UC Berkeley EECS
Adj. Ass. Prof.
Dr. Gerald Friedland

Computational Structures in Data Science

Lecture #4: Recursion II and Higher Order Functions

Hackers steal medical data of US Olympic stars

http://money.cnn.com/2016/09/13/news/wada-hacked-russian-spies/index.html?iid=surge-story-summary

This was: September 16, 2016

Administrative issues

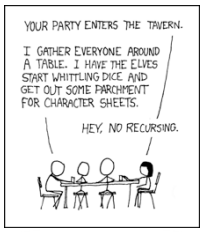
- Waitlist should be cleared.
- Based on your feedback: I will record optional lectures going deeper on data, code, algorithms, information, recursion, decision trees, run"time" complexity and other stuff.
- Speaking of recording: ETS will start capturing.

Computational Concepts today

- More on Recursion
- Runtime (preliminary)
- Higher Order Functions
- Functions as Values
- Functions with functions as argument
- Assignment of function values
- Higher order function patterns
  - Map, Filter, Reduce
- Function factories – create and return functions

Remember: Sanity Check…

- Recursion is ■ Iteration (i.e., loops)
  a) more powerful than
  b) just as powerful as
  c) less powerful than

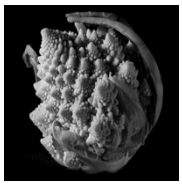


Why Recursion?

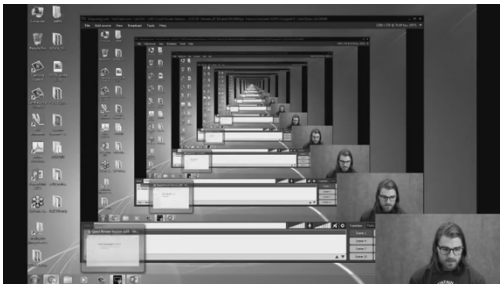
- "After Abstraction, Recursion is probably the 2nd biggest idea in this course"
- "It's tremendously useful when the problem is self-similar"
- "It's no more powerful than iteration, but often leads to more concise & better code"
- "It's more 'mathematical'"
- "It embodies the beauty and joy of computing"
- …

Why Recursion? More Reasons

- Recursive structures exist (sometimes hidden) in nature and therefore in data!
- It's mentally and sometimes computationally more efficient to process recursive structures using recursion.

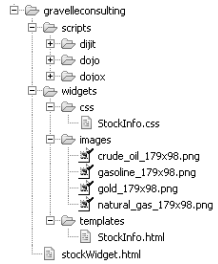


Recursion (unwanted)



Example I

List all items on your hard disk



- **Files**
- **Folders contain**
  - **Files**
  - **Folders**

Recursion!

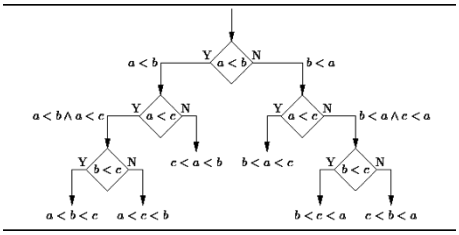
List Files in Python

```
def listfiles(directory):
    content = [os.path.join(directory, x) for x in os.listdir(directory)]

    dirs = sorted([x for x in content if os.path.isdir(x)])
    files = sorted([x for x in content if os.path.isfile(x)])

    for d in dirs:
        print d
        listfiles(d)

    for f in files:
        print f
```

Iterative version about twice as much code and much harder to think about.

Example II

Sort the numbers in a list.



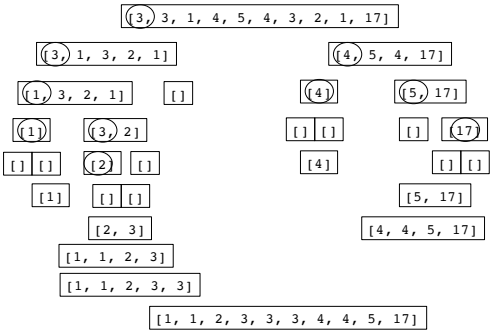
Hidden recursive structure: Decision tree!

Tree Recursion makes Sorting Efficient

Break the problem into multiple smaller sub-problems, and solve them recursively

```
def split(x, s):
    return [i for i in s if i <= x], [i for i in s if i > x]

def qsort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""
    if not s:
        return []
    else:
        pivot = first(s)
        lessor, more = split(pivot, rest(s))
        return qsort(lessor) + [pivot] + qsort(more)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```


QuickSort Example



More on Recursion

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return n**2 + sum_of_squares(n-1)
```

- The sum of no numbers is zero
- The sum of 1^2 through n^2 is n^2 plus the sum of 1^2 through $(n-1)^2$

Recap: Tail Recursion

- All the work happens on the way down the recursion
- On the way back up, just return

```
def sum_up_squares(i, n, accum):
    """Sum the squares from i to n in incr. order"""
    if i > n:
        return accum
    else:
        return sum_up_squares(i+1, n, accum+i**2)

>>> sum_up_squares(1,3,0)
14
```

Base Case

Tail Recursive Case

How much ???

- "Time" is required to compute `sum_of_squares(n)`?
  - Recursively?
  - Iteratively ?
- "Space" is required to compute `sum_of_squares(n)`?
  - Recursively?
  - Iteratively ?

Linear proportional to n
cn for some c

- Count the steps: Recursive is linear, iterative is constant! What about the order of evaluation?
- Careful: As taught traditionally, Computer Science has no measurement units convertible to physical time (*s*) and space (m^3)!

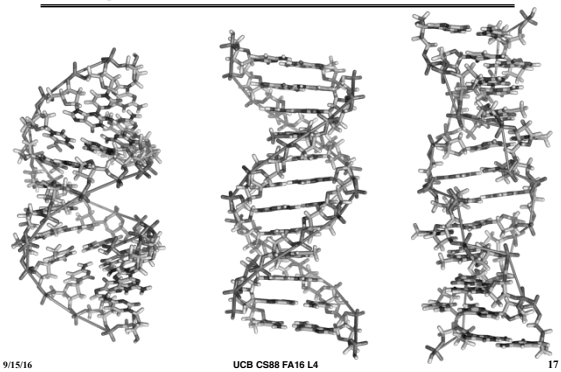
Recap: Defining Functions

`def` <function name> `(`<argument list>`) :`

`return` expression

- Generalizes an expression or set of statements to apply to lots of instances of the problem
- A function should *do one thing well*

Recap: Data or Code?



Higher Order Functions

- Functions that operate on functions
- A function

```
def odd(x):
    return (x%2==1)

>>> odd(3)
True
```

- A function that takes a function arg

```
def filter(fun, s):
    return [x for x in s if fun(x)]

>>> filter(odd, [0,1,2,3,4,5,6,7])
[1, 3, 5, 7]
```

Why is this not 'odd' ?

Higher Order Functions (cont)

- **A function that returns (makes) a function**

```
def leq_maker(c):
    def leq(val):
        return val <= c
    return leq
```

```
>>> leq_maker(3)
<function leq_maker.<locals>.leq at 0x1019d8c80>

>>> leq_maker(3)(4)
False

>>> filter(leq_maker(3), [0,1,2,3,4,5,6,7])
[0, 1, 2, 3]
>>>
```


One more example

- **What does this function do?**

```
def split_fun(p, s):
    """ Returns <you fill this in>."""
    return [i for i in s if p(i)], [i for i in s if not p(i)]
```

```
>>> split_fun(leq_maker(3), [0,1,2,3,4,5,6])
([0, 1, 2, 3], [4, 5, 6])
```


Three super important HOFS

`map(function_to_apply, list_of_inputs)`

Applies function to each element of the list

`filter(condition, list_of_inputs)`

Returns a list of elements for which the condition is true

`reduce(function, list_of_inputs)`

Reduces the list to a result, given the function

Recursion with Higher Order Fun

```
def map(f, s):
    if
    else:
def square(x):
    return x**2

>>> map(square, [2,4,6])
[4, 16, 36]
```

Base Case

Recursive Case

- **Divide and conquer**

Using HOF to preserve interface

```
def sum_of_squares(n):
    def sum_upper(i, accum):
        if i > n:
            return accum
        else:
            return sum_upper(i+1, accum + i*i)

    return sum_upper(1,0)
```

- **What are the globals and locals in a call to `sum_upper`?**
  - Try python tutor
- **Lexical (static) nesting of function def within def - vs**
- **Dynamic nesting of function call within call**

Recap: Quicksort

- **Break the problem into multiple smaller sub-problems, and Solve them recursively**

```
def split(x, s):
    return [i for i in s if i <= x], [i for i in s if i > x]

def qsort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""
    if not s:
        return []
    else:
        pivot = first(s)
        lessor, more = split(pivot, rest(s))
        return qsort(lessor) + [pivot] + qsort(more)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

Quicksort with HOF

```
def qsort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""

    if not s:
        return []
    else:
        pivot = first(s)
        lessor, more = split_fun(leq_maker(pivot), rest(s))
        return qsort(lessor) + [pivot] + qsort(more)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```


How much ???

- **"Time" is required to compute `quicksort(s)`?**
- **"Space" is required?**

Logarithmic to len(s)
c*log(len(s)) for some c

- **Name of this recursion scheme?**
  - **Tree recursion**

Questions?

```
/****************************************************************************\
| Toledo Nanochess (c) Copyright 2009 Oscar Toledo G. All rights reserved   |
| 1257 non-blank characters. Evolution from my winning IOCCC 2005 entry.    |
| o Use D2D4 algebraic style for movements.  biyubi@gmail.com  Nov/20/2009 |
| o On promotion add a number for final piece (3=N, 4=B, 5=R, 6=Q)         |
| o Press Enter alone for computer to play.                                 |
| o Full legal chess moves.                        http://www.nanochess.org |
| o Remove these comments to get 1326 bytes source code (*NIX end-of-line) |
\****************************************************************************/
char*l="ustvrtsuqqqqqqqqyyyyyyyy}{|~z|{}"
"   76Lsabcddcba .pknbrq  PKNBRQ ?A6J57IKJT576,+-48HLSU";
#define F getchar()&z
#define v X(0,0,0,21,
#define Z while(
#define _ ;if(
#define P return--G,y^=8,
B,i,y,u,b,I[411],*G=I,x=10,z=15,M=1e4;X(w,c,h,e,S,s){int t,o,L,E,d,O=e,N=-M*M,K
=78-h<<x,p,*g,n,*m,A,q,r,C,J,a=y?-x:x;y^=8;G++;d=w||s&&s>=h&&v 0,0)>M;do{_ o=I[
p=O]){q=o&z^y _ q<7){A=q--&2?8:4;C=o-9&z?q["& .$  "]:42;do{r=I[p+=C[l]-64]_!w|p
==w){g=q|p+a-S?0:I+S _!r&(q|A<3||g)||(r+1&z^y)>9&&q|A>2){_ m=!(r-2&7))P G[1]=O,
K;J=n=o&z;E=I[p-a]&z;t=q|E-7?n:(n+=2,6^y);Z n<=t){L=r?l[r&7]*9-189-h-q:0 _ s)L
+=(1-q?l[p/x+5]-l[O/x+5]+l[p%x+6]*-~!q-l[O%x+6]+o/16*8:!!m*9)+(q?0:!(I[p-1]^n)+
!(I[p+1]^n)+l[n&7]*9-386+!!g*99+(A<2))+!(E^y^9)_ s>h||1<s&s==h&&L>z|d){p[I]=n,O
[I]=m?*g=*m,*m=0:g?*g=0:0;L-=X(s>h|d?0:p,L-N,h+1,G[1],J=q|A>1?0:p,s)_!(h||s-1|B
-O|i-n|p-b|L<-M))P y^=8,u=J;J=q-1|A<7||m||!s|d|r|o<z||v 0,0)>M;O[I]=o;p[I]=r;m?
*m=*g,*g=0:g?*g=9^y:0;}_ L>N){*G=O _ s>1){_ h&&c-L<0)P L _!h)i=n,B=O,b=p;}N=L;}
n+=J||(g=I+p,m=p<O?g-3:g+2,*m<z|m[O-p]||I[p+=p-O]);}}}}Z!r&q>2||(p=O,q|A>2|o>z&
!r&&++C*--A));}}}Z++O>98?O=20:e-O);P N+M*M&&N>-K+1924|d?N:0;}main(){Z++B<121)*G
++=B/x%x<2|B%x<2?7:B/x&4?0:*l++&31;Z B=19){Z B++<99)putchar(B%x?l[B[I]|16]:x)_
x-(B=F)){i=I[B+=(x-F)*x]&z;b=F;b+=(x-F)*x;Z x-(*G=F))i=*G^8^y;}else v u,5);v u,
1);}}
```