Recent Results

Alex Karlovitz

In this document, I collect results from tests using the faster, better organized version of the code.

Note: all of the tests in this document which used the disk method failed because we were scaling the constant term of the Fourier expansion to 0**.**

Nov 15 2019

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (50, 30, 200, 0.5, 0.1, 1)$$

Best ν found: $9.5337i$
Correct!

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (50, 30, 150, 0.5, 0.1, 1)$$

Best ν found: $9.5337i$
Correct!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (50, 50, 150, 0.666666666667, 0.9, 1)$$

Best ν found: $9.3888i$
Incorrect!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 300, 0.666666666667, 0.9, 1)$$

Best ν found: $9.3266i$
Incorrect!

Nov 17 2019

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (50, 30, 100, 0.5, 0.1, 1)$$

Best ν found: $9.5337i$
Correct!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 150, 500, 0.666666666667, 0.9, 1)$$

Best ν found: $9.8455i$
Incorrect!

Nov 18 2019

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (50, 30, 61, 0.5, 0.1, 1)$$

Best ν found: $9.5337i$
Correct!

Nov 22 2019

UHP model with

$$(D, M_0, N, Y_1, Y_2, R) = (50, 50, 200, 0.333333333333, 0.05, 0.707106781186547)$$

Best ν found: $7.2209i$
Correct!

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (50, 50, 200, 0.666666666667, 0.9, 0.707106781186547)$$

Best ν found: $7.0784i$
Incorrect!

Ideas to Test the Disk Model

I have been attempting tests to determine what is going wrong with the disk model. Here are some tests that might give some information.

1. Run both models (upper half plane and disk) with a correct eigenvalue as input. The linear system should thus return approximately correct Fourier coefficients. Use these coefficients to evaluate the Maass form at some points. Just checking at one point is not enough, since the Maass form is only unique up to scaling. So check the ratio at two points, and check if the two models give the same value.

2. Run the disk model with the correct eigenvalue as input. Check that the Fourier coefficients given by the two sets of test points (at different ρ values) are close to each other.

3. Use the coefficients from the upper half plane model (which I know are correct, since they match those found on Strömbergsson's website) to evaluate the Maass form at a collection of test points. Set up a linear system for the expansion in the disk model to solve for those Fourier coefficients. This should give the correct Foruier coefficients, since the function values are verified to be approximately correct.

4. Put the correct coefficients obtained from test 3 into the linear system for the disk model. They should solve the linear system.

Nov 25 2019

I attempted test 3 described above with two different inputs. I used the ratio idea from test 1 to verify that the Fourier coefficients found were correct.

Using $M_0 = 50$ and $D = 50$, I took $N = 2M_0$ test points equally spaced around a circle of radius $9/10$ to set up the system. The ratio test did **not** give the same values.

Using $M_0 = 100$ and $D = 100$, I took $N = 2M_0$ test points equally spaced around a circle of radius $9/10$ to set up the system. The ratio test did give the same values, but only to a precision of about 2×10^{-4}. I

am starting to wonder if the expansion in the disk model requires much higher precision than that of the upper half plane model to work.

Using $M_0 = 50$ and $D = 50$, I took $N = 2M_0$ test points equally spaced around a circle of radius $1/2$ to set up the system. The ratio test did work, to a precision of about 10^{-14}. It appears that the system is much better conditioned when the radius is relatively far from 1.

Nov 27 2019

Disk model with

$$(D, M_0, N, P_1, P_2, R) = (100, 100, 250, 0.5, 0.666666666667, 1)$$

Best ν found: $10.0555i$

Incorrect!

I tried the ratio test (test 1 described above) with various inputs, and none of them were working. However, I noticed that for all my tests I was using a radius of $2/3$ for the test points for the disk model. Inspired by the tests on Nov 25, I reduced the radius of those test points.

I tried the ratio test with the following inputs:

$$d = 100, M_0 = 100, N = 250, Y = \frac{1}{3}, \rho = \frac{1}{2}$$

The outputs were as follows:

Ratio for UHP:

$$(-0.15371 + 0.00000000000000029035i)$$

Ratio for Disk:

$$(-0.14936 + 0.0046816i)$$

This is the first time the ratio test came even remotely close to working.

I tried the same test as the previous one with $\rho = \frac{1}{3}$, and the results were again wildly off.