Numeric types in Python:

```
>>> type(2)
<class 'int'>
>>> type(1.5)
<class 'float'>
>>> type(1+1j)
<class 'complex'>
```

Represents integers exactly

Represents real numbers approximately

Rational implementation using functions:

```
def rational(n, d):
    def select(name):
        if name == 'n':
            return n
        elif name == 'd':
            return d
    return select

def numer(x):
    return x('n')

def denom(x):
    return x('d')
```

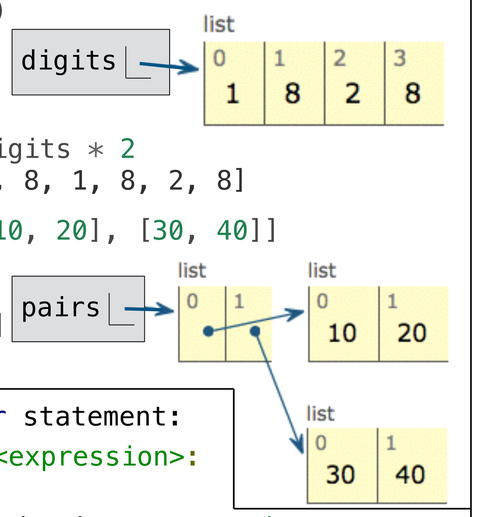
This function represents a rational number

Constructor is a higher-order function

Selector calls x

Lists:

```
>>> digits = [1, 8, 2, 8]
>>> len(digits)
4
>>> digits[3]
8
>>> [2, 7] + digits * 2
[2, 7, 1, 8, 2, 8, 1, 8, 2, 8]
>>> pairs = [[10, 20], [30, 40]]
>>> pairs[1]
[30, 40]
>>> pairs[1][0]
30
```



Executing a `for` statement:

```
for <name> in <expression>:
    <suite>
```

1. Evaluate the header <expression>, which must yield an iterable value (a sequence)
2. For each element in that sequence, in order:
   A. Bind <name> to that element in the current frame
   B. Execute the <suite>

Unpacking in a for statement:

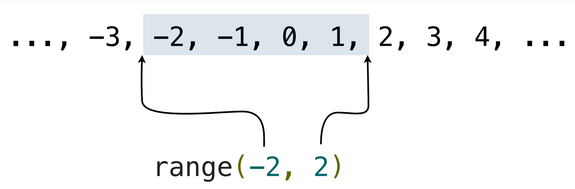
A sequence of fixed-length sequences

```
>>> pairs=[[1, 2], [2, 2], [3, 2], [4, 4]]
>>> same_count = 0
```

A name for each element in a fixed-length sequence

```
>>> for x, y in pairs:
...     if x == y:
...         same_count = same_count + 1

>>> same_count
2
```



Length: ending value − starting value

Element selection: starting value + index

```
>>> list(range(-2, 2))
[-2, -1, 0, 1]
```

List constructor

```
>>> list(range(4))
[0, 1, 2, 3]
```

Range with a 0 starting value

List comprehensions:

[<map exp> for <name> in <iter exp> if <filter exp>]

Short version: [<map exp> for <name> in <iter exp>]

A combined expression that evaluates to a list using this evaluation procedure:

1. Add a new frame with the current frame as its parent
2. Create an empty *result list* that is the value of the expression
3. For each element in the iterable value of <iter exp>:
   A. Bind <name> to that element in the new frame from step 1
   B. If <filter exp> evaluates to a true value, then add the value of <map exp> to the result list

The result of calling **repr** on a value is what Python prints in an interactive session

The result of calling **str** on a value is what Python prints using the **print** function

```
>>> 12e12
12000000000000.0
>>> print(repr(12e12))
12000000000000.0
```

```
>>> print(today)
2014-10-13
```

str and **repr** are both polymorphic; they apply to any object
repr invokes a zero-argument method __repr__ on its argument

```
>>> today.__repr__()
'datetime.date(2014, 10, 13)'
>>> today.__str__()
'2014-10-13'
```

```
>>> suits = ['coin', 'string', 'myriad']
>>> suits.pop()
'myriad'
>>> suits.remove('string')
>>> suits.append('cup')
>>> suits.extend(['sword', 'club'])
>>> suits[2] = 'spade'
>>> suits
['coin', 'cup', 'spade', 'club']
>>> suits[0:2] = ['diamond']
>>> suits
['diamond', 'spade', 'club']
>>> suits.insert(0, 'heart')
>>> suits
['heart', 'diamond', 'spade', 'club']
```

Remove and return the last element

Remove a value

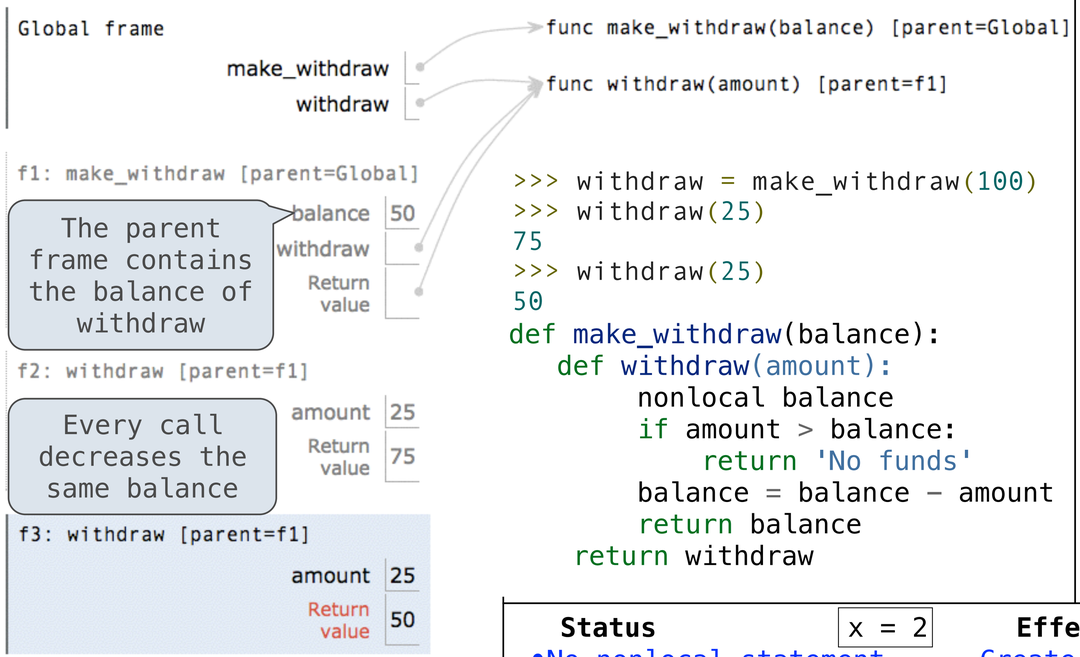
Add all values

Replace a slice with values

Add an element at an index

Identity:
<exp0> **is** <exp1>
evaluates to True if both <exp0> and <exp1> evaluate to the same object
Equality:
<exp0> **==** <exp1>
evaluates to True if both <exp0> and <exp1> evaluate to equal values
Identical objects are always equal values

You can **copy** a list by calling the list constructor or slicing the list from the beginning to the end.




```
>>> withdraw = make_withdraw(100)
>>> withdraw(25)
75
>>> withdraw(25)
50
def make_withdraw(balance):
    def withdraw(amount):
        nonlocal balance
        if amount > balance:
            return 'No funds'
        balance = balance - amount
        return balance
    return withdraw
```

List & dictionary mutation:

```
>>> a = [10]
>>> b = a
>>> a == b
True
>>> a.append(20)
>>> a == b
True
>>> a
[10, 20]
>>> b
[10, 20]
```

```
>>> a = [10]
>>> b = [10]
>>> a == b
True
>>> b.append(20)
>>> a
[10]
>>> b
[10, 20]
>>> a == b
False
```

```
>>> nums = {'I': 1.0, 'V': 5, 'X': 10}
>>> nums['X']
10
>>> nums['I'] = 1
>>> nums['L'] = 50
>>> nums
{'X': 10, 'L': 50, 'V': 5, 'I': 1}
>>> sum(nums.values())
66
>>> dict([(3, 9), (4, 16), (5, 25)])
{3: 9, 4: 16, 5: 25}
>>> nums.get('A', 0)
0
>>> nums.get('V', 0)
5
>>> {x: x*x for x in range(3,6)}
{3: 9, 4: 16, 5: 25}
```

Strings as sequences:

```
>>> city = 'Berkeley'
>>> len(city)
8
>>> city[3]
'k'
>>> 'here' in "Where's Waldo?"
True
>>> 234 in [1, 2, 3, 4, 5]
False
>>> [2, 3, 4] in [1, 2, 3, 4]
False
```

Membership:

```
>>> digits = [1, 8, 2, 8]
>>> 2 in digits
True
>>> 1828 not in digits
True
```

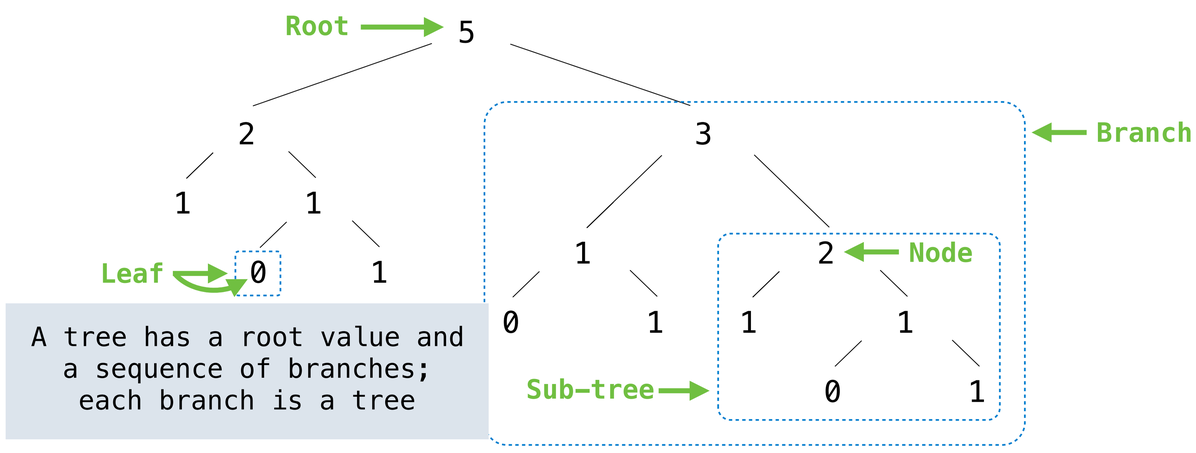
Slicing:

```
>>> digits[0:2]
[1, 8]
>>> digits[1:]
[8, 2, 8]
```

Slicing creates a new object

Status	x = 2	Effect
•No nonlocal statement •"x" **is not** bound locally		Create a new binding from name "x" to number 2 in the first frame of the current environment
•No nonlocal statement •"x" **is** bound locally		Re-bind name "x" to object 2 in the first frame of the current environment
•nonlocal x •"x" **is** bound in a non-local frame		Re-bind "x" to 2 in the first non-local frame of the current environment in which "x" is bound
•nonlocal x •"x" **is not** bound in a non-local frame		SyntaxError: no binding for nonlocal 'x' found
•nonlocal x •"x" **is** bound in a non-local frame •"x" also bound locally		SyntaxError: name 'x' is parameter and nonlocal

Tree data abstraction:



A tree has a root value and a sequence of branches; each branch is a tree

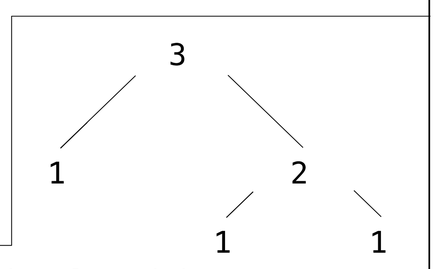
```
def tree(root, branches=[]):
    for branch in branches:
        assert is_tree(branch)
    return [root] + list(branches)
def root(tree):
    return tree[0]
def branches(tree):
    return tree[1:]
def is_tree(tree):
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
def is_leaf(tree):
    return not branches(tree)
def leaves(tree):
    """The leaf values in tree.

    >>> leaves(fib_tree(5))
    [1, 0, 1, 0, 1, 1, 0, 1]
    """
    if is_leaf(tree):
        return [root(tree)]
    else:
        return sum([leaves(b) for b in branches(tree)], [])
```

Verifies the tree definition

Creates a list from a sequence of branches

Verifies that tree is bound to a list




```
>>> tree(3, [tree(1),
...          tree(2, [tree(1),
...                   tree(1)])])
[3, [1], [2, [1], [1]]]
```

```
def fib_tree(n):
    if n == 0 or n == 1:
        return tree(n)
    else:
        left = fib_tree(n-2),
        right = fib_tree(n-1)
        fib_n = root(left) + root(right)
        return tree(fib_n, [left, right])
```

```
class Tree:
    def __init__(self, entry, branches=()):
        self.entry = entry
        for branch in branches:
            assert isinstance(branch, Tree)
        self.branches = list(branches)
    def is_leaf(self):
        return not self.branches
def leaves(tree):
    if tree.is_leaf():
        return [tree.entry]
    else:
        return sum([leaves(b) for b in tree.branches], [])
```

Built-in isinstance function: returns True if branch has a class that *is* **or** *inherits from* Tree

```
def fib_Tree(n):
    if n == 0 or n == 1:
        return Tree(n)
    else:
        left = fib_Tree(n-2)
        right = fib_Tree(n-1)
        fib_n = left.entry+right.entry
        return Tree(fib_n,[left, right])
```

```
class Link:
    empty = ()
    def __init__(self, first, rest=empty):
        self.first = first
        self.rest = rest
    def __getitem__(self, i):
        if i == 0:
            return self.first
        else:
            return self.rest[i-1]
    def __len__(self):
        return 1 + len(self.rest)
    def __repr__(self):
        if self.rest:
            rest_str = ', ' + repr(self.rest)
        else:
            rest_str = ''
        return 'Link({0}{1})'.format(self.first, rest_str)
def extend_link(s, t):
    """Return a Link with the
    elements of s followed by
    those of t.
    """
    if s is Link.empty:
        return t
    else:
        return Link(s.first, extend_link(s.rest, t))
def map_link(f, s):
    if s is Link.empty:
        return s
    else:
        return Link(f(s.first), map_link(f, s.rest))
```

Some zero length sequence

Sequence abstraction special names:

`__getitem__`	Element selection []
`__len__`	Built-in len function

Yes, this call is recursive

Contents of the repr string of a Link instance

```
>>> s = Link(3, Link(4))
>>> extend_link(s, s)
Link(3, Link(4, Link(3, Link(4))))
>>> square = lambda x: x * x
>>> map_link(square, s)
Link(9, Link(16))
```

Python object system:

Idea: All bank accounts have a **balance** and an account **holder**; the **Account** class should add those attributes to each of its instances

A new instance is created by calling a class

```
>>> a = Account('Jim')
>>> a.holder
'Jim'
>>> a.balance
0
```

An account instance

balance: 0 holder: 'Jim'

When a class is called:
1. A new instance of that class is created:
2. The `__init__` method of the class is called with the new object as its first argument (named `self`), along with any additional arguments provided in the call expression.

`__init__` is called a constructor

```
class Account:
    def __init__(self, account_holder):
        self.balance = 0
        self.holder = account_holder
    def deposit(self, amount):
        self.balance = self.balance + amount
        return self.balance
    def withdraw(self, amount):
        if amount > self.balance:
            return 'Insufficient funds'
        self.balance = self.balance - amount
        return self.balance
```

self should always be bound to an instance of the Account class or a subclass of Account

Function call: all arguments within parentheses

Method invokation: One object before the dot and other arguments within parentheses

```
>>> type(Account.deposit)
<class 'function'>
>>> type(a.deposit)
<class 'method'>
>>> Account.deposit(a, 5)
10
>>> a.deposit(2)
12
```

Dot expression

Call expression

`<expression> . <name>`

The `<expression>` can be any valid Python expression.
The `<name>` must be a simple name.
Evaluates to the value of the attribute looked up by `<name>` in the object that is the value of the `<expression>`.

To evaluate a dot expression:
1. Evaluate the `<expression>` to the left of the dot, which yields the object of the dot expression
2. `<name>` is matched against the instance attributes of that object; if an attribute with that name exists, its value is returned
3. If not, `<name>` is looked up in the class, which yields a class attribute value
4. That value is returned unless it is a function, in which case a bound method is returned instead

Assignment statements with a dot expression on their left-hand side affect attributes for the object of that dot expression
- If the object is an instance, then assignment sets an instance attribute
- If the object is a class, then assignment sets a class attribute

Account class attributes

interest: ~~0.02~~ ~~0.04~~ 0.05
(withdraw, deposit, __init__)

Instance attributes of jim_account

balance: 0
holder: 'Jim'
interest: 0.08

Instance attributes of tom_account

balance: 0
holder: 'Tom'

```
>>> jim_account = Account('Jim')
>>> tom_account = Account('Tom')
>>> tom_account.interest
0.02
>>> jim_account.interest
0.02
>>> Account.interest = 0.04
>>> tom_account.interest
0.04
>>> jim_account.interest
0.04
```

```
>>> jim_account.interest = 0.08
>>> jim_account.interest
0.08
>>> tom_account.interest
0.04
>>> Account.interest = 0.05
>>> tom_account.interest
0.05
>>> jim_account.interest
0.08
```

```
class CheckingAccount(Account):
    """A bank account that charges for withdrawals."""
    withdraw_fee = 1
    interest = 0.01
    def withdraw(self, amount):
        return Account.withdraw(self, amount + self.withdraw_fee)
        # or
        return super().withdraw(amount + self.withdraw_fee)
```

To look up a name in a class:
1. If it names an attribute in the class, return the attribute value.
2. Otherwise, look up the name in the base class, if there is one.

```
>>> ch = CheckingAccount('Tom')  # Calls Account.__init__
>>> ch.interest     # Found in CheckingAccount
0.01
>>> ch.deposit(20)  # Found in Account
20
>>> ch.withdraw(5)  # Found in CheckingAccount
14
```