Recentish Results

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In this document, I collect results from tests using the faster, better organized version of the Python code.

Tests for Hejhal's Algorithm

The code assumes we are working on a Hecke triangle group generated by $z \mapsto z+1$ and $z \mapsto -R^2/z$ for some $R > 0$. Here are a few values of R and corresponding ν values which are known to come from a true Maass form.

- $R = 1$, $\nu = i9.5336952613\ldots$
  - this is the group $SL(2,\mathbb{Z})$
- $R = 1/\sqrt{2}$, $\nu = i7.220872\ldots$
  - this is the congruence group Γ_4
- $R = 1/\sqrt{2}$, $\nu = i11.317680\ldots$
  - this is also the congruence group Γ_4, just a different eigenvalue
- $R = 7/20$, $\nu = 0.26705241700910205677150208864259506276668(7)\ldots$
  - note that this is an infinite volume fundamental domain, since $7/20 < 1/2$
  - note that ν is real in this example

Ideas to Test the Disk Model

I have been attempting tests to determine what is going wrong with the disk model. Here are some tests that might give some information.

1. Run both models (upper half plane and disk) with a correct eigenvalue as input. The linear system should thus return approximately correct Fourier coefficients. Use these coefficients to evaluate the Maass form at some points. Just checking at one point is not enough, since the Maass form is only unique up to scaling. So check the ratio at two points, and check if the two models give the same value.

2. Run the disk model with the correct eigenvalue as input. Check that the Fourier coefficients given by the two sets of test points (at different ρ values) are close to each other.

3. Use the coefficients from the upper half plane model (which I know are correct, since they match those found on Strömbergsson's website) to evaluate the Maass form at a collection of test points. Set up a linear system for the expansion in the disk model to solve for those Fourier coefficients. This should give the correct Foruier coefficients, since the function values are verified to be approximately correct.

4. Put the correct coefficients obtained from test 3 into the linear system for the disk model. They should solve the linear system.

Dec 8 2019

I ran the ratio test (test 1 above) with the following input:

$$(D, M_0, N, R, Y_1, \rho_1) = (100, 100, 250, 1, 1/3, 1/2)$$

The results were: Ratio for UHP:

$$(-0.1537167192 + 0.000000000000000290359490i)$$

Ratio for Disk:

$$(-0.1502925464 + 0.005967181881i)$$

So these are reasonably close!

I ran the test again with the following input:

$$(D, M_0, N, R, Y_1, \rho_1) = (100, 100, 250, 1, 1/3, 1/2)$$

(so I just changed the radius ρ_1). The results were: Ratio for UHP:

$$(-0.1537167192 + 0.0000000000000002903594904i)$$

Ratio for Disk:

$$(-0.1537167191 - 0.0000000001309977764i)$$

This did a lot better!

Dec 9 2018

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.666666666667, 0.9, 1)$$

Best ν found: $9.1873i$
Incorrect!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.5, 0.666666666667, 1)$$

Best ν found: $9.5328i$
Partially correct. Correct to 2 decimal places.

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.5, 0.9, 1)$$

Best ν found: $9.5328i$
Partially correct. Correct to 2 decimal places.

Dec 11 2019

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.333333333333, 0.666666666667, 1)$$

Best ν found: $9.5596i$
Incorrect!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.333333333333, 0.9, 1)$$

Best ν found: $9.5596i$
Incorrect!

Dec 12 2019

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 50, 150, 0.5, 0.9, 1)$$

Best ν found: $8.95237i$

Incorrect!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (200, 50, 150, 0.5, 0.9, 1)$$

Best ν found: $8.94445i$

Incorrect!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (200, 100, 150, 0.5, 0.9, 1)$$

Best ν found: $8.94445i$

Incorrect!

Thoughts

It appears that decreasing N, the number of test points, caused the algorithm in the disk model to fail. So the next test will be to try higher values of N.

Dec 16 2019

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (50, 50, 250, 0.5, 0.9, 1)$$

Best ν found: $8.94445i$

Incorrect!.

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 50, 250, 0.5, 0.9, 1)$$

Best ν found: $8.94445i$

Incorrect!.

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.5, 0.9, 1)$$

Best ν found: $9.5328i$

Partially correct. Correct to 2 decimal places.

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 500, 0.5, 0.9, 1)$$

Best ν found: $8.94445i$

Incorrect!.

Dec 19 2019

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.5, 0.9, 0.707106781186547)$$

Best ν found: $7.03346i$
Incorrect!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.5, 0.9, 0.35)$$

Best ν found: 0.20445
Incorrect!

Dec 27 2019

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (32, 10, 20, 0.5, 0.1, 1)$$

Best ν found: $9.53369i$
Correct!

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (64, 15, 30, 0.5, 0.1, 1)$$

Best ν found: $9.5337i$
Correct!

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (32, 10, 20, 0.5, 0.1, 0.707106781186547\dots)$$

Best ν found: $7.22087i$
Correct!

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (32, 10, 20, 0.5, 0.1, 0.707106781186547\dots)$$

Best ν found: $11.31769i$
Partially correct. Correct to 4 decimal places.

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (64, 15, 30, 0.5, 0.1, 0.707106781186547\dots)$$

Best ν found: $7.22087i$
Correct!

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (64, 15, 30, 0.5, 0.1, 0.707106781186547\dots)$$

Best ν found: $11.31768i$
Correct!

Jan 8 2020

UHP model (with flares)

$$(D, M_0, N, R) = (32, 10, 20, 1)$$

Best ν found: $9.8526i$

Incorrect!

UHP model (with flares)

$$(D, M_0, N, R) = (64, 10, 20, 1)$$

Best ν found: $9.8526i$

Incorrect!

UHP model (with flares)

$$(D, M_0, N, R) = (64, 15, 30, 1)$$

Best ν found: $9.5337i$

Correct!

UHP model (with flares)

$$(D, M_0, N, R) = (64, 25, 50, 1)$$

Best ν found: $9.5337i$

Correct!

Jan 12 2018

UHP model (with flares)

$$(D, M_0, N, R) = (64, 10, 21, 0.35)$$

Best ν found: 0.3142

Incorrect!