Sequent Calculus Rules

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$$\frac{}{\alpha,\ \Gamma\ \Rightarrow\ \alpha,\ \Delta}\ \mathit{Axiom}$$

$$\frac{\Gamma,\ \neg\alpha \vee \beta\ \Rightarrow\ \Delta}{\Gamma,\ \alpha\ \rightarrow\ \beta\ \Rightarrow\ \Delta}\ [\rightarrow\ \Rightarrow r.w.] \qquad \frac{\Gamma\ \Rightarrow\ \neg\alpha \vee \beta,\ \Delta}{\Gamma\ \Rightarrow\ \alpha\ \rightarrow\ \beta,\ \Delta}\ [\Rightarrow\ \rightarrow r.w.]$$

$$\frac{\alpha,\ \beta,\ \Gamma\ \Rightarrow\ \Delta}{(\alpha \wedge \beta),\ \Gamma\ \Rightarrow\ \Delta}\ [\wedge\Rightarrow] \qquad \frac{\Gamma\Rightarrow\ \alpha,\ \Delta\quad \mathit{and}\quad \Gamma\Rightarrow\beta,\ \Delta}{\Gamma\ \Rightarrow(\alpha \wedge \beta),\ \Delta}\ [\Rightarrow\wedge]$$

$$\frac{\alpha,\Gamma\Rightarrow\Delta\quad \mathit{and}\quad \beta,\Gamma\Rightarrow\Delta}{(\alpha \vee \beta),\ \Gamma\ \Rightarrow\ \Delta}\ [\vee\Rightarrow] \qquad \frac{\Gamma\ \Rightarrow\ \alpha,\ \beta,\ \Delta}{\Gamma\ \Rightarrow\ (\alpha\ \vee\ \beta),\ \Delta}\ [\Rightarrow\ \vee\ ]$$

$$\frac{\Gamma\ \Rightarrow\ \alpha,\ \Delta}{\neg\alpha,\ \Gamma\ \Rightarrow\ \Delta}\ [\neg\Rightarrow] \qquad \frac{\Gamma,\ \alpha\ \Rightarrow\ \Delta}{\Gamma\ \Rightarrow\ \neg\alpha,\ \Delta}\ [\Rightarrow\neg]$$

$$\frac{\forall x[\Phi(x)],\ \Phi(k),\ \Gamma\ \Rightarrow\ \Delta}{\forall x[\Phi(x)],\ \Gamma\ \Rightarrow\ \Delta}\ [\forall\Rightarrow] \qquad \frac{\Gamma\ \Rightarrow\ \Phi(k),\Delta}{\Gamma\ \Rightarrow\ \forall x[\Phi(x)],\ \Delta}\ [\Rightarrow\forall]\ \dagger$$

† where κ cannot occur anywhere in the lower sequent.

$$\frac{\neg\forall x[\neg\Phi(x)],\ \Gamma\ \Rightarrow\ \Delta}{\exists x[\Phi(x)],\ \Gamma\ \Rightarrow\ \Delta}\ [\exists\Rightarrow r.w.] \qquad \frac{\Gamma\ \Rightarrow\ \neg\forall x[\neg\Phi(x)],\ \Delta}{\Gamma\ \Rightarrow\ \exists x[\Phi(x)],\ \Delta}\ [\Rightarrow\exists r.w.]$$