hsls_els

1 = strongly disagree 2 = disagree 3 = agree 4 = strongly agree

i1 = Teen (9th / 11th grader) confident can do excellent job on (fall 2009 / spring 2012) math tests

i2 = Teen (9th / 11th grader) certain can understand (fall 2009 / spring 2012) math textbook

i3 = Can understand difficult math class (ELS ONLY!)

i4 = Teen confident can do an excellent job on math assignments

i5 = Teen certain can master skills in math course

Can understand difficult math class

```
library(dplyr)
```

```
Attaching package: 'dplyr'

The following objects are masked from 'package:stats':

    filter, lag

The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

```
library(lavaan)
```

```
This is lavaan 0.6-12
lavaan is FREE software! Please report any bugs.
```

```
library(ltm)
```

```
Warning: package 'ltm' was built under R version 4.3.2
```

```
Loading required package: MASS
```

```

Attaching package: 'MASS'
```

```
The following object is masked from 'package:dplyr':

    select
```

```
Loading required package: msm
```

```
Warning: package 'msm' was built under R version 4.3.2
```

```
Loading required package: polycor
```

```
Warning: package 'polycor' was built under R version 4.3.2
```

```
library(sjlabelled)
```

```
Warning: package 'sjlabelled' was built under R version 4.3.3
```

```

Attaching package: 'sjlabelled'
```

```
The following object is masked from 'package:dplyr':

    as_label
```

```
library(kableExtra)
```

```
Warning: package 'kableExtra' was built under R version 4.3.3
```

```
Attaching package: 'kableExtra'

The following object is masked from 'package:dplyr':

    group_rows
```

```
library(sirt)
```

```
Warning: package 'sirt' was built under R version 4.3.3

- sirt 4.1-15 (2024-02-06 00:05:40)
```

```
library(mirt)
```

```
Warning: package 'mirt' was built under R version 4.3.3

Loading required package: stats4

Loading required package: lattice


Attaching package: 'mirt'

The following object is masked from 'package:ltm':

    Science
```

```
#source("code/download_data.R")
source("F:/Users/alex/OneDrive/Documents/data/prepare_data.R")
```

```
Warning: package 'here' was built under R version 4.3.3

here() starts at F:/Users/alex/OneDrive/Documents/data

Joining with `by = join_by(stu_id, sch_id, STRAT_ID, psu, sex, i1, i2, i4, i5,
i1_2, i2_2, i4_2, i5_2, sample)`

Warning in rm(dat_full, els, els_02_12_byf3pststu_v1_0, hsls,
hsls_17_student_pets_sr_v1_0, : object 'no_nas_hsls5' not found
```

```r
m_items <- paste0("i", 1:5)
m_items_2 <- paste0(m_items, "_2")
# get subset of relevant variables
dat <- dat[, c("stu_id", "sample", "sex", "dropout", m_items, m_items_2)]

dat$mean_score <- c(rowMeans(dat[dat$sample == "ELS", m_items], na.rm = TRUE),
                    rowMeans(dat[dat$sample == "HSLS", m_items[-3]], na.rm = TRUE))
dat$mean_score_2 <- c(rowMeans(dat[dat$sample == "ELS", m_items_2], na.rm = TRUE),
                    rowMeans(dat[dat$sample == "HSLS", m_items_2[-3]], na.rm = TRUE))
```

```r
# Creating only HSLS
hsls <- subset(dat, sample == "HSLS")

hsls_1 <- hsls[, c("i1", "i2", "i4", "i5")]
head(hsls_1)
```

```
      i1 i2 i4 i5
16198  4  3  4  3
16199  3  3  4  3
16200  4  2  4  3
16201  3  3  3  3
16202  3  3  3  3
16203  4  4  4  3
```

```r
hsls_2 <- hsls[, c("i1_2", "i2_2", "i4_2", "i5_2")]
head(hsls_2)
```

```
      i1_2 i2_2 i4_2 i5_2
16198    4    4    4    4
16199    2    2    3    2
16200    2    1    3    3
16201   NA   NA   NA   NA
16202   NA   NA   NA   NA
16203    3    3    3    3
```

```r
hsls_1_noNA <- na.omit(hsls_1)
hsls_2_noNA <- na.omit(hsls_2)
```

```r
# Creating only ELS
els <- subset(dat, sample == "ELS")

els_1 <- els[, c("i1", "i2", "i3", "i4", "i5")]
head(els_1)
```

```
  i1 i2 i3 i4 i5
1  2  1  2  2  1
2  4  3  4  4  4
3  3  2  2  3  2
4  4  3  3  3  4
5  2  2  3  3  3
6  2  2  3  3 NA
```

```r
els_2 <- els[, c("i1_2", "i2_2", "i3_2", "i4_2", "i5_2")]
head(els_2)
```

```
  i1_2 i2_2 i3_2 i4_2 i5_2
1    3    2    2    2    3
2    3    3    3    3    3
3    2    3    3    2    2
4    3    2    2    3    4
5    3    3    3    4    4
6   NA   NA   NA   NA   NA
```

```r
els_1_noNA <- na.omit(els_1)
els_2_noNA <- na.omit(els_2)
```

```r
cfa_config <-  '
  group: ELS
  math =~ NA   * i1 +
          el2_1 * i2 +
          el3_1 * i3 +
          el4_1 * i4 +
          el5_1 * i5

  # Naming the intercepts!
  i1 ~ nu1_1 * 1
  i2 ~ nu2_1 * 1
```

```
i3 ~ nu3_1 * 1
i4 ~ nu4_1 * 1
i5 ~ nu5_1 * 1

# Naming the residual variances!
i1 ~~ theta1_1 * i1
i2 ~~ theta2_1 * i2
i3 ~~ theta3_1 * i3
i4 ~~ theta4_1 * i4
i5 ~~ theta5_1 * i5

# Adding the covariances
i1 ~~ i2
i2 ~~ i3

# Fixing latent variance to 1, as we freed first factor loading
math ~~ 1 * math

# Fixing latent mean to 0 for identification?
math ~ 0 * 1


group: HSLS
math =~ NA   * i1 +
        hl2_2 * i2 +
        hl4_2 * i4 +
        hl5_2 * i5

# Naming the intercepts!
i1 ~ nu1_2 * 1
i2 ~ nu2_2 * 1
i4 ~ nu4_2 * 1
i5 ~ nu5_2 * 1

# Naming the residual variances!
i1 ~~ theta1_2 * i1
i2 ~~ theta2_2 * i2
i4 ~~ theta4_2 * i4
i5 ~~ theta5_2 * i5

# Adding the covariances
  #i1 ~~ i2
```

```
    i2 ~~ i4

  # Fixing latent variance to 1, as we freed first factor loading
  math ~~ 1 * math

  # Fixing latent mean to 0 for identification?
  math ~ 0 * 1
'

fit_config  <- cfa(cfa_config, data = dat, group = "sample",
                   estimator = "MLR", missing = "FIML", se = "robust.mlr")
```

```
Warning in lav_data_full(data = data, group = group, cluster = cluster, : lavaan WARNING: so
  24 32 33 34 35 36 37 38 39 40 44 45 47 50 51 56 57 65 66 67 69 72 73 77 78 87 99 101 110 1
```

```
Warning in lav_data_full(data = data, group = group, cluster = cluster, : lavaan WARNING: so
  16206 16211 16212 16222 16223 16226 16227 16229 16235 16241 16250 16252 16268 16273 16274
```

```
s_config <- summary(fit_config, fit.measures = TRUE, standardized = TRUE)


mod_indices <- modindices(fit_config, sort. = TRUE, free.remove = FALSE)
head(mod_indices)
```

```
   lhs op rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
35  i1 ~~  i3     1   ELS     1 81.621  0.033   0.033    0.119    0.119
42  i4 ~~  i5     1   ELS     1 81.621  0.038   0.038    0.218    0.218
37  i1 ~~  i5     1   ELS     1 76.182 -0.027  -0.027   -0.113   -0.113
40  i3 ~~  i4     1   ELS     1 27.064 -0.018  -0.018   -0.090   -0.090
39  i2 ~~  i5     1   ELS     1 12.584  0.011   0.011    0.043    0.043
38  i2 ~~  i4     1   ELS     1 12.584 -0.011  -0.011   -0.045   -0.045
```

```
# Just for ELS
mod_indices_els <- mod_indices[mod_indices$group == "ELS", ]
head(mod_indices_els)
```

```
   lhs op rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
35  i1 ~~  i3     1   ELS     1 81.621  0.033   0.033    0.119    0.119
42  i4 ~~  i5     1   ELS     1 81.621  0.038   0.038    0.218    0.218
```

```
37  i1 ~~  i5     1   ELS     1 76.182 -0.027  -0.027   -0.113   -0.113
40  i3 ~~  i4     1   ELS     1 27.064 -0.018  -0.018   -0.090   -0.090
39  i2 ~~  i5     1   ELS     1 12.584  0.011   0.011    0.043    0.043
38  i2 ~~  i4     1   ELS     1 12.584 -0.011  -0.011   -0.045   -0.045
```

```
# Just for HSLS
mod_indices_hsls <- mod_indices[mod_indices$group == "HSLS", ]
head(mod_indices_hsls)
```

```
    lhs op rhs block group level    mi    epc sepc.lv sepc.all sepc.nox
47   i4 ~~  i5     2  HSLS     1 0.044 -0.001  -0.001   -0.004   -0.004
43   i1 ~~  i2     2  HSLS     1 0.044 -0.001  -0.001   -0.003   -0.003
46   i2 ~~  i5     2  HSLS     1 0.044  0.001   0.001    0.003    0.003
44   i1 ~~  i4     2  HSLS     1 0.044  0.001   0.001    0.004    0.004
25   i2 ~1         2  HSLS     1 0.000  0.000   0.000    0.000    0.000
23 math =~  i5     2  HSLS     1 0.000  0.000   0.000    0.000    0.000
```

#CONFIG ELS + HSLS

```
config_comb <-  '
  # ELS
  group: 1

  # Time point 1
  math_t1 =~ NA   * i1 +
          el2_1 * i2 +
          el3_1 * i3 +
          el4_1 * i4 +
          el5_1 * i5

  # Naming the intercepts!
  i1 ~ enu1_1 * 1
  i2 ~ enu2_1 * 1
  i3 ~ enu3_1 * 1
  i4 ~ enu4_1 * 1
  i5 ~ enu5_1 * 1

  # Naming the residual variances!
  i1 ~~ etheta1_1 * i1
  i2 ~~ etheta2_1 * i2
  i3 ~~ etheta3_1 * i3
```

```
i4 ~~ etheta4_1 * i4
i5 ~~ etheta5_1 * i5

# Fixing latent variance to 1, as we freed first factor loading
math_t1 ~~ 1 * math_t1

# Fixing latent mean to 0 for identification?
math_t1 ~ 0 * 1

# Time point 2
 math_t2 =~ NA   * i1_2 +
           el2_2 * i2_2 +
           el3_2 * i3_2 +
           el4_2 * i4_2 +
           el5_2 * i5_2

# Naming the intercepts!
i1_2 ~ enu1_2 * 1
i2_2 ~ enu2_2 * 1
i3_2 ~ enu3_2 * 1
i4_2 ~ enu4_2 * 1
i5_2 ~ enu5_2 * 1

# Naming the residual variances!
i1_2 ~~ etheta1_2 * i1_2
i2_2 ~~ etheta2_2 * i2_2
i3_2 ~~ etheta3_2 * i3_2
i4_2 ~~ etheta4_2 * i4_2
i5_2 ~~ etheta5_2 * i5_2


## Adding the covariances ##
i1 ~~ i2
i1_2 ~~ i2_2

i1 ~~ i3
i1_2 ~~ i3_2

i2 ~~ i3
i2_2 ~~ i3_2

i4 ~~ i5
```

```
 i4_2 ~~ i5_2


 # Fixing latent variance to 1, as we freed first factor loading
 math_t2 ~~ 1 * math_t2

 # Fixing latent mean to 0 for identification
 math_t2 ~ 0 * 1

 # Correlations across time
 math_t1 ~~ math_t2
 i1 ~~ i1_2
 i2 ~~ i2_2
 i3 ~~ i3_2
 i4 ~~ i4_2
 i5 ~~ i5_2

# HSLS
group: 2
 # Time Point 1
 math_t1 =~ NA   * i1 +
            hl2_1 * i2 +
            #hl3_1 * i3 +
            hl4_1 * i4 +
            hl5_1 * i5

 # Naming the intercepts!
 i1 ~ hnu1_1 * 1
 i2 ~ hnu2_1 * 1
 #i3 ~ hnu3_1 * 1
 i4 ~ hnu4_1 * 1
 i5 ~ hnu5_1 * 1

 # Naming the residual variances!
 i1 ~~ htheta1_1 * i1
 i2 ~~ htheta2_1 * i2
 #i3 ~~ htheta3_1 * i3
 i4 ~~ htheta4_1 * i4
 i5 ~~ htheta5_1 * i5
```

```
# Fixing latent variance to 1, as we freed first factor loading
math_t1 ~~ 1 * math_t1

# Fixing latent mean to 0 for identification
math_t1 ~ 0 * 1

# Time Point 2
math_t2 =~ NA   * i1_2 +
           hl2_2 * i2_2 +
           #hl3_2 * i3_2 +
           hl4_2 * i4_2 +
           hl5_2 * i5_2

# Naming the intercepts!
i1_2 ~ hnu1_2 * 1
i2_2 ~ hnu2_2 * 1
#i3_2 ~ hnu3_2 * 1
i4_2 ~ hnu4_2 * 1
i5_2 ~ hnu5_2 * 1

# Naming the residual variances!
i1_2 ~~ htheta1_2 * i1_2
i2_2 ~~ htheta2_2 * i2_2
#i3_2 ~~ htheta3_1 * i3_2
i4_2 ~~ htheta4_2 * i4_2
i5_2 ~~ htheta5_2 * i5_2

## Adding the covariances ##
i1 ~~ i2
i1_2 ~~ i2_2

i1 ~~ i4
i1_2 ~~ i4_2

i2 ~~ i4
i2_2 ~~ i4_2

i1 ~~ i5
i1_2 ~~ i5_2

# Fixing latent variance to 1, as we freed first factor loading
```

```
  math_t2 ~~ 1 * math_t2

  # Fixing latent mean to 0 for identification
  math_t2 ~ 0 * 1


  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  #i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2

'


fit_config_comb  <- cfa(config_comb, data = dat, group = "sample",
                    estimator = "MLR", missing = "FIML", se = "robust.mlr")
```

```
Warning in lav_data_full(data = data, group = group, cluster = cluster, : lavaan WARNING: so
  24 33 38 44 47 50 56 65 66 67 77 87 99 101 110 119 123 124 130 139 140 164 166 173 177 185
```

```
Warning in lav_data_full(data = data, group = group, cluster = cluster, : lavaan WARNING: so
  16211 16227 16369 16424 16479 16500 16511 16581 16588 16613 16647 16649 16662 16686 16697
```

```
fit_config_comb
```

```
lavaan 0.6-12 ended normally after 98 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50
```

```
Model Test User Model:
                                                      Standard      Robust
  Test Statistic                                       274.812     235.207
  Degrees of freedom                                        28          28
  P-value (Chi-square)                                   0.000       0.000
  Scaling correction factor                                          1.168
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                                218.655     187.143
    HSLS                                                56.158      48.065
```

```r
head(modindices(fit_config_comb, sort. = TRUE, free.remove = FALSE))
```

```
        lhs op  rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
97  math_t2 =~   i3     1     1     1 47.242 -0.068  -0.068   -0.071   -0.071
91  math_t1 =~ i2_2     1     1     1 38.012  0.050   0.050    0.056    0.056
90  math_t1 =~ i1_2     1     1     1 35.233 -0.057  -0.057   -0.064   -0.064
109      i2 ~~ i3_2     1     1     1 33.516  0.022   0.022    0.078    0.078
101      i1 ~~   i5     1     1     1 32.875 -0.018  -0.018   -0.071   -0.071
126    i1_2 ~~ i4_2     1     1     1 30.409  0.022   0.022    0.088    0.088
```

```r
s_config_comb <- summary(fit_config_comb, fit.measures = TRUE)
```

WEAK ELS + HSLS

```r
# Only the first latent variance is set to 1, the rest are free.
# Latent means all 0.

weak_comb <- '

##############################################################################
##                            ELS (Group 1)                                 ##
##############################################################################
group: ELS

  ######################
  # Time Point 1
  ######################
```

```
math_t1 =~ NA   * i1 +
           l2_1 * i2 +
           l3_1 * i3 +
           l4_1 * i4 +
           l5_1 * i5

# Intercepts
i1 ~ enu1_1 * 1
i2 ~ enu2_1 * 1
i3 ~ enu3_1 * 1
i4 ~ enu4_1 * 1
i5 ~ enu5_1 * 1

# Residual variances
i1 ~~ etheta1_1 * i1
i2 ~~ etheta2_1 * i2
i3 ~~ etheta3_1 * i3
i4 ~~ etheta4_1 * i4
i5 ~~ etheta5_1 * i5

# Fix latent variance to 1 for identification, fix mean to 0
math_t1 ~~ 1 * math_t1
math_t1 ~ 0 * 1


######################
# Time Point 2
######################
math_t2 =~ NA   * i1_2 +
           l2_2 * i2_2 +
           l3_2 * i3_2 +
           l4_2 * i4_2 +
           l5_2 * i5_2

# Intercepts
i1_2 ~ enu1_2 * 1
i2_2 ~ enu2_2 * 1
i3_2 ~ enu3_2 * 1
i4_2 ~ enu4_2 * 1
i5_2 ~ enu5_2 * 1

# Residual variances
```

```
  i1_2 ~~ etheta1_2 * i1_2
  i2_2 ~~ etheta2_2 * i2_2
  i3_2 ~~ etheta3_2 * i3_2
  i4_2 ~~ etheta4_2 * i4_2
  i5_2 ~~ etheta5_2 * i5_2

  # Covariances among items
  i1 ~~ i2
  i1_2 ~~ i2_2

  i1 ~~ i3
  i1_2 ~~ i3_2

  i2 ~~ i3
  i2_2 ~~ i3_2

  i4 ~~ i5
  i4_2 ~~ i5_2

  # Fix latent variance to 1 for identification, free mean
  math_t2 ~~ 1 * math_t2
  math_t2 ~ NA * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2


################################################################################
##                            HSLS (Group 2)                                  ##
################################################################################
group: HSLS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ NA   *i1 +
```

```
            l2_1 * i2 +   # Same label as ELS
            # no i3 in HSLS
            l4_1 * i4 +   # Same label as ELS
            l5_1 * i5     # Same label as ELS

# Intercepts
i1 ~ hnu1_1 * 1
i2 ~ hnu2_1 * 1
# i3 ~ hnu3_1 * 1 (item not in HSLS)
i4 ~ hnu4_1 * 1
i5 ~ hnu5_1 * 1

# Residual variances
i1 ~~ htheta1_1 * i1
i2 ~~ htheta2_1 * i2
# i3 ~~ htheta3_1 * i3 (item not in HSLS)
i4 ~~ htheta4_1 * i4
i5 ~~ htheta5_1 * i5

# Free latent variance and free latent mean
math_t1 ~~ var_hsls * math_t1
math_t1 ~ NA * 1


######################
# Time Point 2
######################
math_t2 =~ NA   * i1_2 +
            l2_2 * i2_2 +   # Same label as ELS
            # no i3_2 in HSLS at Time 2
            l4_2 * i4_2 +   # Same label as ELS
            l5_2 * i5_2     # Same label as ELS

# Intercepts
i1_2 ~ hnu1_2 * 1
i2_2 ~ hnu2_2 * 1
# i3_2 ~ hnu3_2 * 1 (item not in HSLS)
i4_2 ~ hnu4_2 * 1
i5_2 ~ hnu5_2 * 1

# Residual variances
```

```
  i1_2 ~~ htheta1_2 * i1_2
  i2_2 ~~ htheta2_2 * i2_2
  # i3_2 ~~ htheta3_2 * i3_2 (item not in HSLS)
  i4_2 ~~ htheta4_2 * i4_2
  i5_2 ~~ htheta5_2 * i5_2

  # Covariances among items
  i1 ~~ i2
  i1_2 ~~ i2_2

  i1 ~~ i4
  i1_2 ~~ i4_2

  i2 ~~ i4
  i2_2 ~~ i4_2

  i1 ~~ i5
  i1_2 ~~ i5_2

  # Free latent variance and free latent mean
  math_t2 ~~ var_hsls_t2 * math_t2
  math_t2 ~ NA * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  # i3 ~~ i3_2 (item not in HSLS)
  i4 ~~ i4_2
  i5 ~~ i5_2

'


fit_weak_comb  <- cfa(weak_comb, data = dat, group = "sample",
                      estimator = "MLR", missing = "FIML", se = "robust.mlr")
```

```
Warning in lav_data_full(data = data, group = group, cluster = cluster, : lavaan WARNING: so
  24 33 38 44 47 50 56 65 66 67 77 87 99 101 110 119 123 124 130 139 140 164 166 173 177 185
```

```
Warning in lav_data_full(data = data, group = group, cluster = cluster, : lavaan WARNING: so
  16211 16227 16369 16424 16479 16500 16511 16581 16588 16613 16647 16649 16662 16686 16697
```

```
Warning in lav_model_vcov(lavmodel = lavmodel, lavsamplestats = lavsamplestats, : lavaan WARN
    The variance-covariance matrix of the estimated parameters (vcov)
    does not appear to be positive definite! The smallest eigenvalue
    (= 1.885309e-19) is close to zero. This may be a symptom that the
    model is not identified.
```

```
fit_weak_comb
```

```
lavaan 0.6-12 ended normally after 88 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        86
  Number of equality constraints                     6

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50

Model Test User Model:
                                              Standard      Robust
  Test Statistic                               364.936     288.592
  Degrees of freedom                                29          29
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.265
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                        259.106     204.901
    HSLS                                       105.830      83.691
```

```
head(modindices(fit_weak_comb, sort. = TRUE, free.remove = FALSE))
```

```
        lhs op  rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
103 math_t2 =~   i3     1   ELS     1 70.861 -0.076  -0.076   -0.079   -0.079
141 math_t1 =~ i5_2     2  HSLS     1 49.758 -0.046  -0.037   -0.048   -0.048
107      i1 ~~   i5     1   ELS     1 34.540 -0.019  -0.019   -0.074   -0.074
97  math_t1 =~ i2_2     1   ELS     1 33.184  0.040   0.040    0.045    0.045
```

```
96  math_t1 =~ i1_2     1   ELS     1 31.078 -0.049  -0.049   -0.055   -0.055
106      i1 ~~   i4     1   ELS     1 30.684  0.018   0.018    0.070    0.070
```

```r
s_weak_comb <- summary(fit_weak_comb, fit.measures = TRUE, standardized = TRUE)

lavTestLRT(fit_config_comb, fit_weak_comb)
```

```
Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")

lavaan NOTE:
    The "Chisq" column contains standard test statistics, not the
    robust test that should be reported per model. A robust difference
    test is a function of two standard (not robust) statistics.

                Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit_config_comb 28 474900 475590 274.81
fit_weak_comb   29 474988 475669 364.94     22.776       1   1.82e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

STRONG ELS + HSLS

```r
strong_comb <- '

################################################################################
##                            ELS (Group 1)                                   ##
################################################################################
group: ELS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ NA    * i1 +
             l2_1 * i2 +
             l3_1 * i3 +
             l4_1 * i4 +
             l5_1 * i5

  # Intercepts
  # Same labels for both groups!
```

```
i1 ~ nu1_1 * 1
i2 ~ nu2_1 * 1
i3 ~ nu3_1 * 1
i4 ~ nu4_1 * 1
i5 ~ nu5_1 * 1

# Residual variances
i1 ~~ etheta1_1 * i1
i2 ~~ etheta2_1 * i2
i3 ~~ etheta3_1 * i3
i4 ~~ etheta4_1 * i4
i5 ~~ etheta5_1 * i5

# Fix latent variance to 1 for identification, fix mean to 0
math_t1 ~~ 1 * math_t1
math_t1 ~ 0 * 1


######################
# Time Point 2
######################
math_t2 =~ 1*i1_2 +
           l2_2 * i2_2 +
           l3_2 * i3_2 +
           l4_2 * i4_2 +
           l5_2 * i5_2

# Intercepts
# Same labels for both groups!
i1_2 ~ nu1_2 * 1
i2_2 ~ nu2_2 * 1
i3_2 ~ nu3_2 * 1
i4_2 ~ nu4_2 * 1
i5_2 ~ nu5_2 * 1

# Residual variances
i1_2 ~~ etheta1_2 * i1_2
i2_2 ~~ etheta2_2 * i2_2
i3_2 ~~ etheta3_2 * i3_2
i4_2 ~~ etheta4_2 * i4_2
i5_2 ~~ etheta5_2 * i5_2
```

```
  # Covariances among items
  i1 ~~ i2
  i1_2 ~~ i2_2

  i1 ~~ i3
  i1_2 ~~ i3_2

  i2 ~~ i3
  i2_2 ~~ i3_2

  i4 ~~ i5
  i4_2 ~~ i5_2

  # Fix latent variance to 1 for identification, free mean
  math_t2 ~~ var_els_t2 * math_t2
  math_t2 ~ NA * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2


################################################################################
##                            HSLS (Group 2)                                  ##
################################################################################
group: HSLS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ NA   * i1 +
             l2_1 * i2 +   # Same label as ELS
             # no i3 in HSLS
             l4_1 * i4 +   # Same label as ELS
             l5_1 * i5     # Same label as ELS

  # Intercepts
```

```
 # Same labels for both groups!
 i1 ~ nu1_1 * 1
 i2 ~ nu2_1 * 1
 # i3 ~ hnu3_1 * 1 (item not in HSLS)
 i4 ~ nu4_1 * 1
 i5 ~ nu5_1 * 1

 # Residual variances
 i1 ~~ htheta1_1 * i1
 i2 ~~ htheta2_1 * i2
 # i3 ~~ htheta3_1 * i3 (item not in HSLS)
 i4 ~~ htheta4_1 * i4
 i5 ~~ htheta5_1 * i5

# Free latent variance and free latent mean
 math_t1 ~~ var_hsls * math_t1
 math_t1 ~ NA * 1


 ######################
 # Time Point 2
 ######################
 math_t2 =~ NA   * i1_2 +
            l2_2 * i2_2 +   # Same label as ELS
            # no i3_2 in HSLS at Time 2
            l4_2 * i4_2 +   # Same label as ELS
            l5_2 * i5_2     # Same label as ELS

 # Intercepts
 # Same labels for both groups!

 i1_2 ~ nu1_2 * 1
 i2_2 ~ nu2_2 * 1
 # i3_2 ~ hnu3_2 * 1 (item not in HSLS)
 i4_2 ~ nu4_2 * 1
 i5_2 ~ nu5_2 * 1

 # Residual variances
 i1_2 ~~ htheta1_2 * i1_2
 i2_2 ~~ htheta2_2 * i2_2
 # i3_2 ~~ htheta3_2 * i3_2 (item not in HSLS)
```

```
  i4_2 ~~ htheta4_2 * i4_2
  i5_2 ~~ htheta5_2 * i5_2

  # Covariances among items
  i1 ~~ i2
  i1_2 ~~ i2_2

  i1 ~~ i4
  i1_2 ~~ i4_2

  i2 ~~ i4
  i2_2 ~~ i4_2

  i1 ~~ i5
  i1_2 ~~ i5_2

  # Free latent variance and free latent mean
  math_t2 ~~ var_hsls_t2 * math_t2
  math_t2 ~ NA * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  # i3 ~~ i3_2 (item not in HSLS)
  i4 ~~ i4_2
  i5 ~~ i5_2

'


fit_strong_comb  <- cfa(strong_comb, data = dat, group = "sample",
                        estimator = "MLR", missing = "FIML", se = "robust.mlr")
```

```
Warning in lav_data_full(data = data, group = group, cluster = cluster, : lavaan WARNING: so
  24 33 38 44 47 50 56 65 66 67 77 87 99 101 110 119 123 124 130 139 140 164 166 173 177 185

Warning in lav_data_full(data = data, group = group, cluster = cluster, : lavaan WARNING: so
  16211 16227 16369 16424 16479 16500 16511 16581 16588 16613 16647 16649 16662 16686 16697
```

```
fit_strong_comb
```

```
lavaan 0.6-12 ended normally after 172 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        86
  Number of equality constraints                    14

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50

Model Test User Model:
                                                Standard      Robust
  Test Statistic                                794.745     705.843
  Degrees of freedom                                 37          37
  P-value (Chi-square)                            0.000       0.000
  Scaling correction factor                                   1.126
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                           521.566     463.223
    HSLS                                          273.178     242.620
```

```r
head(modindices(fit_strong_comb, sort. = TRUE, free.remove = FALSE))
```

```
        lhs op rhs block group level      mi    epc sepc.lv sepc.all sepc.nox
112 math_t2 =~  i4     1   ELS     1 237.142  0.027   0.021    0.022    0.022
152 math_t2 =~  i4     2  HSLS     1 222.944 -0.025  -0.017   -0.023   -0.023
153 math_t2 =~  i5     2  HSLS     1 168.010  0.022   0.015    0.021    0.021
113 math_t2 =~  i5     1   ELS     1 166.953 -0.024  -0.018   -0.019   -0.019
9        i4 ~1         1   ELS     1 122.263 -0.053  -0.053   -0.056   -0.056
10       i5 ~1         1   ELS     1  92.968  0.047   0.047    0.051    0.051
```

```r
s_strong_comb <- summary(fit_strong_comb, fit.measures = TRUE, standardized = TRUE)

lavTestLRT(fit_weak_comb, fit_strong_comb)
```

```
Scaled Chi-Squared Difference Test (method = "satorra.bentler.2001")
```

```
lavaan NOTE:
    The "Chisq" column contains standard test statistics, not the
    robust test that should be reported per model. A robust difference
    test is a function of two standard (not robust) statistics.

                Df    AIC    BIC  Chisq Chisq diff Df diff Pr(>Chisq)
fit_weak_comb   29 474988 475669 364.94
fit_strong_comb 37 475402 476015 794.74     689.28       8  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Strong invariance holds! What if across groups AND time?

HSLS ONLY

```
# hsls_time <-  '
#   math_T1 =~ NA  * i1 +
#              l2_1 * i2 +
#              l4_1 * i4 +
#              l5_1 * i5
#
#
#   # Fixing latent variance to 1, as we freed first factor loading
#   math_T1 ~~ 1 * math_T1
#
#   # Fixing latent mean to 0 for identification?
#   math_T1 ~ 0 * 1
#
#   # Time Point 2
#   math_T2 =~ NA  * i1_2 +
#              l2_2 * i2_2 +
#              l4_2 * i4_2 +
#              l5_2 * i5_2
#
#
#
#   # Adding the covariances
#   i1_2 ~~ i2_2
#   i2_2 ~~ i4_2
```

```
#
#   i1 ~~ i5
#   i1_2 ~~ i5_2
#
#   # Fixing latent variance to 1, as we freed first factor loading
#   math_T2 ~~ 1 * math_T2
#
#   # Fixing latent mean to 0 for identification?
#   math_T2 ~ 0 * 1
#
#   # Correlations across time
#   math_T1 ~~ math_T2
#   i1 ~~ i1_2
#   i2 ~~ i2_2
#   i4 ~~ i4_2
#   i5 ~~ i5_2
# '
#
# fit_hsls_time  <- cfa(hsls_time, data = hsls,
#                       estimator = "MLR", missing = "FIML", se = "robust.mlr")
# fit_hsls_time
# head(modindices(fit_hsls_time, sort. = TRUE, free.remove = FALSE))
#
#
# s_hsls_time <- summary(fit_hsls_time, fit.measures = TRUE)
#
#
```

ELS ONLY

```
#
# els_time <-  '
#   math_T1 =~ NA  * i1 +
#              l2_1 * i2 +
#              l3_1 * i3 +
#              l4_1 * i4 +
#              l5_1 * i5
#
#
#   # Fixing latent variance to 1, as we freed first factor loading
```

```
#   math_T1 ~~ 1 * math_T1
#
#   # Fixing latent mean to 0 for identification?
#   math_T1 ~ 0 * 1
#
#   # Time Point 2
#   math_T2 =~ NA  * i1_2 +
#              l2_2 * i2_2 +
#              l3_2 * i3_2 +
#              l4_2 * i4_2 +
#              l5_2 * i5_2
#
#   # Adding the covariances
#   i1 ~~ i2
#   i1_2 ~~ i2_2
#
#   i2 ~~ i3
#   i2_2 ~~ i3_2
#
#   i4 ~~ i5
#   i4_2 ~~ i5_2
#
#
#
#
#   # Fixing latent variance to 1, as we freed first factor loading
#   math_T2 ~~ 1 * math_T2
#
#   # Fixing latent mean to 0 for identification?
#   math_T2 ~ 0 * 1
#
#   # Correlations across time
#   math_T1 ~~ math_T2
#   i1 ~~ i1_2
#   i2 ~~ i2_2
#   i3 ~~ i3_2
#   i4 ~~ i4_2
#   i5 ~~ i5_2
# '
#
# fit_els_time  <- cfa(els_time, data = els,
```

```
#                       estimator = "MLR", missing = "FIML", se = "robust.mlr")
# fit_els_time
# head(modindices(fit_els_time, sort. = TRUE, free.remove = FALSE))
#
# s_els_time <- summary(fit_hsls_time, fit.measures = TRUE)
```

HSLS ONLY WEAK

```
#
# hsls_weak <-  '
#   math_T1 =~ 1  * i1 +
#              l2 * i2 +
#              l4 * i4 +
#              l5 * i5
#
#
#   # Freeing latent variance, as we restrict first factor loading instead
#   math_T1 ~~ NA * math_T1
#
#   # Fixing latent mean to 0 for identification?
#   math_T1 ~ 0 * 1
#
#   # Time Point 2
#   math_T2 =~ 1  * i1_2 +
#              l2 * i2_2 +
#              l4 * i4_2 +
#              l5 * i5_2
#
#
#
#   # Adding the covariances
#   i1 ~~ i2
#   i1_2 ~~ i2_2
#
#   i2 ~~ i4
#   i2_2 ~~ i4_2
#
#   i1 ~~ i5
#   i1_2 ~~ i5_2
#
```

```
#   # Freeing latent variance, as we restrict first factor loading
#   math_T2 ~~ NA * math_T2
#
#   # Fixing latent mean to 0 for identification?
#   math_T2 ~ 0 * 1
#
#   # Correlations across time
#   math_T1 ~~ math_T2
#   i1 ~~ i1_2
#   i2 ~~ i2_2
#   i4 ~~ i4_2
#   i5 ~~ i5_2
# '
#
# fit_hsls_weak  <- cfa(hsls_weak, data = hsls,
#                       estimator = "MLR", missing = "FIML", se = "robust.mlr")
# fit_hsls_weak
# head(modindices(fit_hsls_weak, sort. = TRUE, free.remove = FALSE))
#
#
# s_hsls_weak <- summary(fit_hsls_weak, fit.measures = TRUE)
#
```