## Factory Planning Algorithm

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Given a collection of recipes, create a matrix M where each column of M is a recipe, and each row corresponds to an item. For instance, if mining creates two iron per second and smelting consumes five iron to create one steel,

$$M = \begin{pmatrix} 2 & -5 \\ 0 & 1 \end{pmatrix}.$$

Multiplying by a column of recipes, R, yields a column of outputs produced, P. Mining two times and smelting once produces one steel with a deficit of one iron:

$$R = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad P = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad MR = P.$$

The user can specify up to rank M values of P, however, they must be chosen such that the submatrix of corresponding rows has linearly independent rows. If they specify fewer, the remaining choices can be assumed to be zero, to maximize self-sufficiency of the production line. Our goal then is to solve for R.

If the rank of M is less than the number of columns of M then several recipes are linearly dependent, so we can choose rank M linearly independent columns.

The system now has a unique solution, so we rewrite the equations so that it can be solved by a matrix inversion. Conceptually, this is done by adding a sink for every unspecified item.