Relativistic Dispersion Relation for the Free Electron

Alejandro Legarda

April 22, 2016

Abstract

We investigate the energy-momentum relation for the free electron over a wide range of energies, using data from gamma photon Compton scattering and the photoelectric effect. We show that a relativistic model is appropriate for the dispersion relation.

Contents

1 Method

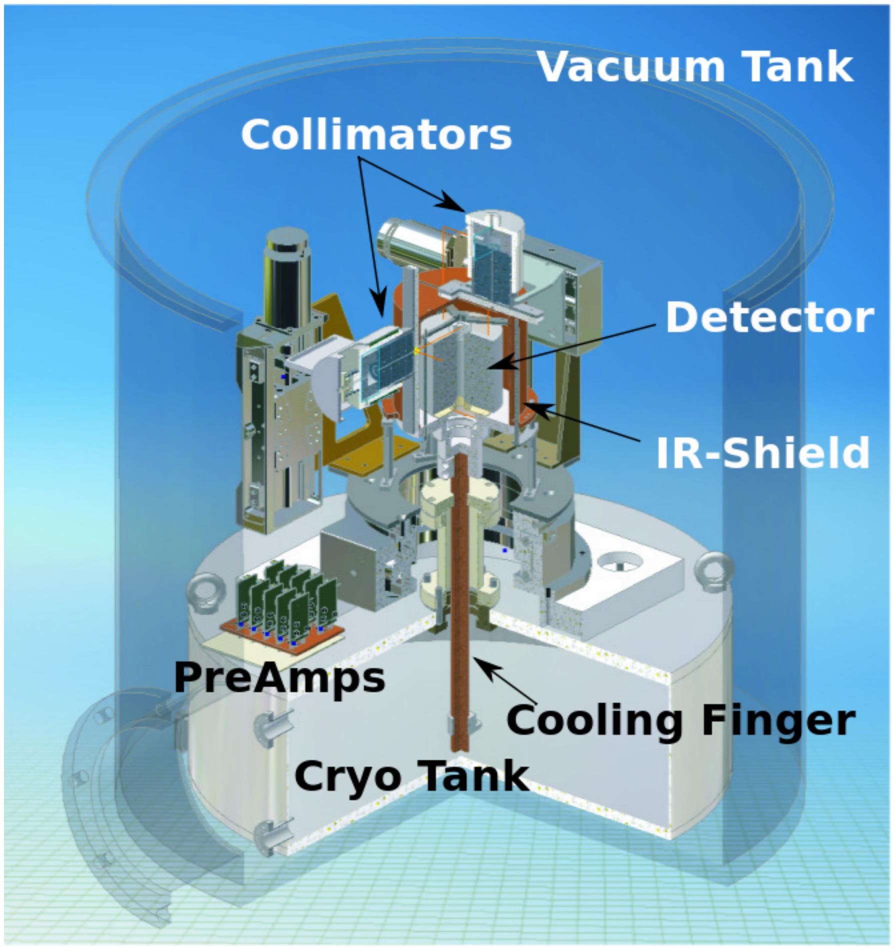

Figure 1: The Germanium detector. This detector is connected to a pre-amp, which feeds into an amplifier and finally passes the signal on to a PHA, allowing the computer software to produce a histogram based on channel number (energy).

We aim to measure both gamma energies and Compton edges precisely. We use a Germanium detector because of its better energy resolution than sodium-iodide detectors. Gamma radiation from our source enters the Germanium detector and Compton scatters with electrons (or, eventually, undergoes the photoelectric effect), producing electron-hole pairs. This separated charge is collected by the application of a high voltage. The quantity of charge collected is proportional to the energy deposited by the gamma in the crystal. A charge-sensitive pre-amplifier produces pulse heights proportional to the collected charge, an amplifier amplifies the signal, and a pulse height analyzer (PHA) digitizes the pulse heights and displays a histogram of the energies. By measuring features of the resulting spectrum and using momentum and energy conservation, we can independently identify the kinetic energy and momentum of the electrons.

From the PHA's spectrum we are able to extract the energy of the incoming photon, E_γ, which is represented bu the full energy photopeak. This peak is due to the full energy transfer from the gamma photon to the detector. The process is usually composed of several Compton scatters, but the final interaction is the photoelectric effect, and thus the totality of the photon energy is dumped into the detector. We also extract the kinetic energy, T, given to an electron at a 180 degree scatter. This is observed as the Compton Edge on the spectrum. This is equal to the maximum energy which can be deposited in the detector by a single Compton scatter.

2 Spectrum Identification and Calibration

Since it is hard to see from the plots due to their scale, we provide a table of Compton edges and full energy peaks. Sources marked with an asterisk (*) indicate that they were used for analysis or calibration (if Compton edge is not present in the row, that indicates it was used for calibration).
Note how we were unable to discern any Compton Edges from Ba-133's PHA spectrum - this was due to the complexity of Barium's nuclear decay scheme, which makes it hard to discern the locations of peaks let alone Compton Edges.

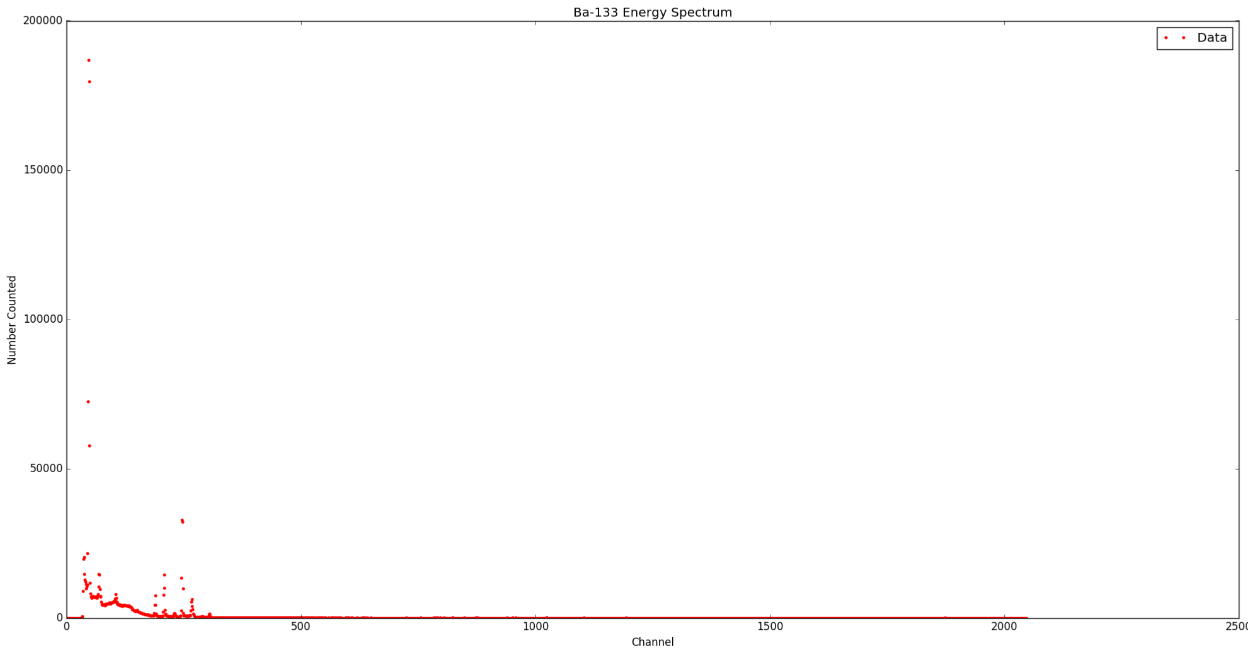

Figure 2: PHA spectrum for Ba-133

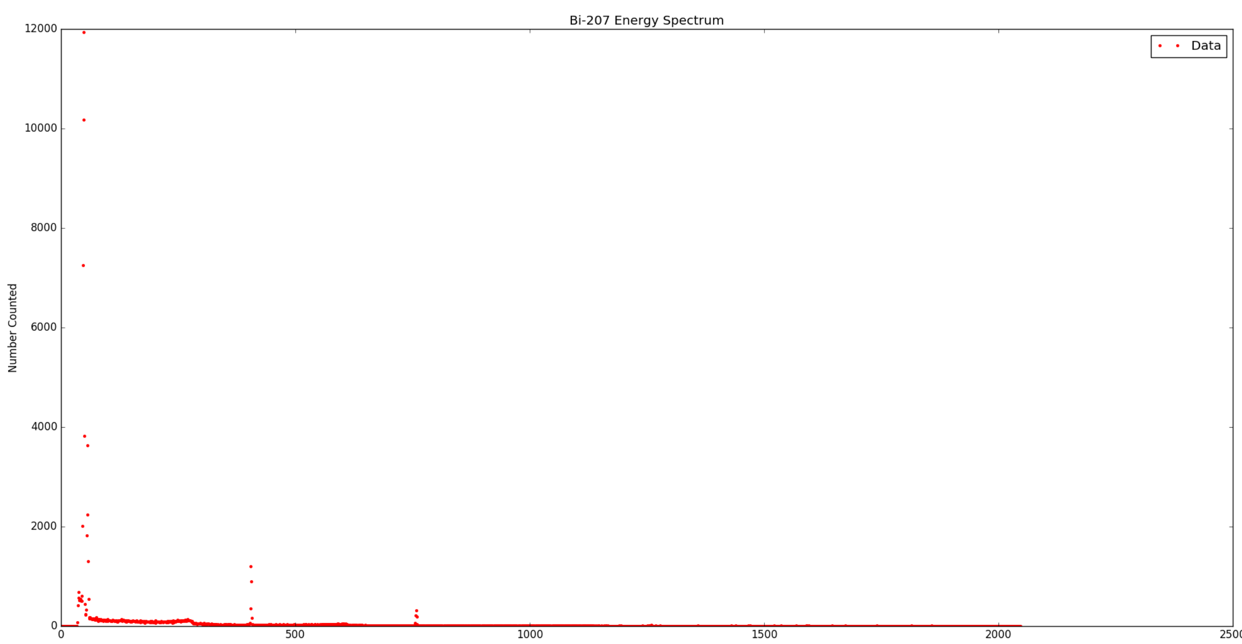

Figure 3: PHA spectrum for Bi-207

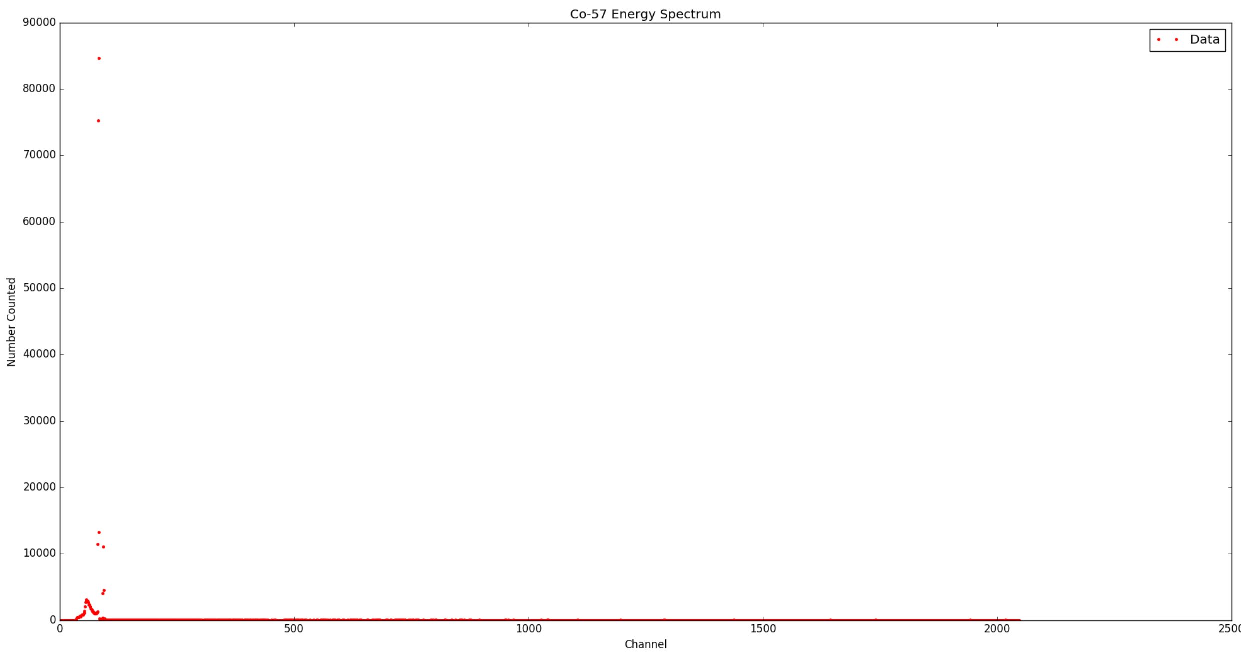

Figure 4: PHA spectrum for Co-57

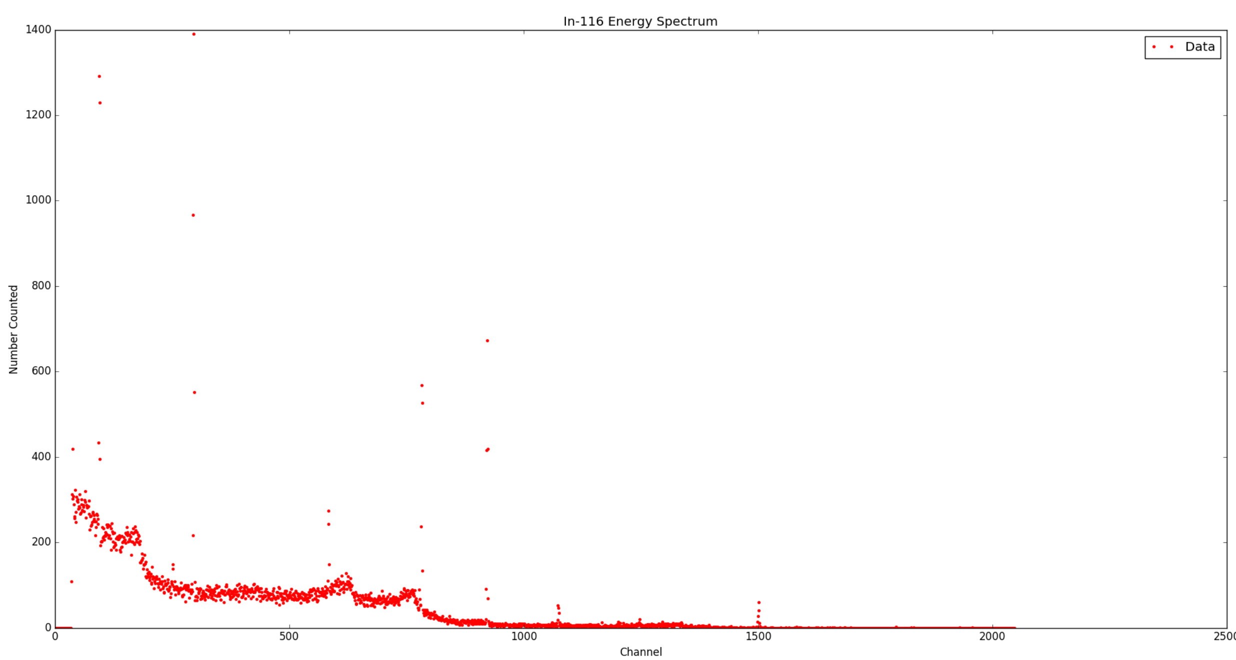

Figure 5: PHA spectrum for In-116

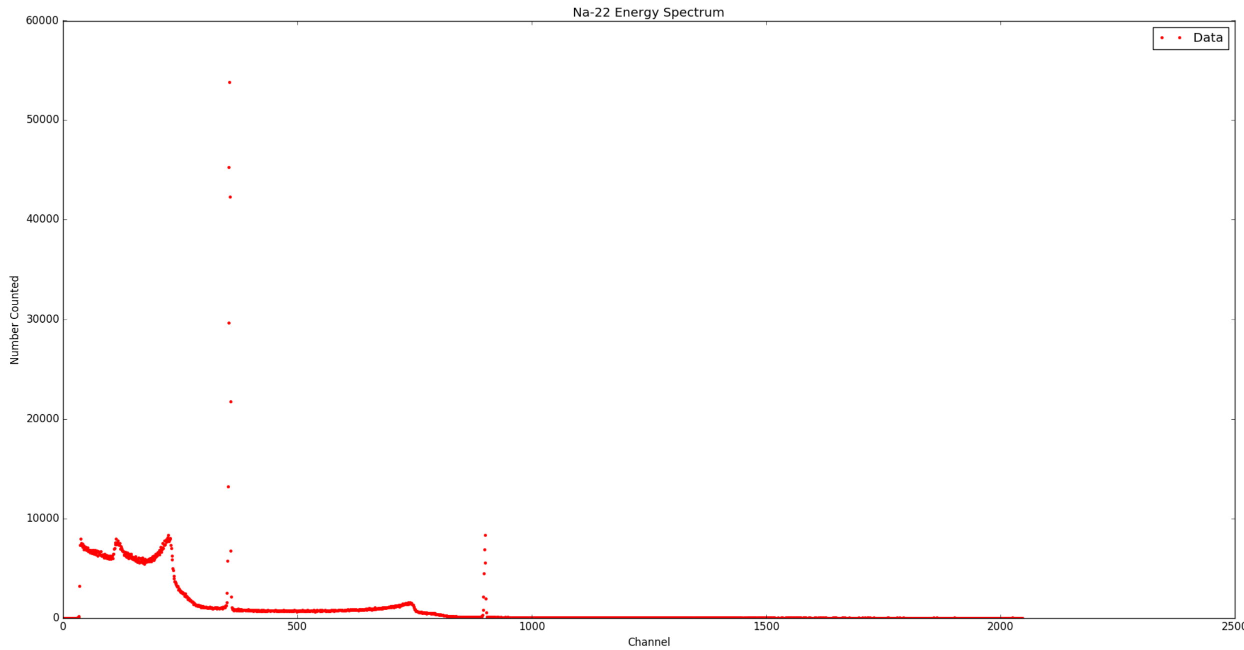

Figure 6: PHA spectrum for Na-22

Table 1: Compton Edges and Full Energy Peaks

Source	Compton Edge Channel	Uncertainty	Full Energy Peak	Uncertainty
*Cs-137	338	3	469	2
*Na-22	234	3	353	2
*Na-22	749	3	900	2
Bi-207			58	2
*Bi-207	279	3	405	2
*Bi-207	614	3	758	2
*Bi-207			1259	2
Co-57	65	3	83	2
Co-57			93	2
In-116			94	2
*In-116	186	4	296	2
In-116			584	2
*In-116	634	3	782	2
*In-116	773	4	922	2
In-116			1073	2
In-116			1501	2
*Ba-133			46	2
Ba-133			69	2
Ba-133			105	2
Ba-133			189	2
Ba-133			208	2
Ba-133			246	2
Ba-133			267	2
Ba-133			289	2
*Ba-133			305	2

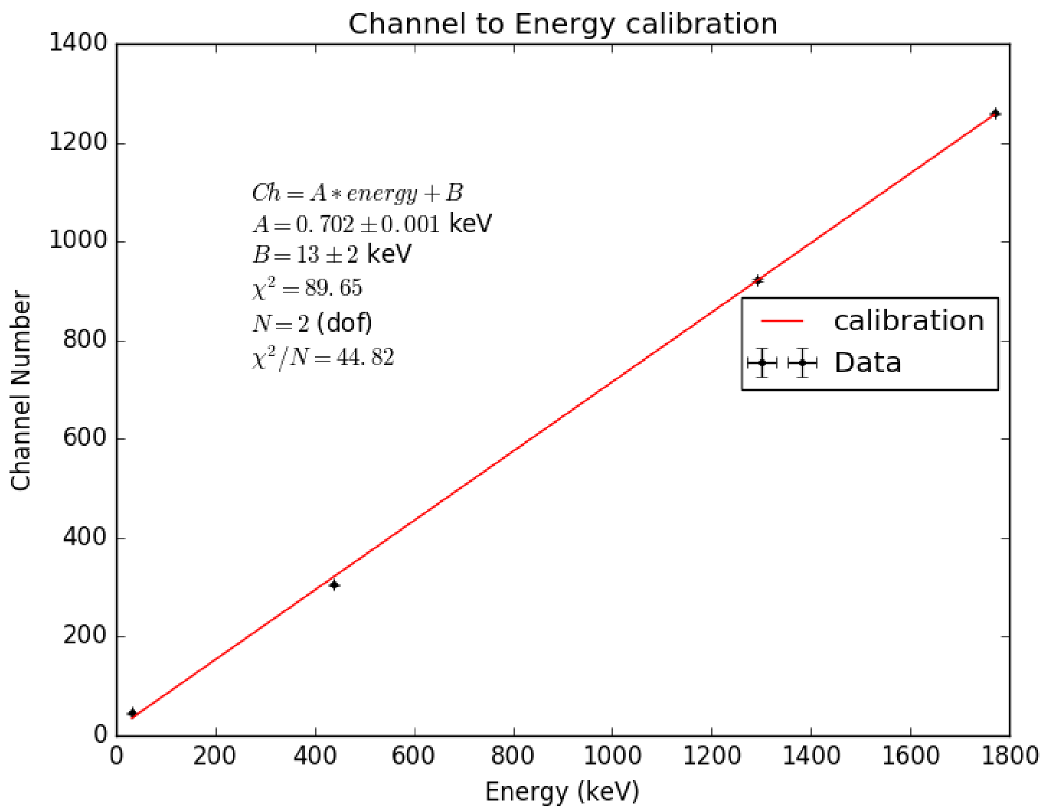

Figure 7: Channel/Energy Calibration

3 Analysis

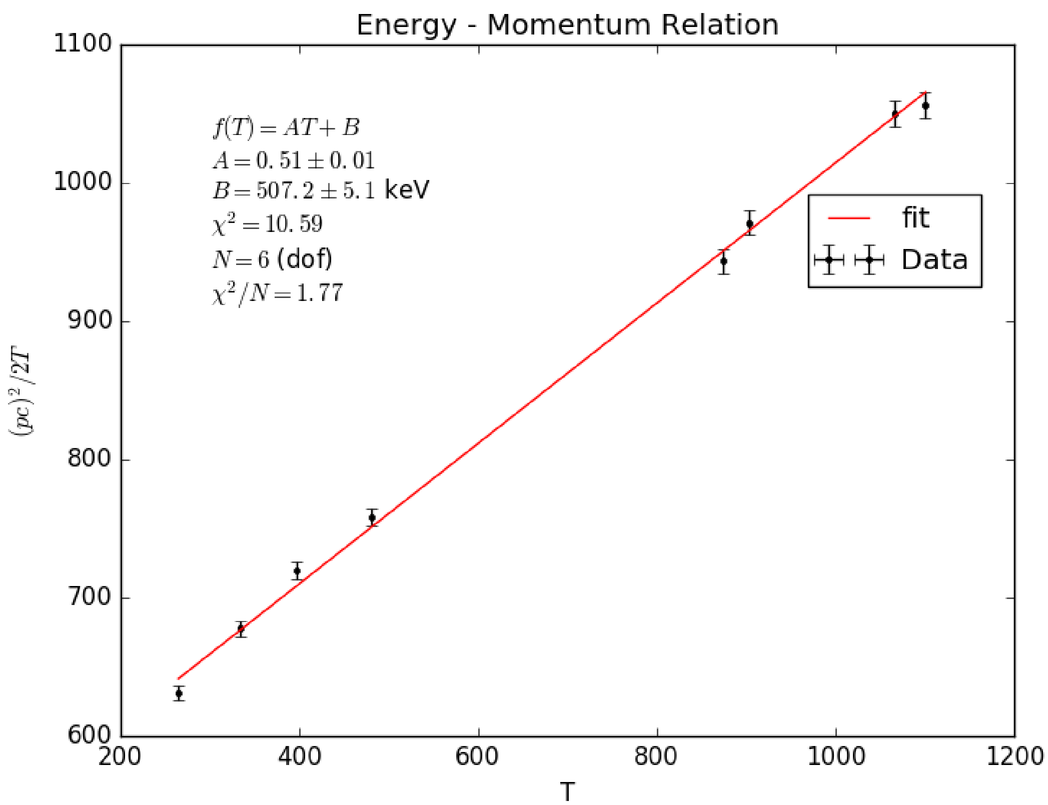

Figure 8: Energy/Momentum Relation

From the energy vs. momentum plot ($(pc)^2/2T$ against T), we get a function, $g(T)$ that describes their relation. Note that errors were calculated by propagating from the original measurements of E_γ and T. From equation (15) in the lab manual, we rearrange to show equivalency:

$$(pc)^2 = 2m_0c^2T + T^2 \tag{1}$$
$$\frac{(pc)^2}{2T} = m_0c^2 + \frac{T^2}{2} \tag{2}$$
$$\tag{3}$$

From the fit we have the relation $g(T)$ $\frac{T}{2}$ to satisfaction. Our fit then gives us an electron rest mass value of $m_0c^2 = 507 \pm 5keV$, which agrees well with the literature value of $511keV$. We see that at low T ($T << 2m_0c^2$), the constant term dominates and therefore we recover the classical equation

$$(pc)^2 = 2m_0c^2T \tag{4}$$
$$\frac{p^2}{2T} = m_0. \tag{5}$$
$$\tag{6}$$

From direct rest mass calculation we obtain an average of $m_0c^2 = 512 \pm 7keV$, where in this case the error was calculated using a simple standard deviation form the mean. This agrees well with the literature value of $511keV$.

Plotting either momentum or energy against β (v/c), we can see that as $\beta \to 1$, both momentum and energy diverge exponentially. This is because no system with mass can achieve $\beta = 1$, i.e., an infinite quantity of energy/momentum would be required to do so. Insight about the form of these plots can be acquired by examining the real mass energy, equation, which includes $\gamma = \frac{1}{\sqrt{1-\beta^2}}$.

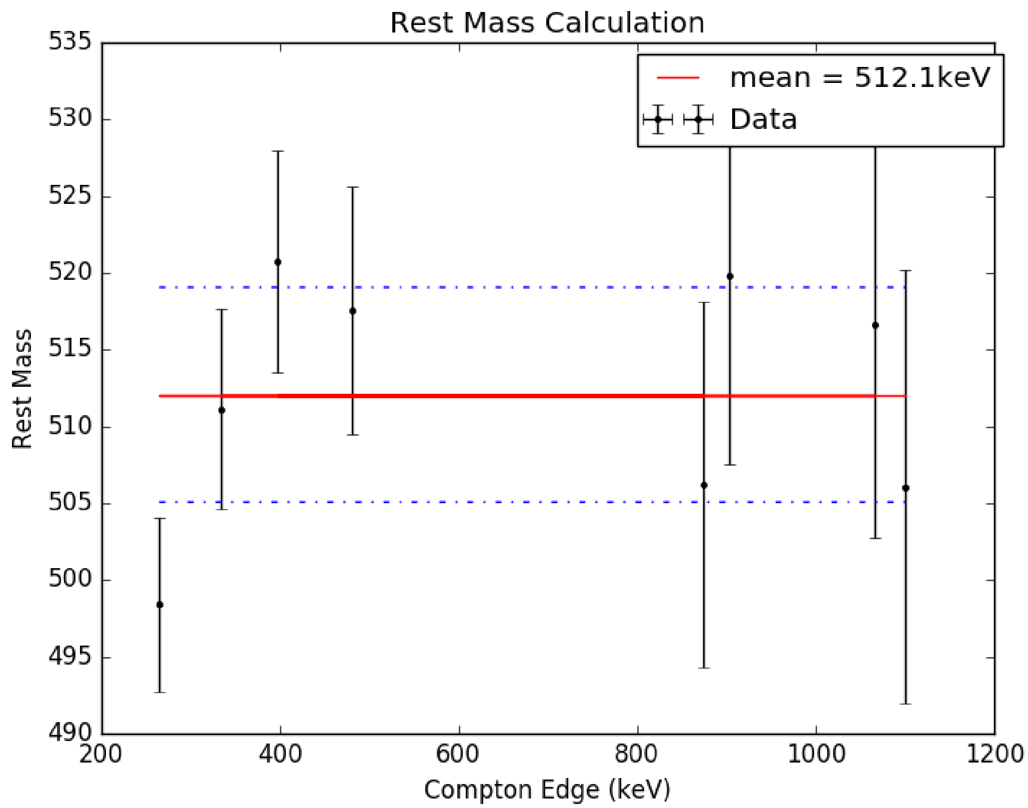

Figure 9: Electron Rest Mass

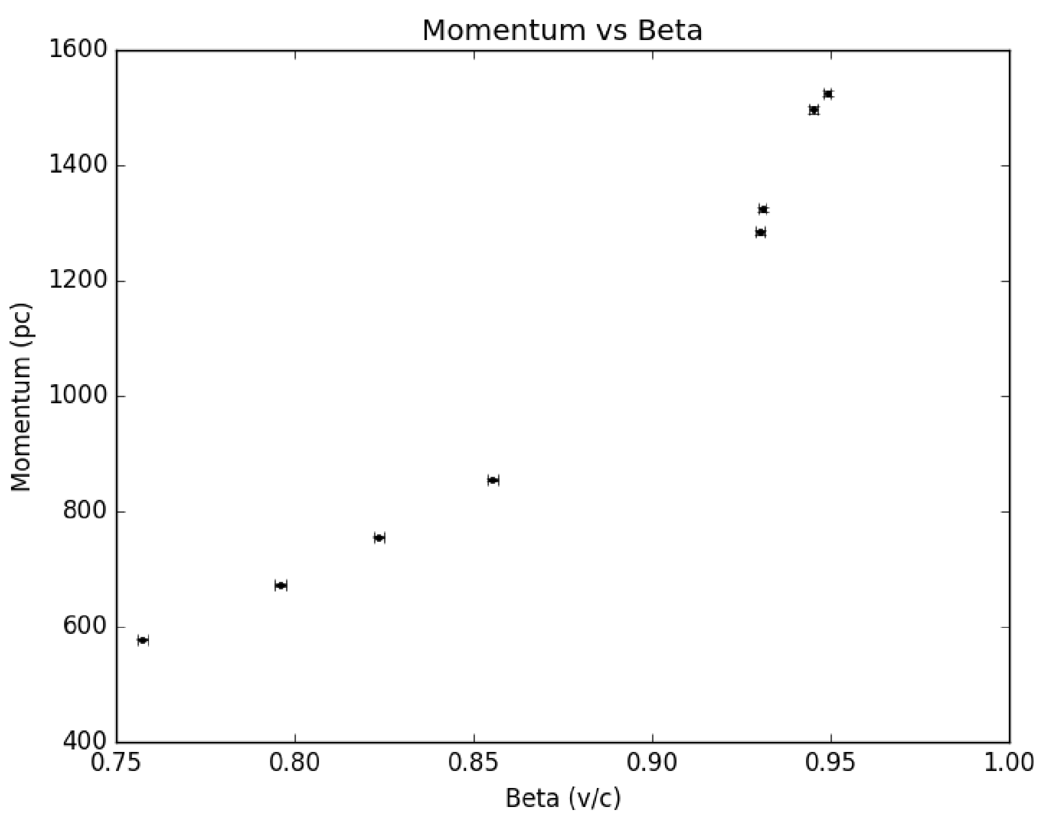

Figure 10: Electron Rest Mass

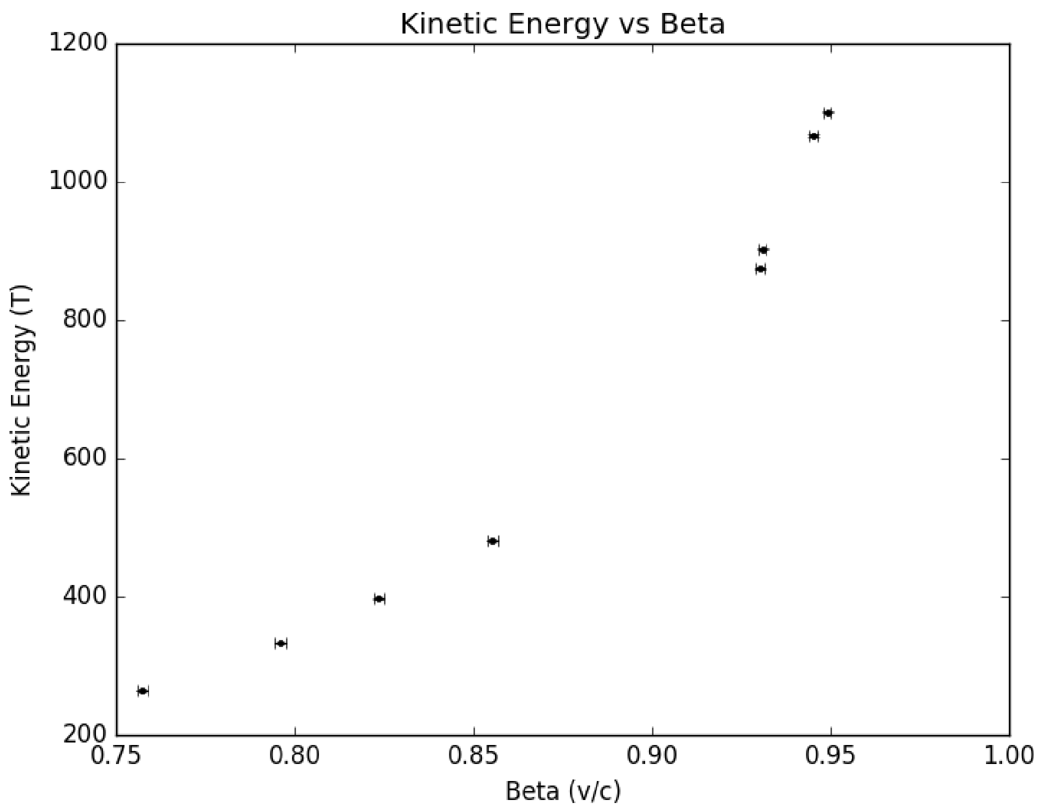

Figure 11: Electron Rest Mass

4 Discussion

It is clear that if we wish to have a relationship that tolerates any value of T, we must use the special relativistic model of the dispersion relation. A classical model would have produced a terrible fit since it is evident from our plot that $\frac{(pc)^2}{2T^2}$ does not equal a constant. For high-energy particle physics it is essential that relativity be taken into account.
We found that our calculated electron rest masses agree well with the literature values. The largest source of error for these was the uncertainty in the channel number for the Compton Edges, since they are hard to pinpoint and interpolation is necessary (to try and figure out the center of the edge).
As explained earlier, as $\beta \to 1$, momentum and (therefore) energy will diverge to infinity, as $\beta = 1$ is a fundamental spacetime limit only achieved by massless particles.

References

[1] University of Chicago Department of Physics. "Relativistic Dispersion Relation for the Free Electron" https://wiki.uchicago.edu/display/P211manuals/Relativistic+Dispersion+Relation+for+the+Free+Electron. (Accessed March, 2016)

[2] Taylor, John. *An Introduction to Error Analysis*. Sausalito: University Science Books, 1997.