hsls_els

1 = strongly disagree 2 = disagree 3 = agree 4 = strongly agree

i1 = Teen (9th / 11th grader) confident can do excellent job on (fall 2009 / spring 2012) math tests

i2 = Teen (9th / 11th grader) certain can understand (fall 2009 / spring 2012) math textbook

i3 = Can understand difficult math class (ELS ONLY!)

i4 = Teen confident can do an excellent job on math assignments

i5 = Teen certain can master skills in math course

Can understand difficult math class

Turning warnings off (bad idea)

```
options(warn = -1)
```

```
library(dplyr)
```

```
Attaching package: 'dplyr'

The following objects are masked from 'package:stats':

    filter, lag

The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

```r
library(lavaan)
```

```
This is lavaan 0.6-19
lavaan is FREE software! Please report any bugs.
```

```r
library(ltm)
```

```
Loading required package: MASS


Attaching package: 'MASS'

The following object is masked from 'package:dplyr':

    select

Loading required package: msm

Loading required package: polycor
```

```r
library(sjlabelled)
```

```
Attaching package: 'sjlabelled'

The following object is masked from 'package:dplyr':

    as_label
```

```r
library(kableExtra)
```

```
Attaching package: 'kableExtra'

The following object is masked from 'package:dplyr':

    group_rows
```

```
library(sirt)
```

```
- sirt 4.1-15 (2024-02-06 00:05:40)
```

```
library(mirt)
```

```
Loading required package: stats4
```

```
Loading required package: lattice
```

```

Attaching package: 'mirt'
```

```
The following object is masked from 'package:ltm':

    Science
```

```
#source("code/download_data.R")
source("F:/Users/alex/OneDrive/Documents/data/prepare_data.R")
```

```
here() starts at F:/Users/alex/OneDrive/Documents/data
```

```
Joining with `by = join_by(stu_id, sch_id, STRAT_ID, psu, sex, i1, i2, i4, i5,
i1_2, i2_2, i4_2, i5_2, sample)`
```

```
m_items <- paste0("i", 1:5)
m_items_2 <- paste0(m_items, "_2")
# get subset of relevant variables
dat <- dat[, c("stu_id", "sample", "sex", "dropout", m_items, m_items_2)]

dat$mean_score <- c(rowMeans(dat[dat$sample == "ELS", m_items], na.rm = TRUE),
                    rowMeans(dat[dat$sample == "HSLS", m_items[-3]], na.rm = TRUE))
dat$mean_score_2 <- c(rowMeans(dat[dat$sample == "ELS", m_items_2], na.rm = TRUE),
                    rowMeans(dat[dat$sample == "HSLS", m_items_2[-3]], na.rm = TRUE))
```

```r
# Creating only HSLS
hsls <- subset(dat, sample == "HSLS")

hsls_1 <- hsls[, c("i1", "i2", "i4", "i5")]
head(hsls_1)
```

```
      i1 i2 i4 i5
16198  4  3  4  3
16199  3  3  4  3
16200  4  2  4  3
16201  3  3  3  3
16202  3  3  3  3
16203  4  4  4  3
```

```r
hsls_2 <- hsls[, c("i1_2", "i2_2", "i4_2", "i5_2")]
head(hsls_2)
```

```
      i1_2 i2_2 i4_2 i5_2
16198    4    4    4    4
16199    2    2    3    2
16200    2    1    3    3
16201   NA   NA   NA   NA
16202   NA   NA   NA   NA
16203    3    3    3    3
```

```r
hsls_1_noNA <- na.omit(hsls_1)
hsls_2_noNA <- na.omit(hsls_2)


# Creating only ELS
els <- subset(dat, sample == "ELS")

els_1 <- els[, c("i1", "i2", "i3", "i4", "i5")]
head(els_1)
```

```
  i1 i2 i3 i4 i5
1  2  1  2  2  1
2  4  3  4  4  4
3  3  2  2  3  2
```

```
4  4  3  3  3  4
5  2  2  3  3  3
6  2  2  3  3 NA
```

```r
els_2 <- els[, c("i1_2", "i2_2", "i3_2", "i4_2", "i5_2")]
head(els_2)
```

```
  i1_2 i2_2 i3_2 i4_2 i5_2
1    3    2    2    2    3
2    3    3    3    3    3
3    2    3    3    2    2
4    3    2    2    3    4
5    3    3    3    4    4
6   NA   NA   NA   NA   NA
```

```r
els_1_noNA <- na.omit(els_1)
els_2_noNA <- na.omit(els_2)
```

```r
cfa_config <-  '
  group: ELS
  math =~ NA   * i1 +
          el2_1 * i2 +
          el3_1 * i3 +
          el4_1 * i4 +
          el5_1 * i5

  # Naming the intercepts!
  i1 ~ nu1_1 * 1
  i2 ~ nu2_1 * 1
  i3 ~ nu3_1 * 1
  i4 ~ nu4_1 * 1
  i5 ~ nu5_1 * 1

  # Naming the residual variances!
  i1 ~~ theta1_1 * i1
  i2 ~~ theta2_1 * i2
  i3 ~~ theta3_1 * i3
  i4 ~~ theta4_1 * i4
  i5 ~~ theta5_1 * i5
```

```
  # Adding the covariances
  i1 ~~ i2
  i2 ~~ i3

  # Fixing latent variance to 1, as we freed first factor loading
  math ~~ 1 * math

  # Fixing latent mean to 0 for identification?
  math ~ 0 * 1


  group: HSLS
  math =~ NA   * i1 +
          hl2_2 * i2 +
          hl4_2 * i4 +
          hl5_2 * i5

  # Naming the intercepts!
  i1 ~ nu1_2 * 1
  i2 ~ nu2_2 * 1
  i4 ~ nu4_2 * 1
  i5 ~ nu5_2 * 1

  # Naming the residual variances!
  i1 ~~ theta1_2 * i1
  i2 ~~ theta2_2 * i2
  i4 ~~ theta4_2 * i4
  i5 ~~ theta5_2 * i5

  # Adding the covariances
    #i1 ~~ i2
    i2 ~~ i4

  # Fixing latent variance to 1, as we freed first factor loading
  math ~~ 1 * math

  # Fixing latent mean to 0 for identification?
  math ~ 0 * 1
'

fit_config  <- cfa(cfa_config, data = dat, group = "sample",
```

```
                    estimator = "MLR", missing = "FIML", se = "robust.mlr")


s_config <- summary(fit_config, fit.measures = TRUE, standardized = TRUE)


mod_indices <- modindices(fit_config, sort. = TRUE, free.remove = FALSE)
head(mod_indices)
```

```
   lhs op rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
35  i1 ~~  i3     1   ELS     1 81.621  0.033   0.033    0.119    0.119
42  i4 ~~  i5     1   ELS     1 81.621  0.038   0.038    0.218    0.218
37  i1 ~~  i5     1   ELS     1 76.182 -0.027  -0.027   -0.113   -0.113
40  i3 ~~  i4     1   ELS     1 27.064 -0.018  -0.018   -0.090   -0.090
39  i2 ~~  i5     1   ELS     1 12.584  0.011   0.011    0.043    0.043
38  i2 ~~  i4     1   ELS     1 12.584 -0.011  -0.011   -0.045   -0.045
```

```
# Just for ELS
mod_indices_els <- mod_indices[mod_indices$group == "ELS", ]
head(mod_indices_els)
```

```
   lhs op rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
35  i1 ~~  i3     1   ELS     1 81.621  0.033   0.033    0.119    0.119
42  i4 ~~  i5     1   ELS     1 81.621  0.038   0.038    0.218    0.218
37  i1 ~~  i5     1   ELS     1 76.182 -0.027  -0.027   -0.113   -0.113
40  i3 ~~  i4     1   ELS     1 27.064 -0.018  -0.018   -0.090   -0.090
39  i2 ~~  i5     1   ELS     1 12.584  0.011   0.011    0.043    0.043
38  i2 ~~  i4     1   ELS     1 12.584 -0.011  -0.011   -0.045   -0.045
```

```
# Just for HSLS
mod_indices_hsls <- mod_indices[mod_indices$group == "HSLS", ]
head(mod_indices_hsls)
```

```
    lhs op rhs block group level    mi    epc sepc.lv sepc.all sepc.nox
47   i4 ~~  i5     2  HSLS     1 0.044 -0.001  -0.001   -0.004   -0.004
43   i1 ~~  i2     2  HSLS     1 0.044 -0.001  -0.001   -0.003   -0.003
46   i2 ~~  i5     2  HSLS     1 0.044  0.001   0.001    0.003    0.003
44   i1 ~~  i4     2  HSLS     1 0.044  0.001   0.001    0.004    0.004
25   i2 ~1         2  HSLS     1 0.000  0.000   0.000    0.000    0.000
23 math =~  i5     2  HSLS     1 0.000  0.000   0.000    0.000    0.000
```

#CONFIG ELS + HSLS

```
config_comb <-  '
  # ELS
  group: 1

  # Time point 1
  math_t1 =~ NA   * i1 +
          el2_1 * i2 +
          el3_1 * i3 +
          el4_1 * i4 +
          el5_1 * i5

  # Naming the intercepts!
  i1 ~ enu1_1 * 1
  i2 ~ enu2_1 * 1
  i3 ~ enu3_1 * 1
  i4 ~ enu4_1 * 1
  i5 ~ enu5_1 * 1

  # Naming the residual variances!
  i1 ~~ etheta1_1 * i1
  i2 ~~ etheta2_1 * i2
  i3 ~~ etheta3_1 * i3
  i4 ~~ etheta4_1 * i4
  i5 ~~ etheta5_1 * i5

  # Fixing latent variance to 1, as we freed first factor loading
  math_t1 ~~ 1 * math_t1

  # Fixing latent mean to 0 for identification?
  math_t1 ~ 0 * 1

  # Time point 2
   math_t2 =~ NA   * i1_2 +
            el2_2 * i2_2 +
            el3_2 * i3_2 +
            el4_2 * i4_2 +
            el5_2 * i5_2

  # Naming the intercepts!
  i1_2 ~ enu1_2 * 1
```

```
 i2_2 ~ enu2_2 * 1
 i3_2 ~ enu3_2 * 1
 i4_2 ~ enu4_2 * 1
 i5_2 ~ enu5_2 * 1

 # Naming the residual variances!
 i1_2 ~~ etheta1_2 * i1_2
 i2_2 ~~ etheta2_2 * i2_2
 i3_2 ~~ etheta3_2 * i3_2
 i4_2 ~~ etheta4_2 * i4_2
 i5_2 ~~ etheta5_2 * i5_2


 ## Adding the covariances ##
 i1 ~~ i2
 i1_2 ~~ i2_2

 i1 ~~ i3
 i1_2 ~~ i3_2

 i2 ~~ i3
 i2_2 ~~ i3_2

 i4 ~~ i5
 i4_2 ~~ i5_2


 # Fixing latent variance to 1, as we freed first factor loading
 math_t2 ~~ 1 * math_t2

 # Fixing latent mean to 0 for identification
 math_t2 ~ 0 * 1

 # Correlations across time
 math_t1 ~~ math_t2
 i1 ~~ i1_2
 i2 ~~ i2_2
 i3 ~~ i3_2
 i4 ~~ i4_2
 i5 ~~ i5_2

# HSLS
```

```
group: 2
 # Time Point 1
 math_t1 =~ NA   * i1 +
            hl2_1 * i2 +
            #hl3_1 * i3 +
            hl4_1 * i4 +
            hl5_1 * i5

 # Naming the intercepts!
 i1 ~ hnu1_1 * 1
 i2 ~ hnu2_1 * 1
 #i3 ~ hnu3_1 * 1
 i4 ~ hnu4_1 * 1
 i5 ~ hnu5_1 * 1

 # Naming the residual variances!
 i1 ~~ htheta1_1 * i1
 i2 ~~ htheta2_1 * i2
 #i3 ~~ htheta3_1 * i3
 i4 ~~ htheta4_1 * i4
 i5 ~~ htheta5_1 * i5


 # Fixing latent variance to 1, as we freed first factor loading
 math_t1 ~~ 1 * math_t1

 # Fixing latent mean to 0 for identification
 math_t1 ~ 0 * 1

 # Time Point 2
 math_t2 =~ NA   * i1_2 +
            hl2_2 * i2_2 +
            #hl3_2 * i3_2 +
            hl4_2 * i4_2 +
            hl5_2 * i5_2

 # Naming the intercepts!
 i1_2 ~ hnu1_2 * 1
 i2_2 ~ hnu2_2 * 1
 #i3_2 ~ hnu3_2 * 1
 i4_2 ~ hnu4_2 * 1
```

```
  i5_2 ~ hnu5_2 * 1

  # Naming the residual variances!
  i1_2 ~~ htheta1_2 * i1_2
  i2_2 ~~ htheta2_2 * i2_2
  #i3_2 ~~ htheta3_1 * i3_2
  i4_2 ~~ htheta4_2 * i4_2
  i5_2 ~~ htheta5_2 * i5_2

  ## Adding the covariances ##
  i1 ~~ i2
  i1_2 ~~ i2_2

  i1 ~~ i4
  i1_2 ~~ i4_2

  i2 ~~ i4
  i2_2 ~~ i4_2

  i1 ~~ i5
  i1_2 ~~ i5_2

  # Fixing latent variance to 1, as we freed first factor loading
  math_t2 ~~ 1 * math_t2

  # Fixing latent mean to 0 for identification
  math_t2 ~ 0 * 1


  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  #i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2

'


fit_config_comb  <- cfa(config_comb, data = dat, group = "sample",
```

```
                      estimator = "MLR", missing = "FIML", se = "robust.mlr")
fit_config_comb
```

```
lavaan 0.6-19 ended normally after 98 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50

Model Test User Model:
                                                  Standard      Scaled
  Test Statistic                                   274.812     235.207
  Degrees of freedom                                    28          28
  P-value (Chi-square)                               0.000       0.000
  Scaling correction factor                                      1.168
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                            187.143     187.143
    HSLS                                            48.065      48.065
```

```
head(modindices(fit_config_comb, sort. = TRUE, free.remove = FALSE))
```

```
        lhs op  rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
97  math_t2 =~   i3     1     1     1 47.242 -0.068  -0.068   -0.071   -0.071
91  math_t1 =~ i2_2     1     1     1 38.012  0.050   0.050    0.056    0.056
90  math_t1 =~ i1_2     1     1     1 35.233 -0.057  -0.057   -0.064   -0.064
109      i2 ~~ i3_2     1     1     1 33.516  0.022   0.022    0.078    0.078
101      i1 ~~   i5     1     1     1 32.875 -0.018  -0.018   -0.071   -0.071
126    i1_2 ~~ i4_2     1     1     1 30.409  0.022   0.022    0.088    0.088
```

```
fitmeasures(fit_config_comb, c("aic", "bic", "cfi", "df"))
```

```
       aic        bic        cfi         df
474900.288 475589.784      0.999     28.000
```

```r
s_config_comb <- summary(fit_config_comb, fit.measures = TRUE)
parTable(fit_config_comb)
```

```
   id     lhs op     rhs user block group free ustart exo     label plabel
1   1 math_t1 =~      i1    1     1     1    1     NA   0             .p1.
2   2 math_t1 =~      i2    1     1     1    2     NA   0     el2_1   .p2.
3   3 math_t1 =~      i3    1     1     1    3     NA   0     el3_1   .p3.
4   4 math_t1 =~      i4    1     1     1    4     NA   0     el4_1   .p4.
5   5 math_t1 =~      i5    1     1     1    5     NA   0     el5_1   .p5.
6   6      i1 ~1            1     1     1    6     NA   0    enu1_1   .p6.
7   7      i2 ~1            1     1     1    7     NA   0    enu2_1   .p7.
8   8      i3 ~1            1     1     1    8     NA   0    enu3_1   .p8.
9   9      i4 ~1            1     1     1    9     NA   0    enu4_1   .p9.
10 10      i5 ~1            1     1     1   10     NA   0    enu5_1  .p10.
11 11      i1 ~~      i1    1     1     1   11     NA   0 etheta1_1  .p11.
12 12      i2 ~~      i2    1     1     1   12     NA   0 etheta2_1  .p12.
13 13      i3 ~~      i3    1     1     1   13     NA   0 etheta3_1  .p13.
14 14      i4 ~~      i4    1     1     1   14     NA   0 etheta4_1  .p14.
15 15      i5 ~~      i5    1     1     1   15     NA   0 etheta5_1  .p15.
16 16 math_t1 ~~ math_t1    1     1     1    0      1   0             .p16.
17 17 math_t1 ~1            1     1     1    0      0   0             .p17.
18 18 math_t2 =~    i1_2    1     1     1   16     NA   0             .p18.
19 19 math_t2 =~    i2_2    1     1     1   17     NA   0     el2_2   .p19.
20 20 math_t2 =~    i3_2    1     1     1   18     NA   0     el3_2   .p20.
21 21 math_t2 =~    i4_2    1     1     1   19     NA   0     el4_2   .p21.
22 22 math_t2 =~    i5_2    1     1     1   20     NA   0     el5_2   .p22.
23 23    i1_2 ~1            1     1     1   21     NA   0    enu1_2   .p23.
24 24    i2_2 ~1            1     1     1   22     NA   0    enu2_2   .p24.
25 25    i3_2 ~1            1     1     1   23     NA   0    enu3_2   .p25.
26 26    i4_2 ~1            1     1     1   24     NA   0    enu4_2   .p26.
27 27    i5_2 ~1            1     1     1   25     NA   0    enu5_2   .p27.
28 28    i1_2 ~~    i1_2    1     1     1   26     NA   0 etheta1_2  .p28.
29 29    i2_2 ~~    i2_2    1     1     1   27     NA   0 etheta2_2  .p29.
30 30    i3_2 ~~    i3_2    1     1     1   28     NA   0 etheta3_2  .p30.
31 31    i4_2 ~~    i4_2    1     1     1   29     NA   0 etheta4_2  .p31.
32 32    i5_2 ~~    i5_2    1     1     1   30     NA   0 etheta5_2  .p32.
33 33      i1 ~~      i2    1     1     1   31     NA   0             .p33.
34 34    i1_2 ~~    i2_2    1     1     1   32     NA   0             .p34.
```

```
35 35      i1 ~~      i3    1     1     1   33     NA   0           .p35.
36 36    i1_2 ~~    i3_2    1     1     1   34     NA   0           .p36.
37 37      i2 ~~      i3    1     1     1   35     NA   0           .p37.
38 38    i2_2 ~~    i3_2    1     1     1   36     NA   0           .p38.
39 39      i4 ~~      i5    1     1     1   37     NA   0           .p39.
40 40    i4_2 ~~    i5_2    1     1     1   38     NA   0           .p40.
41 41 math_t2 ~~ math_t2    1     1     1    0      1   0           .p41.
42 42 math_t2 ~1            1     1     1    0      0   0           .p42.
43 43 math_t1 ~~ math_t2    1     1     1   39     NA   0           .p43.
44 44      i1 ~~    i1_2    1     1     1   40     NA   0           .p44.
45 45      i2 ~~    i2_2    1     1     1   41     NA   0           .p45.
46 46      i3 ~~    i3_2    1     1     1   42     NA   0           .p46.
47 47      i4 ~~    i4_2    1     1     1   43     NA   0           .p47.
48 48      i5 ~~    i5_2    1     1     1   44     NA   0           .p48.
49 49 math_t1 =~      i1    1     2     2   45     NA   0           .p49.
50 50 math_t1 =~      i2    1     2     2   46     NA   0     hl2_1 .p50.
51 51 math_t1 =~      i4    1     2     2   47     NA   0     hl4_1 .p51.
52 52 math_t1 =~      i5    1     2     2   48     NA   0     hl5_1 .p52.
53 53      i1 ~1            1     2     2   49     NA   0    hnu1_1 .p53.
54 54      i2 ~1            1     2     2   50     NA   0    hnu2_1 .p54.
55 55      i4 ~1            1     2     2   51     NA   0    hnu4_1 .p55.
56 56      i5 ~1            1     2     2   52     NA   0    hnu5_1 .p56.
57 57      i1 ~~      i1    1     2     2   53     NA   0 htheta1_1 .p57.
58 58      i2 ~~      i2    1     2     2   54     NA   0 htheta2_1 .p58.
59 59      i4 ~~      i4    1     2     2   55     NA   0 htheta4_1 .p59.
60 60      i5 ~~      i5    1     2     2   56     NA   0 htheta5_1 .p60.
61 61 math_t1 ~~ math_t1    1     2     2    0      1   0           .p61.
62 62 math_t1 ~1            1     2     2    0      0   0           .p62.
63 63 math_t2 =~    i1_2    1     2     2   57     NA   0           .p63.
64 64 math_t2 =~    i2_2    1     2     2   58     NA   0     hl2_2 .p64.
65 65 math_t2 =~    i4_2    1     2     2   59     NA   0     hl4_2 .p65.
66 66 math_t2 =~    i5_2    1     2     2   60     NA   0     hl5_2 .p66.
67 67    i1_2 ~1            1     2     2   61     NA   0    hnu1_2 .p67.
68 68    i2_2 ~1            1     2     2   62     NA   0    hnu2_2 .p68.
69 69    i4_2 ~1            1     2     2   63     NA   0    hnu4_2 .p69.
70 70    i5_2 ~1            1     2     2   64     NA   0    hnu5_2 .p70.
71 71    i1_2 ~~    i1_2    1     2     2   65     NA   0 htheta1_2 .p71.
72 72    i2_2 ~~    i2_2    1     2     2   66     NA   0 htheta2_2 .p72.
73 73    i4_2 ~~    i4_2    1     2     2   67     NA   0 htheta4_2 .p73.
74 74    i5_2 ~~    i5_2    1     2     2   68     NA   0 htheta5_2 .p74.
75 75      i1 ~~      i2    1     2     2   69     NA   0           .p75.
76 76    i1_2 ~~    i2_2    1     2     2   70     NA   0           .p76.
77 77      i1 ~~      i4    1     2     2   71     NA   0           .p77.
```

```
78 78    i1_2 ~~    i4_2    1     2     2   72     NA   0           .p78.
79 79      i2 ~~      i4    1     2     2   73     NA   0           .p79.
80 80    i2_2 ~~    i4_2    1     2     2   74     NA   0           .p80.
81 81      i1 ~~      i5    1     2     2   75     NA   0           .p81.
82 82    i1_2 ~~    i5_2    1     2     2   76     NA   0           .p82.
83 83 math_t2 ~~ math_t2    1     2     2    0      1   0           .p83.
84 84 math_t2 ~1            1     2     2    0      0   0           .p84.
85 85 math_t1 ~~ math_t2    1     2     2   77     NA   0           .p85.
86 86      i1 ~~    i1_2    1     2     2   78     NA   0           .p86.
87 87      i2 ~~    i2_2    1     2     2   79     NA   0           .p87.
88 88      i4 ~~    i4_2    1     2     2   80     NA   0           .p88.
89 89      i5 ~~    i5_2    1     2     2   81     NA   0           .p89.
   start    est    se
1  0.791  0.775 0.008
2  0.803  0.759 0.009
3  0.842  0.868 0.008
4  0.798  0.800 0.008
5  0.799  0.792 0.008
6  2.541  2.541 0.008
7  2.357  2.358 0.009
8  2.462  2.462 0.009
9  2.623  2.623 0.009
10 2.650  2.650 0.009
11 0.428  0.261 0.010
12 0.436  0.293 0.010
13 0.464  0.175 0.011
14 0.444  0.248 0.010
15 0.439  0.255 0.010
16 1.000  1.000 0.000
17 0.000  0.000 0.000
18 0.734  0.752 0.008
19 0.745  0.726 0.009
20 0.766  0.752 0.009
21 0.691  0.682 0.008
22 0.760  0.750 0.008
23 2.564  2.565 0.009
24 2.367  2.369 0.009
25 2.444  2.445 0.009
26 2.849  2.850 0.008
27 2.740  2.742 0.009
28 0.394  0.223 0.009
29 0.400  0.271 0.010
30 0.421  0.277 0.010
```

```
31 0.375  0.286 0.008
32 0.425  0.290 0.010
33 0.000  0.076 0.009
34 0.000  0.006 0.009
35 0.000 -0.044 0.009
36 0.000 -0.024 0.009
37 0.000  0.017 0.009
38 0.000  0.069 0.009
39 0.000  0.082 0.009
40 0.000  0.049 0.008
41 1.000  1.000 0.000
42 0.000  0.000 0.000
43 0.000  0.532 0.010
44 0.000  0.010 0.004
45 0.000  0.024 0.004
46 0.000  0.003 0.004
47 0.000  0.010 0.004
48 0.000  0.009 0.004
49 0.640  0.654 0.011
50 0.636  0.640 0.007
51 0.613  0.607 0.007
52 0.612  0.624 0.007
53 2.974  2.974 0.005
54 2.720  2.720 0.006
55 3.066  3.066 0.005
56 2.980  2.979 0.005
57 0.287  0.145 0.013
58 0.334  0.256 0.008
59 0.258  0.149 0.007
60 0.266  0.142 0.006
61 1.000  1.000 0.000
62 0.000  0.000 0.000
63 0.730  0.675 0.011
64 0.654  0.686 0.008
65 0.650  0.667 0.007
66 0.650  0.631 0.007
67 2.769  2.769 0.006
68 2.542  2.542 0.006
69 2.912  2.912 0.005
70 2.848  2.848 0.005
71 0.334  0.214 0.014
72 0.394  0.317 0.010
73 0.297  0.150 0.008
```

```
74 0.303  0.208 0.008
75 0.000 -0.003 0.008
76 0.000  0.010 0.009
77 0.000 -0.001 0.008
78 0.000  0.028 0.008
79 0.000 -0.011 0.006
80 0.000 -0.038 0.008
81 0.000 -0.032 0.007
82 0.000  0.046 0.007
83 1.000  1.000 0.000
84 0.000  0.000 0.000
85 0.000  0.405 0.009
86 0.000  0.007 0.002
87 0.000  0.030 0.003
88 0.000  0.007 0.002
89 0.000  0.013 0.002
```

BETWEEN WEAK ELS + HSLS

```
weak_comb <- '

################################################################################
##                             ELS (Group 1)                                  ##
################################################################################
group: ELS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1_1 * i1 +
             l2_1 * i2 +
             l3_1 * i3 +
             l4_1 * i4 +
             l5_1 * i5

  # Intercepts
  i1 ~ 0 * 1
  i2 ~ enu2_1 * 1
  i3 ~ enu3_1 * 1
  i4 ~ enu4_1 * 1
```

```
i5 ~ enu5_1 * 1

# Residual variances
i1 ~~ etheta1_1 * i1
i2 ~~ etheta2_1 * i2
i3 ~~ etheta3_1 * i3
i4 ~~ etheta4_1 * i4
i5 ~~ etheta5_1 * i5

# Free both
math_t1 ~~ var_els_t1 * math_t1
math_t1 ~ mean_els_t1 * 1


######################
# Time Point 2
######################
math_t2 =~ l1_2   * i1_2 +
           l2_2 * i2_2 +
           l3_2 * i3_2 +
           l4_2 * i4_2 +
           l5_2 * i5_2

# Intercepts
i1_2 ~ 0 * 1
i2_2 ~ enu2_2 * 1
i3_2 ~ enu3_2 * 1
i4_2 ~ enu4_2 * 1
i5_2 ~ enu5_2 * 1

# Residual variances
i1_2 ~~ etheta1_2 * i1_2
i2_2 ~~ etheta2_2 * i2_2
i3_2 ~~ etheta3_2 * i3_2
i4_2 ~~ etheta4_2 * i4_2
i5_2 ~~ etheta5_2 * i5_2

# Covariances among items
i1 ~~ i2
i1_2 ~~ i2_2

i1 ~~ i3
```

```
  i1_2 ~~ i3_2

  i2 ~~ i3
  i2_2 ~~ i3_2

  i4 ~~ i5
  i4_2 ~~ i5_2

  # Fix latent variance to 1 for identification, free mean
  math_t2 ~~ var_els_t2 * math_t2
  math_t2 ~ mean_els_t2 * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2


################################################################################
##                            HSLS (Group 2)                                  ##
################################################################################
group: HSLS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1_1 * i1 +
             l2_1 * i2 +   # Same label as ELS
             # no i3 in HSLS
             l4_1 * i4 +   # Same label as ELS
             l5_1 * i5     # Same label as ELS

  # Intercepts
  i1 ~ 0 * 1
  i2 ~ hnu2_1 * 1
  # i3 ~ hnu3_1 * 1 (item not in HSLS)
  i4 ~ hnu4_1 * 1
  i5 ~ hnu5_1 * 1
```

```
# Residual variances
i1 ~~ htheta1_1 * i1
i2 ~~ htheta2_1 * i2
# i3 ~~ htheta3_1 * i3 (item not in HSLS)
i4 ~~ htheta4_1 * i4
i5 ~~ htheta5_1 * i5

# Free latent variance and free latent mean
math_t1 ~~ var_hsls_t1 * math_t1
math_t1 ~ mean_hsls_t1 * 1


######################
# Time Point 2
######################
math_t2 =~ l1_2  * i1_2 +
           l2_2 * i2_2 +   # Same label as ELS
           # no i3_2 in HSLS at Time 2
           l4_2 * i4_2 +   # Same label as ELS
           l5_2 * i5_2     # Same label as ELS



# Intercepts
i1_2 ~ 0 * 1
i2_2 ~ hnu2_2 * 1
# i3_2 ~ hnu3_2 * 1 (item not in HSLS)
i4_2 ~ hnu4_2 * 1
i5_2 ~ hnu5_2 * 1

# Residual variances
i1_2 ~~ htheta1_2 * i1_2
i2_2 ~~ htheta2_2 * i2_2
# i3_2 ~~ htheta3_2 * i3_2 (item not in HSLS)
i4_2 ~~ htheta4_2 * i4_2
i5_2 ~~ htheta5_2 * i5_2

# Covariances among items
i1 ~~ i2
i1_2 ~~ i2_2

i1 ~~ i4
```

```
  i1_2 ~~ i4_2

  i2 ~~ i4
  i2_2 ~~ i4_2

  i1 ~~ i5
  i1_2 ~~ i5_2

  # Free latent variance and free latent mean
  math_t2 ~~ var_hsls_t2 * math_t2
  math_t2 ~ mean_hsls_math_t2 * 1  # free mean


  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  # i3 ~~ i3_2 (item not in HSLS)
  i4 ~~ i4_2
  i5 ~~ i5_2

'


fit_weak_comb  <- sem(weak_comb, data = dat, group = "sample",
                      estimator = "MLR", missing = "FIML", se = "robust.mlr")
fit_weak_comb
```

```
lavaan 0.6-19 ended normally after 131 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81
  Number of equality constraints                     6

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50
```

```
Model Test User Model:
                                                      Standard      Scaled
  Test Statistic                                       368.014     323.188
  Degrees of freedom                                        34          34
  P-value (Chi-square)                                   0.000       0.000
  Scaling correction factor                                          1.139
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                                228.778     228.778
    HSLS                                                94.411      94.411
```

```
head(modindices(fit_weak_comb, sort. = TRUE, free.remove = FALSE))
```

```
        lhs op  rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
103 math_t2 =~   i3     1   ELS     1 72.712 -0.102  -0.076   -0.079   -0.079
141 math_t1 =~ i5_2     2  HSLS     1 39.492 -0.049  -0.032   -0.041   -0.041
107      i1 ~~   i5     1   ELS     1 35.681 -0.019  -0.019   -0.075   -0.075
97  math_t1 =~ i2_2     1   ELS     1 30.437  0.048   0.038    0.043    0.043
115      i2 ~~ i3_2     1   ELS     1 29.842  0.021   0.021    0.079    0.079
139 math_t1 =~ i2_2     2  HSLS     1 28.588  0.048   0.031    0.035    0.035
```

```
s_weak_comb <- summary(fit_weak_comb, fit.measures = TRUE, standardized = TRUE)
fitMeasures(fit_weak_comb, c("rmsea", "chisq.scaled", "cfi", "tli", "df"))
```

```
     rmsea chisq.scaled          cfi          tli           df
     0.023      323.188        0.998        0.996       34.000
```

```
#s_weak_comb
#lavTestLRT(fit_config_comb, fit_weak_comb)
```

BETWEEN STRONG ELS + HSLS

```
strong_between <- '

################################################################################
##                            ELS (Group 1)                                   ##
################################################################################
```

```
group: ELS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1_1    * i1 +
             l2_1 * i2 +
             l3_1 * i3 +
             l4_1 * i4 +
             l5_1 * i5

  # Intercepts
  # Same labels for both groups!
  i1 ~ 0 * 1
  i2 ~ nu2_1 * 1
  i3 ~ nu3_1 * 1
  i4 ~ nu4_1 * 1
  i5 ~ nu5_1 * 1

  # Residual variances
  i1 ~~ etheta1_1 * i1
  i2 ~~ etheta2_1 * i2
  i3 ~~ etheta3_1 * i3
  i4 ~~ etheta4_1 * i4
  i5 ~~ etheta5_1 * i5

  # Free both
  math_t1 ~~ var_els_t1 * math_t1
  math_t1 ~ mean_els_t1 * 1


  ######################
  # Time Point 2
  ######################
  math_t2 =~ l1_2   * i1_2 +
             l2_2 * i2_2 +
             l3_2 * i3_2 +
             l4_2 * i4_2 +
             l5_2 * i5_2

  # Intercepts
```

```
  # Same labels for both groups!
  i1_2 ~ 0 * 1
  i2_2 ~ nu2_2 * 1
  i3_2 ~ nu3_2 * 1
  i4_2 ~ nu4_2 * 1
  i5_2 ~ nu5_2 * 1

  # Residual variances
  i1_2 ~~ etheta1_2 * i1_2
  i2_2 ~~ etheta2_2 * i2_2
  i3_2 ~~ etheta3_2 * i3_2
  i4_2 ~~ etheta4_2 * i4_2
  i5_2 ~~ etheta5_2 * i5_2

  # Covariances among items
  i1 ~~ i2
  i1_2 ~~ i2_2

  i1 ~~ i3
  i1_2 ~~ i3_2

  i2 ~~ i3
  i2_2 ~~ i3_2

  i4 ~~ i5
  i4_2 ~~ i5_2

  # Fix latent variance to 1 for identification, free mean
  math_t2 ~~ var_els_t2 * math_t2
  math_t2 ~ mean_els_t2 * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2


################################################################################
```

```
##                            HSLS (Group 2)                                  ##
################################################################################
group: HSLS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1_1   * i1 +
             l2_1 * i2 +   # Same label as ELS
             # no i3 in HSLS
             l4_1 * i4 +   # Same label as ELS
             l5_1 * i5     # Same label as ELS

  # Intercepts
  # Same labels for both groups!
  i1 ~ 0 * 1
  i2 ~ nu2_1 * 1
  # i3 ~ hnu3_1 * 1 (item not in HSLS)
  i4 ~ nu4_1 * 1
  i5 ~ nu5_1 * 1

  # Residual variances
  i1 ~~ htheta1_1 * i1
  i2 ~~ htheta2_1 * i2
  # i3 ~~ htheta3_1 * i3 (item not in HSLS)
  i4 ~~ htheta4_1 * i4
  i5 ~~ htheta5_1 * i5

 # Free both
  math_t1 ~~ var_hsls_t1 * math_t1
  math_t1 ~ mean_hsls_t1 * 1


  ######################
  # Time Point 2
  ######################
  math_t2 =~ l1_2   * i1_2 +
             l2_2 * i2_2 +   # Same label as ELS
             # no i3_2 in HSLS at Time 2
             l4_2 * i4_2 +   # Same label as ELS
             l5_2 * i5_2     # Same label as ELS
```

```
# Intercepts
# Same labels for both groups!

i1_2 ~ 0 * 1
i2_2 ~ nu2_2 * 1
# i3_2 ~ hnu3_2 * 1 (item not in HSLS)
i4_2 ~ nu4_2 * 1
i5_2 ~ nu5_2 * 1

# Residual variances
i1_2 ~~ htheta1_2 * i1_2
i2_2 ~~ htheta2_2 * i2_2
# i3_2 ~~ htheta3_2 * i3_2 (item not in HSLS)
i4_2 ~~ htheta4_2 * i4_2
i5_2 ~~ htheta5_2 * i5_2

# Covariances among items
i1 ~~ i2
i1_2 ~~ i2_2

i1 ~~ i4
i1_2 ~~ i4_2

i2 ~~ i4
i2_2 ~~ i4_2

i1 ~~ i5
i1_2 ~~ i5_2

# Free latent variance and free latent mean
math_t2 ~~ var_hsls_t2 * math_t2
math_t2 ~ mean_hsls_t2 * 1

# Correlations across time
math_t1 ~~ math_t2
i1 ~~ i1_2
i2 ~~ i2_2
# i3 ~~ i3_2 (item not in HSLS)
i4 ~~ i4_2
i5 ~~ i5_2
```

```
'

fit_strong_between  <- sem(strong_between, data = dat, group = "sample",
                      estimator = "MLR", missing = "FIML", se = "robust.mlr")
fit_strong_between
```

```
lavaan 0.6-19 ended normally after 112 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81
  Number of equality constraints                    12

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                               979.397     878.847
  Degrees of freedom                                40          40
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.114
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                          612.296     612.296
    HSLS                                         266.550     266.550
```

```
head(modindices(fit_strong_between, sort. = TRUE, free.remove = FALSE))
```

```
        lhs op  rhs block group level      mi    epc sepc.lv sepc.all sepc.nox
102 math_t1 =~ i1_2     1   ELS     1 176.902 -0.033  -0.027   -0.030   -0.030
111 math_t2 =~   i5     1   ELS     1 170.928  0.030   0.023    0.025    0.025
67     i1_2 ~1          2  HSLS     1 156.308  0.085   0.085    0.103    0.103
23     i1_2 ~1          1   ELS     1 156.308 -0.085  -0.085   -0.095   -0.095
110 math_t2 =~   i4     1   ELS     1 149.746 -0.028  -0.021   -0.022   -0.022
63  math_t2 =~ i1_2     2  HSLS     1 149.526  0.030   0.021    0.026    0.026
```

```r
s_strong_between <- summary(fit_strong_between, fit.measures = TRUE, standardized = TRUE)
#s_strong_between
fitMeasures(fit_strong_between, c("rmsea", "chisq.scaled", "cfi", "tli", "df"))
```

```
     rmsea chisq.scaled          cfi          tli           df
     0.036      878.847        0.995        0.990       40.000
```

```r
fit_weak_comb
```

```
lavaan 0.6-19 ended normally after 131 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81
  Number of equality constraints                     6

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                               368.014     323.188
  Degrees of freedom                                34          34
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.139
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                        228.778     228.778
    HSLS                                        94.411      94.411
```

```r
#lavTestLRT(fit_weak_comb, fit_strong_between)
```

WITHIN STRONG ELS+HSLS

```
strong_within <- '

##############################################################################
##                            ELS (Group 1)                                 ##
##############################################################################
group: ELS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ el1    * i1 +
             el2 * i2 +
             el3 * i3 +
             el4 * i4 +
             el5 * i5

  # Intercepts
  # Same labels for both times!
  i1 ~ 0 * 1
  i2 ~ nu2_2 * 1
  i3 ~ nu3_3 * 1
  i4 ~ nu4_4 * 1
  i5 ~ nu5_5 * 1

  # Residual variances
  i1 ~~ etheta1_1 * i1
  i2 ~~ etheta2_1 * i2
  i3 ~~ etheta3_1 * i3
  i4 ~~ etheta4_1 * i4
  i5 ~~ etheta5_1 * i5

  # Fix latent variance to 1 for identification, fix mean to 0
  math_t1 ~~ 1 * math_t1
  math_t1 ~ mean_els_t1 * 1


  ######################
  # Time Point 2
  ######################
```

```
math_t2 =~ el1 * i1_2 +
           el2 * i2_2 +
           el3 * i3_2 +
           el4 * i4_2 +
           el5 * i5_2

# Intercepts
# Same labels for both groups!
i1_2 ~ 0 * 1
i2_2 ~ nu2_2 * 1
i3_2 ~ nu3_2 * 1
i4_2 ~ nu4_2 * 1
i5_2 ~ nu5_2 * 1

# Residual variances
i1_2 ~~ etheta1_2 * i1_2
i2_2 ~~ etheta2_2 * i2_2
i3_2 ~~ etheta3_2 * i3_2
i4_2 ~~ etheta4_2 * i4_2
i5_2 ~~ etheta5_2 * i5_2

# Covariances among items
i1 ~~ i2
i1_2 ~~ i2_2

i1 ~~ i3
i1_2 ~~ i3_2

i2 ~~ i3
i2_2 ~~ i3_2

i4 ~~ i5
i4_2 ~~ i5_2

# Fix latent variance to 1 for identification, free mean
math_t2 ~~ var_els_t2 * math_t2
math_t2 ~ mean_els_t2 * 1

# Correlations across time
math_t1 ~~ math_t2
i1 ~~ i1_2
```

```
  i2 ~~ i2_2
  i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2


################################################################################
##                            HSLS (Group 2)                                  ##
################################################################################
group: HSLS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ hl1   * i1 +
             hl2 * i2 +   # Same label as ELS
             # no i3 in HSLS
             hl4 * i4 +   # Same label as ELS
             hl5 * i5     # Same label as ELS

  # Intercepts
  # Same labels for both times!
  i1 ~ 0 * 1
  i2 ~ hnu2_1 * 1
  # i3 ~ hnu3_1 * 1 (item not in HSLS)
  i4 ~ hnu4_1 * 1
  i5 ~ hnu5_1 * 1

  # Residual variances
  i1 ~~ htheta1_1 * i1
  i2 ~~ htheta2_1 * i2
  # i3 ~~ htheta3_1 * i3 (item not in HSLS)
  i4 ~~ htheta4_1 * i4
  i5 ~~ htheta5_1 * i5

 # Free latent variance and free latent mean
  math_t1 ~~ var_hsls * math_t1
  math_t1 ~ mean_hsls_t1 * 1


  ######################
  # Time Point 2
```

```
#####################
math_t2 =~ hl1   * i1_2 +
           hl2 * i2_2 +   # Same label as ELS
           # no i3_2 in HSLS at Time 2
           hl4 * i4_2 +   # Same label as ELS
           hl5 * i5_2     # Same label as ELS

# Intercepts
# Same labels for both times!

i1_2 ~ 0 * 1
i2_2 ~ hnu2_2 * 1
# i3_2 ~ hnu3_2 * 1 (item not in HSLS)
i4_2 ~ hnu4_2 * 1
i5_2 ~ hnu5_2 * 1

# Residual variances
i1_2 ~~ htheta1_2 * i1_2
i2_2 ~~ htheta2_2 * i2_2
# i3_2 ~~ htheta3_2 * i3_2 (item not in HSLS)
i4_2 ~~ htheta4_2 * i4_2
i5_2 ~~ htheta5_2 * i5_2

# Covariances among items
i1 ~~ i2
i1_2 ~~ i2_2

i1 ~~ i4
i1_2 ~~ i4_2

i2 ~~ i4
i2_2 ~~ i4_2

i1 ~~ i5
i1_2 ~~ i5_2

# Free latent variance and free latent mean
math_t2 ~~ var_hsls_t2 * math_t2
math_t2 ~ mean_hsls_t2 * 1

# Correlations across time
```

```
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  # i3 ~~ i3_2 (item not in HSLS)
  i4 ~~ i4_2
  i5 ~~ i5_2

'

fit_strong_within  <- cfa(strong_within, data = dat, group = "sample",
                    estimator = "MLR", missing = "FIML", se = "robust.mlr",
                    std.lv = TRUE)
fit_strong_within
```

```
lavaan 0.6-19 ended normally after 123 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81
  Number of equality constraints                    10

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                               571.360     515.496
  Degrees of freedom                                38          38
  P-value (Chi-square)                           0.000       0.000
  Scaling correction factor                                  1.108
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                        416.217     416.217
    HSLS                                        99.278      99.278
```

```
head(modindices(fit_strong_within, sort. = TRUE, free.remove = FALSE))
```

```
        lhs op     rhs block group level     mi    epc sepc.lv sepc.all
16  math_t1 ~~ math_t1     1   ELS     1 90.352  0.183   1.000    1.000
41  math_t2 ~~ math_t2     1   ELS     1 90.352 -0.183  -1.000   -1.000
103 math_t1 =~    i4_2     1   ELS     1 52.783 -0.053  -0.053   -0.058
111      i1 ~~      i5     1   ELS     1 49.761 -0.022  -0.022   -0.088
102 math_t1 =~    i3_2     1   ELS     1 38.385 -0.045  -0.045   -0.047
61  math_t1 ~~ math_t1     2  HSLS     1 38.010 -0.097  -1.000   -1.000
    sepc.nox
16     1.000
41    -1.000
103   -0.058
111   -0.088
102   -0.047
61    -1.000
```

```r
s_strong_within <- summary(fit_strong_within, fit.measures = TRUE, standardized = TRUE)
#s_strong_within
fitMeasures(fit_strong_within, c("rmsea", "chisq.scaled", "cfi", "tli", "df"))
```

```
     rmsea chisq.scaled          cfi          tli           df
     0.028      515.496        0.997        0.994       38.000
```

WITHIN+BETWEEN WEAK

```r
# Latent variance and mean set to 1 and 0 in ELS time 1,
# free variance, mean set to 0 everywhere else

both_weak_comb <- '

################################################################################
##                            ELS (Group 1)                                   ##
################################################################################
group: ELS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1 * i1 +
             l2 * i2 +
             l3 * i3 +
```

```
            l4 * i4 +
            l5 * i5

# Intercepts
i1 ~ 0 * 1
i2 ~ enu2_1 * 1
i3 ~ enu3_1 * 1
i4 ~ enu4_1 * 1
i5 ~ enu5_1 * 1

# Residual variances
i1 ~~ etheta1_1 * i1
i2 ~~ etheta2_1 * i2
i3 ~~ etheta3_1 * i3
i4 ~~ etheta4_1 * i4
i5 ~~ etheta5_1 * i5

# Free both
math_t1 ~~ var_els_t1 * math_t1
math_t1 ~ mean_els_t1 * 1


######################
# Time Point 2
######################
math_t2 =~ l1 * i1_2 +
           l2 * i2_2 +
           l3 * i3_2 +
           l4 * i4_2 +
           l5 * i5_2

# Intercepts
i1_2 ~ 0 * 1
i2_2 ~ enu2_2 * 1
i3_2 ~ enu3_2 * 1
i4_2 ~ enu4_2 * 1
i5_2 ~ enu5_2 * 1

# Residual variances
i1_2 ~~ etheta1_2 * i1_2
i2_2 ~~ etheta2_2 * i2_2
```

```
  i3_2 ~~ etheta3_2 * i3_2
  i4_2 ~~ etheta4_2 * i4_2
  i5_2 ~~ etheta5_2 * i5_2

  # Covariances among items

  i1 ~~ i5
  i1_2 ~~ i5_2

  i1 ~~ i3
  i1_2 ~~ i3_2

  i1 ~~ i4
  i1_2 ~~ i4_2

  i2 ~~ i3
  i2_2 ~~ i3_2

  i4 ~~ i5
  i4_2 ~~ i5_2


  # Free both
  math_t2 ~~ var_els_t2 * math_t2
  math_t2 ~ mean_els_t2 * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2


################################################################################
##                            HSLS (Group 2)                                  ##
################################################################################
group: HSLS

  ######################
  # Time Point 1
```

```
#####################
math_t1 =~ l1 * i1 +
           l2 * i2 +   # Same label as ELS
           # no i3 in HSLS
           l4 * i4 +   # Same label as ELS
           l5 * i5     # Same label as ELS

# Intercepts
i1 ~ 0 * 1
i2 ~ hnu2_1 * 1
# i3 ~ hnu3_1 * 1 (item not in HSLS)
i4 ~ hnu4_1 * 1
i5 ~ hnu5_1 * 1

# Residual variances
i1 ~~ htheta1_1 * i1
i2 ~~ htheta2_1 * i2
# i3 ~~ htheta3_1 * i3 (item not in HSLS)
i4 ~~ htheta4_1 * i4
i5 ~~ htheta5_1 * i5

# Free both
math_t1 ~~ var_hsls_t1 * math_t1
math_t1 ~ mean_hsls_t1 * 1


#####################
# Time Point 2
#####################
math_t2 =~ l1 * i1_2 +
           l2 * i2_2 +   # Same label as ELS
           # no i3_2 in HSLS at Time 2
           l4 * i4_2 +   # Same label as ELS
           l5 * i5_2     # Same label as ELS

# Intercepts
i1_2 ~ 0 * 1
i2_2 ~ hnu2_2 * 1
# i3_2 ~ hnu3_2 * 1 (item not in HSLS)
i4_2 ~ hnu4_2 * 1
i5_2 ~ hnu5_2 * 1
```

```
  # Residual variances
  i1_2 ~~ htheta1_2 * i1_2
  i2_2 ~~ htheta2_2 * i2_2
  # i3_2 ~~ htheta3_2 * i3_2 (item not in HSLS)
  i4_2 ~~ htheta4_2 * i4_2
  i5_2 ~~ htheta5_2 * i5_2

  # Covariances among items


  i1 ~~ i5
  i1_2 ~~ i5_2

  # i1 ~~ i3
  # i1_2 ~~ i3_2

  i1 ~~ i4
  i1_2 ~~ i4_2

  i4 ~~ i5
  i4_2 ~~ i5_2

  # Free latent variance and free latent mean
  math_t2 ~~ var_hsls_t2 * math_t2
  math_t2 ~ mean_hsls_t2 * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  # i3 ~~ i3_2 (item not in HSLS)
  i4 ~~ i4_2
  i5 ~~ i5_2

'

fit_both_weak_comb  <- sem(both_weak_comb, data = dat, group = "sample",
                     estimator = "MLR", missing = "FIML", se = "robust.mlr")
fit_both_weak_comb
```

```
lavaan 0.6-19 ended normally after 112 iterations
```

```
  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81
  Number of equality constraints                    10

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50

Model Test User Model:
                                                  Standard      Scaled
  Test Statistic                                   402.536     368.810
  Degrees of freedom                                    38          38
  P-value (Chi-square)                               0.000       0.000
  Scaling correction factor                                      1.091
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                            295.386     295.386
    HSLS                                            73.424      73.424
```

```
fitMeasures(fit_both_weak_comb, c("rmsea", "chisq.scaled", "cfi", "tli", "df", "aic", "bic
```

```
      rmsea chisq.scaled          cfi          tli           df          aic
      0.023      368.810        0.998        0.996       38.000   475008.011
        bic
  475612.385
```

```
head(modindices(fit_both_weak_comb, sort. = TRUE, free.remove = FALSE))
```

```
       lhs op  rhs block group level     mi    epc sepc.lv sepc.all sepc.nox
108 math_t2 =~   i4     1   ELS     1 50.106  0.059   0.043    0.047    0.047
104 math_t1 =~ i5_2     1   ELS     1 40.568  0.050   0.042    0.046    0.046
101 math_t1 =~ i2_2     1   ELS     1 35.772  0.046   0.038    0.044    0.044
112      i1 ~~ i3_2     1   ELS     1 34.023 -0.021  -0.021   -0.103   -0.103
102 math_t1 =~ i3_2     1   ELS     1 33.877 -0.051  -0.042   -0.045   -0.045
121      i3 ~~   i4     1   ELS     1 24.866  0.017   0.017    0.088    0.088
```

```r
s_both_weak_comb <- summary(fit_both_weak_comb, fit.measures = TRUE, standardized = TRUE)
#s_both_weak_comb
```

WITHIN+BETWEEN STRONG

```r
both_strong_comb <- '

################################################################################
##                              ELS (Group 1)                                 ##
################################################################################
group: ELS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1 * i1 +
             l2 * i2 +
             l3 * i3 +
             l4 * i4 +
             l5 * i5

  ############
  # Equal Intercepts #
  ############
   i1 ~ 0 * 1
   i2 ~ nu2 * 1
   i3 ~ nu3 * 1
   i4 ~ nu4 * 1
   i5 ~ nu5 * 1

  # Residual variances
  i1 ~~ etheta1_1 * i1
  i2 ~~ etheta2_1 * i2
  i3 ~~ etheta3_1 * i3
  i4 ~~ etheta4_1 * i4
  i5 ~~ etheta5_1 * i5

  # Free latent mean and variance
  math_t1 ~~ var_els_t1 * math_t1
```

```
math_t1 ~ mean_els_t1 * 1


#####################
# Time Point 2
#####################
math_t2 =~ l1 * i1_2 +
           l2 * i2_2 +
           l3 * i3_2 +
           l4 * i4_2 +
           l5 * i5_2

###########
# Equal Intercepts #
###########
# Intercepts
i1_2 ~ 0 * 1
i2_2 ~ nu2 * 1
i3_2 ~ nu3 * 1
i4_2 ~ nu4 * 1
i5_2 ~ nu5 * 1

# Residual variances
i1_2 ~~ etheta1_2 * i1_2
i2_2 ~~ etheta2_2 * i2_2
i3_2 ~~ etheta3_2 * i3_2
i4_2 ~~ etheta4_2 * i4_2
i5_2 ~~ etheta5_2 * i5_2

# Covariances among items

i1 ~~ i5
i1_2 ~~ i5_2

i1 ~~ i3
i1_2 ~~ i3_2

i1 ~~ i4
i1_2 ~~ i4_2

i2 ~~ i3
```

```
  i2_2 ~~ i3_2

  i4 ~~ i5
  i4_2 ~~ i5_2


  # Free latent variance, free mean
  math_t2 ~~ var_els_t2 * math_t2
  math_t2 ~ mean_els_t2 * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2


################################################################################
##                            HSLS (Group 2)                                  ##
################################################################################
group: HSLS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1 * i1 +
             l2 * i2 +   # Same label as ELS
             # no i3 in HSLS
             l4 * i4 +   # Same label as ELS
             l5 * i5     # Same label as ELS

  ############
  # Equal Intercepts #
  ############

   i1 ~ 0 * 1
   i2 ~ nu2 * 1
  #i3 ~ nu3 * 1 (item not in HSLS)
   i4 ~ nu4 * 1
   i5 ~ nu5 * 1
```

```
# Residual variances
i1 ~~ htheta1_1 * i1
i2 ~~ htheta2_1 * i2
# i3 ~~ htheta3_1 * i3 (item not in HSLS)
i4 ~~ htheta4_1 * i4
i5 ~~ htheta5_1 * i5

# Free latent variance and mean
math_t1 ~~ var_hsls_t1 * math_t1
math_t1 ~ mean_hsls_t1 * 1


######################
# Time Point 2
######################
math_t2 =~ l1 * i1_2 +
           l2 * i2_2 +   # Same label as ELS
           # no i3_2 in HSLS at Time 2
           l4 * i4_2 +   # Same label as ELS
           l5 * i5_2     # Same label as ELS

############
# Equal Intercepts #
############

 i1_2 ~ 0 * 1
 i2_2 ~ nu2 * 1
#i3_2 ~ nu3 * 1 (item not in HSLS)
 i4_2 ~ nu4 * 1
 i5_2 ~ nu5 * 1

# Residual variances
i1_2 ~~ htheta1_2 * i1_2
i2_2 ~~ htheta2_2 * i2_2
# i3_2 ~~ htheta3_2 * i3_2 (item not in HSLS)
i4_2 ~~ htheta4_2 * i4_2
i5_2 ~~ htheta5_2 * i5_2

# Covariances among items
```

```
  i1 ~~ i5
  i1_2 ~~ i5_2

  # i1 ~~ i3
  # i1_2 ~~ i3_2

  i1 ~~ i4
  i1_2 ~~ i4_2

  i4 ~~ i5
  i4_2 ~~ i5_2

  # Free latent variance
  math_t2 ~~ var_hsls_t2 * math_t2
  math_t2 ~ mean_hsls_t2 * 1 #free mean

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  # i3 ~~ i3_2 (item not in HSLS)
  i4 ~~ i4_2
  i5 ~~ i5_2

'

fit_both_strong_comb  <-
  sem(both_strong_comb, data = dat, group = "sample",
                    estimator = "MLR", missing = "FIML", se = "robust.mlr")

fit_both_strong_comb
```

```
lavaan 0.6-19 ended normally after 97 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81
  Number of equality constraints                    20

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
```

```
  Number of missing patterns per group:
    ELS                                            89
    HSLS                                           50

Model Test User Model:
                                              Standard      Scaled
  Test Statistic                              1542.603    1436.560
  Degrees of freedom                            48          48
  P-value (Chi-square)                          0.000       0.000
  Scaling correction factor                                 1.074
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                       1191.896    1191.896
    HSLS                                       244.665     244.665
```

```
fitMeasures(fit_both_strong_comb, c("rmsea", "chisq.scaled", "cfi", "tli", "df", "aic", "b
```

```
      rmsea chisq.scaled          cfi          tli           df          aic
      0.041     1436.560        0.992        0.987       48.000   476128.078
        bic
  476647.328
```

```
head(modindices(fit_both_strong_comb, sort. = TRUE, free.remove = FALSE))
```

```
        lhs op  rhs block group level      mi    epc sepc.lv sepc.all sepc.nox
9        i4 ~1          1   ELS     1 378.675 -0.095  -0.095   -0.102   -0.102
118 math_t2 =~   i4     1   ELS     1 375.201 -0.039  -0.029   -0.031   -0.031
4   math_t1 =~   i4     1   ELS     1 306.319 -0.032  -0.027   -0.029   -0.029
26     i4_2 ~1          1   ELS     1 299.280  0.097   0.097    0.109    0.109
113 math_t1 =~ i4_2     1   ELS     1 297.412  0.039   0.032    0.037    0.037
21  math_t2 =~ i4_2     1   ELS     1 239.919  0.032   0.024    0.027    0.027
```

```
s_both_strong_comb <- summary(fit_both_strong_comb, fit.measures = TRUE, standardized = TR
#s_both_strong_comb
```

W/B Partial Strong

```
both_strong_partial_ei4 <- '

################################################################################
##                            ELS (Group 1)                                   ##
################################################################################
group: ELS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1 * i1 +
             l2 * i2 +
             l3 * i3 +
             l4 * i4 +
             l5 * i5

  ############
  # Equal Intercepts #
  ############
   i1 ~ 0 * 1
   i2 ~ nu2 * 1
   i3 ~ nu3 * 1

   # Unique i4 intercept
   i4 ~ nu4_1 * 1

   i5 ~ nu5 * 1

  # Residual variances
  i1 ~~ etheta1_1 * i1
  i2 ~~ etheta2_1 * i2
  i3 ~~ etheta3_1 * i3
  i4 ~~ etheta4_1 * i4
  i5 ~~ etheta5_1 * i5

  # Free latent mean and variance
  math_t1 ~~ var_els_t1 * math_t1
  math_t1 ~ mean_els_t1 * 1
```

```
######################
# Time Point 2
######################
math_t2 =~ l1 * i1_2 +
           l2 * i2_2 +
           l3 * i3_2 +
           l4 * i4_2 +
           l5 * i5_2

############
# Equal Intercepts #
############
# Intercepts
i1_2 ~ 0 * 1
i2_2 ~ nu2 * 1
i3_2 ~ nu3 * 1
i4_2 ~ nu4 * 1
i5_2 ~ nu5 * 1

# Residual variances
i1_2 ~~ etheta1_2 * i1_2
i2_2 ~~ etheta2_2 * i2_2
i3_2 ~~ etheta3_2 * i3_2
i4_2 ~~ etheta4_2 * i4_2
i5_2 ~~ etheta5_2 * i5_2

# Covariances among items

i1 ~~ i5
i1_2 ~~ i5_2

i1 ~~ i3
i1_2 ~~ i3_2

i1 ~~ i4
i1_2 ~~ i4_2

i2 ~~ i3
i2_2 ~~ i3_2

i4 ~~ i5
```

```
  i4_2 ~~ i5_2


  # Free latent variance, free mean
  math_t2 ~~ var_els_t2 * math_t2
  math_t2 ~ mean_els_t2 * 1

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  i3 ~~ i3_2
  i4 ~~ i4_2
  i5 ~~ i5_2


################################################################################
##                            HSLS (Group 2)                                  ##
################################################################################
group: HSLS

  ######################
  # Time Point 1
  ######################
  math_t1 =~ l1 * i1 +
             l2 * i2 +   # Same label as ELS
             # no i3 in HSLS
             l4 * i4 +   # Same label as ELS
             l5 * i5     # Same label as ELS

  ############
  # Equal Intercepts #
  ############
   i1 ~ 0 * 1
   i2 ~ nu2 * 1
  #i3 ~ nu3 * 1 (item not in HSLS)
   i4 ~ nu4 * 1
   i5 ~ nu5 * 1

  # Residual variances
  i1 ~~ htheta1_1 * i1
```

```
i2 ~~ htheta2_1 * i2
# i3 ~~ htheta3_1 * i3 (item not in HSLS)
i4 ~~ htheta4_1 * i4
i5 ~~ htheta5_1 * i5

# Free latent variance and mean
math_t1 ~~ var_hsls_t1 * math_t1
math_t1 ~ mean_hsls_t1 * 1

######################
# Time Point 2
######################
math_t2 =~ l1 * i1_2 +
           l2 * i2_2 +   # Same label as ELS
           # no i3_2 in HSLS at Time 2
           l4 * i4_2 +   # Same label as ELS
           l5 * i5_2     # Same label as ELS

############
# Equal Intercepts #
############

 i1_2 ~ 0 * 1
 i2_2 ~ nu2 * 1
#i3_2 ~ nu3 * 1 (item not in HSLS)
 i4_2 ~ nu4 * 1
 i5_2 ~ nu5 * 1

# Residual variances
i1_2 ~~ htheta1_2 * i1_2
i2_2 ~~ htheta2_2 * i2_2
# i3_2 ~~ htheta3_2 * i3_2 (item not in HSLS)
i4_2 ~~ htheta4_2 * i4_2
i5_2 ~~ htheta5_2 * i5_2

# Covariances among items


i1 ~~ i5
i1_2 ~~ i5_2

# i1 ~~ i3
```

```
  # i1_2 ~~ i3_2

  i1 ~~ i4
  i1_2 ~~ i4_2

  i4 ~~ i5
  i4_2 ~~ i5_2

  # Free latent variance
  math_t2 ~~ var_hsls_t2 * math_t2
  math_t2 ~ mean_hsls_t2 * 1 #free mean

  # Correlations across time
  math_t1 ~~ math_t2
  i1 ~~ i1_2
  i2 ~~ i2_2
  # i3 ~~ i3_2 (item not in HSLS)
  i4 ~~ i4_2
  i5 ~~ i5_2

'

fit_both_strong_partial_ei4  <- sem(both_strong_partial_ei4, data = dat, group = "sample",
                     estimator = "MLR", missing = "FIML", se = "robust.mlr")
fit_both_strong_partial_ei4
```

```
lavaan 0.6-19 ended normally after 99 iterations

  Estimator                                         ML
  Optimization method                           NLMINB
  Number of model parameters                        81
  Number of equality constraints                    19

  Number of observations per group:               Used       Total
    ELS                                          13926       16197
    HSLS                                         22839       23503
  Number of missing patterns per group:
    ELS                                             89
    HSLS                                            50

Model Test User Model:
                                                Standard      Scaled
```

```
  Test Statistic                                1049.413     977.026
  Degrees of freedom                              47           47
  P-value (Chi-square)                          0.000        0.000
  Scaling correction factor                                  1.074
    Yuan-Bentler correction (Mplus variant)
  Test statistic for each group:
    ELS                                         731.286     731.286
    HSLS                                        245.739     245.739
```

```
fitMeasures(fit_both_strong_partial_ei4, c("rmsea", "chisq.scaled", "cfi", "tli", "df", "a
```

```
     rmsea chisq.scaled          cfi          tli           df          aic
     0.034      977.026        0.994        0.991       47.000   475636.889
       bic
476164.652
```

```
head(modindices(fit_both_strong_partial_ei4, sort. = TRUE, free.remove = FALSE))
```

```
       lhs op  rhs block group level      mi    epc sepc.lv sepc.all sepc.nox
55      i1 ~1          2  HSLS     1 203.047  0.063   0.063    0.083    0.083
153 math_t2 =~   i1     2  HSLS     1 181.936  0.021   0.015    0.019    0.019
51  math_t1 =~   i1     2  HSLS     1 180.790  0.020   0.013    0.018    0.018
112 math_t1 =~ i4_2     1   ELS     1 175.604  0.030   0.025    0.029    0.029
109 math_t1 =~ i1_2     1   ELS     1 170.958 -0.029  -0.025   -0.027   -0.027
26     i4_2 ~1          1   ELS     1 169.082  0.072   0.072    0.082    0.082
```

```
s_both_strong_comb <- summary(fit_both_strong_partial_ei4, fit.measures = TRUE, standardiz
```

HSLS ONLY

```
# hsls_time <-  '
#   math_T1 =~ NA  * i1 +
#              l2_1 * i2 +
#              l4_1 * i4 +
#              l5_1 * i5
#
#
```

```
#   # Fixing latent variance to 1, as we freed first factor loading
#   math_T1 ~~ 1 * math_T1
#
#   # Fixing latent mean to 0 for identification?
#   math_T1 ~ 0 * 1
#
#   # Time Point 2
#   math_T2 =~ NA  * i1_2 +
#              l2_2 * i2_2 +
#              l4_2 * i4_2 +
#              l5_2 * i5_2
#
#
#
#   # Adding the covariances
#   i1_2 ~~ i2_2
#   i2_2 ~~ i4_2
#
#   i1 ~~ i5
#   i1_2 ~~ i5_2
#
#   # Fixing latent variance to 1, as we freed first factor loading
#   math_T2 ~~ 1 * math_T2
#
#   # Fixing latent mean to 0 for identification?
#   math_T2 ~ 0 * 1
#
#   # Correlations across time
#   math_T1 ~~ math_T2
#   i1 ~~ i1_2
#   i2 ~~ i2_2
#   i4 ~~ i4_2
#   i5 ~~ i5_2
# '
#
# fit_hsls_time  <- cfa(hsls_time, data = hsls,
#                       estimator = "MLR", missing = "FIML", se = "robust.mlr")
# fit_hsls_time
# head(modindices(fit_hsls_time, sort. = TRUE, free.remove = FALSE))
#
#
```

```
# s_hsls_time <- summary(fit_hsls_time, fit.measures = TRUE)
#
#
```

ELS ONLY

```
# els_time <-  '
#   math_T1 =~ l1_1 * NA  * i1 +
#              l2_1 * i2 +
#              l3_1 * i3 +
#              l4_1 * i4 +
#              l5_1 * i5
#
#
#   # Fixing latent variance to 1, as we freed first factor loading
#   math_T1 ~~ NA * math_T1
#
#   # Fixing latent mean to 0 for identification?
#   math_T1 ~ 0 * 1
#
#   # Time Point 2
#   math_T2 =~ NA  * i1_2 +
#              l2_2 * i2_2 +
#              l3_2 * i3_2 +
#              l4_2 * i4_2 +
#              l5_2 * i5_2
#
#   # Adding the covariances
#   i1 ~~ i2
#   i1_2 ~~ i2_2
#
#   i2 ~~ i3
#   i2_2 ~~ i3_2
#
#   i4 ~~ i5
#   i4_2 ~~ i5_2
#
#
#
#
```

```
#   # Fixing latent variance to 1, as we freed first factor loading
#   math_T2 ~~ NA * math_T2
#
#   # Fixing latent mean to 0 for identification?
#   math_T2 ~ 0 * 1
#
#   # Correlations across time
#   math_T1 ~~ math_T2
#   i1 ~~ i1_2
#   i2 ~~ i2_2
#   i3 ~~ i3_2
#   i4 ~~ i4_2
#   i5 ~~ i5_2
# '
#
# fit_els_time  <- cfa(els_time, data = els,
#                      estimator = "MLR", missing = "FIML", se = "robust.mlr")
# fit_els_time
# head(modindices(fit_els_time, sort. = TRUE, free.remove = FALSE))
#
# s_els_time <- summary(fit_els_time, fit.measures = TRUE, standardized = TRUE)
```

K-fold validation

```
library(caret)
```

```
Loading required package: ggplot2
```

```

Attaching package: 'ggplot2'
```

```
The following object is masked from 'package:sjlabelled':

    as_label
```

```
k <- 10  # Number of folds
set.seed(42)
folds <- createFolds(dat$sample, k = k, list = TRUE)
```

```r
cfi_list <- numeric(k)
rmsea_list <- numeric(k)
srmr_list <- numeric(k)
chisq_list <- numeric(k)

for (i in seq_along(folds)) {
  # Split into training and validation
  validation_indices <- folds[[i]]
  train_data <- dat[-validation_indices, ]
  validation_data <- dat[validation_indices, ]

  # Fit model on training set
  fit_train <- fit_train <- sem(both_strong_partial_ei4, data = train_data,
                                group = "sample",
                                estimator = "MLR",
                                missing = "FIML",
                                se = "robust.mlr")

  # Fit model on validation set using training constraints
  fit_validation <- fit_validation <- sem(both_strong_partial_ei4,
                                          data = validation_data,
                                          group = "sample",
                                          estimator = "MLR",
                                          missing = "FIML", se = "robust.mlr")

  # Extract fit measures for the validation set
  cfi_list[i] <- fitMeasures(fit_validation, "cfi")
  rmsea_list[i] <- fitMeasures(fit_validation, "rmsea")
  srmr_list[i] <- fitMeasures(fit_validation, "srmr")
  chisq_list[i] <- fitMeasures(fit_validation, "chisq.scaled")
}

fit_measures_summary <- data.frame(
  Fold = seq_len(k),
  CFI = cfi_list,
  RMSEA = rmsea_list,
  SRMR = srmr_list,
  chisquare = chisq_list
)

fit_measures_summary <- rbind(
```

```
  fit_measures_summary,
  data.frame(
    Fold = "Mean",
    CFI = mean(cfi_list),
    RMSEA = mean(rmsea_list),
    SRMR = mean(srmr_list),
    chisquare = mean(chisq_list)
  ),
  data.frame(
    Fold = "SD",
    CFI = sd(cfi_list),
    RMSEA = sd(rmsea_list),
    SRMR = sd(srmr_list),
    chisquare = sd(chisq_list)
  )
)

# Display fit measures summary
print(fit_measures_summary)
```

```
   Fold          CFI       RMSEA        SRMR chisquare
1     1 0.9934423414 0.036906512 0.023831975 154.28390
2     2 0.9940840973 0.035169185 0.022386103 142.96924
3     3 0.9947781295 0.032514042 0.022549146 126.08007
4     4 0.9927581038 0.038744826 0.023552413 161.83597
5     5 0.9947909686 0.033259938 0.021451555 135.07740
6     6 0.9958110729 0.029014786 0.019990899 113.89821
7     7 0.9949440057 0.032159835 0.021995136 130.04654
8     8 0.9930883801 0.037331820 0.024971764 158.72218
9     9 0.9941189973 0.034335904 0.023643486 139.13529
10   10 0.9943338072 0.034375132 0.022724958 141.25962
11 Mean 0.9942149904 0.034381198 0.022709744 140.33084
12   SD 0.0009283821 0.002852367 0.001397406  15.05467
```

```
k <- 10  # Number of folds
set.seed(42)
folds_2 <- createFolds(dat$sample, k = k, list = TRUE)

cfi_list_2 <- numeric(k)
rmsea_list_2 <- numeric(k)
srmr_list_2 <- numeric(k)
```

```r
chisq_list_2 <- numeric(k)

for (i in seq_along(folds_2)) {
  # Split into training and validation
  validation_indices_2 <- folds_2[[i]]
  train_data_2 <- dat[-validation_indices_2, ]
  validation_data_2 <- dat[validation_indices_2
                           , ]

  # Fit model on training set
  fit_train_2 <- sem(both_strong_comb, data = train_data,
                                group = "sample",
                                estimator = "MLR",
                                missing = "FIML",
                                se = "robust.mlr")

  # Fit model on validation set using training constraints
  fit_validation_2 <- sem(both_strong_comb,
                                           data = validation_data_2,
                                           group = "sample",
                                           estimator = "MLR",
                                           missing = "FIML", se = "robust.mlr")

  # Extract fit measures for the validation set
  cfi_list_2[i] <- fitMeasures(fit_validation_2, "cfi")
  rmsea_list_2[i] <- fitMeasures(fit_validation_2, "rmsea")
  srmr_list_2[i] <- fitMeasures(fit_validation_2, "srmr")
  chisq_list_2[i] <- fitMeasures(fit_validation_2, 'chisq.scaled')
}

fit_measures_summary_2 <- data.frame(
  Fold = seq_len(k),
  CFI = cfi_list_2,
  RMSEA = rmsea_list_2,
  SRMR = srmr_list_2,
  chisquare =  chisq_list_2
)

# Calculate mean and standard deviation of fit measures across folds
fit_measures_summary_2 <- rbind(
  fit_measures_summary_2,
```

```
  data.frame(
    Fold = "Mean",
    CFI = mean(cfi_list_2),
    RMSEA = mean(rmsea_list_2),
    SRMR = mean(srmr_list_2),
    chisquare = mean(chisq_list_2)
  ),
  data.frame(
    Fold = "SD",
    CFI = sd(cfi_list_2),
    RMSEA = sd(rmsea_list_2),
    SRMR = sd(srmr_list_2),
    chisquare = sd(chisq_list_2)
  )
)

# Display fit measures summary
print(fit_measures_summary_2)
```

```
   Fold         CFI       RMSEA        SRMR chisquare
1     1 0.990991231 0.042804521 0.024319173 196.07829
2     2 0.991997652 0.040475182 0.023497891 179.29671
3     3 0.992362988 0.038908811 0.023790774 165.49312
4     4 0.989604868 0.045933643 0.025101776 214.26489
5     5 0.991708560 0.041522764 0.023012711 189.75043
6     6 0.993863180 0.034751052 0.021054695 147.09210
7     7 0.991884270 0.040318338 0.022852693 182.78645
8     8 0.990979591 0.042201770 0.026034085 194.37595
9     9 0.990069483 0.044150645 0.025314445 205.96086
10   10 0.991194644 0.042403381 0.024350190 195.62530
11 Mean 0.991465647 0.041347011 0.023932843 187.07241
12   SD 0.001202646 0.003058123 0.001434509  19.58996
```