Bus schedule

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Suppose that a bus is scheduled to arrive at the stop on the hour, at 15 minutes past the hour, at 30 minutes past the hour, and at 45 minutes past the hour. However, due to random fluctuations, it arrives anywhere between 1 minute early and 2 minutes late with uniform distribution ($\mathcal{U}(-1, 2)$). Assume that the arrivals of different buses are independent and identically distributed (IID).

Parameters

```
t_int = 15; % time interval between stops
dlo = -1;   % low end of delay
dhi = 2;    % high end of delay
rep = 1e6;  % number of repetitions
tol = 1/60; % tolerance
mu = 7.55;  % true mean wait time
```

Mean estimation

The Monte-Carlo simulation will assume people arrive at the bus stop randomly, one person every time a bus arrives ($\mathcal{U}(0, 15)$). Passengers will enter the first bus that arrives. There is no problem of space inside the bus. Furthermore, we will use the inherent symmetry of the problem to rule out one of the posibilities: the passenger arrives after the first bus has already passed. Time is centered on the passenger. Monte Carlo methods are very versatile. In this case, we present two possibilities to estimate the mean waiting time for a bus. The first one is defined by the number of runs performed while the second one is constrained by the maximum error allowed (99% of the time). The `meanMC_CLT()` function is part of the GAIL package.

```
disp(['The true mean is ' num2str(mu)])

span_d = dhi - dlo;

tic;
x = rand(rep, 3);
waitt = f(x, t_int, span_d, dhi, dlo); % function definition at the
 end
muhat = mean(waitt);
toc;

disp(['Estimated mean with ' num2str(rep) ' runs: ' num2str(muhat)])
disp(' ')

tic;
y = @(n)f(rand(n, 3), t_int, span_d, dhi, dlo);
```

```matlab
[muhat, out] = meanMC_CLT(y, tol);
toc;
err = abs(muhat-mu);

disp(['The error is ' num2str(err, '%e')])
disp(['It should be bounded by ' num2str(out.errBd, '%e')])
disp([num2str(out.nSample) ' samples used'])
disp(['Therefore, the estimated mean is ' num2str(muhat)])

The true mean is 7.55
```

Confidence interval estimation

Since an estimated quantity is unlikely to be exact, sometimes is more useful to compute a range of values within the true answer lies with some probability. That is called a confidence interval. In this example, a confidence interval for the probability that the time to wait of a bus is greater than 8 minutes will be constructed. The function `binomialCI` is implemented in the main directory.

```matlab
tic;
y = f(rand(rep, 3), t_int, span_d, dhi, dlo);
ci = binomialCI(rep, sum(y > 8));
toc;

disp('The confidence interval is')
disp(ci)
```

Function

```matlab
function [y] = f(x, t_int, span_d, dhi, dlo)
% function for waiting time (y)

x_p = x(:, 1) * t_int;
x_2 = x(:, 2) * span_d + t_int + dlo - dhi;
x_3 = x(:, 3) * span_d + 2*t_int + dlo - dhi;

y = (x_2 - x_p).*(x_2 >= x_p) + (x_3 - x_p).*(x_2 < x_p);
end

Elapsed time is 0.087531 seconds.
Estimated mean with 1000000 runs: 7.553

Elapsed time is 0.071740 seconds.
The error is 3.840138e-03
It should be bounded by 1.666666e-02
673396 samples used
Therefore, the estimated mean is 7.5462
```

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