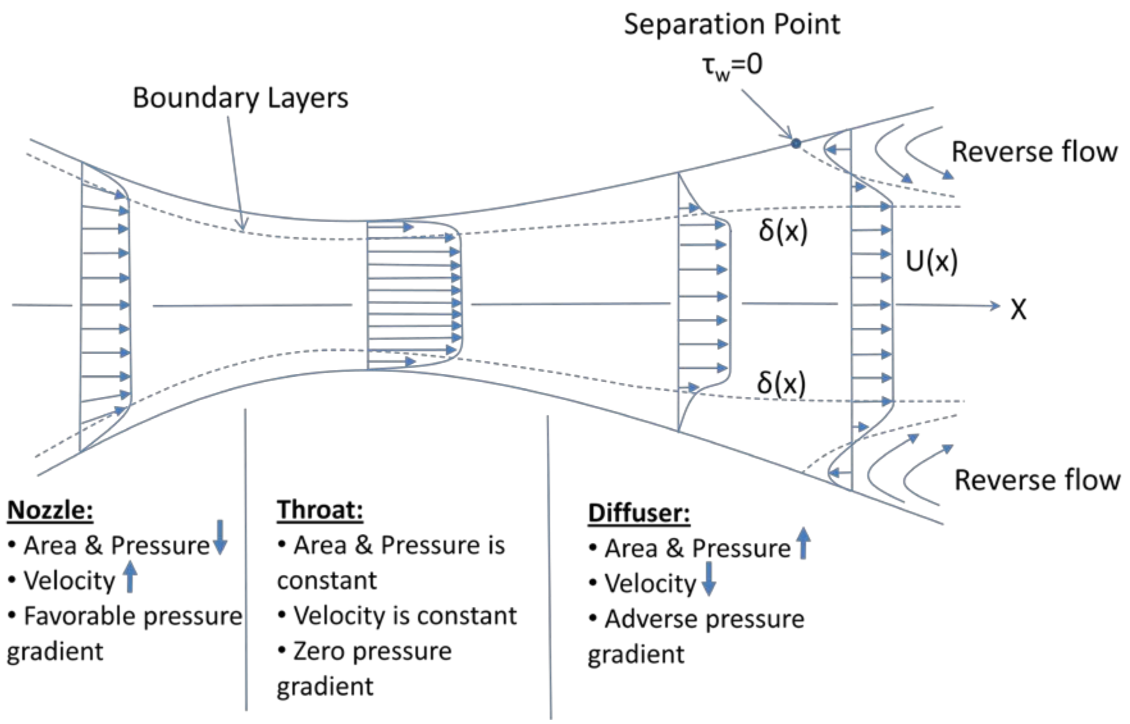
FLOW SEPARATION

The flow separation phenomena can be studied very well with the Interactive Experiment. By using various flow models and by varying the speed of the flow the separation effects can be observed visually.

The flow models that we use can simulate a nozzle, throat or a diffuser. The flow phenomena that we can observe can be summarized in the diagram below[2]:



Flow separation occurs because of excessive momentum loss in the boundary layer near a wall. This loss can be initiated by an adverse pressure gradient where dp/dx>0. Flow separation can occur in a diffuser or a sudden expansion. In the diffuser flow separation will occur at one or both walls if the diffuser angle is too large leading to excessive adverse pressure gradient. Flow separation will result in reverse flow, increased losses and poor pressure recovery. This is called a diffuser stall.

In a favorable pressure gradient – like in a nozzle – where dp/dx <0 flow separation can never occur. Separation occurs when $\delta u/\delta y=0$ (or $\tau_w=0$) where τ_w is the wall shear stress.

The boundary layer may become turbulent once the laminar layer separates. Separation streamline is the line of zero velocity dividing the forward and reverse flow, and it starts from the separation point. The reverse flow causes large irregular eddies. These eddies are undesirable because of energy loss. The separated boundary layer curls, and the disturbed flow region continues downstream. You will very clearly observe this during the experiments. The pressure downstream remains approximately the same as at the separation point because the energy is dissipated as heat[1].

Both laminar and turbulent boundary layers separate, but laminar layers tend to separate more easily. This is because the laminar flow velocity gradient from the wall is lower and the adverse pressure gradient can more rapidly halt the slow moving fluid near the wall. A turbulent boundary layer is more resistant to adverse pressure gradient. However, greater the adverse pressure gradient quicker the separation for both laminar and turbulent flows. The boundary layer, $\delta(x)$, thickens rapidly in an adverse pressure gradient, and one can no longer assume that $\delta(x)$ is small[1].

The boundary layer separation greatly affects the flow as a whole. A wake of disturbed flow downstream is formed which radically alters the flow pattern. The effective boundary of the flow is an unknown shape – which also includes the zone of separation - instead of the wall. The altered flow pattern may cause the position of the minimum pressure to move upstream. This may result in the point of separation moving upstream[1].

Flow separation becomes very important in the design of aerodynamics. For example, flow separation increases drag in racing cars or airplanes. You will also observe in the Interactive Experiment that sharp edges always cause flow separation. So this is why it is so important to streamline the surfaces.

There is no exact theory to predict the separation point. However, we can use the momentum equation to obtain an approximate prediction for laminar flows. The momentum integral equation can be expressed as[1]:

$$\tau_0 = \rho \frac{d}{dx}\left(u_m^2 \theta\right) + \rho \frac{du_m}{dx} u_m \delta^* \text{......................Equation 1}$$

Where τ_0 is the shear stress at the boundary, ρ is density, u_m is the mainstream velocity, δ^* is the displacement thickness and θ is the momentum thickness. Displacement thickness is given by[1]:

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{u_m}\right) dy \text{...........................Equation 2}$$

The concept of displacement thickness allows us to consider the main flow as that of a frictionless fluid past a displaced surface instead of the actual flow past the actual surface[1]. Similarly the momentum thickness is defined as:

$$\theta = \int_0^\infty \frac{u}{u_m}\left(1 - \frac{u}{u_m}\right) dy \text{...........................Equation 3}$$

Rearranging Equation 1 and multiplying by $2\theta/\mu u_m$ we get:

$$\frac{u_m}{\nu}\frac{d}{dx}\left(\theta^2\right) = \frac{2\theta\tau_0}{\mu u_m} - 2\left(2 + \frac{\delta^*}{\theta}\right)\frac{\theta^2}{\nu}\frac{du_m}{dx} \text{................Equation 4}$$

Where ν is the kinematic viscosity. Also for laminar flow:

$$\frac{\tau_0}{\mu} = \left(\frac{\partial u}{\partial y}\right)_{y=0} \text{.......................……...Equation 5}$$

Equation 5 can be obtained from the velocity distribution. The velocity distribution in laminar layers for various pressure gradients has been studied. The analysis has shown that the right hand side of equation 4 can be approximated to:

$$\frac{u_m}{\nu}\frac{d}{dx}(\theta^2) = 0.45 - \frac{6\theta^2}{\nu}\frac{du_m}{dx} \text{..................Equation 6}$$

If we rearrange, multiply by νu_m^5 and integrate equation 6, we get:

$$\theta^2 = \theta_0^2 + \frac{0.45\nu}{u_m^6}\int_0^x u_m^5 dx \text{.....................Equation 7}$$

If x is measured from the upstream stagnation point, then $\theta_0 = 0$. We can thus calculate variation of u_m with x and θ, for a given pressure gradient.

EXAMPLE[1]

Problem: If we assume $u_m = a(1 + bx)^{-1}$ and $\theta_0 = 0$, where a and b are positive constants, determine if the boundary layer will separate. It is shown experimentally that if boundary layer separates, separation occurs at approximately:

$$\frac{\theta^2}{\nu}\frac{du_m}{dx} = -0.09$$

Solution: Since $\frac{du_m}{dx} = \frac{-ab}{(1+bx)^2}$ is negative, there is an adverse pressure gradient. So the boundary layer will separate. If we substitute u_m into equation 7 we get:

$$\theta^2 = \frac{0.45\nu}{a^6(1+bx)^{-6}}\int_0^x a^5(1+bx)^{-5}dx = \frac{0.45\nu}{4ab}\{(1+bx)^6 - (1+bx)^2\}$$

So, $\frac{0.45\nu}{4ab}\{(1+bx)^6 - (1+bx)^2\} = \frac{0.09\nu(1+bx)^2}{ab}$

Thus, $x = \frac{0.1583}{b}$

References:

1. "Mechanics of Fluids," B. S. Massey, Chapman & Hall, ISBN 0 412 34280 4
2. "Fluid Mechanics," Frank M. White, McGraw-Hill Book Company, ISBN 0 07 069673X