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# Measuring acoustic glitches in the modes of solar-like oscillators using Gaussian processes

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## ABSTRACT

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*Keywords:* Asteroseismology (73), Astronomy software (1855), Helioseismology (709), Stellar interior (1500)

## 1. INTRODUCTION

Test citation (Lund et al. 2017).

A Gaussian process is a prior over function space, which allows us to fit any function as a sum of Gaussians, constrained by the choice of mean and kernel functions. The function describing the mode frequencies comprises several features including an approximately regular spacing in frequency, $\Delta\nu$, curvature caused by the smoothy varying stellar structure, and glitches due to sharp variation in stellar structure. The latter is fairly well understood (Houdek & Gough 2007), whereas the smooth curvature is not. In most work studying acoustic glitches, a polynomial is typically fit to the smooth background. However, this has its drawbacks; the flexibility of a polynomial increases with its order, and regularisation is required to combat over-fitting. A Gaussian process offers more flexibility than a polynomial with fewer free parameters. Furthermore, these parameters are priors over the flexibility of the function, such as a typical length scale or variance.

## 2. METHOD

### 2.1. *The model*

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The radial mode frequencies follow some function of radial order, $\nu_{n,l=0} = f(n)$ where $f$ can be described by a Gaussian process,

$$f(\boldsymbol{n}) \sim \mathcal{GP}\left(m(\boldsymbol{n}), k(\boldsymbol{n}, \boldsymbol{n}')\right), \tag{1}$$

for array $\boldsymbol{n}$ with some mean function, $m(\boldsymbol{n})$ and kernel function, $k(\boldsymbol{n}, \boldsymbol{n}')$. In other words, the function $f$ is drawn from a GP characterised by $m$ and $k$.

The mean function comprises a background frequency as a function of $n$, $f_{\rm bkg}(n)$, and the glitches due to the second ionisation of helium, $\delta_{\rm He}(\nu)$ and base of the convective zone $\delta_{\rm BCZ}(\nu_{\rm bkg})$,

$$m(\boldsymbol{n}) = f_{\rm bkg}(\boldsymbol{n}) + \delta_{\rm He}(\boldsymbol{\nu}_{\rm bkg}) + \delta_{\rm BCZ}(\boldsymbol{\nu}_{\rm bkg}), \tag{2}$$

where $\boldsymbol{\nu}_{\rm bkg} = f_{\rm bkg}(\boldsymbol{n})$. For the background, we adopt the asymptotic approximation to first order in $n$,

$$f_{\rm bkg}(n) = \Delta\nu(n + \epsilon), \tag{3}$$

where $\Delta\nu$ is the large frequency separation and $\epsilon$ is some offset. We define the small change in mode frequency caused by the second ionisation of helium, $\delta\nu_{\rm He} = \delta_{\rm He}(\nu)$ as,

$$\delta_{\rm He}(\nu) = a_{\rm He}\nu e^{-b_{\rm He}\nu^2} \sin\left(2\pi\tau_{\rm He}\nu + \phi_{\rm He}\right), \tag{4}$$

where $a_{\rm He}$ is the frequency-dependent amplitude and $b_{\rm He}$ relates to the acoustic width of the glitch. We define the small change in frequency due to the base of the convective zone, $\delta\nu_{\rm BCZ} = \delta_{\rm BCZ}(\nu)$ as,

$$\delta_{\rm BCZ}(\nu) = \frac{a_{\rm BCZ}}{\nu^2} \sin\left(2\pi\tau_{\rm BCZ}\nu + \phi_{\rm BCZ}\right), \tag{5}$$

where $a_{\rm BCZ}$ is the frequency-dependent amplitude. For both glitches, parameters $\tau$ and $\phi$ relate to the acoustic depth and phase of the glitch respectively.

The kernel function describes the covariance between modes at different $n$. We expect consecutive modes to correlate more strongly than modes further apart. Therefore, we adopt the squared-exponential (or radial basis) kernel function,

$$k(\boldsymbol{n}, \boldsymbol{n}') = \sigma^2 \exp\left\{-\frac{\|\boldsymbol{n} - \boldsymbol{n}'\|^2}{2\lambda^2}\right\}, \tag{6}$$

where $\sigma^2$ is the variance and *lambda* is the length scale.

The model described above comprises 11 parameters,

$$\boldsymbol{\theta} = \{\Delta\nu, \epsilon, a_{\rm He}, b_{\rm He}, \tau_{\rm He}, \phi_{\rm He}, a_{\rm BCZ}, \tau_{\rm BCZ}, \phi_{\rm BCZ}, \sigma^2, \lambda\}. \tag{7}$$

Using Bayes' theorem, we write the posterior probability density function for the model given some observation of radial mode frequencies $\boldsymbol{\nu}_n$,

$$p(\boldsymbol{\theta} \mid \boldsymbol{\nu}_n) \propto p(\boldsymbol{\theta})\, p(\boldsymbol{\nu}_n \mid \boldsymbol{\theta}), \tag{8}$$

where $\Pi(\boldsymbol{\theta})$ is the a priori probability of the model and $\mathcal{L}(\boldsymbol{\nu}_n \mid \boldsymbol{\theta})$ is the likelihood of the data given the model.

### 2.2. *The prior*

The prior for the model can be separated into a product of the priors for each individual parameter,

$$p(\boldsymbol{\theta}) = \prod_{i=0}^{10} p(\theta_i). \tag{9}$$

In the following sections we write our choice of prior for each of $\theta_i$.

#### 2.2.1. *Background parameters*

The prior for $\Delta\nu$ is provided as the location and scale of a normal distribution, $\Delta\nu \sim \mathcal{N}(\mu_{\Delta\nu}, \sigma_{\Delta\nu})$. The prior for $\epsilon$ is chosen to be a log-normal distribution, $\epsilon \sim \ln\mathcal{N}(\ln(1.4), 0.4)$.

#### 2.2.2. *Glitch parameters*

The glitch parameters' priors must convey at least two things; the first is that the glitch amplitudes are approximately 0.1 to 1.0 µHz, and the second is that the acoustic depths are in physically sensible places. The former comes from reviewing previous measurements of the glitches in the literature and can be enforced by inspecting the sensibility of a grid of glitch amplitude parameters as a function of $\nu_{\max}$ (see Appendix...). The latter is less straightforward to determine, and will be defined later in this section.

- Stellar model grid
- Extract acoustic depths
- Fit Gaussian mixture to $(\log\nu_{\max}, T_{\text{eff}}, \log\tau_{\text{He}}, \log\tau_{\text{BCZ}})$
- Sample from the conditional distribution of $(\log\tau_{\text{He}}, \log\tau_{\text{BCZ}})$ given $(\log\nu_{\max}, T_{\text{eff}})$
- This gives the prior for $(\log\tau_{\text{He}}, \log\tau_{\text{BCZ}})$

#### 2.2.3. *Kernel parameters*

We chose to fix the kernel parameters to $\lambda = 5.0$ and $\sigma^2 = 0.1\,\mu_{\Delta\nu}$. The length scale ensures that the GP does not fit to the glitch, and reflects the typical scale of curvature of the radial mode frequencies with respect to $n$. The variance is fixed at 10 % of $\Delta\nu$ which corresponds to the approximate amplitude of the mode curvature.

### 2.3. *The likelihood*

The GP likelihood of function for the true frequency $\boldsymbol{f}_n = f(\boldsymbol{n})$ given $\boldsymbol{\theta}$ is a normal distribution,

$$\boldsymbol{f}_n \mid \boldsymbol{\theta} \sim \mathcal{N}\left[m_\theta(\boldsymbol{n}), k_\theta(\boldsymbol{n}, \boldsymbol{n})\right], \tag{10}$$

and the likelihood of some noisy observation $\boldsymbol{\nu}_n$ of $\boldsymbol{f}_n$ is,

$$\boldsymbol{\nu}_n \mid \boldsymbol{f}_n \sim \mathcal{N}(\boldsymbol{f}_n, \boldsymbol{\mathcal{I}} \odot \boldsymbol{\sigma}_n^2), \tag{11}$$

where $\boldsymbol{\mathcal{I}}$ is the $N \times N$ identity matrix, $\boldsymbol{\sigma}_n^2$ is the array of variances corresponding to each of $\boldsymbol{\nu}_n$, and $\odot$ is the element-wise product operator.

The marginal likelihood of $\boldsymbol{\nu}_n$ given the model is,

$$p(\boldsymbol{\nu}_n \mid \boldsymbol{\theta}) = \int p(\boldsymbol{\nu}_n \mid \boldsymbol{f}_n, \boldsymbol{\theta})\, p(\boldsymbol{f}_n \mid \boldsymbol{\theta})\, \mathrm{d}\boldsymbol{f}_n \tag{12}$$

which given equations above is,

$$p(\boldsymbol{\nu}_n \mid \boldsymbol{\theta}) = \mathcal{N}\left(\boldsymbol{\mu}_n, \boldsymbol{K}_n + \boldsymbol{\mathcal{I}} \odot (\boldsymbol{\sigma}_n^2 + \delta)\right) \tag{13}$$

where $\boldsymbol{\mu}_n = m_\theta(\boldsymbol{n})$, $\boldsymbol{K}_n = k_\theta(\boldsymbol{n}, \boldsymbol{n})$, and $\delta = 1 \times 10^{-6}$ is some small value to ensure that the covariance is positive semidefinite.

The conditional distribution for the GP at new radial orders $\boldsymbol{n}_\star$ is derived from the joint distribution,

$$\begin{bmatrix} \boldsymbol{\nu}_n \\ \boldsymbol{f}_{n_\star} \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} \boldsymbol{\mu}_n \\ \boldsymbol{\mu}_{n_\star} \end{bmatrix}, \begin{bmatrix} \boldsymbol{K}_n + \boldsymbol{\mathcal{I}} \odot (\boldsymbol{\sigma}_n^2 + \delta) & \boldsymbol{K}_{n,n_\star} \\ \boldsymbol{K}_{n_\star,n} & \boldsymbol{K}_{n_\star} \end{bmatrix} \right). \tag{14}$$

We marginalise over

## 3. RESULTS

### 3.1. *Model stars*

### 3.2. *Sun-as-a-star*

### 3.3. 16 *Cygni*

### 3.4. *KIC* 4448777

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*Software:* astropy (Astropy Collaboration et al. 2018), jax (Bradbury et al. 2018), jaxns (Albert 2020), numpy (Harris et al. 2020), numpyro (Phan et al. 2019; Bingham et al. 2019), PBjam (Nielsen et al. 2021)

## APPENDIX

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