Image Point Motion

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1 Image Point Motion

Consider a 3-D point $\mathbf{X} = (X, Y, Z)^T$ and 2 cameras $P = K[I\ \mathbf{0}]$ and $P' = K[R\ \mathbf{t}]$. The projections of the 3-D point onto the image planes are:

$$\mathbf{x} = P[\mathbf{X}^T 1]^T = K\mathbf{X}. \tag{1}$$

$$\mathbf{x}' = P'[\mathbf{X}^T 1]^T = K[R\ \mathbf{t}][\mathbf{X}^T 1]^T. \tag{2}$$

K is invertible, thus

$$\mathbf{X} = K^{-1}\mathbf{x}. \tag{3}$$

Substitute 3 into the 2 and simplify:

$$\mathbf{x}' = K[R\ \mathbf{t}][(K^{-1}\mathbf{x})^T 1]^T \tag{4}$$

$$= [KR\ K\mathbf{t}][(K^{-1}\mathbf{x})^T 1]^T \tag{5}$$

$$= KRK^{-1}\mathbf{x} + K\mathbf{t}. \tag{6}$$

Consider the case of pure translation: $(R = I)$ Eq. 6 becomes:

$$\mathbf{x}' = \mathbf{x} + K\mathbf{t}. \tag{7}$$

Note: $\mathbf{x}, \mathbf{x}'$ are projective quantities (not the pixel coordinates). Converting them into the pixel coordinates yields (assuming $K = diag(f, f, 1), \mathbf{t} = (t_1, t_2, t_3)^T$):

$$(x, y)^T = (fX/Z, fY/Z)^T. \tag{8}$$

$$(x', y')^T = (f(X + t_1)/(Z + t_3), f(Y + t_2)/(Z + t_3)). \tag{9}$$

The equation (9.6) in the book states:

$$\mathbf{x}' = \mathbf{x} + K\mathbf{t}/Z \tag{10}$$

In my notation it becomes:

$$\mathbf{x}' = (x, y, 1)^T + K\mathbf{t}/Z \tag{11}$$

$$Z\mathbf{x}' = (fX, fY, Z)^T + K\mathbf{t} \tag{12}$$