


Measuring acoustic glitches in the modes of solar-like oscillators using Gaussian processes

ALEXANDER J. LYTTLE ^{1, 2, 3} AND GUY R. DAVIES^{1, 2, 3}

¹ *School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, United Kingdom*

² *The Alan Turing Institute, British Library, 96 Euston Rd, London NW1 2DB, United Kingdom*

³ *Stellar Astrophysics Center, University of Aarhus, Nordre Ringgade 1, 8000 Aarhus C, Denmark*

ABSTRACT

Put abstract here.

Keywords: Asteroseismology (73), Astronomy software (1855), Helioseismology (709), Stellar interior (1500)

1. INTRODUCTION

Test citation (Lund et al. 2017).

A Gaussian process is a prior over function space, which allows us to fit any function as a sum of Gaussians, constrained by the choice of mean and kernel functions. The function describing the mode frequencies comprises several features including an approximately regular spacing in frequency, $\Delta\nu$, curvature caused by the smooth varying stellar structure, and glitches due to sharp variation in stellar structure. The latter is fairly well understood (Houdek & Gough 2007), whereas the smooth curvature is not. In most work studying acoustic glitches, a polynomial is typically fit to the smooth background. However, this has its drawbacks; the flexibility of a polynomial increases with its order, and regularisation is required to combat over-fitting. A Gaussian process offers more flexibility than a polynomial with fewer free parameters. Furthermore, these parameters are priors over the flexibility of the function, such as a typical length scale or variance.

2. METHOD

2.1. The model

The radial mode frequencies follow some function of radial order, $\nu_{n,l=0} = f(n)$ where f can be described by a Gaussian process,

$$f(\mathbf{n}) \sim \mathcal{GP}(m(\mathbf{n}), k(\mathbf{n}, \mathbf{n}')), \quad (1)$$

for array \mathbf{n} with some mean function, $m(\mathbf{n})$ and kernel function, $k(\mathbf{n}, \mathbf{n}')$.

The mean function comprises a background frequency as a function of n , $f_{\text{bkg}}(n)$, and the glitches due to the second ionisation of helium, $\delta_{\text{He}}(\nu)$ and base of the convective zone $\delta_{\text{BCZ}}(\nu_{\text{bkg}})$,

$$m(\mathbf{n}) = f_{\text{bkg}}(\mathbf{n}) + \delta_{\text{He}}(\boldsymbol{\nu}_{\text{bkg}}) + \delta_{\text{BCZ}}(\boldsymbol{\nu}_{\text{bkg}}), \quad (2)$$

where $\boldsymbol{\nu}_{\text{bkg}} = f_{\text{bkg}}(\mathbf{n})$. For the background, we adopt the asymptotic approximation to first order in n ,

$$f_{\text{bkg}}(n) = \Delta\nu(n + \epsilon), \quad (3)$$

where $\Delta\nu$ is the large frequency separation and ϵ is some offset. We define the small change in mode frequency caused by the second ionisation of helium, $\delta\nu_{\text{He}} = \delta_{\text{He}}(\nu)$ as,

$$\delta_{\text{He}}(\nu) = a_{\text{He}}\nu e^{-b_{\text{He}}\nu^2} \sin(2\pi\tau_{\text{He}}\nu + \phi_{\text{He}}), \quad (4)$$

where a_{He} is the frequency-dependent amplitude and b_{He} relates to the acoustic width of the glitch. We define the small change in frequency due to the base of the convective zone, $\delta\nu_{\text{BCZ}} = \delta_{\text{BCZ}}(\nu)$ as,

$$\delta_{\text{BCZ}}(\nu) = \frac{a_{\text{BCZ}}}{\nu^2} \sin(2\pi\tau_{\text{BCZ}}\nu + \phi_{\text{BCZ}}), \quad (5)$$

where a_{BCZ} is the frequency-dependent amplitude. For both glitches, parameters τ and ϕ relate to the acoustic depth and phase of the glitch respectively.

The kernel function describes the covariance between modes at different n . We expect consecutive modes to correlate more strongly than modes further apart. Therefore, we adopt the squared-exponential (or radial basis) kernel function,

$$k(\mathbf{n}, \mathbf{n}') = \sigma^2 \exp\left\{-\frac{\|\mathbf{n} - \mathbf{n}'\|^2}{2\lambda^2}\right\}, \quad (6)$$

where σ^2 is the variance and λ is the length scale.

The model described above comprises 11 parameters,

$$\boldsymbol{\theta} = \{\Delta\nu, \epsilon, a_{\text{He}}, b_{\text{He}}, \tau_{\text{He}}, \phi_{\text{He}}, a_{\text{BCZ}}, \tau_{\text{BCZ}}, \phi_{\text{BCZ}}, \sigma^2, \lambda\}. \quad (7)$$

Using Bayes' theorem, we write the posterior probability density function for the model given some observation of radial mode frequencies $\boldsymbol{\nu}_n$,

$$p(\boldsymbol{\theta} \mid \boldsymbol{\nu}_n) \sim \Pi(\boldsymbol{\theta}) \mathcal{L}(\boldsymbol{\nu}_n \mid \boldsymbol{\theta}), \quad (8)$$

where $\Pi(\boldsymbol{\theta})$ is the a priori probability of the model and $\mathcal{L}(\boldsymbol{\nu}_n \mid \boldsymbol{\theta})$ is the likelihood of the data given the model.

2.2. The prior

The prior for the model can be separated into a product of the priors for each individual parameter,

$$\Pi(\boldsymbol{\theta}) = \prod_{i=0}^{10} \Pi(\theta_i). \quad (9)$$

In the following sections we write our choice of prior for each of θ_i .

2.2.1. Background parameters

The prior for $\Delta\nu$ is provided as the location and scale of a normal distribution, $\Delta\nu \sim \mathcal{N}(\mu_{\Delta\nu}, \sigma_{\Delta\nu})$. The prior for ϵ is chosen to be a log-normal distribution, $\epsilon \sim \ln\mathcal{N}(\ln(1.4), 0.4)$.

2.2.2. Glitch parameters

The glitch parameters' priors must convey at least two things; the first is that the glitch amplitudes are approximately 0.1 to 1.0 μHz , and the second is that the acoustic depths are in physically sensible places. The former comes from reviewing previous measurements of the glitches in the literature and can be enforced by inspecting the sensibility of a grid of glitch amplitude parameters as a function of ν_{max} (see Appendix...). The latter is less straightforward to determine, and will be defined later in this section.

- Stellar model grid
- Extract acoustic depths
- Fit Gaussian mixture to $(\log \nu_{\text{max}}, T_{\text{eff}}, \log \tau_{\text{He}}, \log \tau_{\text{BCZ}})$
- Sample from the conditional distribution of $(\log \tau_{\text{He}}, \log \tau_{\text{BCZ}})$ given $(\log \nu_{\text{max}}, T_{\text{eff}})$
- This gives the prior for $(\log \tau_{\text{He}}, \log \tau_{\text{BCZ}})$

2.2.3. Kernel parameters

We chose to fix the kernel parameters to $\lambda = 5.0$ and $\sigma^2 = 0.1 \mu_{\Delta\nu}$. The length scale ensures that the GP does not fit to the glitch, and reflects the typical scale of curvature of the radial mode frequencies with respect to n . The variance is fixed at 10 % of $\Delta\nu$ which corresponds to the approximate amplitude of the mode curvature.

3. RESULTS

3.1. Model stars

3.2. Sun-as-a-star

3.3. 16 Cygni

3.4. KIC 4448777

Put acknowledgments here.

Software: astropy (Astropy Collaboration et al. 2018), jax (Bradbury et al. 2018),
jaxns (Albert 2020), numpy (Harris et al. 2020), numpyro (Phan et al. 2019; Bingham
et al. 2019), PBjam (Nielsen et al. 2021)

APPENDIX

REFERENCES

- Albert, J. G. 2020, arXiv e-prints,
arXiv:2012.15286.
<https://arxiv.org/abs/2012.15286>
- Astropy Collaboration, Price-Whelan,
A. M., Sipőcz, B. M., et al. 2018, The
Astronomical Journal, 156, 123,
doi: [10.3847/1538-3881/aabc4f](https://doi.org/10.3847/1538-3881/aabc4f)
- Bingham, E., Chen, J. P., Jankowiak, M.,
et al. 2019, J. Mach. Learn. Res., 20,
28:1
- Bradbury, J., Frostig, R., Hawkins, P.,
et al. 2018, JAX: Composable
Transformations of Python+NumPy
Programs
- Harris, C. R., Millman, K. J., van der
Walt, S. J., et al. 2020, Nature, 585,
357, doi: [10.1038/s41586-020-2649-2](https://doi.org/10.1038/s41586-020-2649-2)
- Houdek, G., & Gough, D. O. 2007,
Monthly Notices of the Royal
Astronomical Society, 375, 861,
doi: [10.1111/j.1365-2966.2006.11325.x](https://doi.org/10.1111/j.1365-2966.2006.11325.x)
- Lund, M. N., Silva Aguirre, V., Davies,
G. R., et al. 2017, The Astrophysical
Journal, 835, 172,
doi: [10.3847/1538-4357/835/2/172](https://doi.org/10.3847/1538-4357/835/2/172)
- Nielsen, M. B., Davies, G. R., Ball, W. H.,
et al. 2021, The Astronomical Journal,
161, 62, doi: [10.3847/1538-3881/abcd39](https://doi.org/10.3847/1538-3881/abcd39)
- Phan, D., Pradhan, N., & Jankowiak, M.
2019, arXiv preprint arXiv:1912.11554.
<https://arxiv.org/abs/1912.11554>