					✓	✓
PRPO-6/24/2020	✓	✓	✓	✓					✓	✓					✓	✓
PRTH-6/16/2020	✓	✓	✓	✓						✓					✓	
PRTH-8/20/2020		✓	✓						✓							✓
PRVB-6/10/2019	✓		✓	✓					✓	✓				✓		✓
PTPI-5/17/2021	✓	✓	✓	✓					✓	✓						
PTPI-12/3/2021	✓	✓	✓	✓					✓	✓						✓
PZG-1/29/2021	✓	✓	✓	✓					✓	✓						✓
QES-3/20/2020	✓		✓						✓	✓					✓	
QLGN-11/29/2021			✓	✓		✓			✓	✓			✓	✓	✓	✓
QLGN-4/12/2022		✓							✓	✓						✓
REI-3/9/2020	✓	✓	✓						✓	✓					✓	✓
RIBT-4/27/2021	✓		✓						✓							✓
RMBL-5/20/2020	✓		✓	✓					✓	✓					✓	✓
RNAZ-12/8/2022	✓	✓	✓	✓					✓	✓						
SAI-8/9/2022	✓	✓	✓	✓					✓	✓					✓	✓
SALM-10/22/2021			✓	✓					✓	✓			✓			✓
SBT-3/19/2020	✓	✓	✓	✓						✓						✓
SCPH-3/16/2020	✓	✓	✓	✓						✓						✓
SD-3/20/2020	✓	✓	✓	✓						✓					✓	✓
SFT-6/23/2022	✓		✓						✓				✓			✓
SGRP-7/26/2021	✓	✓	✓						✓	✓		✓				✓
SIEB-1/29/2021	✓		✓						✓	✓			✓			✓
SLGG-1/14/2020									✓	✓	✓					✓
SLNG-9/21/2022									✓							✓
SLNO-3/15/2019	✓	✓	✓	✓						✓	✓			✓	✓	✓

SLNO-12/26/2019	✓		✓						✓						✓	✓
SLS-7/3/2019	✓	✓	✓	✓					✓	✓			✓	✓		✓
SNES-2/5/2020	✓		✓	✓					✓							✓
SNSS-4/15/2020	✓								✓				✓		✓	✓
SOHO-3/12/2020	✓	✓	✓	✓					✓	✓					✓	✓
SOHO-3/20/2020	✓		✓						✓	✓			✓		✓	✓
SONM-3/20/2020	✓	✓	✓	✓					✓	✓			✓			✓
SONM-4/22/2020	✓								✓							✓
SONN-7/31/2020	✓		✓						✓				✓			✓
SPRO-9/6/2022	✓	✓	✓	✓					✓	✓					✓	✓
SPRO-9/22/2022	✓	✓	✓	✓					✓	✓					✓	✓
SPRT-8/27/2021	✓	✓	✓	✓					✓	✓					✓	✓
SRRK-6/17/2022	✓	✓	✓	✓					✓	✓	✓		✓		✓	✓
SSNT-11/15/2021	✓	✓	✓						✓	✓		✓				✓
SSNT-1/5/2022	✓		✓						✓	✓						✓
STIM-6/23/2020	✓	✓	✓	✓					✓	✓						✓
STKS-3/27/2020		✓	✓	✓					✓	✓						✓
STRM-3/16/2021	✓	✓	✓	✓					✓	✓			✓			✓
STSA-9/10/2020	✓	✓	✓	✓					✓	✓			✓	✓	✓	✓
SUNW-1/8/2020			✓						✓	✓						✓
SVRA-3/16/2021	✓	✓	✓	✓					✓	✓			✓	✓		✓
SYBX-1/2/2018	✓	✓	✓	✓						✓					✓	✓
SYBX-11/25/2020	✓	✓	✓	✓					✓	✓		✓		✓	✓	✓
SYPR-8/15/2019		✓		✓												
TAT-5/31/2019									✓	✓		✓			✓	
TNXP-6/1/2022	✓	✓	✓	✓					✓	✓						✓
TRKA-10/22/2021									✓	✓						✓
TRVI-3/31/2020	✓	✓	✓	✓					✓	✓					✓	✓
TRVI-8/11/2020		✓							✓				✓			✓
TRVI-3/2/2022	✓	✓	✓	✓					✓	✓			✓		✓	✓
TRVN-11/2/2018	✓	✓	✓	✓					✓	✓						✓
TTPH-5/7/2020									✓				✓	✓		✓
TXMD-1/4/2022	✓		✓						✓				✓			✓
TYME-2/3/2021									✓							✓
UONEK-6/16/2020	✓	✓	✓	✓					✓	✓					✓	✓
UONE-5/31/2018	✓	✓	✓	✓					✓						✓	✓
UONE-6/7/2018		✓	✓	✓					✓	✓						✓
UONE-8/6/2018	✓		✓						✓	✓				✓		
UONE-6/12/2020	✓		✓						✓	✓						✓

UONE-6/16/2020	✓	✓	✓	✓					✓	✓			✓		✓	✓
UONE-6/18/2020	✓	✓	✓	✓					✓	✓					✓	✓
UONE-6/19/2020	✓		✓						✓	✓						
USWS-4/2/2020	✓		✓						✓	✓				✓	✓	✓
USWS-4/9/2020			✓	✓					✓	✓			✓			✓
VCNX-11/10/2020	✓	✓	✓	✓					✓	✓				✓	✓	✓
VCNX-2/19/2021	✓	✓	✓						✓	✓						✓
VCNX-8/8/2022				✓					✓				✓			✓
VERB-7/21/2020									✓							
VERY-4/8/2021	✓	✓	✓						✓	✓			✓			✓
VIEW-10/27/2022	✓	✓							✓	✓				✓		✓
VNCE-12/3/2020	✓	✓	✓	✓					✓	✓						
VTL-9/12/2018	✓		✓						✓	✓				✓		✓
VTSI-7/10/2019	✓	✓	✓						✓							✓
VTVT-4/17/2018	✓	✓	✓	✓					✓	✓					✓	✓
VTVT-10/3/2018	✓	✓	✓	✓						✓	✓		✓		✓	✓
VTVT-12/28/2018	✓	✓	✓	✓					✓	✓			✓	✓		✓
VUZI-3/31/2020			✓	✓										✓	✓	✓
VVOS-1/26/2022	✓	✓	✓						✓	✓						✓
VVOS-12/20/2022	✓								✓	✓					✓	✓
VVPR-10/22/2018	✓		✓						✓	✓					✓	
VXRT-10/4/2018	✓	✓	✓	✓					✓	✓						✓
VXRT-3/19/2019	✓	✓	✓						✓	✓		✓				✓
VXRT-9/26/2019		✓	✓	✓	✓				✓	✓					✓	✓
WATT-4/21/2020	✓	✓	✓						✓	✓			✓			
WISA-7/22/2021	✓		✓						✓	✓						✓
WRAP-6/24/2022			✓	✓					✓	✓				✓	✓	✓
WVE-8/11/2022	✓	✓	✓	✓					✓	✓						✓
XELA-4/30/2020	✓		✓	✓					✓	✓			✓			✓
XELA-7/8/2021	✓	✓	✓	✓					✓	✓	✓			✓	✓	✓
XLO-7/5/2022	✓	✓	✓	✓					✓	✓					✓	✓
YCBD-4/8/2020		✓	✓	✓					✓	✓					✓	✓
ZOM-6/13/2019	✓		✓	✓					✓	✓					✓	✓
ZY-8/11/2022	✓		✓						✓			✓	✓			✓

APPENDIX D: ADDITIONAL REGRESSIONS

Specification	***17***	***18***
Dependent Variable	**Log_OpenHigh**	**BoxCox_OpenHigh**
Constant	0.432	0.348
Low Float (<=25th percnetile)	-0.132	-0.170
30-day Volume to TSO Ratio	-0.218	-0.229
High Short (>=75th percentile)	0.100	0.105
Retail Volume to Total Volume, 1-Day Lag	0.365	0.450
Low Price (<=$5)	-0.248**	-0.279**
Log Market Cap	-0.047	-0.050
Pre Covid Dummy	-0.143	-0.194
News Filing Dummy	0.136	0.164
VIX	-0.004	-0.006*
Russell Microcap Index Return	1.590	1.712
NYSE Dummy	0.153	0.205*
Average Effective Spread (5-day lag)	1.971*	1.892
60-Day Momentum	-0.036	-0.043
1-year EPS Growth	-0.001	0.002
30-Day Variance of Returns	0.830	0.574
10-year T-Bill	0.002	0.022
Monthly CPI Rate	-0.002	-0.006
Communications Sector	0.257*	0.270
Consumer Discretionary	-0.026	0.015
Consumer Staples	0.227	0.409
Energy	-0.054	-0.054
Financials	0.036	0.051
Industrials	-0.124	-0.126
Information Technology	0.137	0.192
Materials	-0.065	-0.082
Real Estate	-0.286	-0.409
Number of Observations	286	286
R^2	0.139	0.132
Adjusted R^2	0.052	0.044
F-Statistic	1.603 (.0356)	1.508 (.0584)

Note: Standard errors are used in both specifications as heteroskedasticity is not present.
Unstandardized Beta significance levels are denoted as *** p<0.01, ** p<0.05, * p<0.1.