

HOMework 9

Task 1: Taylor Series

Introduction

I am using MATLAB to create a function that approximates a square wave function using a Fourier series. I will define a new function for the Fourier square wave with the given equations. I will then plot the square wave graph and the Fourier series with varying values of N. These Fourier series graphs should approximate the square wave graph.

Model and Theory

$$1) c_k = \begin{cases} \frac{4}{n\pi} & \text{if } n \text{ is odd} \\ 0 & \text{otherwise} \end{cases}$$

$$2) f(x) = 2[H(\frac{x}{L}) - H(\frac{x}{L} - 1)] - 1$$

$$3) g_N(x) = \sum_{k=0}^N c_k \cdot \sin \frac{n\pi x}{L}$$

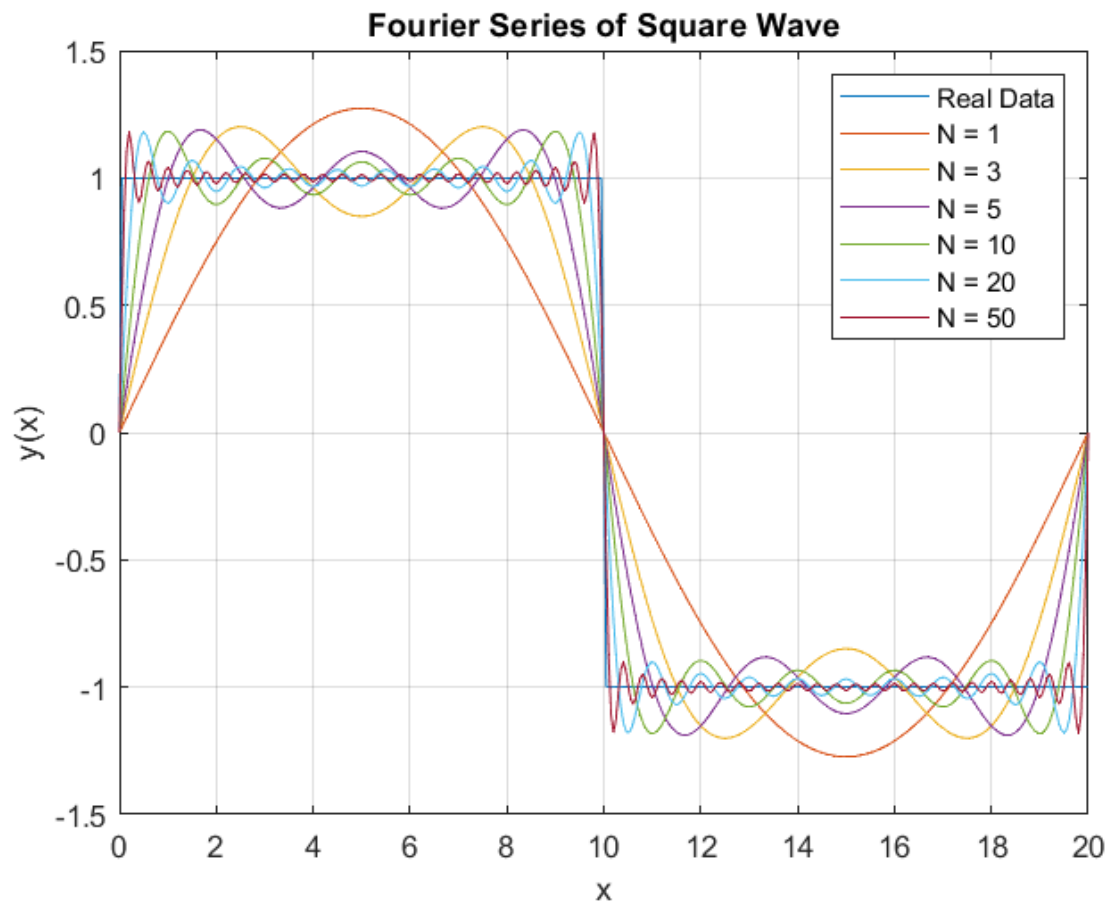
Methodology

I first cleared the workspace and command window. I then defined variables, such as L, delta x, and the values of N. I then created a range of x values from 0 to 2L that are equally spaced at intervals of delta x. I then created a range of f values by inputting my x values into equation 2. I then plotted the f values against the x values using plot() and heaviside(). I then created my FourierSquareWave() function. This function takes in inputs N, L, and x. It outputs y. Within my function, I set y to the initial value of zero. Then, using a for-loop, I iterated through every integer from 0 to N, checked if it was an odd number using a conditional statement, and depending if it was or not, I would set ck to its corresponding value as shown in equation 1. I then used equation 3 to calculate for y. Then the loop iterates with the next N value until all of them have been used. Going back to the main code, I began another for-loop, iterating through every value of N. Within the loop, I created a vector g that is the same size as vector x, and then iterated through every value of x and passed x in the FourierSquareWave() function I made. I set the output to the corresponding index in the g vector and plotted x versus g on the plot. I repeated this until all values of N were used. I then added a title, axis labels, and a legend (using legend()). Lastly, I added comments to make my code easier to understand for users.

Calculations and results

See next page.

4)



The plot shows the square wave function and the Fourier series approximations with varying values of N.

Discussions and Conclusions

The graph I created on MATLAB (4) matches the graph in the instructions document. As N increases, the corresponding graph follows the square wave function closer. When N=1, the Fourier series approximation is not as strong. When N=50, the Fourier series approximation is much stronger since it follows the square wave function better.

Task 2: Sinusoidal Fit

Introduction

I am using MATLAB to fit the data of the sales amount of a certain product in a store by month as a sinusoidal function. I will solve for the Fourier coefficients and I will create a plot of my resultant model equation, showing the relationship between the number of items sold and the time in days. Using my resultant model equation, I will then predict the number of items sold on May 15th.

Model and Theory

$$1) f(t) = A_0 + A_1 \cdot \cos(\omega_0 \cdot t) + B_1 \cdot \sin(\omega_0 \cdot t)$$

$$2) A_0 = \frac{\sum_{i=1}^N y}{N}$$

$$3) A_1 = \frac{2}{N} \sum_{i=1}^N y \cdot \cos(\omega_0 t)$$

$$4) B_1 = \frac{2}{N} \sum_{i=1}^N y \cdot \sin(\omega_0 t)$$

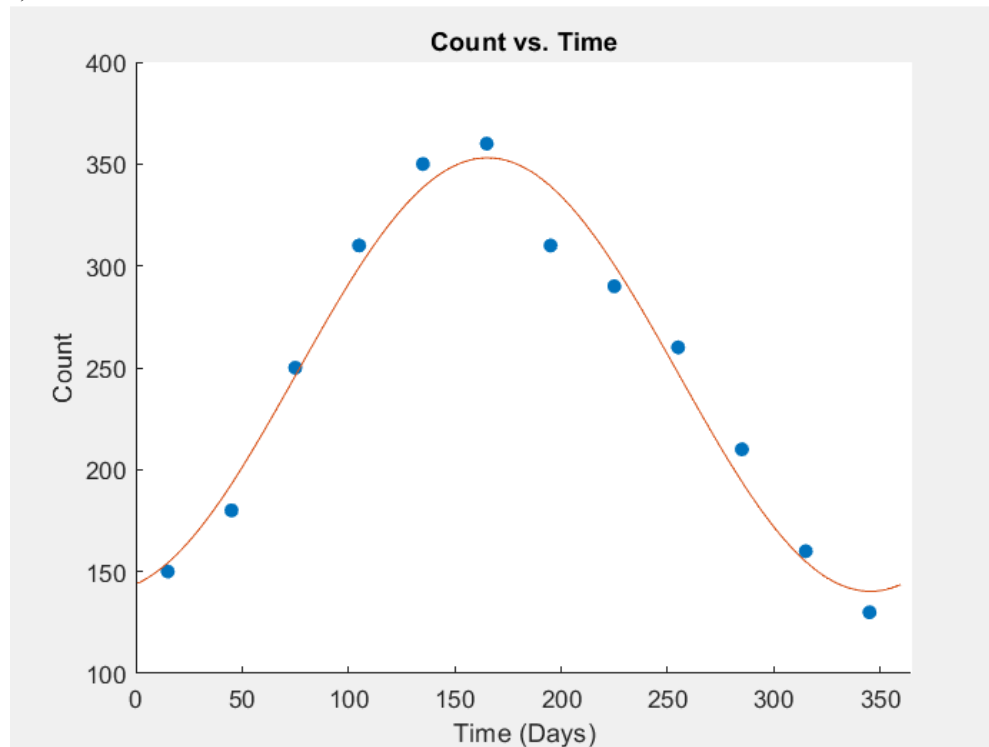
Methodology

I first cleared the workspace and command window. I then inputted the count data in a vector. I then created a vector representing the months using linspace(). This vector has 12 elements evenly spaced from 15 to 345. The values are spaced from 15 to 345 because I want my month data to represent the middle of the month, and approximating each month to have 30 days, I want my first point to be 0 plus half a month, and my last point to be 360 minus half a month. I set the N, which represents the number of occurrences, to the length of my count vector. I also set the angular frequency to 2 times pi divided by 360 (since we are approximating a year to have 360 days, I want one cycle of my sinusoid to be 360 days long; 360 days is the period). I then solved for the Fourier coefficients using equations 2, 3, and 4, where y is equal to the count. Those coefficients correspond to equation 1. Afterwards, I plotted my sinusoidal model. I created a range of time values from 1 to 360 with increments of 1. I then created the corresponding y values using equation 1 and the coefficients I found. I then plotted the original data points using my month vector, my count vector, and scatter(). I then plotted my sinusoidal model using my range of values I created and plot(). I also added a title, axis labels, and axis limits. Then, I predicted the amount of items sold on May 15th. I set a variable called May15 equal to 30 times 5 minus 15 (this number represents the middle of May). I then used equation 1 and the coefficient I found to calculate the number of items sold on May 15th, inputting t with May15. I then outputted the result. Lastly, I added comments to make my code easier to understand for users.

Calculations and results

See next page.

5)



6)

A0 =
 246.6667
 A1 =
 -102.8855
 B1 =
 26.9365
 Prediction =
 338.4647

The plot shows the sinusoidal model I developed plotted against the actual data points. The Fourier coefficient, as well as the number of items sold on May 15th prediction, is also shown above.

Discussions and Conclusions

My sinusoidal model (5) is a good fit for the data because the curve follows the data points closely and there are roughly equal numbers of points above and below the curve. My prediction for the number of items sold on May 15th (6) is also a good prediction, because comparing it to the actual number of items sold in May (350), it is only off by about 11.5353.