STREAMLINES

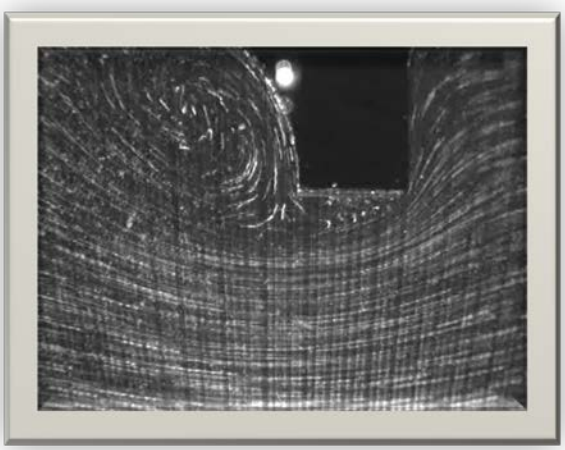
The flow of a real fluid is very complex and, as a result, complete solution of problems can seldom be obtained without recourse to experiment[1]. Fluid mechanics is a highly visual subject[2]. While using the Interactive Experiments you will learn a great deal about the flow qualitatively and quantitatively using particle image velocimetry (PIV). The most common mathematical method for flow visualization is the streamline pattern.

Flow patterns can be described by lines and there are several types of lines:

1. **Streamline**: this is an imaginary curve across which – at that instant – no fluid is flowing. It can also be called a flowline or line of flow. At this instant in time the direction of the velocity of every single particle on this line is along this line. The pattern, which several streamlines form, gives a very good description of the flow. Since the streamlines are describing an instant of time the patterns they form can be considered to be an instantaneous photograph of the flow. This is exactly what you see when you use the Interactive Experiment and take snapshots of the particles moving fast through the flow model.
2. **Pathline**: An individual particle in the flow does not necessarily follow the flow. So the actual path that a given fluid particle follows is called a Pathline. If one considers a streamline as an instantaneous photograph of the flow, a pathline is time exposure of the path of the particle at successive instants of time.
3. **Streakline**: This line is the locus of particles which have passed through a prescribed point. Another term used for this line is filament line. Traditionally a streakline can be produced experimentally by the continuous release of marked particles such as dye, smoke or bubbles[2]. In our experiment we are producing streaklines using solid particles which are illuminated by a laser.

Figure 1 shows an example of what you may see during your experiments.

Figure 1. Streamlines



A region bounded by streamlines is called a streamtube. Because the streamlines are tangent to the flow velocity, fluid that is inside a stream tube must remain forever within that same stream tube. A scalar function whose contours define the streamlines is known as the stream function.

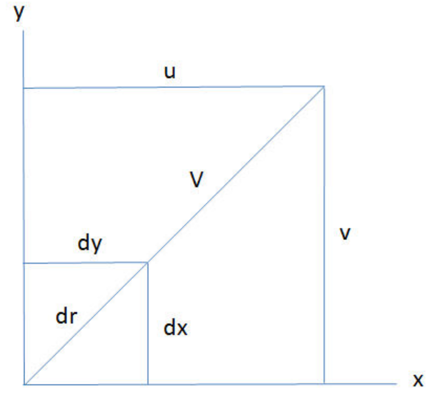
In general, streamlines, pathlines, and streaklines are not the same. But in steady flow all three lines coincide. In our experiments you will observe variation of flow parameters both in time and space. If the flow properties vary over time, the flow is said to be unsteady. But the flow can be steady to one observer and not to another. This is because all motion is relative. If the flow properties vary over location, the flow is said to be non-uniform. Fluid flow analysis is simplified greatly if we assume steady uniform flow. It is important to understand these terms that describe the flow variation in time and space:

1. **Steady uniform flow**: This is considered to be fluid that is flowing at a high constant rate through a straight long pipe of constant cross section, disregarding flow close to the walls. There is always non-uniformity near the walls. This is because fluid has viscosity which reduces the velocity to zero at the wall boundary, i.e. the non-slip condition.
2. **Steady non-uniform flow**: This is fluid flowing in a nozzle or a venturi, i.e. a tapering pipe.
3. **Non-steady uniform flow**: This is an accelerating or decelerating flow through a long straight pipe of constant cross section, disregarding close to the walls.
4. **Non-steady, non-uniform flow**: This is an accelerating or decelerating flow in a nozzle or a venturi, i.e. a tapering pipe.

There is another element and that is the dimension. In general all fluid flow is three dimensional. So the velocity, pressure and other fluid parameters vary in all three coordinates, x, y and z. In the Interactive Experiment we use only one camera and observe the flow only in the x and y plane. So we approximate the flow to a two dimensional flow. If we were to use two cameras then we can get the z component of velocity. Conventional PIV utilizes a single camera oriented orthogonally to the object plane, resulting in the loss of the out-of-plane component. Stereoscopic PIV uses two cameras to obtain the full three-dimensional vector. Two camera-axes are rotated inward such that they intersect on the object plane. Considerable simplification in analysis is achieved by not having appreciable variation of fluid parameters in the z coordinate. The assumption we have to make is that the streamlines are the same at any instant of time in the z plane. So the two dimensional flow are functions of time and two rectangular space coordinates, x and y.

We can now discuss the mathematical definitions of streamlines, pathlines, and streaklines. Figure 2 shows the geometric relations for defining a streamline.

Figure 2. Streamline Geometric Relations



Every vector arc length dr along a streamline must be tangent to V, so the respective components must have the same proportion:

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dr}{V} \quad \text{.......................................Equation 1}$$

Thus the streamline passing through (x_0,y_0,t_0) can be calculated by integrating equation 1, assuming we know u and v as functions of position and time.

The pathline is defined by integrating the relation between velocity and displacement.

$$\frac{dx}{dt} = u(x, y, t) \quad \text{....................................Equation 2}$$

$$\frac{dy}{dt} = v(x, y, t) \quad \text{....................................Equation 3}$$

The pathline function f(x,y,t) is determined by integrating equations 2 and 3 with respect to time with the initial condition (x_0,y_0,t_0), and then eliminating time.

The streakline function is determined by taking the integrated result of equation 2 & 3 but retaining the time as a parameter, finding the integration constants for (x_0,y_0) for a sequence of times σ<t, and then eliminating σ from the result to obtain the streaklines.

EXAMPLE[2]

Problem: Given the velocity distribution: u=Kx and v=-Ky w=0 where, K is constant, compute and plot the streamlines of the flow, including directions, and give some possible interpretations of the pattern. Also,

Flow is incompressible:

$$\frac{du}{dx} + \frac{dv}{dy} = 0$$

And, flow is irrotational:

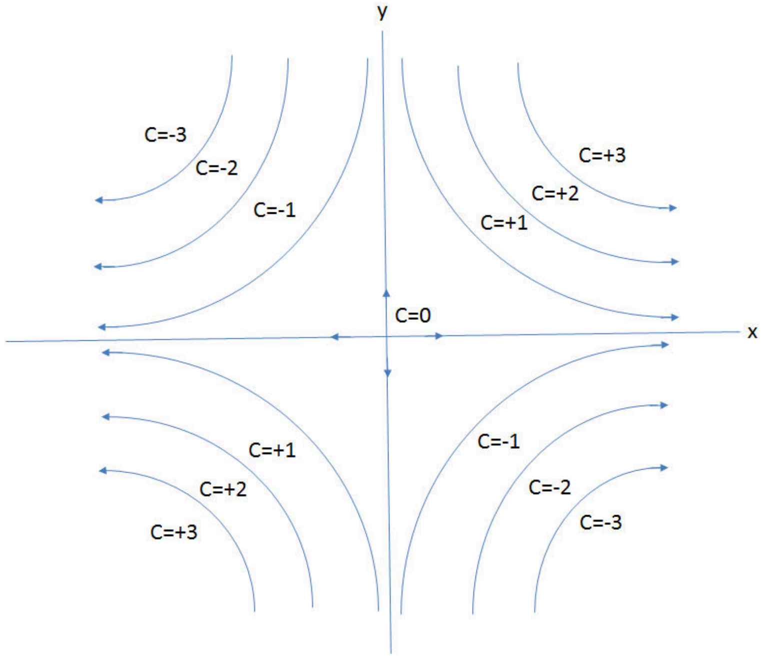
$$\begin{pmatrix} i & j \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \\ u & v \end{pmatrix} = 0$$

Solution: Since time does not appear explicitly in the given equation, the motion is steady, so that streamlines, pathlines, and streaklines coincide. Since w=0 everywhere, the motion is two dimensional, in the x-y plane. The motion is also incompressible and irrotational. The streamlines can be computed by substituting the expressions for u and v into equation 1.

$$\frac{dx}{Kx} = -\frac{dy}{Ky}$$

Integrating, we obtain ln x = -ln y +ln C or

$$xy=C$$



This is the general expression for streamlines, which are hyperbolas. The streamline pattern is entirely dependent on the value of K. It could represent the impingement of two opposing streams, or the upper half could simulate the flow of a single downward stream against a flat wall. The stagnation point (i.e. u=v=0) occurs when two opposing streamlines intersect each other, where C=0.

References:

1. “Mechanics of Fluids,” B. S. Massey, Chapman & Hall, ISBN 0 412 34280 4
2. “Fluid Mechanics,” Frank M. White, McGraw-Hill Book Company, ISBN 0 07 069673 X