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Measuring acoustic glitches in the modes of solar-like oscillators using Gaussian processes

Alexander J. Lyttle[1,2,3] and Guy R. Davies[1,2,3]

[1] *School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, United Kingdom*
[2] *The Alan Turing Institute, British Library, 96 Euston Rd, London NW1 2DB, United Kingdom*
[3] *Stellar Astrophysics Center, University of Aarhus, Nordre Ringgade 1, 8000 Aarhus C, Denmark*

ABSTRACT

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1. INTRODUCTION

Glitches in the mode frequencies of solar-like oscillators arise due to sharp changes in the sound-speed profile of the star. These perturb the modes from their regular spacing $\Delta\nu$ by some small amount of the order 0.1 to 1 µHz. Such glitches have been observed in the Sun this (**TODO:** citation needed), and in several other high signal-to-noise stars (**TODO:** citation needed). The two dominant glitches in these stars are due to the second ionisation of helium near the stellar surface and the base of the convective zone. Fitting these glitches provide insight into the helium abundance and convective boundary respectively.

Compare the different methods already gone. Early work measuring the glitch fits a model to the second differences, $\Delta_2\nu = \nu_{n-1} - 2\nu_n + \nu_{n+1}$, which removes first-order effects and amplifies the glitches. What are the issues with this? Alternatively, the glitch can be fit to the modes directly. Typically, both methods involve fitting a polynomial to the smoothly varying component of the modes. Assuming a fourth order polynomial (e.g. **TODO:** citation needed), this model has 13 parameters. The

In this work we apply a new method for fitting the glitches. Our method only requires radial mode observations and fits directly to the frequencies. Using a GP, we are able to reduce the model parameters from previous work, without sacrificing flexibility.

Corresponding author: Alexander J. Lyttle
ajl573@bham.ac.uk

A Gaussian process is a prior over function space, which allows us to fit any function as a sum of Gaussian distributions, constrained by the choice of mean and kernel functions. The function describing the mode frequencies comprises several features including an approximately regular spacing in frequency, $\Delta\nu$, curvature caused by the smoothy varying stellar structure, and glitches due to sharp variation in stellar structure. The latter is fairly well understood (Houdek & Gough 2007), whereas the smooth curvature is not. In most work studying acoustic glitches, a polynomial is typically fit to the smooth background. However, this has its drawbacks; the flexibility of a polynomial increases with its order, and regularisation is required to combat over-fitting. A Gaussian process offers more flexibility than a polynomial with fewer free parameters. Furthermore, these parameters are priors over the flexibility of the function, such as a typical length scale or variance.

2. METHOD

2.1. *The model*

A radial mode frequency is some function of radial order, $\nu_{n,l=0} = f(n)$ where f can be described by a Gaussian process,

$$f(\boldsymbol{n}) \sim \mathcal{GP}\left[m(\boldsymbol{n}), k(\boldsymbol{n}, \boldsymbol{n}')\right], \tag{1}$$

for array $\boldsymbol{n}$ with some mean function, $m(\boldsymbol{n})$ and kernel function, $k(\boldsymbol{n}, \boldsymbol{n}')$. In other words, the function $f(\boldsymbol{n})$ is drawn from a GP characterised by m and k.

The mean function comprises a background frequency as a function of n, $f_{\mathrm{bkg}}(n)$, and the glitches due to the second ionisation of helium, $\delta_{\mathrm{He}}(\nu)$ and base of the convective zone $\delta_{\mathrm{BCZ}}(\nu_{\mathrm{bkg}})$,

$$m(n) = f_{\mathrm{bkg}}(n) + \delta_{\mathrm{He}}(\nu_{\mathrm{bkg}}) + \delta_{\mathrm{BCZ}}(\nu_{\mathrm{bkg}}), \tag{2}$$

where $\nu_{\mathrm{bkg}} = f_{\mathrm{bkg}}(n)$. For the background, we adopt the asymptotic approximation to first order in n,

$$f_{\mathrm{bkg}}(n) = \Delta\nu(n + \epsilon), \tag{3}$$

where $\Delta\nu$ is the large frequency separation and ϵ is some offset. We define the small change in mode frequency caused by the second ionisation of helium, $\delta\nu_{\mathrm{He}} = \delta_{\mathrm{He}}(\nu)$ as,

$$\delta_{\mathrm{He}}(\nu) = a_{\mathrm{He}}\nu e^{-b_{\mathrm{He}}\nu^2} \sin\left(2\pi\tau_{\mathrm{He}}\nu + \phi_{\mathrm{He}}\right), \tag{4}$$

where a_{He} is the frequency-dependent amplitude and b_{He} relates to the acoustic width of the glitch. We define the small change in frequency due to the base of the convective zone, $\delta\nu_{\mathrm{BCZ}} = \delta_{\mathrm{BCZ}}(\nu)$ as,

$$\delta_{\mathrm{BCZ}}(\nu) = \frac{a_{\mathrm{BCZ}}}{\nu^2} \sin\left(2\pi\tau_{\mathrm{BCZ}}\nu + \phi_{\mathrm{BCZ}}\right), \tag{5}$$

where a_{BCZ} is the frequency-dependent amplitude. For both glitches, parameters τ and ϕ relate to the acoustic depth and phase of the glitch respectively.

The kernel function describes the covariance between modes at different n. We expect consecutive modes to correlate more strongly than modes further apart. Therefore, we adopt the squared-exponential (or radial basis) kernel function,

$$k(\boldsymbol{n}, \boldsymbol{n}') = \sigma^2 \exp\left(-\frac{\|\boldsymbol{n} - \boldsymbol{n}'\|^2}{2\lambda^2}\right), \tag{6}$$

where σ^2 is the variance and λ is the length scale.

The model described above comprises 11 parameters,

$$\boldsymbol{\theta} = [\Delta\nu, \epsilon, a_{\mathrm{He}}, b_{\mathrm{He}}, \tau_{\mathrm{He}}, \phi_{\mathrm{He}}, a_{\mathrm{BCZ}}, \tau_{\mathrm{BCZ}}, \phi_{\mathrm{BCZ}}, \sigma^2, \lambda]. \tag{7}$$

Using Bayes' theorem, we write the posterior probability density function for the model given some observation of radial mode frequencies $\boldsymbol{\nu}_n$,

$$p(\boldsymbol{\theta} \mid \boldsymbol{\nu}_n) \propto p(\boldsymbol{\theta})\, p(\boldsymbol{\nu}_n \mid \boldsymbol{\theta}), \tag{8}$$

where $\Pi(\boldsymbol{\theta})$ is the a priori probability of the model and $\mathcal{L}(\boldsymbol{\nu}_n \mid \boldsymbol{\theta})$ is the likelihood of the data given the model.

2.2. *The prior*

The prior for the model can be separated into a product of the priors for each individual parameter,

$$p(\boldsymbol{\theta}) = \prod_{i=0}^{10} p(\theta_i). \tag{9}$$

In the following sections we write our choice of prior for each of θ_i.

2.2.1. *Background parameters*

The prior for $\Delta\nu$ is provided as the location and scale of a normal distribution, $\Delta\nu \sim \mathcal{N}(\mu_{\Delta\nu}, \sigma_{\Delta\nu})$. The prior for ϵ is chosen to be a log-normal distribution, $\epsilon \sim \ln\mathcal{N}(\ln(1.4), 0.4)$.

2.2.2. *Glitch parameters*

The glitch parameters' priors must convey at least two things; the first is that the glitch amplitudes are approximately 0.1 to 1.0 µHz, and the second is that the acoustic depths are in physically sensible places. The former comes from reviewing previous measurements of the glitches in the literature and can be enforced by inspecting the sensibility of a grid of glitch amplitude parameters as a function of $\nu_{\max}$ (see Appendix . . .) The latter is less straightforward to determine, and will be defined later in this section.

- Stellar model grid
- Extract acoustic depths

- Fit Gaussian mixture to $(\log \nu_{\max}, T_{\text{eff}}, \log \tau_{\text{He}}, \log \tau_{\text{BCZ}})$
- Sample from the conditional distribution of $(\log \tau_{\text{He}}, \log \tau_{\text{BCZ}})$ given $(\log \nu_{\max}, T_{\text{eff}})$
- This gives the prior for $(\log \tau_{\text{He}}, \log \tau_{\text{BCZ}})$

2.2.3. *Kernel parameters*

We chose to fix the kernel parameters to $\lambda = 5.0$ and $\sigma^2 = 0.1\,\mu_{\Delta\nu}$. **TODO:** Explain why these values. The length scale ensures that the GP does not fit to the glitch, and reflects the typical scale of curvature of the radial mode frequencies with respect to n. The variance is fixed at 10 % of $\Delta\nu$ which corresponds to the approximate amplitude of the mode curvature.

2.2.4. *Average amplitude*

The average amplitude of the glitches has been used in previous work as a proxy for the helium abundance in the star. The average amplitude between frequencies ν_{low} and ν_{high} is,

$$\langle A \rangle = \frac{\int_{\nu_{\text{low}}}^{\nu_{\text{high}}} A(\nu)\,\mathrm{d}\nu}{\int_{\nu_{\text{low}}}^{\nu_{\text{high}}} \mathrm{d}\nu} \tag{10}$$

where $A(\nu)$ is the amplitude of the glitch at frequency ν. We derive this for the helium glitch,

$$\langle A_{\text{He}} \rangle = \frac{a_{\text{He}} \left(e^{-b_{\text{He}}\,\nu_{\text{low}}} - e^{-b_{\text{He}}\,\nu_{\text{high}}} \right)}{2 b_{\text{He}} (\nu_{\text{high}} - \nu_{\text{low}})}, \tag{11}$$

and BCZ glitch,

$$\langle A_{\text{BCZ}} \rangle = \frac{a_{\text{BCZ}}}{\nu_{\text{low}}\,\nu_{\text{high}}}. \tag{12}$$

We expect the average amplitude of the helium glitch to be greater than that of the convective zone glitch most of the time. **TODO:** Explain why this is. To encode this knowledge into the prior, we introduce the parameter for the logarithm of the ratio between the average amplitudes of the two glitches. $\log r_A = \log\langle A_{\text{He}} \rangle - \log\langle A_{\text{BCZ}} \rangle$.

$$\log r_A \sim \mathcal{N}(0.6, 0.3). \tag{13}$$

The mean is chosen such that the mean lies where $\langle A_{\text{He}} \rangle = 4\langle A_{\text{BCZ}} \rangle$. The variance is chosen such that $\langle A_{\text{He}} \rangle < \langle A_{\text{BCZ}} \rangle$ and $\langle A_{\text{He}} \rangle > 16\langle A_{\text{BCZ}} \rangle$ 5 % of the time.

2.3. *The likelihood*

The GP likelihood of function for the true frequency $\boldsymbol{\nu}_n = f(\boldsymbol{n})$ given $\boldsymbol{\theta}$ is a normal distribution,

$$\boldsymbol{\nu}_n \mid \boldsymbol{\theta} \sim \mathcal{N}\left[m_\theta(\boldsymbol{n}), k_\theta(\boldsymbol{n}, \boldsymbol{n})\right]. \tag{14}$$

The likelihood of some noisy observation $\boldsymbol{\nu}_n^{\text{obs}}$ of $\boldsymbol{\nu}_n$ is,

$$\boldsymbol{\nu}_n^{\text{obs}} \mid \boldsymbol{\nu}_n \sim \mathcal{N}\left[\boldsymbol{\nu}_n, \operatorname{diag}(\boldsymbol{\sigma}_n^2)\right], \tag{15}$$

where $\boldsymbol{\sigma}_n^2$ is the array of variances corresponding to each of $\boldsymbol{\nu}_n^{\mathrm{obs}}$, characterising independent, Gaussian noise.

The marginal likelihood of $\boldsymbol{\nu}_n^{\mathrm{obs}}$ given the model is thus,

$$\mathcal{L}(\boldsymbol{\nu}_n^{\mathrm{obs}} \mid \boldsymbol{\theta}) = \int p(\boldsymbol{\nu}_n^{\mathrm{obs}} \mid \boldsymbol{\nu}_n, \boldsymbol{\theta})\, p(\boldsymbol{\nu}_n \mid \boldsymbol{\theta})\, \mathrm{d}\boldsymbol{\nu}_n \tag{16}$$

which given equations above is,

$$\mathcal{L}(\boldsymbol{\nu}_n^{\mathrm{obs}} \mid \boldsymbol{\theta}) = \mathcal{N}\left[\boldsymbol{\mu}_\theta, \boldsymbol{K}_\theta + \mathrm{diag}(\boldsymbol{\sigma}_n^2) + \delta\right] \tag{17}$$

where $\boldsymbol{\mu}_\theta = m_\theta(\boldsymbol{n})$, $\boldsymbol{K}_\theta = k_\theta(\boldsymbol{n}, \boldsymbol{n})$, and $\delta = 1 \times 10^{-6}$ is some small value to ensure that the covariance is positive semidefinite.

See C. E. Rasmussen & C. K. I. Williams for GP conditional

3. RESULTS

3.1. *Model stars*

3.2. *Sun-as-a-star*

3.3. 16 *Cygni*

3.4. *KIC* 4448777

Put acknowledgments here.

Software: astropy (Astropy Collaboration et al. 2018), jax (Bradbury et al. 2018), jaxns (Albert 2020), numpy (Harris et al. 2020), numpyro (Phan et al. 2019; Bingham et al. 2019), PBjam (Nielsen et al. 2021)

APPENDIX

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