CS 613 - Machine Learning

Assignment 3 - Classification)
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1 Theory

1. Consider the following set of training examples for an unknown target function: $(x_1, x_2) \rightarrow y$:

Y	x_1	x_2	Count
+	T	T	3
+	T	F	4
+	F	T	4
+	F	F	1
-	T	T	0
-	T	F	1
-	F	T	3
-	F	F	5

(a) What is the sample entropy, $H(Y)$ from this training data (using log base 2) (2pts)?

Number of samples with '+' class (p): 12
Number of samples with '-' class (n): 9
Total number of samples (p+n): 21

$$\begin{aligned} H(\frac{p}{p+n}, \frac{n}{p+n}) &= H(\frac{12}{21}, \frac{9}{21}) \\ &= (-\frac{12}{21} * log_2(\frac{12}{21}) + (-\frac{9}{21} * log_2 \frac{9}{21})) \\ &= (-0.57 * log_2(0.57)) + (-0.43 log_2(-0.43)) \\ &= (-0.57 * -0.81) + (-0.43 * -1.22) \\ &= 0.46 + 0.52 \\ &= \mathbf{0.98} \end{aligned}$$

(b) What are the information gains for branching on variables x_1 and x_2 (4pts)?

We'll first compute the information gain on x_1 and then on x_2.
The count for each class when split on x_1:

$$p_T = 3 + 4 = 7 \qquad n_T = 0 + 1 = 1 \qquad p_T + n_T = 8$$
$$p_F = 4 + 1 = 5 \qquad n_F = 3 + 5 = 8 \qquad p_F + n_F = 13$$
$$p + n = 21$$

$$\begin{aligned} remainder(x_1) &= \frac{p_T + n_T}{p+n} * H(\frac{p_T}{p_T + n_T}, \frac{n_T}{p_T + n_T}) + \frac{p_F + n_F}{p+n} * H(\frac{p_F}{p_F + n_F}, \frac{n_F}{p_F + n_F}) \\ &= \frac{8}{21} * H(\frac{7}{8}, \frac{1}{8}) + \frac{13}{21} * H(\frac{5}{13}, \frac{8}{13}) \\ &= \frac{8}{21} * (-\frac{7}{8} * log_2(\frac{7}{8}) - \frac{1}{8} * log_2(\frac{1}{8})) + \frac{13}{21} * (-\frac{5}{13} * log_2(\frac{5}{13}) - \frac{8}{13} * log_2(\frac{8}{13})) \\ &= \frac{8}{21} * (0.17 + 0.38) + \frac{13}{21} * (0.52 + 0.43) \\ &= 0.21 + 0.59 = \mathbf{0.8} \end{aligned}$$

$$IG(x_1) = 0.98 - 0.8 = \mathbf{0.18}$$

The count for each class when split on x_2:

$$p_T = 3 + 4 = 7 \qquad n_T = 0 + 3 = 3 \qquad p_T + n_T = 10$$
$$p_F = 4 + 1 = 5 \qquad n_F = 1 + 5 = 6 \qquad p_F + n_F = 11$$
$$p + n = 21$$

$$\begin{aligned} remainder(x_2) &= \frac{p_T + n_T}{p+n} * H(\frac{p_T}{p_T + n_T}, \frac{n_T}{p_T + n_T}) + \frac{p_F + n_F}{p+n} * H(\frac{p_F}{p_F + n_F}, \frac{n_F}{p_F + n_F}) \\ &= \frac{10}{21} * H(\frac{7}{10}, \frac{3}{10}) + \frac{11}{21} * H(\frac{5}{11}, \frac{6}{11}) \\ &= \frac{10}{21} * (-\frac{7}{10} * log_2(\frac{7}{10}) - \frac{3}{10} * log_2(\frac{3}{10})) + \frac{11}{21} * (-\frac{5}{11} * log_2(\frac{5}{11}) - \frac{6}{11} * log_2(\frac{6}{11})) \\ &= \frac{10}{21} * (0.36 + 0.54) + \frac{11}{21} * (0.51 + 0.48) \\ &= 0.43 + 0.52 = \mathbf{0.85} \end{aligned}$$

$$IG(x_2) = 0.98 - 0.85 = \mathbf{0.13}$$

(c) Draw the decision tree that would be learned by the ID3 algorithm without pruning from this training data (5pts)?

Figure 1: Initial Decision Tree, where leaf nodes are not collapsed

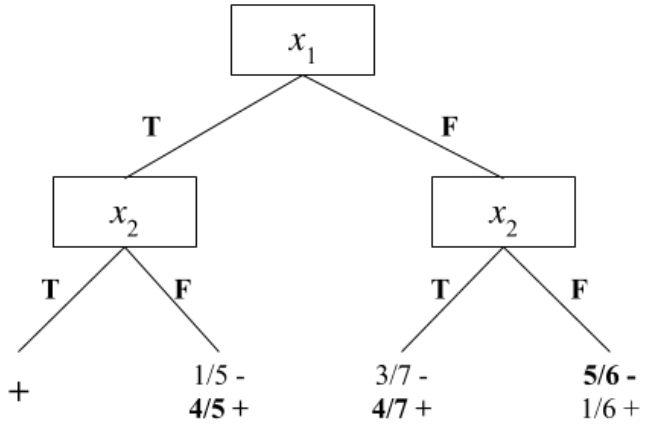
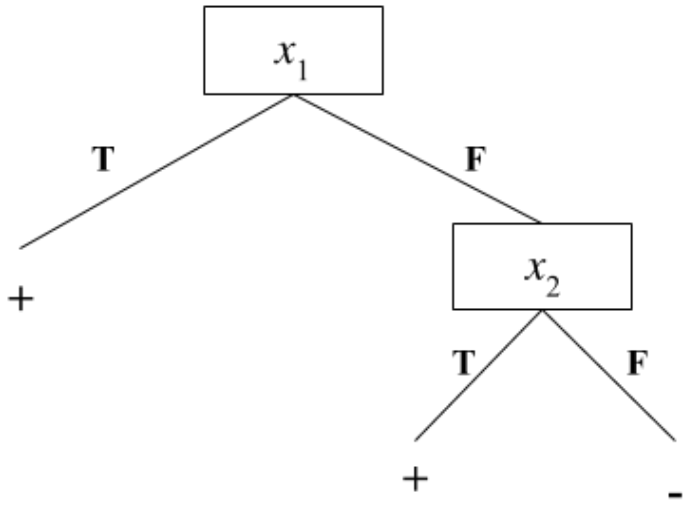


Figure 2: Final Decision Tree, collapsed left node, select most probable outcome for other nodes.



2. We decided that maybe we can use the number of characters and the average word length an essay to determine if the student should get an A in a class or not. Below are five samples of this data:

# of Chars	Average Word Length	Give an A
216	5.68	Yes
69	4.78	Yes
302	2.31	No
60	3.16	Yes
393	4.2	No

   (a) What are the class priors, $P(A = Yes), P(A = No)$? (1pt)

   (b) Find the parameters of the Gaussians necessary to do Gaussian Naive Bayes classification on this decision to give an A or not. Standardize the features first over all the data together so that there is no unfair bias towards the features of different scales (5pts).

   (c) Using your response from the prior question, determine if an essay with 242 characters and an average word length of 4.56 should get an A or not (5pts).

3. Consider the following questions pertaining to a k-Nearest Neighbors algorithm (1pt each = 3pts):

   (a) How could you use a *validation set* to determine the user-defined parameter k?

   (b) Why shouldn't you use the training set to determine this?

   (c) Why shouldn't you use the testing set to determine this?

4. The Linear Kernel is commonly used if we already are working in high feature space. It is defined as $k(x, y) = \sum_{d=1}^{D} x_d y_d$. If your observations have three features ($D = 3$), what is the function $\phi(u)$ such that $k(x, y) = \phi(x)\phi(y)$? Show your work (4pts).

5. True or false: A Gaussian Kernel is better than a linear kernel when there are many features but few training samples (1pt).

2 Naive Bayes Classifier

Let's train and test a *Naive Bayes Classifier* to classifiy Spam or Not from the Spambase Dataset.

First download the dataset *spambase.data* from Blackboard. As mentioned in the Datasets area, this dataset contains 4601 rows of data, each with 57 continuous valued features followed by a binary class label (0=not-spam, 1=spam). There is no header information in this file and the data is comma separated. As always, your code should work on any dataset that lacks header information and has several comma-separated continuous-valued features followed by a class id $\in 0, 1$.

Write a script that:

1. Reads in the data.
2. Randomizes the data.
3. Selects the first 2/3 (round up) of the data for training and the remaining for testing
4. Standardizes the data (except for the last column of course) using the training data
5. Divides the training data into two groups: Spam samples, Non-Spam samples.
6. Creates Normal models for each feature for each class.
7. Classify each testing sample using these models and choosing the class label based on which class probability is higher.
8. Computes the following statistics using the testing data results:
   (a) Precision
   (b) Recall
   (c) F-measure
   (d) Accuracy

Implementation Details

1. Seed the random number generate with zero prior to randomizing the data
2. If you decide to work in log space, realize that Matlab interprets $0log0$ as NaN (not a number). You should identify this situation and consider it to be a value of zero.
3. Although Naive Bayes Classifiers can do multi-class classification directly, you may assume binary classification in your implementation.

In your report you will need:

1. The statistics requested for your Naive Bayes classifier run.

3 Multi-Class Support Vector Machines

Many learning algorithms are designed to only perform binary classification out-of-the-box. Support vector machines are one such algorithm. If we want to perform multi-class classification we can either use a *one vs all* or *one vs one* approach. In this section we will explore both.

For simplicy you **ARE** allowed to use a support vector machine library and/or functions. In Matlab these are *fitcsvm* to train and *predict* to predict the class labels.

Write a script that:

1. Reads in the data from the Cardiotocography set provided on Blackboard. Read about how we'll use this dataset in the Datasets section.
2. Randomizes the data.
3. Selects the first 2/3 (round up) of the data for training and the remaining for testing
4. Standardizes the data (except for the last column of course) using the training data
5. First trains and evaluates using a *One vs All* approach:
   (a) Train K binary classifiers, one per class. When training classifier i, set all instances pertaining to class $i's$ label to one, and set the label of all other instances to zero.
   (b) For each test sample, run it through each of the K classifiers to see which class(es) it belongs to vs the "others". In other words, if when testing a sample using classifier i you get back a label of one, then this sample was thought to belong to class i. If you got back zero, then it was not. If there's multiple classes it belongs to, select one of those classes at random as the predicted class label.
   (c) Since there's more than one class, the concept of "Positive" and "Negative" don't really apply. Therefore just compute the *accuracy* as the percentage of samples classified correctly
6. Trains and evaluates using a *One vs One* approach:
   (a) Train $\frac{K(K-1)}{2}$ one-vs-one binary classifiers where you only use the training samples from the relevant classes. That is, if you are training a classifier that labels observations as either class i or class k, then only use observations with labels i and j (discard the rest).
   (b) For each test sample, run it through each of the $K(K-1)/2$ classifiers to see which class(es) "beat" the others the most. Choose that class as the your observation's label. Again if there is a tie among several classes, choose at random the predicted label from those classes.
   (c) Since there's more than one class, the concept of "Positive" and "Negative" don't really apply. Therefore just compute the *accuracy* as the percentage of samples classified correctly confident.

Implementation Details

1. Seed the random number generate with zero prior to randomizing the data
2. In the case of ties, just choose a class at random (from the ties).
3. Your code should work for an arbitrary number of classes.

In your report you will need:

1. The accuracy for your two approaches.