mediation_graphs

```
# Load required packages
library(MASS)
library(lavaan)
```

```
This is lavaan 0.6-19
lavaan is FREE software! Please report any bugs.
```

```
library(glue)
library(dplyr)
```

```

Attaching package: 'dplyr'

The following object is masked from 'package:MASS':

    select

The following objects are masked from 'package:stats':

    filter, lag

The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

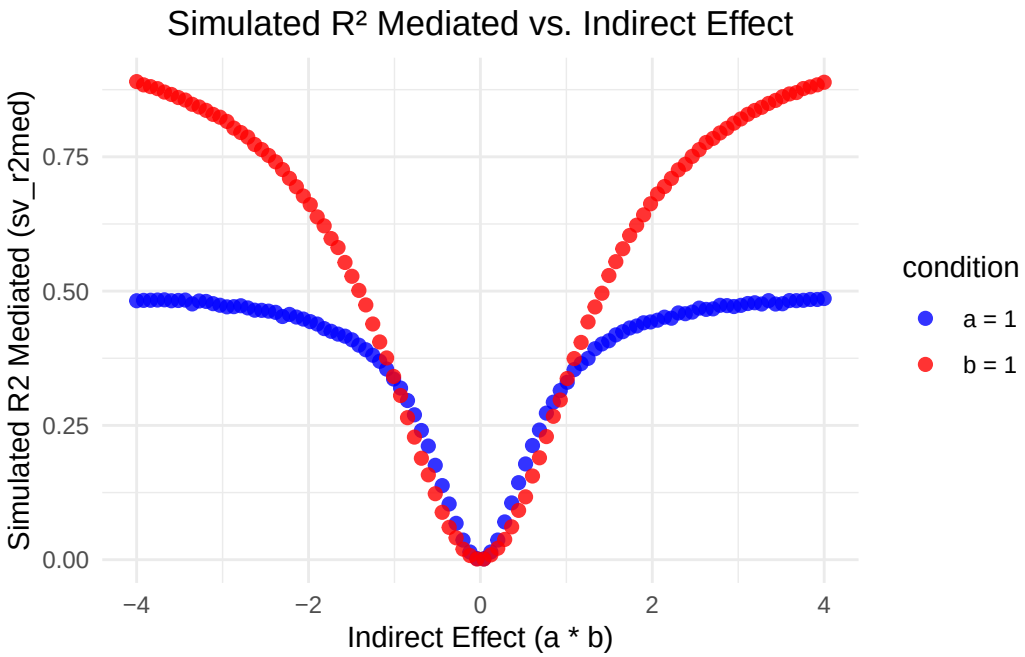
```
library(ggplot2)
rsquare_med <- function(data, x, m, y) {
```

```r
# Compute correlations among the variables
rxm <- cor(data[x], data[[m]])
rxy <- cor(data[[x]], data[[y]])
rmy <- cor(data[[m]], data[[y]])

# Regression: m ~ x (to get alpha, first indirect path)
# Equation 2 in Fairchild, et al
model1 <- lm(as.formula(paste(m, "~", x)), data = data)
alpha <- coef(model1)[[x]]

# Regression: y ~ x + m (to get 'tau_prime' and 'beta')
# Equation 1 in Fairchild, et al
model2 <- lm(as.formula(paste(y, "~", x, "+", m)), data = data)

tau_prime <- coef(model2)[[x]]
beta <- coef(model2)[[m]]

# Compute total effect of x on y: tau = tau_prime + (alpha*beta)
total <- tau_prime + (alpha*beta)


# Compute effect-size measures
mediatedeffect <- alpha * beta       # Indirect effect of x on y via M = alpha*beta
rxmsquared <- rxm^2                  # squared correlation between x and m
partialrxy_msquared <- ((rxy - rmy * rxm) / sqrt((1 - rmy^2) * (1 - rxmsquared)))^2
partialrmy_xsquared <- ((rmy - rxy * rxm) / sqrt((1 - rxy^2) * (1 - rxmsquared)))^2
overallrsquared <- (((rxy^2) + (rmy^2)) - (2 * rxy * rmy * rxm)) / (1 - rxmsquared)
rsquaredmediated <- (rmy^2) - (overallrsquared - (rxy^2))
proportionmediated <- if (total != 0) mediatedeffect / total else NA

# Create a list of results
results <- list(
  alpha = alpha,
  beta = beta,
  tau_prime = tau_prime,
  total = total,
  mediatedeffect = mediatedeffect,
  rxm = rxm,
  rxmsquared = rxmsquared,
  rxy = rxy,
  rmy = rmy,
```

```
    partialrxy_msquared = partialrxy_msquared,
    partialrmy_xsquared = partialrmy_xsquared,
    overallrsquared = overallrsquared,
    rsquaredmediated = rsquaredmediated,
    proportionmediated = proportionmediated
  )

  return(results)
}

# Define a simulation function that takes an indirect effect value and a condition.
simulate_indirect_condition <- function(indirect, condition, sample_size = 1000, num_reps

  # Set parameters and assign a condition label based on the input.
  if (condition == "blue") {
    # Blue condition: alpha = 1, beta = indirect --> Label: "a = 1"
    pop_alpha  <- 1
    pop_beta   <- indirect
    cond_label <- "a = 1"
  } else if (condition == "red") {
    # Red condition: alpha = indirect, beta = 1 --> Label: "b = 1"
    pop_alpha  <- indirect
    pop_beta   <- 1
    cond_label <- "b = 1"
  } else {
    stop("Unknown condition. Choose 'blue' or 'red'.")
  }

  pop_tau_prime <- 0  # No direct effect

  # Create a "fake" dataset (used only to define the lavaan model)
  d_fake <- data.frame(
    x = rnorm(sample_size),
    m = rnorm(sample_size),
    y = rnorm(sample_size)
  )

  # Build the lavaan model string
  model_string <- glue("
    # Equation for y: note the direct effect of x is set to 0
    y ~ {pop_tau_prime}*x + {pop_beta}*m
```

```
  # Equation for m
  m ~ {pop_alpha}*x
  # Fix variances to 1
  x ~~ 1*x
  y ~~ 1*y
  m ~~ 1*m
")

# Fit the model using lavaan to extract the implied covariance matrix
fit <- lavaan::lavaan(model = model_string, data = d_fake)
pop_cov <- lavaan::lavInspect(fit, "cov.all")

# Generate a "population" dataset (empirical = TRUE)
pop_data <- as.data.frame(
  MASS::mvrnorm(n = sample_size,
                mu = rep(0, 3),
                Sigma = pop_cov,
                empirical = TRUE)
)

# Compute the "true" values using your rsquare_med() function
pop_rs <- rsquare_med(data = pop_data, x = "x", m = "m", y = "y")

# Run simulation replications
sim_matrix <- replicate(num_reps, {
  sim_data <- as.data.frame(
    MASS::mvrnorm(n = sample_size,
                  mu = rep(0, 3),
                  Sigma = pop_cov,
                  empirical = FALSE)
  )
  unlist(rsquare_med(data = sim_data, x = "x", m = "m", y = "y"))
})

# Calculate the average estimates over replications
sim_means <- rowMeans(sim_matrix)
sim_means["proportionmediated"] <- if (sim_means["total"] != 0) {
  sim_means["mediatedeffect"] / sim_means["total"]
} else NA

# Return a data frame with the results and the condition label
```

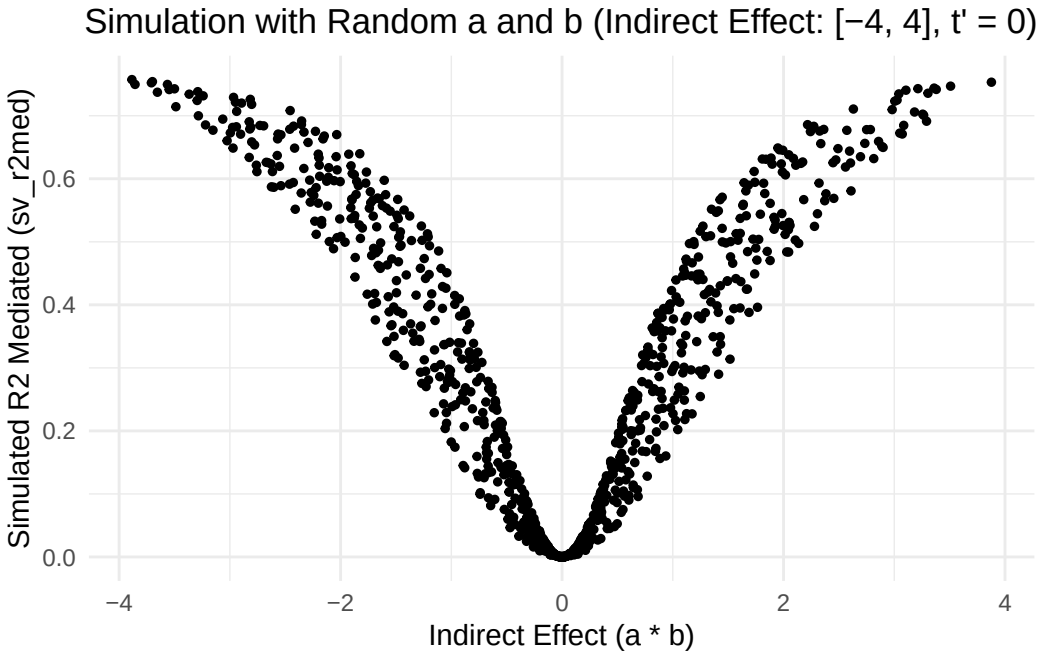
```r
  data.frame(
    indirect_effect = indirect,
    sv_r2med        = sim_means["rsquaredmediated"],
    pop_r2med       = pop_rs$rsquaredmediated,
    condition       = cond_label,
    stringsAsFactors = FALSE
  )
}

# Create a grid of indirect effect values from -4 to 4.
indirect_values <- seq(-4, 4, length.out = 100)

# Run simulations for each condition:
# "blue" will produce condition label "a = 1"
a_results <- lapply(indirect_values, function(x) {
  simulate_indirect_condition(indirect = x, condition = "blue",
                              sample_size = 1000, num_reps = 100)
})
# "red" will produce condition label "b = 1"
b_results <- lapply(indirect_values, function(x) {
  simulate_indirect_condition(indirect = x, condition = "red",
                              sample_size = 1000, num_reps = 100)
})

# Combine the simulation results
sim_results_indirect <- bind_rows(a_results, b_results)

# Create the scatterplot with custom colors and condition labels in the legend.
ggplot(sim_results_indirect, aes(x = indirect_effect, y = sv_r2med, color = condition)) +
  geom_point(size = 2, alpha = 0.8) +
  scale_color_manual(values = c("a = 1" = "blue", "b = 1" = "red")) +
  labs(x = "Indirect Effect (a * b)",
       y = "Simulated R2 Mediated (sv_r2med)",
       title = "Simulated R² Mediated vs. Indirect Effect") +
  theme_minimal() +
  theme(plot.title = element_text(hjust = 0.5))
```




```
simulate_random_model <- function(sample_size = 1000, num_reps = 100) {
  # Randomly draw   and   from Uniform(-2, 2)
  pop_alpha <- runif(1, min = -2, max = 2)
  pop_beta  <- runif(1, min = -2, max = 2)

  # Compute the indirect effect (  *  )
  indirect_effect <- pop_alpha * pop_beta

  # Set    (tau prime) to 0
  pop_tau_prime <- 0

  # Create a "fake" dataset for lavaan (its only purpose is to help define the model)
  d_fake <- data.frame(
    x = rnorm(sample_size),
    m = rnorm(sample_size),
    y = rnorm(sample_size)
  )

  # Build the lavaan model string using glue()
  model_string <- glue("
    # Equation for y (direct effect from x is 0)
```

```
  y ~ {pop_tau_prime}*x + {pop_beta}*m
  # Equation for m
  m ~ {pop_alpha}*x
  # Fix variances of x, m, and y to 1
  x ~~ 1*x
  y ~~ 1*y
  m ~~ 1*m
")

# Fit the model to extract the implied covariance matrix
fit <- lavaan::lavaan(model = model_string, data = d_fake)
pop_cov <- lavaan::lavInspect(fit, "cov.all")

# Generate a "population" dataset using empirical = TRUE
pop_data <- as.data.frame(
  MASS::mvrnorm(n = sample_size,
                mu = rep(0, 3),
                Sigma = pop_cov,
                empirical = TRUE)
)
pop_rs <- rsquare_med(data = pop_data, x = "x", m = "m", y = "y")

# Run simulation replications (with empirical = FALSE)
sim_matrix <- replicate(num_reps, {
  sim_data <- as.data.frame(
    MASS::mvrnorm(n = sample_size,
                  mu = rep(0, 3),
                  Sigma = pop_cov,
                  empirical = FALSE)
  )
  unlist(rsquare_med(data = sim_data, x = "x", m = "m", y = "y"))
})
sim_means <- rowMeans(sim_matrix)
sim_means["proportionmediated"] <- if (sim_means["total"] != 0) {
  sim_means["mediatedeffect"] / sim_means["total"]
} else NA

# Return a data frame with the random parameters and simulation results
data.frame(
  pop_alpha       = pop_alpha,
  pop_beta        = pop_beta,
```

```
    indirect_effect = indirect_effect,
    pop_r2med       = pop_rs$rsquaredmediated,
    sv_r2med        = sim_means["rsquaredmediated"],
    stringsAsFactors = FALSE
  )
}

# Set the number of simulations
n_sim <- 1000

# Run the simulations and combine the results into one data frame
random_results <- do.call(rbind, lapply(1:n_sim, function(i) {
  simulate_random_model(sample_size = 1000, num_reps = 100)
}))

# Create a scatterplot of the indirect effect vs. the simulated R2 mediated.
ggplot(random_results, aes(x = indirect_effect, y = sv_r2med)) +
  geom_point(size = 1, alpha = 1) +
  labs(x = "Indirect Effect (a * b)",
       y = "Simulated R2 Mediated (sv_r2med)",
       title = "Simulation with Random a and b (Indirect Effect: [-4, 4], t' = 0)") +
  theme_minimal() +
  theme(plot.title = element_text(hjust = 0.5))
```


Simulation with Random a and b (Indirect Effect: [−4, 4], t' = 0)
Simulated R2 Mediated (sv_r2med)
0.0
0.2
0.4
0.6
−4
−2
0
2
4
Indirect Effect (a * b)