Testing the Random Walk Hypothesis and seeking excess returns using a Black Swan trading strategy

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A dissertation submitted in partial fulfillment

of the requirements for the degree of

Masters of Science

of

University College London.

Department of Computer Science

University College London

April 5, 2021

Acknowledgements

I would like to thank my professors at UCL for their insights throughout the year. To Dr Guido Germano I would thank him for his guidance and continuous support. To Dr Riaz Ahmad for the useful critiques and for providing me with both deep academic and industrial knowledge throughout his courses and this master thesis. Lastly, I would like to thank my friends and family for their encouragement to pursue further studies and complete this programme.

Abstract

There a numerous studies claiming that the markets do not follow the so-called random walk hypothesis. Many market participants believe the markets are momentum trending or mean-reverting. This thesis aims to further explore both of these behaviours and determine what process the S&P 500 follows.

This thesis also evaluates a long/short equity trading strategy by investing in the top and bottom 10% of the stocks listed in the S&P 500 exploiting market's reaction after unpredictable Black Swan events. This will also test the efficient market hypothesis and if the beta is a valuable tool for portfolio selection. The active strategy yields a 91.2% increase while buying and holding the S&P 500 yielded 85.2% between 2007 and 2018. Other results include that neither the daily returns of the S&P 500 and the active strategy were represented by the normal distribution.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

Before the discovery of Australia, there was no reason for people to believe that black swans existed. After the discovery of this new continent in 1697 a theory that had been validated for millennia was broken down bringing a psychological paradigm that you should not rule out a black swan existed because you had not observed any. [1]

This metaphor is now used to describe highly unpredictable events, bring an extreme impact and humans believe they can explain why it happened only after the event. This expression has been widely used in all types of social, cultural, technological, historical and economic events. Certain examples include the rise of the internet, personal computers, the first great war and the 9/11 attacks on the world trade center. Nassim Nicholas Taleb also noted that a black swan depends on the observer. For instance, on Thanksgiving a turkey can see the act of getting slaughtered as a black swan, however, this is not the case for the butcher.

Nassim Nicholas Taleb first wrote about this concept in his book Fooled by Randomness, published in 2001. Here he discussed the impact it had concerning financial events, both positive and negative. This metaphor was extended in his second book The Black Swan: The impact of the Highly Improbable where he defined black swan events to have the following three features: [2]

1. The event is highly unpredictable to the observer.

2. Such events will have a significant impact, either positive or negative.

3. After one of these events, the observer will attempt to provide an explainable answer stating it could have been predictable.

Analysing the effect of black swans by eliminating the worst and best performing days provide remarkable insights. The Standard & Poors 500 Stock Index increased from 17 USD to 1540 USD between 1950 and 2008, almost a 9,000% increase. If the best 40 days were to be excluded the investment would have yielded a 70% loss. Even more impressive, if an investor was to identify and exclude the worst 40 days investing in the index would have yielded an impressive 66,000%, collecting 11,235 USD in 2008. [3]

Reflecting on this, renowned investor Nassim Taleb has assured on numerous occasions that the best way to face these events is to take into account they will happen repeatedly throughout time. To hedge for this systematic tail risk he proposed that assigning a certain proportion of your investment on far out the money put options. Investing in these cheap derivative contracts will almost surely provide consistent small losses in the hope that a negative black swan occurs providing large positive returns. Therefore an investor can assign less of his portfolio to safer assets and instead exploit the behaviour of black swans by using these cheap derivative contracts.

1.1 Purpose

Such Black Swan events are known to be non-computational and models fail to properly apply scientific methods to capture the tail risk exposure of their portfolio. In this report, we aim to set up a strategy that does not account for these events to be an anomaly, but as the starting point on how our strategy is set up.

This thesis also investigates properties of financial returns such as deviations from random walks and testing if returns follow a normal distribution as assumed in many stochastic models used in quantitative finance. Finally this thesis aims to validate previous studies on mean reversion of stock returns and exploit such behaviour to create a strategy that outperforms our benchmark index, the S&P 500.

A basket of stocks is created taking all listed companies in the S&P 500 ranging from 2006 up to December 2018. The S&P 500 index was chosen given that previous research has explored similar studies on global indices (MSCI World index). This thesis takes a slightly different approach and attempts to find similar behaviour with an index from a localised developed country. This provides us with specific characteristics such as:

1. There is a risk associated with not getting filled at the best bid/ask when placing a market buy or sell order. Therefore, only highly liquid stocks are used to minimise this associated risk.

2. It is using the 500 largest market cap companies in the USA, being a developed economy that is subject to similar systematic risks. Some of these companies are also listed in other indices such as NASDAQ, Dow Jones and the NYSE

3. A wide variety of stocks from different sectors and industries are listed in the S&P 500; such as Energy, TMT, Financial Services, Real Estate and Utility companies. This benefits investing in such ETF since it reduces idiosyncratic risk thanks to the diversification of the portfolio.

1.2 Emperical Evidence of Black Swans

Black Swans have caused a significant market impact in the past, causing investors to use a long-only strategy to lose a substantial amount of money. Relating how Nassim Taleb defined such events, Black Monday and the 2008 financial crisis were Black Swan given the rarity, extremeness and retrospective predictability of the event. [1]

On October 19 1987, stock prices around the world started to crash. This started when markets opened in Hong Kong, subsequently hitting European and American markets. Losses in the Dow Jones Industrial Average (DJIA) fell 508 points, being a 22.61% [4]. collapse in one day while markets in New Zealand collapsed around 60% from its 1987 peak, taking many years to recover. [5]. These daily returns brought volatility levels that were previously unseen having approximately 21 standard deviations above the mean. To put this in context discoveries in

Range	P(inside range)	P(outside range)
$\mu + 0.5\sigma$	38.29%	2 in 3
$\mu + \sigma$	68.27%	1 in 3
$\mu + 3\sigma$	86.64%	1 in 370
$\mu + 5\sigma$	99.99994%	1 in 1744278
$\mu + x\sigma$	$erf(x/\sqrt{2}) * 100\%$	1 - $erf(x/\sqrt{2})$

Table 1.1: Probability of outcome at defined standard deviations from the mean.

the field of high-energy particle physics the discovery of a new particle is published when the confidence level is above 5 standard deviations. A breakdown of the approximate frequency of events deviating from the mean under a normal distribution is outlined in table 1.

As shown in table 1 an event of 1 and 5 standard deviations would happen approximately twice a week and once every 5,000 years respectively. Therefore it can be concluded that an event of the magnitude of Black Monday would have been nonexistent in recorded history, resulting in a Black Swan event.

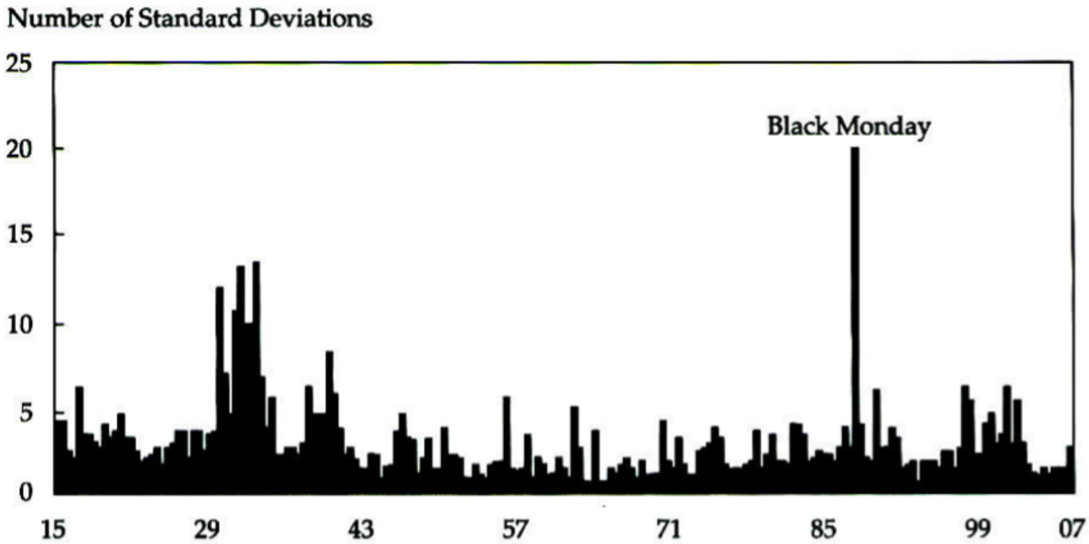


Figure 1.1: Daily standard deviation of the returns for the Dow Jones 1915-2017 [3]

1.3 Mean-Reverting vs Momentum based Strategies

Trading strategies are built to seek alpha and get excess returns from the markets. Therefore if the markets behave as a random walk trading would be pointless, since accounting for transaction costs will result in losses. An analogy with this is if an investor intends to bet on a fair coin. These events will also follow a random walk creating a Markov Chain, so investing repeatedly over a long period will not provide either profits or losses given the unpredictability of rolling the fair dice. In the case that the gambler has to pay a small fee every time he plays the game, he will suffer consistent losses. However, in the case of the markets behaving as a random walk, there would still be market participants which will benefit from trading activities. These would include market makers, (to profit from the bid-ask spread and providing liquidity) and participants that seek to reduce their risk and hedge their exposure by buying or selling derivative contracts such as options, futures and swaps.

When dealing with mean-reverting time series data investors will attempt to identify when the price of the security is relatively low for some reference and purchase it to sell it at a higher price. On the contrary, if the investor believes that the current price is relatively high he could take a short-selling position to make a profit when the price of the security falls.

Another property that the time series data can express being momentum trending towards a higher (bull) or lower (bear) price. For instance, in the scenario of a bear market, an investor will expect the price to drop in the future so many will short sell it and buy at an even lower price. In the case of a bull market, the opposite is true. This leads to a common phrase heard around trading floors such as Bull or Bear markets

As discussed in chapter 2, determining how the financial markets behave has been a conflict in academic research for decades. In fact, at different time horizons, the markets tend to express different behaviours making this an interesting field to investigate. Some research has also described that prices can express mean-reverting and momentum trending at the same time as the ”fractal” nature of stock prices. [29]

1.4 Research Proposals

Two main hypotheses are laid out in this thesis.

1.4.1 Financial market returns follow the so-called random walk hypothesis

- H_0: The markets follow a random walk.
- H_1: The markets are mean-reverting.
- H_2: The markets are momentum trending.

1.4.2 Beta as a tool for portfolio selection

- H_0: It is not possible to use beta to construct a strategy that consistently outperforms the market over a long period.
- H_1: It is possible to use beta for portfolio selection to create a strategy that outperforms the market over a long period.

Chapter 2

Literature Review

There are three main philosophies to consider when studying the behaviour of stock market returns. Some quantitative driven hedge funds have been able to build models based on these theories to develop actionable strategies that have been providing consistent returns over the years [8] [9]

2.1 Random Walk and Efficient Market hypothesis

It was first postulated that future stock market returns could not be predicted since returns followed the so-called random walk hypothesis. This follows the efficient market hypothesis formulated by American economist Eugene F. Fama in 1970, stating that prices and returns reflect all available information.[10] Therefore the only feasible way to predict future market movements is through the information that has not been released to the public. In that case, using historical data or other types of information will be worthless, since it will not be able to predict future price movements at any time scales. This is commonly modelled in quantitative finance using a continuous-time stochastic process as shown in equation 2.1

$$S_t = S_0 e^{((\mu - \frac{\sigma^2}{2})t + \sigma W_t)} \tag{2.1}$$

[6]

2.1.1 Mean Reversion

An alternative hypothesis is that price fluctuation does not follow the so-called random walk hypothesis. Market returns can be predicted based on past information since there is a tendency for market returns to express mean-reversion. There is empirical evidence that this behaviour is true, where stock prices tend to rise prices then fall towards the mean reversion and vice-versa. This theory suggests that asset prices and returns return to the mean or average of the entire dataset. [11] Eugene Fama and Kenneth French were able to provide empirical evidence to this theory in 1988, testing it with the New York Stock Exchange (NYSE) between 1926 and 1985. This study showed there was a weak autocorrelation for the daily and weekly holding periods, following the efficient market hypothesis. However, when this was tested for longer time horizons large negative autocorrelation of returns were observed providing evidence for mean reversion theory at specific time lags. [12]

There is still much debate around the theory of mean reversion across different asset classes and local markets. This is largely due to the different ways studies have previously attempted to validate this theory. These include:

1. Autocorrelation of the returns.
2. Augmented Dickey-Fuller test (ADF).
3. Providing a value for the Hurst Exponent

2.1.2 Momentum Trending

Stock market returns can also be momentum trending as shown by previous studies [13]. This thesis will also determine if the time series is momentum trending using the same tests as for mean reversion as shown in section 2.1.1. Market participants use this theory to perform technical analysis on time-series data and attempt to outperform the market seeking excess returns.

2.2 Beta as a measure of risk and portfolio selection

The Capital Asset Pricing Model (CAPM) is an investment theory that has been widely used in industry since the early 1960s. It shows the relationship between the expected returns from an investment and its associated market risk. The CAPM relies on a single important factor, Beta, which captures the exposure to a regional broad benchmark market portfolio. For this research, the S&P 500 will be used.[14]

The CAPM and the beta are seen as controversial model and measure of risk since they were first proposed. It is one of the most debated topics in finance and research that aims to validate or reject empirical evidence that will have conflictive results.

The empirical record may indicate that markets are more complex than posited by the simple CAPM. But it seems highly unlikely that expected returns are unrelated to the risks of doing badly in bad times. In this broader sense, announcement of the death of beta appears to be highly premature. [15]

Assuming that market participants are risk-averse when making portfolio selection the Capital Asset Pricing Model states that investors will: [16]

1. Minimize the variance in their portfolio.
2. Maximize the expected returns of the portfolio.

If a portfolio is mean-variance-efficient further assumptions are added to the model. Investors agree to a joint distribution of stock returns during the timeframe of the investment and investors can borrow unlimited amounts of capital at a determined risk-free rate [17]. Thereby investors will create a portfolio at time t-1 producing some non-constant return at time t [18]

The risk exposure is divided into systematic and stock-specific risk. The stock-specific risk may be reduced by diversification, however, systematic risk can only be reduced using hedging techniques such as managing long/short strategies adding derivative products to the portfolio.

The expected return for asset i can be determined using the CAPM as:

$$E(r_i) = r_f + \beta_i(E(r_M) - r_f) \tag{2.2}$$

where $E(r_i)$ is the expected return, r_f is the risk free rate, β_i is the beta value for stock i and $E(r_M)$ is the expected market return.

The beta is a measure of asset correlation with the market. A beta value of 1 will have a perfect correlation, -1 will have a perfect negative correlation and 0 will not correlate with the market. If the magnitude of beta is bigger than 1 this asset will have higher volatility than the market.

Stock specific beta is computed by computing the covariance between market returns and stock specific returns, divided by the market returns variance, as shown in equation 2.3.

$$\beta_i = \frac{cov(E(r_M), E(r_i))}{var(E(r_M))} \tag{2.3}$$

This paper does not aim to validate or reject what has been studied and used for decades, however, we do aim to investigate if the beta is a useful tool for portfolio selection and as a suitable measure of risk.

The S&P 500 is required to mean-revert for this strategy to gain excess returns. According to mean reversion theory, after a Black Swan hits the market, it will revert towards a long-term mean [12]. Hence, a basket containing high and low beta values will be created after every Black Swan event to seek excess returns. Since the is constantly changing this thesis uses the last twelve months of data to calculate the rolling beta, excluding the day of the Black Swan.

2.3 Coping with Black Swan Events using Options- An Approach by Universa Capital

Universa Investments has been using the philosophy of tail risk hedging ever since its inception in 2007. Mark Spitzanel, CIO of Universa Investments discusses how clients seek excess returns with low risk and minimal drawdown. Traditional asset allocation tackles this by having a balance of stocks and bonds. If a portfolio is seen as a pie, it can be sliced into different pieces, typically with 60% stocks and 40% bonds. Common portfolio theory suggests these slices are balancing each other in terms of risk, as they normally have a negative correlation, or at least they dont collapse together. Investors further enhance by adding assets such as credit spreads, real estate and emerging market funds, fine-tuning the intricate balance of asset allocation. However, this may not work out in times of stress since assets may be inflated as central banks reduce interest rates and further stimulate the economy with quantitative easing.

Universa Investments studied this approach and introduced new methods to achieve better performance in the event of negative black swan and bear markets. They decided to invest a small slice of their pie in far out the money put options providing large returns in the event of a sharp decline in stock valuations. Equation 2.4 shows the payoff of a put option contract, where losses are reduced in such conditions thanks to the trade-off of investing in cheap far out the money puts.

$$\text{Payoff Put Option} = \max(S_t - X) \tag{2.4}$$

[20]

Having such a tiny slice of the pie of such cheap contracts, (around 1% of total investment) they do not signify a large consistent loss when stocks rise. Universa then allocates a larger slice into stocks benefiting from a bulls market, gaining excess returns. Such an approach reduces the tail risk of the portfolio and can provide excess returns.

A recent example of when such a strategy would have been optimal was on August 24th 2015. The stock market experienced a flash crash causing the Dow Jones to drop by more than 1,000 points due to margin calls and lack of liquidity. Many investors lost a significant amount of capital, however, this investment strategy allowed Mark Spitznagels and Nassim Taleb's fund to yield a record of 1 billion USD. [19]

Chapter 3

Methodology and Theory

3.1 Data

The analysis in this chapter evaluates and investigates data for all the companies listed in the Standard & Poor 500 index listed between 2006 and 2018. It consists of daily data for both stock-specific data and index data, pulled from the Thomson Reuters data API. The S&P 500 was chosen as that the constituents are from a localised developed economy being suitable for our test.

The data was populated in a relational database (mySQL) and three tables were created including S&P 500 daily data, stock data and portfolio data. Each row in the stock data represented the 500 active listings of the S&P 500 for a given year allowing the strategy to update the available stocks it could choose from every year.

The python file required to initialise the strategy is under the name *beta-final-bucket.py* and will require several inputs including:

1. Start Date.

2. End Date.

3. % daily return change defining a Black Swan event.

4. Number of months to calculate rolling beta.

After the market close the algorithm checked for a black swan event. If identified it runs the 12-month rolling beta to identify the top 10% highest and lowest beta

```
(env) Alexs-MacBook-Pro-2:stock alexlerner$ python
beta_final_bucket.py
Taking User Input
        Start-date, End-date, X(to identify black swan),
Y(month to calculate beta):

2006-01-01 2019-01-01 0.02 12
<mysql.connector.connection.MySQLConnection object at
0x10d1ba518>
1
Start Date :  2005-1-3
 End Date :  2006-01-03
2

************ portfolio updated ***********

Start Date :  2005-1-4
 End Date :  2006-01-04
3

************ portfolio updated ***********

Start Date :  2005-1-5
 End Date :  2006-01-05
4

************ portfolio updated ***********

Start Date :  2005-1-6
 End Date :  2006-01-06
5

************ portfolio updated ***********

Start Date :  2005-1-9
 End Date :  2006-01-09
6
```

Figure 3.1: Terminal output when running the trading strategy.

stocks. This updates the portfolio, populating the portfolio table with the updated data. A sample of the output can be seen in figure 3.1.

The trading strategy only starts a year after the algorithm initialises since it has to collect data for 12 months and compute the rolling beta in the scenario of a Black Swan event. Hence, the cumulative returns only started to compute after the first trading day of 2007.

3.2 Basic Statistics and Probability Distributions

A proposed question in this thesis is if financial market returns, and more specifically the S&P 500 index, behaves like a random walk as previously postulated. [21] [22]

To test this, a random walk is simulated to compare it with our time series data allowing us to determine what is the behaviour of stock prices.

The financial data from the S&P 500 is analysed on the logarithmic scale. The returns at lag τ are defined as: [23], [24]

$$r(t,\tau) = \log(price(t+\tau)) - log(price(t)) \tag{3.1}$$

The first four moments of the distribution being *mean, variance skewness and kurtosis* are defined as: [25]

$$m_k = E[x^k] \quad \mu_k = E[X - E[X]^k] \quad \gamma_k = \mu_k/\sigma^k \tag{3.2}$$

having m_k as the k^{th} moment, μ_k as the k^{th} as central moment and γ_k as the kk^{th} standardised moment. If a normal distribution is present, the *Excess Kurtosis* will take a value of zero. The excess Kurtosis is defined as: [25]

$$ExcessKurtosis = \gamma_4 - 3 \tag{3.3}$$

It is important to look at the whole body of the distribution and not only and the central moments. Here we discuss the main probability distributions used in financial models.

3.2.1 Normal Distribution

This Normal Distribution is widely used and it plots all of the values in a symmetrical way. Most of the values will be close to the mean and it will take a bell shaped curve. This will be generated by creating pseudo-data from a random number generator and given the Central Limit Theory as discussed in section 3.3 this will evolve towards a *Gaussian Bell Shaped Curve* [25]

$$f(x|\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 \big/ 2\sigma^2} \tag{3.4}$$

where

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

With such distribution, extreme values have a lower frequency than average values. To test if market returns follow a normal distribution, the Skewness and Kurtosis of the time series (S%P 500) are analysed. To determine if this time series suffers from non-normality the Jarque-Bera test will be used as described below.

Skewness: Skewness is defined as the third moment of a distribution. If a distribution is not symmetric it has higher weighting at one either side of the mean. For instance, if the distribution is skewed towards the right it has a longer tail at the right, and consequently skewed to the left represents the opposite. Outliers may cause a distribution to be skewed, where a small number of observations account for the highest values [26]. Hence, risk measures that assume a non-skewed distribution, such as the standard deviation, may overestimate or underestimate the portfolio's exposure and risk [25]. Skewness is described as:

$$skew = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})^3}{n\sigma^3} \tag{3.5}$$

Kurtosis Kurtosis is defined as the fourth moment of the distribution, quantifying if a distribution is thin or fat-tailed. If a normal distribution is used to model fat-tailed time series data it cannot capture tail risk and will underestimate the effect of outliers. Fat-tailed distributions are narrower around the mean and have fat tails capturing extreme events in the market. Kurtosis is defined as: [25]

$$kurtosis = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})^4}{n\sigma^4} \tag{3.6}$$

Jarque-Bera Test for Normality Jarque-Bera test is used to determine if a probability distribution function is normally distributed [26]. It is computed as:

$$JB = n[\frac{skewness^2}{6} + \frac{(kurtosis - 3)^2}{24}] \tag{3.7}$$

If there is a Normal Distribution the Skewness would be zero and Kurtosis is equal to three. Hence, the Jarque-Bera test will have an output of zero.

3.2.2 Large Fluctuation Distributions

A complex system may have large fluctuations occurring with a finite probability. It can also display a skewed dataset, hence it cannot be modelled with a Normal Distribution. We define $P_>(X)$ as the complementary cumulative distribution function.

If the data behaved as a fat-tailed distribution the $P(X) \to 1 - G(X)$ at the tails of the distributions. The tails of the distribution will behave as a power law such that:

$$1 - G(X) \sim ax^{-\alpha} \tag{3.8}$$

having α as the well known *tail exponent* This value of alpha can fall into three main categories:

Thin Tailed Distributions: $\alpha \to \infty$.

All moments in this distributions are well defined.

Fat Tailed Distributions: $0 < \alpha < \infty$.

The complementary cumulative distribution declines as a power-law distribution at the tails. If the time series data follows a fat-tailed distribution it is important to note that only the first k moments with $k < \alpha$ will be defined.

Bounded Distributions: $\alpha < 0$

These distributions will not have tails given that pre-defined limits are set.

3.2.3 T-Student Distribution

Most financial datasets are fat tailed and can be fitted with a t-student distribution. This is defined as: [27]

$$f(x|\mu,\sigma,\nu) = \frac{\Gamma(\frac{\nu+1}{2})}{\sigma\sqrt{\nu\pi}\Gamma(\nu/2)} \frac{\nu + (\frac{x-\mu}{\sigma})^{2}}{\nu}^{-\frac{\nu+1}{2}} \tag{3.9}$$

having μ, σ and ν as the mean, standard deviation and degrees of freedom respectively. As the degrees of freedom $\nu \to \infty$ the distribution will tend towards a normal Gaussian distribution and as $\nu \to 0$ no moments will be defined.

3.3 Central Limit Theorem(s)

In the real world, many systems do follow a Normal distribution.

The Central Limit Theorem states that given a sufficiently large sample taken from a population with well defined mean and variance, the total average will be approximately equal to the mean of the population. Therefore as the sample size increases towards infinity the distribution will become a normal distribution.

In this thesis, we will generate a large set of independently and identically distributed random variables which we expect to evolve towards a normal distribution. By using the addition of uniformly random variables and Poisson statistics this paper aims to provide further validation towards the Central Limit Theory.

The Poisson distribution is defined as:

$$\mathrm{P}(\lambda) = e^{-\lambda}(\lambda^n/n!) \tag{3.10}$$

[25]

having P as the probability of having n events and λ will be the expected result. As the number of λ increases, under the Central Limit Theorem, this distribution will be expected to converge towards a normal distribution as described in section 3.1.1.

3.3.1 Violations of the Central Limit Theorem

The Central Limit Theorem only works when a random process is present. One of the main assumptions of the Central Limit Theory is that there is a summation of independent and identically distributed random variables. At times, this is not possible as two bodies will experience interactions within each other.

Another requirement of the Central Limit Theorem is that the probability distribution must have a finite variance. However, as discussed in section 1.2 there is sufficient evidence that extremely positive and negative returns occur more often than what the normal distribution suggests leading to fat-tailed distributions. If such distribution has α ¿ 2 the second moment, the variance will not be defined violating the Central Limit Theorem (see section 3.1.2).

3.4 Tests for Mean-Reversion and Stationarity

To seek alpha for the proposed trading strategies this thesis attempts to determine if the time series price data is a random walk, mean reverting or trending. Three tests are used to determine this property as shown in sections 3.2.1 - 3.2.3. Mathematically a continuous mean-reverting time series process is modelled using the famous Ornstein-Uhlenbeck stochastic differential equation as shown below: [7]

$$dx_t = \theta(\mu - x_t)dt + \sigma dW_t \tag{3.11}$$

Here θ represents a constant for the rate of reversion of the mean, μ is the mean value of the time series process, σ the square root of the variance of the process and W_t is a Wiener process (ie: Brownian Motion).

This equation shows that the difference of the time series between the current period t and the next period $t+1$ is proportional to the mean μ minus the current price x_t, plus the addition of a *Gaussian noise*.

The following tests will compare the results four different data inputs. This includes the price time series of the S&P 500 used in the trading strategy, and three simulated time series, being:

1. A simulated Geometric Brownian Motion
2. A simulated mean reverting process.
3. A simulated momentum trending process.

This will also be useful to check if the code works appropriately, since values from the three different simulated datasets should provide three different answers to our null hypothesis.

3.4.1 Auto-correlation Function

This is a popular tool that has been used by traders used in technical analysis. The auto-correlation, also known as serial correlation represents the correlation of a signal with respect to a delayed copy of itself. Therefore, it can measure the similarity between observations as a function of time lag between them. This function has been commonly used for two main purposes: [30]

1. Determine if the time series data does not follow a random walk.
2. Provide a measure of persistent trends and self-similar behaviours.

Correlation Coefficient :

To better understand the auto-correlation function it is important to determine a way how similar two time series are. There are different ways to do this, depending on what underlying assumptions your data has. The most common one is the *correlation* as shown below:

$$corr(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} \tag{3.12}$$

where σ represents the standard deviation of X or Y and $cov(X,Y)$ the covariance between X and Y defined as:

$$\sigma_X = \sqrt{\frac{1}{n}\Sigma_{i=1}^{N}[X_i - \overline{X}]^2}$$

and

$$\overline{X} = \frac{1}{N}\Sigma_{i=1}^{N}[X_i]$$

where $\overline{X}$ is the mean and σ is the standard deviation. The mean is simply the average of the full time series, whereas the standard deviation measures how far the data points are from the mean. This is achieved by taking the square root of the variance, being the squared distance from the mean.

$$var(X) = \sigma_X^2$$

If the variance has a value of zero, this would mean all data points are at the mean. If a high variance is detected the data points are more random being scattered around. The covariance between X and Y is a generalisation of the concept of variance between two time series datasets instead of one. This shows how these two time series data change together, being a symmetric coefficient such that:

$$corr(X,Y) = corr(Y,X)$$

It is important to analyse the correlation coefficient with care, since this does not necessarily mean that the change in variable X will cause the change of variable

Y. This is in fact represented by causality, however we will not use it as part of our research. The correlation coefficient is normalised by the variance so that $-1 \leq corr(X,Y) \leq +1$.

Similarly the auto-correlation function will calculate the correlation coefficient with itself, shifted in time. Therefore effects such as periodicity will cause the correlation coefficient to resonate and be higher. The auto-correlation function is defined as:

$$autocorr(X,\tau) = \frac{\Sigma_{i=1}^{N}[X_i - \overline{X}][X_{i-\tau} - \overline{X}]}{\Sigma_{i=1}^{N}[X_i - \overline{X}]^2} \tag{3.13}$$

The auto-correlation function is also plotted at different lags. At lag $\tau = 0$ it will take a value of 1 since the time series will be compared with itself, which will be expected in our results.

3.4.2 Augmented Dickey Fuller Test (ADF)

Test if a unit root is present in a time series sample or more common stationary/trend-stationary. [7] In fact, if the price time series is mean reverting, knowing the current price level will provide us with information on what the next price move will be such that:

1. If the current price level is above the mean, the next price move will be downwards.

2. If the current price level is below the mean, the next price move will be upwards.

An ADF test will therefore focus only on this information. The price changes can be represented using a linear model as shown in equation 3.14:

$$\Delta y(t) = \lambda y(t-1) + \mu + \beta t + \alpha_1 \Delta y(t-1) + ... + \alpha_k \Delta y(t-k) + \varepsilon_t \tag{3.14}$$

[7]

where $\Delta y(t) \equiv y(t) - y(t-1)$, $\Delta y(t-1) \equiv y(t-1) - y(t-2)$ and so on. The ADF test will then determine if $\lambda = 0$ at $y(t-1)$. If this is true, the subsequent price move at $y(t)$ will depend on the previous price, not following a random walk. The test statistic is defined as the ratio of the regression coefficient λ with respect to the standard error of the regression fit: $\lambda/SE(\lambda)$. In this case $y(t-1)$ is the independent variable and $\Delta y(t)$ the dependent variable. Statisticians Fuller and Dickey were able to find the distribution for this test statistic and arranged the key values, allowing research to look up a value of $\lambda/SE(\lambda)$ to determine if the null hypothesis is rejected. (ie: at 95% probability confidence level). The Dickey Fuller test is shown in equation 3.15. [7]

$$DF_\tau = \frac{\lambda}{SE(\lambda)} \tag{3.15}$$

In this paper we will interpret the result by using the *p-value* from the test. Our *p-value* specified threshold will be considered. (ie: at 5%). If our *p-value* is below such threshold the null hypothesis (stationary) will be rejected, otherwise the null hypothesis will be accepted (non-stationary).

3.4.3 Hurst Exponent

This tests determines if a price time series is stationary. This means that the log variance in the log price will increase at a slower, faster or same rate with respect to a geometric random walk. Therefore the variance will be a sublinear function of time, instead of a linear function represented in a geometric random walk. This sublinear function is commonly approximated by τ^{2H}, having τ represent time and H the Hurst exponent. There a three key values the Hurst exponent can take being:

1. $H < 0.5$: Price time series is known to be stationary.
2. $H = 0.5$: Price time series follows the so called Geometric Random Walk.
3. $H > 0.5$: Price time series is momentum trending.

The Variance Ratio test is also used to see if the null hypothesis that the Hurst exponent does deviate from 0.5, not following a random walk.

In fact a price time series that is *stationary* will diffuse from the initial value more slowly than a geometric random walk. The mathematical approach used determines the behaviour of the price time series by measuring the speed of diffusion characterised by the variance as shown in equation 3.16. [7]

$$\mathrm{var}(\tau) = \langle |z(t+\tau) - z(t)|^2 \rangle \tag{3.16}$$

Here z represents the log price time series (ie: $z = \log(t)$), τ is defined lag in days and $\langle - - - \rangle$ represents the mean value for all t's.

It is known that if a price time series follows a geometric random walk the relationship would be :

$$\langle |z(t+\tau) - z(t)|^2 \rangle \sim \tau \tag{3.17}$$

This means that this relationship tends to an equality with a proportionality constant when accounting with a large value for τ, however this is not necessarily true when using a small value for τ. However if the log-price time series is in fact mean reverting or momentum trending, equation 3.17 does not hold and the following relationship is used. [7]

$$\langle |z(t+\tau) - z(t)|^2 \rangle \sim \tau^{2H} \tag{3.18}$$

As previously mentioned if H < 0.5 this price time series is mean reverting and therefore our strategy would be able to achieve excess returns. As this value approximates zero the price time series will be more mean reverting, and as it approximates 1, the price time series will be increasingly trending. Therefore, H also quantifies to what extend the price time series is mean reverting or momentum trending. [7]

Given that we need to know what is the statistical significance to determine if we can reject the null hypothesis that H equals 0.5, the variance ratio test is used as shown below. [31] [7]

$$VR(t,\tau) = \frac{\mathrm{var}(z(t) - z(t-\tau))}{\tau \mathrm{var}(z(t) - z(t-1))} \tag{3.19}$$

3.5 Trading Strategy

This thesis aims to test if the beta is a useful tool for portfolio selection under the assumption that mean reversion is present in our time series data. Many strategies and models take black swan events as being an anomaly. This time we are taking a different approach where we try to exploit market reaction after such events, having black swans as the core foundation of our strategy.

A basket of stocks listed on the S&P 500 is used which is updated every year to check if any constituents have been delisted or listed into the index. Before running the backtest a black swan is defined as a daily 3% returns. Given that no formal definition exists for a black swan we chose this value to show extreme daily returns while assuring we had sufficient events to base the strategy on. A basket of 100,000 USD was created at the start of the test and an empty basket was kept up until it observed the first black swan of the time series.

After a day of extreme negative returns in the S&P 500 index, the twelve-month rolling beta is calculated on all the stocks listed in the index. The strategy will identify the 10% stocks with the highest and lowest beta, where the highest beta stocks will queue for a market buy order and the lowest for a market sell order. In the scenario of a positive black swan, the opposite approach will be taken where placing the market buy orders. (ie: long stocks with low beta and short stocks with the high beta). After every black swan event, the algorithm will take the same approach and rebalance the portfolio.

Consequently, if a mean reversion process is present this strategy will be able to take advantage of it to seek alpha and outperform the benchmark index. For instance, in the case of a negative black swan, the stocks with the highest beta will be able to take advantage of the upwards mean-reverting motion while the stocks with the lowest or negative beta will mean revert at a slower upwards or downwards rate. This will be able to provide further validation in the dispute of whether the markets are mean reverting or follow a Brownian motion as postulated in previous studies. (Estrada & Vargas 2010). The cumulative returns of the strategy and the S&P 500 index are compared for the period of the test. This will also test the efficient market theory where an active strategy can outperform the market. If true this will also provide further validation for beta as a useful tool for portfolio selection and an appropriate risk measure for the S&P 500.

Chapter 4

Results

4.1 Hypothesis I

This thesis aims to determine if the financial markets follow the so called random walk hypothesis. Several tests are laid out including the auto-correlation function, Hurst exponent, Augmented-Dickey Fuller test and the Jarque-Bera test for normality.

4.1.1 Autocorrelation of the Returns

The auto-correlation of the returns are explored for the S&P 500 at different time scales. This allows us to quantify if the time series is mean reverting, random or momentum trending. As described in section 3.4.1 an $auto-correlation < 0$ will be mean reverting, $auto-correlation = 0$ will be a random process and $auto-correlation > 0$ will be a momentum trending process. These results are shown in table 4.1.

Table 4.1 shows that the time series data can have different auto-correlations with different time scales. The data exhibits negative autocorrelations with daily, weekly and yearly returns showing a mean-reverting process is present. However, when assessing the autocorrelation of monthly returns a positive value is observed

Daily Returns	Weekly Returns	Monthly Returns	Yearly Returns
-0.08123	-0.04565	0.08436	-0.07244

Table 4.1: Autocorrelation of the Returns at different time scales

S&P 500	Geometric Brownian Motion	Mean Reversion	Momentum Trending
0.4732	0.5024	0.0001	0.9530

Table 4.2: Hurst exponent for the S&P 500, Geometric Brownian Motion, mean reverting process and momentum trending process

	S&P 500	Mean Reversion	Geometric Brownian Motion
ADF Statistic	0.2461	-315.69	-0.9813
p-value	0.9747	0.0000	0.7600
1% Critical Value	-3.432	-3.430	-3.430
5% Critical Value	-2.862	-2.862	-2.862
10% Critical Value	-2.567	-2.567	-2.567

Table 4.3: Summary Statistics for the ADF test for the S&P, Mean Reversion and Momentum Trending time series data

showing that the time series is both mean-reverting and momentum trending depending on the time scale it is analysed at.

4.1.2 Hurst Exponent

As described in section 3.4.3 The Hurst exponent is computed to test for mean reversion. To be able to assess the accuracy and functionality of this test this is tested for the S&P 500 dataset, a Geometric Brownian motion simulation, a mean reverting process and a momentum trending process. Table 4.2 shows the Hurst exponent for the four tests.

As expected, the Geometric Brownian Motion's Hurst is approximately 0.5, the mean reversion process's tends to zero and momentum trending approaches to one. The Hurst of the S&P is slightly under 0.5 showing that this process is weakly mean reverting.

4.1.3 Augmented-Dickey Fuller Test

Developed in 1979 by David Dickey and Wayne Fuller the Dickey-Fuller test determines if a unit root is present in an autoregressive model. Given the complexity of market data the simple autoregressive model fails to capture the dynamic structure, so the augmented Dickey-Fuller test was chosen. When the ADF test takes a negative number the null hypothesis is rejected that a unit root is present.

Table 4.3 shows the summary statistics for the ADF test for the S&P 500, a mean-reverting process and a Geometric Brownian Motion. The mean-reverting process has a very large negative ADF statistic with a p-value tending to zero. The Geometric Brownian Motion has a slightly negative ADF statistic however the p-value is far beyond the critical values, so the null hypothesis is rejected for the mean reversion process and not the Geometric Brownian motion.

More interestingly, the statistics for the S&P data shows that it is larger than the defined critical values of the test statistic, being at 1, 5 and 10 per cent values respectively. Therefore the null hypothesis of $\lambda = 0$ and therefore makes it less likely that the time series data follows a mean-reversion process.

4.2 Hypothesis II

The second part of this thesis aims to build a trading strategy based on mean reversion and black swan events. As described in section 2.1.1 there is sufficient literature stating the presence of mean reversion and there have been some initial insights previously in this report backing up such theory. The logic of this strategy is laid out in section 3.5.

This will also test if β is a useful tool for portfolio selection, and thereby create a strategy that outperforms the well-diversified S&P 500 index.

Black Swan events are defined at 3% daily change and the strategy starts with $100,000. A total of 3004 trading days are used to backtest the strategy ranging from 03/01/2007 up until 07/12/2018. In this period, a total of 40 positives and 55 negatives were identified, making a total of 95 Black Swan events when defined at 3% daily change. The first black swan event is found on the 27th of February of 2007 is a -3.47% drop in the S&P 500. After this event is where our portfolio starts investing long in the highest beta stocks and shorts the lowest beta. Figure 4.1 shows the cumulative returns of both the passive index investing in the S&P 500 and the active portfolio.

Investing $100,000 in the S&P 500 index on the first trading day of 2007 would have returned $185,193 resulting in an 85.2% increase. If the investor would have chosen the active strategy the investment would have returned $191,111 resulting in

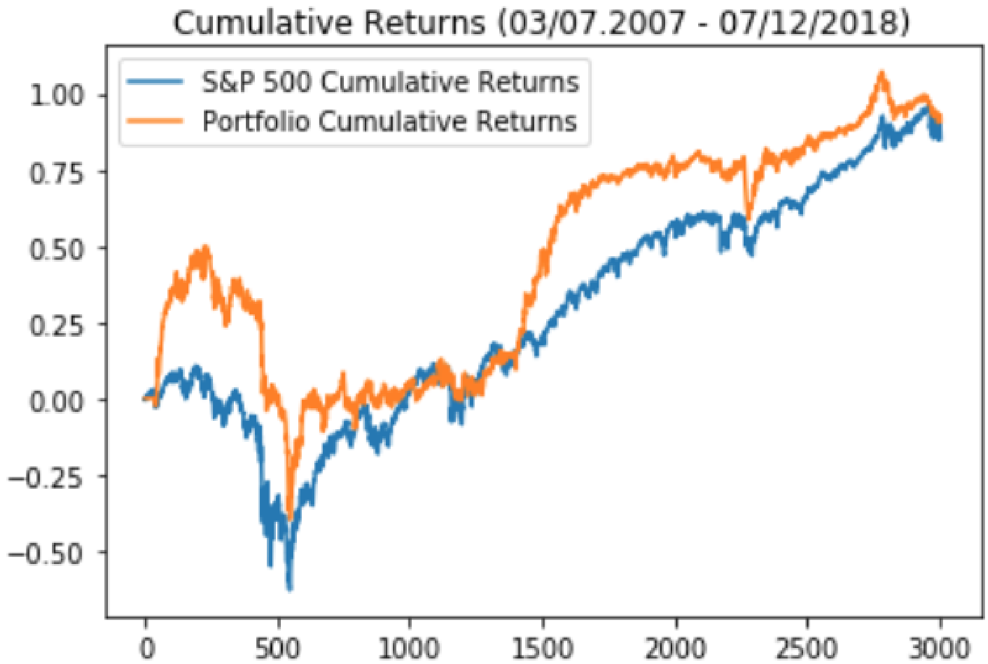


Figure 4.1: Cumulative returns for the active (orange) and passive (blue) strategies

a 91.2% increase. This shows that both the active and passive strategy resulted to be profitable and that the active strategy was able to yield slightly higher returns.

4.2.1 Jarque-Bera test for normality

As described in section 3.3 the Central Limit Theorem is at the core of the statistical theory. This thesis aims to prove this by providing further validation by creating a simulation of random variables as shown in figure 4.2.

This test simulated the addition of uniformly random variables and is tested with 1, 2, 4, 8, 16 and 32 random variables. The results show that as the number of random variables increases the distribution tends towards a normal distribution providing further proof for the Central Limit Theory.

If the markets are random the probability density function of the returns should become a normal distribution. The probability density function of the S&P 500 and the portfolio returns are shown in figure 4.3.

Summary statistics for the mean, variance, Skewness, Kurtosis and Jarque-Bera test for the portfolio returns and S&P returns are shown in table 4.4.

These statistics show that the mean value for the returns was slightly higher

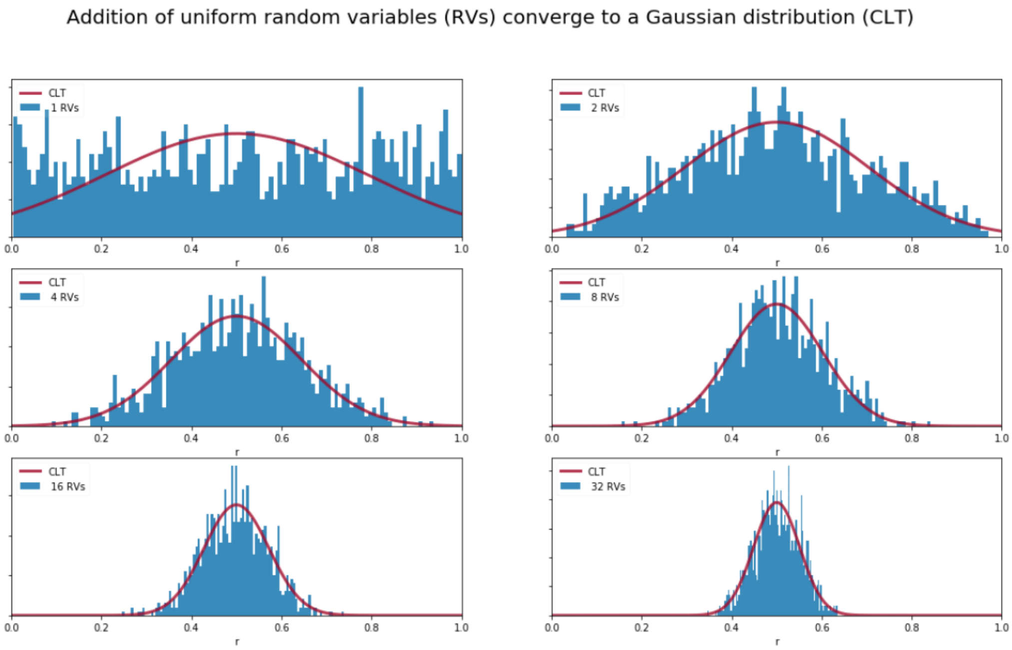
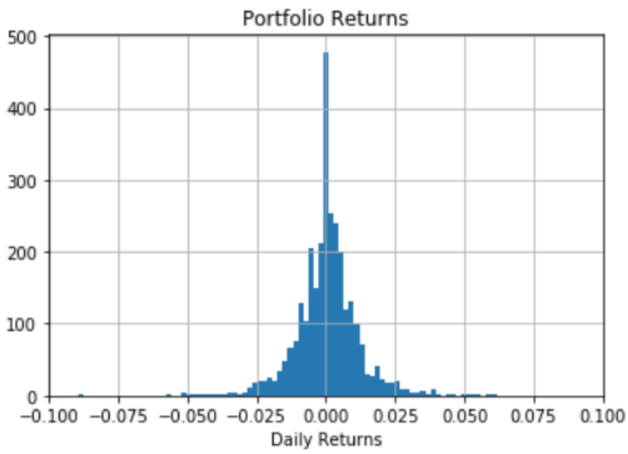


Figure 4.2: Central Limit Theory by the addition of Random Variables

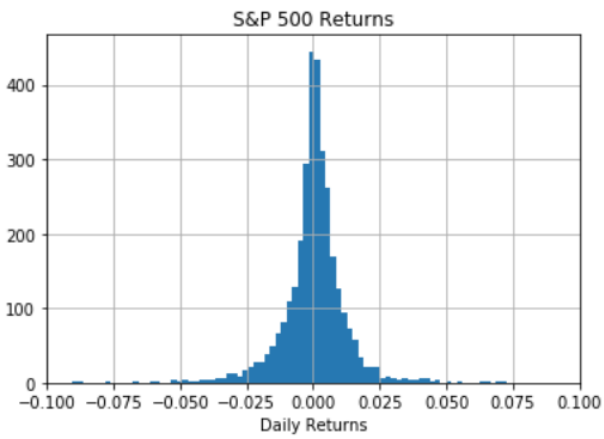
Summary Statistics	**Portfolio Returns**	**S&P 500 Returns**
Mean	0.0003	0.0002
Standard Deviation	0.0116	0.0124
Skewness	-0.0524	-0.1349
Kurtosis	8.4227	11.0437
Jarque-Bera Test	9017	15280

Table 4.4: Summary statistics for the active and passive trading strategy.

than zero for both the trading strategy and the S&P 500. The standard deviation for the passive strategy was higher than the active strategy. This shows that volatility was lower for the active strategy due to the long/short approach instead of only relying on diversification to mitigate risk. The S&P 500 had both a higher magnitude in Skewness and Kurtosis which leads towards a higher value in the Jarque-Bera Test. This shows that both time series deviate from a normal distribution and that it is of a fat-tailed distribution given that $kurtosis > 3$ for both time series. The Skewness appears to be negative for both time series showing that these time series are skewed towards the left. This provides further insights into the structure of the distribution showing that extreme returns are more commonly negative than positive.



(a)



(b)

Figure 4.3: Probability Density Function of the returns for the active and passive strategy.

Chapter 5

General Conclusions

This thesis aims to determine if the S&P 500 index followed the random walk hypothesis, mean reversion process or momentum trending process. This has been a topic of discussion for researchers given that there have been many conflicts in determining this depending on what time scale or what test is used.

This thesis is unable to reject the null hypothesis stating that the markets follow the random walk hypothesis. Three tests were taken including the autocorrelation function of the returns, Hurst exponent and Augmented Dickey-Fuller test. When assessing this as a whole different result are depending on the test performed. When assessing the autocorrelation function, there are initial insights that the markets are mean-reverting when testing for daily, weekly and yearly returns. However monthly returns yield a positive autocorrelation meaning that it is behaving as a momentum trending process. It's also important to note that at none of these time scales a large absolute value for the autocorrelation is present, showing how the null hypothesis cannot be rejected.

The Hurst exponent for the S&P 500 resulted to be slightly under 0.5, being in the region of mean reversion. To validate our function this was also tested using a pseudo-data from a Geometric Brownian Motion, mean-reverting process and momentum trending. This shows how the Hurst exponent the mean-reverting process tends to zero and the Hurst for the momentum trending tends to one.

Lastly, the Augmented Dickey-Fuller test was computed giving a positive ADF statistic for the S&P 500. Interestingly the value is far away from the 1%, 5% and

10% critical values. This means that this test cannot reject the null hypothesis.

Overall our results also conflict with each other showing how hard it is for investors to find and exploit market insights.

The second part of this thesis takes the assumption that markets are mean-reverting. When defining a Black Swan event at 3%, the active equity trading strategy was able to outperform the benchmark index. This shows that beta is a valid tool for portfolio selection and brings further evidence that the markets could be mean reverting. However, these returns are not significantly higher than the benchmark index meaning that the associated transaction costs could make this strategy less profitable than the benchmark index.

We conclude that there still has to be further research to determine if the markets follow the random walk hypothesis. Secondly, there is further evidence that a beta is a valid tool for portfolio selection when testing for our active trading strategy.

Chapter 6

Suggestions For Further Research

It is clear that there is still a lot of work to do to determine what is the behaviour of the market and specifically with the S&P 500. It would also be convenient to determine the exact speed of reversion and what is the relationship of mean reversion after a Black Swan event. This would allow investors to determine a more formal definition for a Black Swan and determine an optimal time length to calculate the rolling beta of the stocks.

This thesis only focuses on U.S. equities. Therefore it would be interesting to see how a similar strategy would behave in a multi-asset scenario such as adding foreign exchange and commodities.

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