Rectangular Partitioning of Rectilinear Polygons

Exercise 15

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Introduction to the problem

Introduction

Exercise 15

Decompose a rectilinear polygon into rectangles, using segments aligned with the edges of the polygon. The algorithm must produce the **minimum number of pieces**, and the output must give a complete description of the partition.

Input

A polygon whose sides meet at right angles.

Output

A **partition** of the polygon into rectangles, with no overlaps.

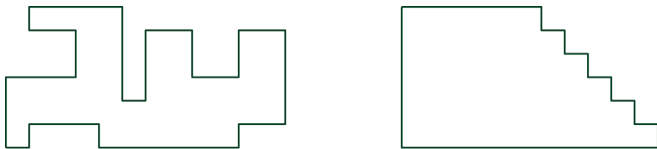We will call this problem *MNC* (minimal nonoverlapping cover).
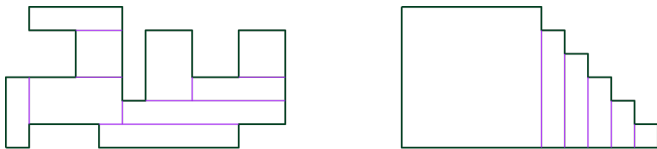
Figure 1: Two rectilinear polygons.

Figure 2: A minimum partition of both.

Preliminaries

Concave and convex vertices

In a rectilinear polygon, we can distinguish between concave and convex vertices.

- A vertex is said to be **concave** if its interior angle is over 180°.
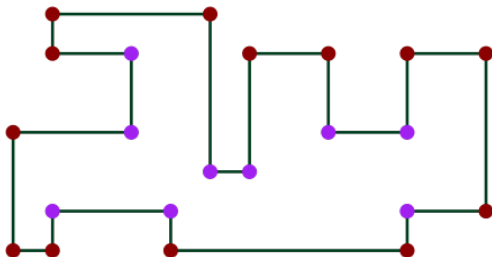- A vertex is said to be **convex** if its interior angle is under 180°.

Figure 3: Convex (red) and concave (purple) vertices.

Co-vertical, co-horizontal and chords

- Two vertices (x_1, y_1), (x_2, y_2) that share no edge are
  co-vertical $\iff y_1 = y_2$ and **co-horizontal** $\iff x_1 = x_2$.
- A **chord** is a segment fully contained inside the polygon that
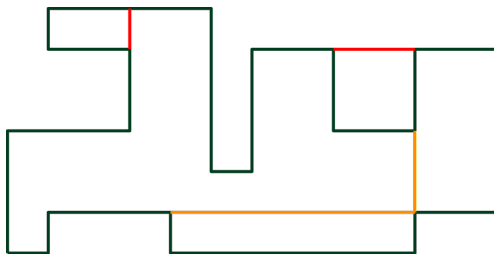 connects two co-horizontal or co-vertical vertices.

Figure 4: Examples of chords (orange) and **not** chords (red).

Chord-less polygons
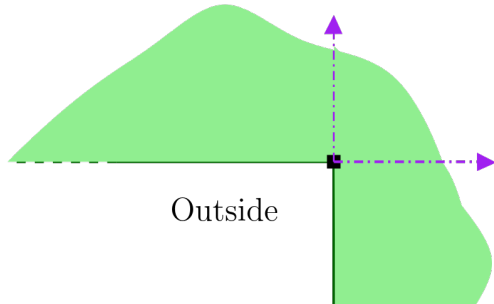
Concave vertices observations

Suppose, for now, that there are no chords in polygon P.

Observation #1

P is a rectangle $\iff$ P has exactly four convex vertices.

Observation #2

A concave vertex must be a vertex of two rectangles of the partition. See example below.

Outside

Algorithm for chord-less polygons

Observation #3

If P is not a rectangle then it must have at least one concave vertex.

Idea to solve the problem

For each concave vertex, extend its vertical edge until it hits another edge. *If it hits a vertex, then that extension is a chord!*
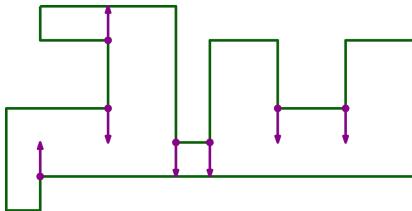

Figure 6: Chord-less polygons optimal solution.

Algorithm for chord-less polygons

Algorithm 1 Chord-less MNC in $O(n \log n)$

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procedure $\mathrm{MNC}(P)$

    $E \leftarrow \emptyset$; $R \leftarrow \emptyset$

    $L \leftarrow$ sorted vertices of P by x-coordinate[1].

    for each vertex v in P **do**

        Insert v to E in position, delete position if duplicate.

        if v is concave **then**

            Determine to extend vertical edge north or south.

            Insert to R the extension of vertical edge from v.

    Return R as the paritioning edges of P.

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[1] If using y-coordinate as tiebreaker, we can read pairs of vertices at a time (vertical edges).

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The problem with polygons with chords

Problem

Suppose there are two co-horizontal vertices which form a chord. We may prefer joining that chord rather than extending the vertical edge.

Conclusion

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- Summarize key points.
- Highlight takeaways.

Thank you for your attention!