# mediation_example

```
library(dplyr)
```

```
Warning: package 'dplyr' was built under R version 4.3.3
```

```
Attaching package: 'dplyr'
```

```
The following objects are masked from 'package:stats':

    filter, lag
```

```
The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

```
library(knitr)
```

```
Warning: package 'knitr' was built under R version 4.3.3
```

```
library(MASS)
```

```
Attaching package: 'MASS'
```

```
The following object is masked from 'package:dplyr':

    select
```

```r
library(parallel)
```

### Creating R square mediated function

```r
rsquare_med <- function(data, x, m, y) {
  # Compute correlations among the variables
  rxm <- cor(data[x], data[[m]])
  rxy <- cor(data[[x]], data[[y]])
  rmy <- cor(data[[m]], data[[y]])

  # Regression: m ~ x (to get alpha, first indirect path)
  # Equation 2 in Fairchild, et al
  model1 <- lm(as.formula(paste(m, "~", x)), data = data)
  alpha <- coef(model1)[[x]]

  # Regression: y ~ x + m (to get 'tau_prime' and 'beta')
  # Equation 1 in Fairchild, et al
  model2 <- lm(as.formula(paste(y, "~", x, "+", m)), data = data)

  tau_prime <- coef(model2)[[x]]
  beta <- coef(model2)[[m]]

  # Compute total effect of x on y: tau = tau_prime + (alpha*beta)
  total <- tau_prime + (alpha*beta)


  # Compute effect-size measures
  mediatedeffect <- alpha * beta       # Indirect effect of x on y via M = alpha*beta
  rxmsquared <- rxm^2                  # squared correlation between x and m
  partialrxy_msquared <- ((rxy - rmy * rxm) / sqrt((1 - rmy^2) * (1 - rxmsquared)))^2
  partialrmy_xsquared <- ((rmy - rxy * rxm) / sqrt((1 - rxy^2) * (1 - rxmsquared)))^2
  overallrsquared <- (((rxy^2) + (rmy^2)) - (2 * rxy * rmy * rxm)) / (1 - rxmsquared)
  rsquaredmediated <- (rmy^2) - (overallrsquared - (rxy^2))

  # Create a list of results
  results <- list(
    alpha = alpha,
    beta = beta,
    tau_prime = tau_prime,
    total = total,
```

```
    mediatedeffect = mediatedeffect,
    rxm = rxm,
    rxmsquared = rxmsquared,
    rxy = rxy,
    rmy = rmy,
    partialrxy_msquared = partialrxy_msquared,
    partialrmy_xsquared = partialrmy_xsquared,
    overallrsquared = overallrsquared,
    rsquaredmediated = rsquaredmediated
  )

  return(results)
}
```

**Set up the simulation parameters.**

```
sample_sizes <- c(50, 100, 200, 500, 1000)
effect_sizes <- c(0.00, 0.14, 0.39, 0.59)  # Null, small, medium, large

# Containing all the conditions
df_params <- expand.grid(
  N = sample_sizes,
  pop_alpha = effect_sizes,
  pop_beta = effect_sizes,
  pop_tau_prime = effect_sizes)

# Making some fake data to feed to lavaan
d_fake <-
  data.frame(x = rnorm(sample_sizes),
             m = rnorm(sample_sizes),
             y = rnorm(sample_sizes))
```

```
# Create the run_simulation function
run_simulation <- function(
    pop_tau_prime,
    pop_alpha,
    pop_beta,
    sample_sizes,
    num_reps
) {
```

```r
# Generate the true values using lavaan
# Create a lavaan model where all values are constrained to
# the population values that we want to use
dgf <- glue::glue("
# Equation 1 from Fairchild et al., without error term
y ~ {pop_tau_prime} * x + {pop_beta} * m

# Equation 2 from Fairchild et al., without error term
m ~ {pop_alpha} * x

x ~~ 1 * x
y ~~ 1 * y
m ~~ 1 * m"
)

# Fit the model, and the implied covariance matrix is the
# population covariance matrix
fit <- lavaan::lavaan(model = dgf, data = d_fake)
summary(fit)
pop_cov <- lavaan::lavInspect(fit, "cov.all")

# Using mvrnorm() with empirical = TRUE gives a data frame that
# will reproduce the population covariance matrix ("true" values).
# Not exact, but VERY close (e.g., -1e-34 instead of 0)
pop_data <-
  MASS::mvrnorm(
    n = sample_sizes,
    mu = c(0, 0, 0),
    Sigma = pop_cov,
    empirical = TRUE
  ) %>% as.data.frame()

pop_results <- rsquare_med(data = pop_data, x = "x", m = "m", y = "y" ) %>%
  as.data.frame() %>%
  t() %>%
  as.data.frame() %>%
  tibble::rownames_to_column("parameter")


sim_results <- lapply(1:num_reps, function(x) {
  # changing empirical to FALSE means we get a sample, (simulated values)
```

```
    sim_results_df <-
      sample_data <-
      MASS::mvrnorm(
        n = sample_sizes,
        mu = c(0, 0, 0),
        Sigma = pop_cov,
        empirical = FALSE
      ) %>% as.data.frame()

    # Getting our results from each replication
    sample_results <- rsquare_med(data = sample_data, x = "x", m = "m", y = "y" ) %>%
      as.data.frame()
  }) %>%
    # Here, everything gets averaged. Instead of num_reps parameters, just the
    # average at the end
    dplyr::bind_rows() %>%
    summarize(across(where(is.numeric), mean)) %>%
    t() %>%
    as.data.frame() %>%
    tibble::rownames_to_column("parameter")

  res_joined <-
    dplyr::full_join(pop_results, sim_results, by = "parameter") %>%
    purrr::set_names("parameter", "pop_r2_med", "sim_r2_med") %>%
    dplyr::mutate(bias = sim_r2_med - pop_r2_med) %>%
    dplyr::filter(parameter == "rsquaredmediated") %>%
    dplyr::mutate(
      pop_alpha = pop_alpha,
      pop_beta = pop_beta,
      pop_tau_prime = pop_tau_prime,
      n = sample_sizes
    ) %>%
    dplyr::select(
      pop_alpha, pop_beta, pop_tau_prime, n,
      pop_r2_med, sim_r2_med, bias
    )
}
```

### Now run_simulation for every row in df_params

```
# Define the function to be applied to each row of df_params
# Running it later, now giving it everything it needs to parallelize

sim_function <- function(params) {
  library(dplyr)

  run_simulation(
    sample_sizes  = params[["N"]],
    pop_alpha = params[["pop_alpha"]],
    pop_beta = params[["pop_beta"]],
    pop_tau_prime = params[["pop_tau_prime"]],
    num_reps = 5000
  )
}

# Set up a cluster for parallel processing
cl <- makeCluster(detectCores() - 1)

# Export necessary variables and functions to the cluster
clusterExport(cl, c("df_params",
                    "run_simulation",
                    "sim_function",
                    "d_fake",
                    "rsquare_med"))

# Use parLapply to parallelize the simulations
sim_res <- parLapply(cl, 1:nrow(df_params), function(i) sim_function(df_params[i, ]))

# Stop the cluster after the computation is done, just in case
stopCluster(cl)

# Make data frame, round bias to nearest thousandth
df_sim_res <- dplyr::bind_rows(sim_res) %>%
  dplyr::mutate(bias = round(bias, 3))

# Looks nicer
knitr::kable(df_sim_res)
```

| pop_alpha | pop_beta | pop_tau_prime | n | pop_r2_med | sim_r2_med | bias |
|---|---|---|---|---|---|---|
| 0.00 | 0.00 | 0.00 | 50 | 0.0000000 | 0.0000106 | 0.000 |
| 0.00 | 0.00 | 0.00 | 100 | 0.0000000 | 0.0000459 | 0.000 |
| 0.00 | 0.00 | 0.00 | 200 | 0.0000000 | -0.0000049 | 0.000 |
| 0.00 | 0.00 | 0.00 | 500 | 0.0000000 | -0.0000002 | 0.000 |
| 0.00 | 0.00 | 0.00 | 1000 | 0.0000000 | -0.0000010 | 0.000 |
| 0.14 | 0.00 | 0.00 | 50 | 0.0000000 | 0.0000135 | 0.000 |
| 0.14 | 0.00 | 0.00 | 100 | 0.0000000 | 0.0000607 | 0.000 |
| 0.14 | 0.00 | 0.00 | 200 | 0.0000000 | 0.0000116 | 0.000 |
| 0.14 | 0.00 | 0.00 | 500 | 0.0000000 | -0.0000014 | 0.000 |
| 0.14 | 0.00 | 0.00 | 1000 | 0.0000000 | 0.0000045 | 0.000 |
| 0.39 | 0.00 | 0.00 | 50 | 0.0000000 | -0.0002480 | 0.000 |
| 0.39 | 0.00 | 0.00 | 100 | 0.0000000 | 0.0001399 | 0.000 |
| 0.39 | 0.00 | 0.00 | 200 | 0.0000000 | 0.0000237 | 0.000 |
| 0.39 | 0.00 | 0.00 | 500 | 0.0000000 | 0.0000282 | 0.000 |
| 0.39 | 0.00 | 0.00 | 1000 | 0.0000000 | 0.0000145 | 0.000 |
| 0.59 | 0.00 | 0.00 | 50 | 0.0000000 | 0.0005146 | 0.001 |
| 0.59 | 0.00 | 0.00 | 100 | 0.0000000 | 0.0000702 | 0.000 |
| 0.59 | 0.00 | 0.00 | 200 | 0.0000000 | 0.0000724 | 0.000 |
| 0.59 | 0.00 | 0.00 | 500 | 0.0000000 | -0.0000470 | 0.000 |
| 0.59 | 0.00 | 0.00 | 1000 | 0.0000000 | -0.0000309 | 0.000 |
| 0.00 | 0.14 | 0.00 | 50 | 0.0000000 | 0.0004106 | 0.000 |
| 0.00 | 0.14 | 0.00 | 100 | 0.0000000 | 0.0001809 | 0.000 |
| 0.00 | 0.14 | 0.00 | 200 | 0.0000000 | 0.0000753 | 0.000 |
| 0.00 | 0.14 | 0.00 | 500 | 0.0000000 | 0.0000305 | 0.000 |
| 0.00 | 0.14 | 0.00 | 1000 | 0.0000000 | 0.0000202 | 0.000 |
| 0.14 | 0.14 | 0.00 | 50 | 0.0003766 | 0.0005482 | 0.000 |
| 0.14 | 0.14 | 0.00 | 100 | 0.0003766 | 0.0007196 | 0.000 |
| 0.14 | 0.14 | 0.00 | 200 | 0.0003766 | 0.0005461 | 0.000 |
| 0.14 | 0.14 | 0.00 | 500 | 0.0003766 | 0.0004327 | 0.000 |
| 0.14 | 0.14 | 0.00 | 1000 | 0.0003766 | 0.0003804 | 0.000 |
| 0.39 | 0.14 | 0.00 | 50 | 0.0029153 | 0.0035336 | 0.001 |
| 0.39 | 0.14 | 0.00 | 100 | 0.0029153 | 0.0027648 | 0.000 |
| 0.39 | 0.14 | 0.00 | 200 | 0.0029153 | 0.0030872 | 0.000 |
| 0.39 | 0.14 | 0.00 | 500 | 0.0029153 | 0.0029941 | 0.000 |
| 0.39 | 0.14 | 0.00 | 1000 | 0.0029153 | 0.0029511 | 0.000 |
| 0.59 | 0.14 | 0.00 | 50 | 0.0066471 | 0.0067062 | 0.000 |
| 0.59 | 0.14 | 0.00 | 100 | 0.0066471 | 0.0069225 | 0.000 |
| 0.59 | 0.14 | 0.00 | 200 | 0.0066471 | 0.0067297 | 0.000 |
| 0.59 | 0.14 | 0.00 | 500 | 0.0066471 | 0.0067333 | 0.000 |
| 0.59 | 0.14 | 0.00 | 1000 | 0.0066471 | 0.0067738 | 0.000 |
| 0.00 | 0.39 | 0.00 | 50 | 0.0000000 | 0.0024578 | 0.002 |

| pop_alpha | pop_beta | pop_tau_prime | n | pop_r2_med | sim_r2_med | bias |
|---|---|---|---|---|---|---|
| 0.00 | 0.39 | 0.00 | 100 | 0.0000000 | 0.0014951 | 0.001 |
| 0.00 | 0.39 | 0.00 | 200 | 0.0000000 | 0.0005041 | 0.001 |
| 0.00 | 0.39 | 0.00 | 500 | 0.0000000 | 0.0002686 | 0.000 |
| 0.00 | 0.39 | 0.00 | 1000 | 0.0000000 | 0.0001175 | 0.000 |
| 0.14 | 0.39 | 0.00 | 50 | 0.0025809 | 0.0046164 | 0.002 |
| 0.14 | 0.39 | 0.00 | 100 | 0.0025809 | 0.0039845 | 0.001 |
| 0.14 | 0.39 | 0.00 | 200 | 0.0025809 | 0.0030876 | 0.001 |
| 0.14 | 0.39 | 0.00 | 500 | 0.0025809 | 0.0027896 | 0.000 |
| 0.14 | 0.39 | 0.00 | 1000 | 0.0025809 | 0.0027279 | 0.000 |
| 0.39 | 0.39 | 0.00 | 50 | 0.0196849 | 0.0208114 | 0.001 |
| 0.39 | 0.39 | 0.00 | 100 | 0.0196849 | 0.0202250 | 0.001 |
| 0.39 | 0.39 | 0.00 | 200 | 0.0196849 | 0.0196567 | 0.000 |
| 0.39 | 0.39 | 0.00 | 500 | 0.0196849 | 0.0193766 | 0.000 |
| 0.39 | 0.39 | 0.00 | 1000 | 0.0196849 | 0.0195368 | 0.000 |
| 0.59 | 0.39 | 0.00 | 50 | 0.0439369 | 0.0439587 | 0.000 |
| 0.59 | 0.39 | 0.00 | 100 | 0.0439369 | 0.0445184 | 0.001 |
| 0.59 | 0.39 | 0.00 | 200 | 0.0439369 | 0.0443608 | 0.000 |
| 0.59 | 0.39 | 0.00 | 500 | 0.0439369 | 0.0440634 | 0.000 |
| 0.59 | 0.39 | 0.00 | 1000 | 0.0439369 | 0.0435899 | 0.000 |
| 0.00 | 0.59 | 0.00 | 50 | 0.0000000 | 0.0052757 | 0.005 |
| 0.00 | 0.59 | 0.00 | 100 | 0.0000000 | 0.0024087 | 0.002 |
| 0.00 | 0.59 | 0.00 | 200 | 0.0000000 | 0.0012859 | 0.001 |
| 0.00 | 0.59 | 0.00 | 500 | 0.0000000 | 0.0005209 | 0.001 |
| 0.00 | 0.59 | 0.00 | 1000 | 0.0000000 | 0.0002662 | 0.000 |
| 0.14 | 0.59 | 0.00 | 50 | 0.0050355 | 0.0095718 | 0.005 |
| 0.14 | 0.59 | 0.00 | 100 | 0.0050355 | 0.0080778 | 0.003 |
| 0.14 | 0.59 | 0.00 | 200 | 0.0050355 | 0.0061935 | 0.001 |
| 0.14 | 0.59 | 0.00 | 500 | 0.0050355 | 0.0054777 | 0.000 |
| 0.14 | 0.59 | 0.00 | 1000 | 0.0050355 | 0.0052063 | 0.000 |
| 0.39 | 0.59 | 0.00 | 50 | 0.0377903 | 0.0415654 | 0.004 |
| 0.39 | 0.59 | 0.00 | 100 | 0.0377903 | 0.0403043 | 0.003 |
| 0.39 | 0.59 | 0.00 | 200 | 0.0377903 | 0.0385732 | 0.001 |
| 0.39 | 0.59 | 0.00 | 500 | 0.0377903 | 0.0381555 | 0.000 |
| 0.39 | 0.59 | 0.00 | 1000 | 0.0377903 | 0.0377832 | 0.000 |
| 0.59 | 0.59 | 0.00 | 50 | 0.0824718 | 0.0825754 | 0.000 |
| 0.59 | 0.59 | 0.00 | 100 | 0.0824718 | 0.0827713 | 0.000 |
| 0.59 | 0.59 | 0.00 | 200 | 0.0824718 | 0.0825938 | 0.000 |
| 0.59 | 0.59 | 0.00 | 500 | 0.0824718 | 0.0826249 | 0.000 |
| 0.59 | 0.59 | 0.00 | 1000 | 0.0824718 | 0.0826908 | 0.000 |
| 0.00 | 0.00 | 0.14 | 50 | 0.0000000 | 0.0004606 | 0.000 |
| 0.00 | 0.00 | 0.14 | 100 | 0.0000000 | 0.0001262 | 0.000 |

| pop_alpha | pop_beta | pop_tau_prime | n | pop_r2_med | sim_r2_med | bias |
|---|---|---|---|---|---|---|
| 0.00 | 0.00 | 0.14 | 200 | 0.0000000 | 0.0001064 | 0.000 |
| 0.00 | 0.00 | 0.14 | 500 | 0.0000000 | 0.0000421 | 0.000 |
| 0.00 | 0.00 | 0.14 | 1000 | 0.0000000 | 0.0000227 | 0.000 |
| 0.14 | 0.00 | 0.14 | 50 | 0.0003695 | 0.0007279 | 0.000 |
| 0.14 | 0.00 | 0.14 | 100 | 0.0003695 | 0.0005098 | 0.000 |
| 0.14 | 0.00 | 0.14 | 200 | 0.0003695 | 0.0004173 | 0.000 |
| 0.14 | 0.00 | 0.14 | 500 | 0.0003695 | 0.0004372 | 0.000 |
| 0.14 | 0.00 | 0.14 | 1000 | 0.0003695 | 0.0003691 | 0.000 |
| 0.39 | 0.00 | 0.14 | 50 | 0.0025378 | 0.0031815 | 0.001 |
| 0.39 | 0.00 | 0.14 | 100 | 0.0025378 | 0.0027314 | 0.000 |
| 0.39 | 0.00 | 0.14 | 200 | 0.0025378 | 0.0025834 | 0.000 |
| 0.39 | 0.00 | 0.14 | 500 | 0.0025378 | 0.0025932 | 0.000 |
| 0.39 | 0.00 | 0.14 | 1000 | 0.0025378 | 0.0025610 | 0.000 |
| 0.59 | 0.00 | 0.14 | 50 | 0.0049637 | 0.0048087 | 0.000 |
| 0.59 | 0.00 | 0.14 | 100 | 0.0049637 | 0.0048231 | 0.000 |
| 0.59 | 0.00 | 0.14 | 200 | 0.0049637 | 0.0048822 | 0.000 |
| 0.59 | 0.00 | 0.14 | 500 | 0.0049637 | 0.0049002 | 0.000 |
| 0.59 | 0.00 | 0.14 | 1000 | 0.0049637 | 0.0048974 | 0.000 |
| 0.00 | 0.14 | 0.14 | 50 | 0.0000000 | 0.0003542 | 0.000 |
| 0.00 | 0.14 | 0.14 | 100 | 0.0000000 | 0.0004768 | 0.000 |
| 0.00 | 0.14 | 0.14 | 200 | 0.0000000 | 0.0001865 | 0.000 |
| 0.00 | 0.14 | 0.14 | 500 | 0.0000000 | 0.0000517 | 0.000 |
| 0.00 | 0.14 | 0.14 | 1000 | 0.0000000 | 0.0000641 | 0.000 |
| 0.14 | 0.14 | 0.14 | 50 | 0.0059794 | 0.0065322 | 0.001 |
| 0.14 | 0.14 | 0.14 | 100 | 0.0059794 | 0.0063052 | 0.000 |
| 0.14 | 0.14 | 0.14 | 200 | 0.0059794 | 0.0060429 | 0.000 |
| 0.14 | 0.14 | 0.14 | 500 | 0.0059794 | 0.0060950 | 0.000 |
| 0.14 | 0.14 | 0.14 | 1000 | 0.0059794 | 0.0060177 | 0.000 |
| 0.39 | 0.14 | 0.14 | 50 | 0.0197233 | 0.0198065 | 0.000 |
| 0.39 | 0.14 | 0.14 | 100 | 0.0197233 | 0.0195452 | 0.000 |
| 0.39 | 0.14 | 0.14 | 200 | 0.0197233 | 0.0192604 | 0.000 |
| 0.39 | 0.14 | 0.14 | 500 | 0.0197233 | 0.0198504 | 0.000 |
| 0.39 | 0.14 | 0.14 | 1000 | 0.0197233 | 0.0197199 | 0.000 |
| 0.59 | 0.14 | 0.14 | 50 | 0.0327473 | 0.0331215 | 0.000 |
| 0.59 | 0.14 | 0.14 | 100 | 0.0327473 | 0.0327409 | 0.000 |
| 0.59 | 0.14 | 0.14 | 200 | 0.0327473 | 0.0326992 | 0.000 |
| 0.59 | 0.14 | 0.14 | 500 | 0.0327473 | 0.0326108 | 0.000 |
| 0.59 | 0.14 | 0.14 | 1000 | 0.0327473 | 0.0325675 | 0.000 |
| 0.00 | 0.39 | 0.14 | 50 | 0.0000000 | 0.0026131 | 0.003 |
| 0.00 | 0.39 | 0.14 | 100 | 0.0000000 | 0.0016514 | 0.002 |
| 0.00 | 0.39 | 0.14 | 200 | 0.0000000 | 0.0007900 | 0.001 |

| pop_alpha | pop_beta | pop_tau_prime | n | pop_r2_med | sim_r2_med | bias |
|---|---|---|---|---|---|---|
| 0.00 | 0.39 | 0.14 | 500 | 0.0000000 | 0.0003341 | 0.000 |
| 0.00 | 0.39 | 0.14 | 1000 | 0.0000000 | 0.0001023 | 0.000 |
| 0.14 | 0.39 | 0.14 | 50 | 0.0156693 | 0.0179573 | 0.002 |
| 0.14 | 0.39 | 0.14 | 100 | 0.0156693 | 0.0167778 | 0.001 |
| 0.14 | 0.39 | 0.14 | 200 | 0.0156693 | 0.0161542 | 0.000 |
| 0.14 | 0.39 | 0.14 | 500 | 0.0156693 | 0.0157834 | 0.000 |
| 0.14 | 0.39 | 0.14 | 1000 | 0.0156693 | 0.0158258 | 0.000 |
| 0.39 | 0.39 | 0.14 | 50 | 0.0552035 | 0.0553383 | 0.000 |
| 0.39 | 0.39 | 0.14 | 100 | 0.0552035 | 0.0556392 | 0.000 |
| 0.39 | 0.39 | 0.14 | 200 | 0.0552035 | 0.0552540 | 0.000 |
| 0.39 | 0.39 | 0.14 | 500 | 0.0552035 | 0.0549213 | 0.000 |
| 0.39 | 0.39 | 0.14 | 1000 | 0.0552035 | 0.0552816 | 0.000 |
| 0.59 | 0.39 | 0.14 | 50 | 0.0949791 | 0.0953293 | 0.000 |
| 0.59 | 0.39 | 0.14 | 100 | 0.0949791 | 0.0939771 | -0.001 |
| 0.59 | 0.39 | 0.14 | 200 | 0.0949791 | 0.0941594 | -0.001 |
| 0.59 | 0.39 | 0.14 | 500 | 0.0949791 | 0.0942539 | -0.001 |
| 0.59 | 0.39 | 0.14 | 1000 | 0.0949791 | 0.0947486 | 0.000 |
| 0.00 | 0.59 | 0.14 | 50 | 0.0000000 | 0.0052933 | 0.005 |
| 0.00 | 0.59 | 0.14 | 100 | 0.0000000 | 0.0023403 | 0.002 |
| 0.00 | 0.59 | 0.14 | 200 | 0.0000000 | 0.0013709 | 0.001 |
| 0.00 | 0.59 | 0.14 | 500 | 0.0000000 | 0.0004275 | 0.000 |
| 0.00 | 0.59 | 0.14 | 1000 | 0.0000000 | 0.0003203 | 0.000 |
| 0.14 | 0.59 | 0.14 | 50 | 0.0216989 | 0.0251475 | 0.003 |
| 0.14 | 0.59 | 0.14 | 100 | 0.0216989 | 0.0239288 | 0.002 |
| 0.14 | 0.59 | 0.14 | 200 | 0.0216989 | 0.0231852 | 0.001 |
| 0.14 | 0.59 | 0.14 | 500 | 0.0216989 | 0.0221871 | 0.000 |
| 0.14 | 0.59 | 0.14 | 1000 | 0.0216989 | 0.0219105 | 0.000 |
| 0.39 | 0.59 | 0.14 | 50 | 0.0807782 | 0.0826194 | 0.002 |
| 0.39 | 0.59 | 0.14 | 100 | 0.0807782 | 0.0816155 | 0.001 |
| 0.39 | 0.59 | 0.14 | 200 | 0.0807782 | 0.0816667 | 0.001 |
| 0.39 | 0.59 | 0.14 | 500 | 0.0807782 | 0.0801911 | -0.001 |
| 0.39 | 0.59 | 0.14 | 1000 | 0.0807782 | 0.0808699 | 0.000 |
| 0.59 | 0.59 | 0.14 | 50 | 0.1410179 | 0.1417873 | 0.001 |
| 0.59 | 0.59 | 0.14 | 100 | 0.1410179 | 0.1406085 | 0.000 |
| 0.59 | 0.59 | 0.14 | 200 | 0.1410179 | 0.1405209 | 0.000 |
| 0.59 | 0.59 | 0.14 | 500 | 0.1410179 | 0.1407387 | 0.000 |
| 0.59 | 0.59 | 0.14 | 1000 | 0.1410179 | 0.1407455 | 0.000 |
| 0.00 | 0.00 | 0.39 | 50 | 0.0000000 | 0.0023527 | 0.002 |
| 0.00 | 0.00 | 0.39 | 100 | 0.0000000 | 0.0014799 | 0.001 |
| 0.00 | 0.00 | 0.39 | 200 | 0.0000000 | 0.0006605 | 0.001 |
| 0.00 | 0.00 | 0.39 | 500 | 0.0000000 | 0.0002657 | 0.000 |

| pop_alpha | pop_beta | pop_tau_prime | n | pop_r2_med | sim_r2_med | bias |
|---|---|---|---|---|---|---|
| 0.00 | 0.00 | 0.39 | 1000 | 0.0000000 | 0.0001329 | 0.000 |
| 0.14 | 0.00 | 0.39 | 50 | 0.0025378 | 0.0050983 | 0.003 |
| 0.14 | 0.00 | 0.39 | 100 | 0.0025378 | 0.0038061 | 0.001 |
| 0.14 | 0.00 | 0.39 | 200 | 0.0025378 | 0.0031400 | 0.001 |
| 0.14 | 0.00 | 0.39 | 500 | 0.0025378 | 0.0026930 | 0.000 |
| 0.14 | 0.00 | 0.39 | 1000 | 0.0025378 | 0.0026604 | 0.000 |
| 0.39 | 0.00 | 0.39 | 50 | 0.0174292 | 0.0187247 | 0.001 |
| 0.39 | 0.00 | 0.39 | 100 | 0.0174292 | 0.0180954 | 0.001 |
| 0.39 | 0.00 | 0.39 | 200 | 0.0174292 | 0.0177131 | 0.000 |
| 0.39 | 0.00 | 0.39 | 500 | 0.0174292 | 0.0174687 | 0.000 |
| 0.39 | 0.00 | 0.39 | 1000 | 0.0174292 | 0.0174918 | 0.000 |
| 0.59 | 0.00 | 0.39 | 50 | 0.0340895 | 0.0334405 | -0.001 |
| 0.59 | 0.00 | 0.39 | 100 | 0.0340895 | 0.0346146 | 0.001 |
| 0.59 | 0.00 | 0.39 | 200 | 0.0340895 | 0.0346935 | 0.001 |
| 0.59 | 0.00 | 0.39 | 500 | 0.0340895 | 0.0341860 | 0.000 |
| 0.59 | 0.00 | 0.39 | 1000 | 0.0340895 | 0.0339727 | 0.000 |
| 0.00 | 0.14 | 0.39 | 50 | 0.0000000 | 0.0028606 | 0.003 |
| 0.00 | 0.14 | 0.39 | 100 | 0.0000000 | 0.0014560 | 0.001 |
| 0.00 | 0.14 | 0.39 | 200 | 0.0000000 | 0.0005915 | 0.001 |
| 0.00 | 0.14 | 0.39 | 500 | 0.0000000 | 0.0002854 | 0.000 |
| 0.00 | 0.14 | 0.39 | 1000 | 0.0000000 | 0.0001296 | 0.000 |
| 0.14 | 0.14 | 0.39 | 50 | 0.0156615 | 0.0176559 | 0.002 |
| 0.14 | 0.14 | 0.39 | 100 | 0.0156615 | 0.0168195 | 0.001 |
| 0.14 | 0.14 | 0.39 | 200 | 0.0156615 | 0.0161825 | 0.001 |
| 0.14 | 0.14 | 0.39 | 500 | 0.0156615 | 0.0159321 | 0.000 |
| 0.14 | 0.14 | 0.39 | 1000 | 0.0156615 | 0.0156051 | 0.000 |
| 0.39 | 0.14 | 0.39 | 50 | 0.0539317 | 0.0538039 | 0.000 |
| 0.39 | 0.14 | 0.39 | 100 | 0.0539317 | 0.0544235 | 0.000 |
| 0.39 | 0.14 | 0.39 | 200 | 0.0539317 | 0.0539494 | 0.000 |
| 0.39 | 0.14 | 0.39 | 500 | 0.0539317 | 0.0538981 | 0.000 |
| 0.39 | 0.14 | 0.39 | 1000 | 0.0539317 | 0.0537913 | 0.000 |
| 0.59 | 0.14 | 0.39 | 50 | 0.0889217 | 0.0880056 | -0.001 |
| 0.59 | 0.14 | 0.39 | 100 | 0.0889217 | 0.0874322 | -0.001 |
| 0.59 | 0.14 | 0.39 | 200 | 0.0889217 | 0.0878476 | -0.001 |
| 0.59 | 0.14 | 0.39 | 500 | 0.0889217 | 0.0887348 | 0.000 |
| 0.59 | 0.14 | 0.39 | 1000 | 0.0889217 | 0.0885502 | 0.000 |
| 0.00 | 0.39 | 0.39 | 50 | 0.0000000 | 0.0030417 | 0.003 |
| 0.00 | 0.39 | 0.39 | 100 | 0.0000000 | 0.0020657 | 0.002 |
| 0.00 | 0.39 | 0.39 | 200 | 0.0000000 | 0.0010384 | 0.001 |
| 0.00 | 0.39 | 0.39 | 500 | 0.0000000 | 0.0002028 | 0.000 |
| 0.00 | 0.39 | 0.39 | 1000 | 0.0000000 | 0.0001832 | 0.000 |

| pop_alpha | pop_beta | pop_tau_prime | n | pop_r2_med | sim_r2_med | bias |
|---|---|---|---|---|---|---|
| 0.14 | 0.39 | 0.39 | 50 | 0.0359269 | 0.0373679 | 0.001 |
| 0.14 | 0.39 | 0.39 | 100 | 0.0359269 | 0.0373381 | 0.001 |
| 0.14 | 0.39 | 0.39 | 200 | 0.0359269 | 0.0367887 | 0.001 |
| 0.14 | 0.39 | 0.39 | 500 | 0.0359269 | 0.0361196 | 0.000 |
| 0.14 | 0.39 | 0.39 | 1000 | 0.0359269 | 0.0361024 | 0.000 |
| 0.39 | 0.39 | 0.39 | 50 | 0.1119334 | 0.1109581 | -0.001 |
| 0.39 | 0.39 | 0.39 | 100 | 0.1119334 | 0.1114357 | 0.000 |
| 0.39 | 0.39 | 0.39 | 200 | 0.1119334 | 0.1118741 | 0.000 |
| 0.39 | 0.39 | 0.39 | 500 | 0.1119334 | 0.1121727 | 0.000 |
| 0.39 | 0.39 | 0.39 | 1000 | 0.1119334 | 0.1118315 | 0.000 |
| 0.59 | 0.39 | 0.39 | 50 | 0.1768152 | 0.1739323 | -0.003 |
| 0.59 | 0.39 | 0.39 | 100 | 0.1768152 | 0.1763423 | 0.000 |
| 0.59 | 0.39 | 0.39 | 200 | 0.1768152 | 0.1762635 | -0.001 |
| 0.59 | 0.39 | 0.39 | 500 | 0.1768152 | 0.1761870 | -0.001 |
| 0.59 | 0.39 | 0.39 | 1000 | 0.1768152 | 0.1764918 | 0.000 |
| 0.00 | 0.59 | 0.39 | 50 | 0.0000000 | 0.0050951 | 0.005 |
| 0.00 | 0.59 | 0.39 | 100 | 0.0000000 | 0.0018014 | 0.002 |
| 0.00 | 0.59 | 0.39 | 200 | 0.0000000 | 0.0011433 | 0.001 |
| 0.00 | 0.59 | 0.39 | 500 | 0.0000000 | 0.0002323 | 0.000 |
| 0.00 | 0.59 | 0.39 | 1000 | 0.0000000 | 0.0002600 | 0.000 |
| 0.14 | 0.59 | 0.39 | 50 | 0.0472014 | 0.0499255 | 0.003 |
| 0.14 | 0.59 | 0.39 | 100 | 0.0472014 | 0.0486659 | 0.001 |
| 0.14 | 0.59 | 0.39 | 200 | 0.0472014 | 0.0480401 | 0.001 |
| 0.14 | 0.59 | 0.39 | 500 | 0.0472014 | 0.0473113 | 0.000 |
| 0.14 | 0.59 | 0.39 | 1000 | 0.0472014 | 0.0472683 | 0.000 |
| 0.39 | 0.59 | 0.39 | 50 | 0.1457352 | 0.1452744 | 0.000 |
| 0.39 | 0.59 | 0.39 | 100 | 0.1457352 | 0.1454842 | 0.000 |
| 0.39 | 0.59 | 0.39 | 200 | 0.1457352 | 0.1464281 | 0.001 |
| 0.39 | 0.59 | 0.39 | 500 | 0.1457352 | 0.1455025 | 0.000 |
| 0.39 | 0.59 | 0.39 | 1000 | 0.1457352 | 0.1457857 | 0.000 |
| 0.59 | 0.59 | 0.39 | 50 | 0.2282044 | 0.2250247 | -0.003 |
| 0.59 | 0.59 | 0.39 | 100 | 0.2282044 | 0.2259637 | -0.002 |
| 0.59 | 0.59 | 0.39 | 200 | 0.2282044 | 0.2284585 | 0.000 |
| 0.59 | 0.59 | 0.39 | 500 | 0.2282044 | 0.2285740 | 0.000 |
| 0.59 | 0.59 | 0.39 | 1000 | 0.2282044 | 0.2281331 | 0.000 |
| 0.00 | 0.00 | 0.59 | 50 | 0.0000000 | 0.0050441 | 0.005 |
| 0.00 | 0.00 | 0.59 | 100 | 0.0000000 | 0.0023211 | 0.002 |
| 0.00 | 0.00 | 0.59 | 200 | 0.0000000 | 0.0013674 | 0.001 |
| 0.00 | 0.00 | 0.59 | 500 | 0.0000000 | 0.0005273 | 0.001 |
| 0.00 | 0.00 | 0.59 | 1000 | 0.0000000 | 0.0002604 | 0.000 |
| 0.14 | 0.00 | 0.59 | 50 | 0.0049637 | 0.0097549 | 0.005 |

| pop_alpha | pop_beta | pop_tau_prime | n | pop_r2_med | sim_r2_med | bias |
|---|---|---|---|---|---|---|
| 0.14 | 0.00 | 0.59 | 100 | 0.0049637 | 0.0075401 | 0.003 |
| 0.14 | 0.00 | 0.59 | 200 | 0.0049637 | 0.0062359 | 0.001 |
| 0.14 | 0.00 | 0.59 | 500 | 0.0049637 | 0.0054284 | 0.000 |
| 0.14 | 0.00 | 0.59 | 1000 | 0.0049637 | 0.0052411 | 0.000 |
| 0.39 | 0.00 | 0.59 | 50 | 0.0340895 | 0.0358833 | 0.002 |
| 0.39 | 0.00 | 0.59 | 100 | 0.0340895 | 0.0356148 | 0.002 |
| 0.39 | 0.00 | 0.59 | 200 | 0.0340895 | 0.0350029 | 0.001 |
| 0.39 | 0.00 | 0.59 | 500 | 0.0340895 | 0.0344925 | 0.000 |
| 0.39 | 0.00 | 0.59 | 1000 | 0.0340895 | 0.0343115 | 0.000 |
| 0.59 | 0.00 | 0.59 | 50 | 0.0666751 | 0.0693202 | 0.003 |
| 0.59 | 0.00 | 0.59 | 100 | 0.0666751 | 0.0676048 | 0.001 |
| 0.59 | 0.00 | 0.59 | 200 | 0.0666751 | 0.0673467 | 0.001 |
| 0.59 | 0.00 | 0.59 | 500 | 0.0666751 | 0.0664215 | 0.000 |
| 0.59 | 0.00 | 0.59 | 1000 | 0.0666751 | 0.0665174 | 0.000 |
| 0.00 | 0.14 | 0.59 | 50 | 0.0000000 | 0.0049697 | 0.005 |
| 0.00 | 0.14 | 0.59 | 100 | 0.0000000 | 0.0024857 | 0.002 |
| 0.00 | 0.14 | 0.59 | 200 | 0.0000000 | 0.0015406 | 0.002 |
| 0.00 | 0.14 | 0.59 | 500 | 0.0000000 | 0.0004756 | 0.000 |
| 0.00 | 0.14 | 0.59 | 1000 | 0.0000000 | 0.0002554 | 0.000 |
| 0.14 | 0.14 | 0.59 | 50 | 0.0217104 | 0.0261715 | 0.004 |
| 0.14 | 0.14 | 0.59 | 100 | 0.0217104 | 0.0236002 | 0.002 |
| 0.14 | 0.14 | 0.59 | 200 | 0.0217104 | 0.0227758 | 0.001 |
| 0.14 | 0.14 | 0.59 | 500 | 0.0217104 | 0.0222822 | 0.001 |
| 0.14 | 0.14 | 0.59 | 1000 | 0.0217104 | 0.0218777 | 0.000 |
| 0.39 | 0.14 | 0.59 | 50 | 0.0789942 | 0.0804358 | 0.001 |
| 0.39 | 0.14 | 0.59 | 100 | 0.0789942 | 0.0797965 | 0.001 |
| 0.39 | 0.14 | 0.59 | 200 | 0.0789942 | 0.0793781 | 0.000 |
| 0.39 | 0.14 | 0.59 | 500 | 0.0789942 | 0.0792671 | 0.000 |
| 0.39 | 0.14 | 0.59 | 1000 | 0.0789942 | 0.0791594 | 0.000 |
| 0.59 | 0.14 | 0.59 | 50 | 0.1319135 | 0.1310580 | -0.001 |
| 0.59 | 0.14 | 0.59 | 100 | 0.1319135 | 0.1314949 | 0.000 |
| 0.59 | 0.14 | 0.59 | 200 | 0.1319135 | 0.1319062 | 0.000 |
| 0.59 | 0.14 | 0.59 | 500 | 0.1319135 | 0.1318653 | 0.000 |
| 0.59 | 0.14 | 0.59 | 1000 | 0.1319135 | 0.1317133 | 0.000 |
| 0.00 | 0.39 | 0.59 | 50 | 0.0000000 | 0.0056816 | 0.006 |
| 0.00 | 0.39 | 0.59 | 100 | 0.0000000 | 0.0025138 | 0.003 |
| 0.00 | 0.39 | 0.59 | 200 | 0.0000000 | 0.0011327 | 0.001 |
| 0.00 | 0.39 | 0.59 | 500 | 0.0000000 | 0.0002136 | 0.000 |
| 0.00 | 0.39 | 0.59 | 1000 | 0.0000000 | 0.0002393 | 0.000 |
| 0.14 | 0.39 | 0.59 | 50 | 0.0472699 | 0.0505522 | 0.003 |
| 0.14 | 0.39 | 0.59 | 100 | 0.0472699 | 0.0492447 | 0.002 |

| pop_alpha | pop_beta | pop_tau_prime | n | pop_r2_med | sim_r2_med | bias |
|---|---|---|---|---|---|---|
| 0.14 | 0.39 | 0.59 | 200 | 0.0472699 | 0.0483298 | 0.001 |
| 0.14 | 0.39 | 0.59 | 500 | 0.0472699 | 0.0472853 | 0.000 |
| 0.14 | 0.39 | 0.59 | 1000 | 0.0472699 | 0.0471201 | 0.000 |
| 0.39 | 0.39 | 0.59 | 50 | 0.1459753 | 0.1445307 | -0.001 |
| 0.39 | 0.39 | 0.59 | 100 | 0.1459753 | 0.1458735 | 0.000 |
| 0.39 | 0.39 | 0.59 | 200 | 0.1459753 | 0.1464928 | 0.001 |
| 0.39 | 0.39 | 0.59 | 500 | 0.1459753 | 0.1460434 | 0.000 |
| 0.39 | 0.39 | 0.59 | 1000 | 0.1459753 | 0.1457736 | 0.000 |
| 0.59 | 0.39 | 0.59 | 50 | 0.2270822 | 0.2244510 | -0.003 |
| 0.59 | 0.39 | 0.59 | 100 | 0.2270822 | 0.2244143 | -0.003 |
| 0.59 | 0.39 | 0.59 | 200 | 0.2270822 | 0.2257762 | -0.001 |
| 0.59 | 0.39 | 0.59 | 500 | 0.2270822 | 0.2267717 | 0.000 |
| 0.59 | 0.39 | 0.59 | 1000 | 0.2270822 | 0.2267709 | 0.000 |
| 0.00 | 0.59 | 0.59 | 50 | 0.0000000 | 0.0045931 | 0.005 |
| 0.00 | 0.59 | 0.59 | 100 | 0.0000000 | 0.0025872 | 0.003 |
| 0.00 | 0.59 | 0.59 | 200 | 0.0000000 | 0.0011867 | 0.001 |
| 0.00 | 0.59 | 0.59 | 500 | 0.0000000 | 0.0006713 | 0.001 |
| 0.00 | 0.59 | 0.59 | 1000 | 0.0000000 | 0.0003215 | 0.000 |
| 0.14 | 0.59 | 0.59 | 50 | 0.0616401 | 0.0636484 | 0.002 |
| 0.14 | 0.59 | 0.59 | 100 | 0.0616401 | 0.0640380 | 0.002 |
| 0.14 | 0.59 | 0.59 | 200 | 0.0616401 | 0.0623105 | 0.001 |
| 0.14 | 0.59 | 0.59 | 500 | 0.0616401 | 0.0619710 | 0.000 |
| 0.14 | 0.59 | 0.59 | 1000 | 0.0616401 | 0.0619443 | 0.000 |
| 0.39 | 0.59 | 0.59 | 50 | 0.1833160 | 0.1808216 | -0.002 |
| 0.39 | 0.59 | 0.59 | 100 | 0.1833160 | 0.1814301 | -0.002 |
| 0.39 | 0.59 | 0.59 | 200 | 0.1833160 | 0.1827790 | -0.001 |
| 0.39 | 0.59 | 0.59 | 500 | 0.1833160 | 0.1831694 | 0.000 |
| 0.39 | 0.59 | 0.59 | 1000 | 0.1833160 | 0.1833205 | 0.000 |
| 0.59 | 0.59 | 0.59 | 50 | 0.2790752 | 0.2742550 | -0.005 |
| 0.59 | 0.59 | 0.59 | 100 | 0.2790752 | 0.2781181 | -0.001 |
| 0.59 | 0.59 | 0.59 | 200 | 0.2790752 | 0.2772806 | -0.002 |
| 0.59 | 0.59 | 0.59 | 500 | 0.2790752 | 0.2785987 | 0.000 |
| 0.59 | 0.59 | 0.59 | 1000 | 0.2790752 | 0.2792297 | 0.000 |