$$b = (X'X)^{-1} X'Y$$

$$b = \left(\frac{1}{n} X'X\right)^{-1} \cdot \frac{1}{n} X'Y$$ normalizing, by taking average.

$$= \left(\frac{1}{n}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_i \end{bmatrix}\begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & y_i \end{bmatrix}\right)^{-1} \cdot \frac{1}{n}\left(\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_i \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_i \end{bmatrix}\right)$$

$$= \left(\begin{bmatrix} \frac{n}{n} & \frac{x_1+x_2+\cdots x_i}{n} \\ \frac{x_1+y_2+\cdots x_i}{n} & \frac{x_1^2+x_2^2+\cdots x_i^2}{n} \end{bmatrix}\right)^{-1} \begin{bmatrix} \frac{y_1+y_2+\cdots y_i}{n} \\ \frac{x_1y_1+x_2y_2+\cdots x_iy_i}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \bar{x} \\ \bar{x} & \overline{x^2} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \bar{y} \\ \overline{xy} \end{bmatrix}$$

$$= \frac{1}{1\cdot\overline{x^2} - (\bar{x})^2}\begin{bmatrix} \overline{x^2} & -\bar{x} \\ -\bar{x} & 1 \end{bmatrix} \cdot \begin{bmatrix} \bar{y} \\ \overline{xy} \end{bmatrix}$$

$$= \begin{array}{cc} \begin{bmatrix} \dfrac{\overline{x^2}\,\bar{y} - \bar{x}\,\overline{xy}}{\overline{x^2} - (\bar{x})^2} \\[2em] \dfrac{-\bar{x}\bar{y} + \overline{xy}}{\overline{x^2} - (\bar{x})^2} \end{bmatrix} & \begin{matrix} b_0 \\[3em] b_1 \end{matrix} \end{array}$$

for b_1:

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y}) \cdot \frac{1}{n}}{\sum (x_i - \bar{x})^2 \cdot \frac{1}{n}}$$

$$= \frac{\frac{\sum x_iy_i - x_i\bar{y} - \bar{x}y_i + \bar{x}\bar{y}}{n}}{\frac{\sum x_i^2 - 2x_i\bar{x} + \bar{x}^2}{n}} = \frac{\overline{xy} - \bar{x}\bar{y} - \bar{x}\bar{y} + \bar{x}\bar{y}}{\overline{x^2} - 2\bar{x}\bar{x} + \bar{x}^2}$$

$$= \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - (\bar{x})^2}$$ same as b_1

$$b_0 = \bar{y} - b_1\bar{x}$$

$$= \bar{y} - \frac{\bar{x}(\overline{xy} - \bar{x}\bar{y})}{\overline{x^2} - (\bar{x})^2} = \frac{\bar{y}\,\overline{x^2} - \bar{y}(\bar{x})^2 - \bar{x}\,\overline{xy} + \bar{x}\bar{x}\bar{y}}{\overline{x^2} - (\bar{x})^2}$$

$$= \frac{\overline{x^2}\,\bar{y} - \bar{x}\,\overline{xy}}{\overline{x^2} - (\bar{x})^2}$$ same as b_0.

∴ both formula is same!