Acceptance-rejection method

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Generate samples of a beta distribution with $\alpha = \beta = 2$ using an acceptance-rejection method.

Beta distribution

The beta distribution, $\mathrm{beta}(\alpha, \beta)$, has the general form,

$$f(x) = \frac{1}{B(\alpha,\beta)} x^{\alpha-1}(1-x)^{\beta-1} \Leftrightarrow B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

with $\alpha = \beta = 2$. One useful property of the gamma function when its argument is an integer is that $\Gamma(n) = (n-1)!$. Therefore,

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} = \frac{\Gamma(2)\Gamma(2)}{\Gamma(4)} = \frac{1!1!}{3!} = \frac{1}{6} \Rightarrow f(x) = 6x(1-x).$$

This makes it evident that the behaviour of $f(x|\alpha,\beta)$ changes its behavior drastically with different values of α and β. That is why the code here is not general and not easy to generalize to every beta distribtuion.

Sampling

The **acceptance-rejection** method implies that samples are generated from a known distribution ($g(x)$) with the same range as the wanted, and that sample distribution is corrected afterwards by the ratio of distributions $f(x)/g(x)$, so that some samples are *rejected*. The final instances of the final distribution are only the accepted samples.

Parameters

```
a = 2;    % alpha
b = 2;    % beta
c = 2/3;  % ratio constant
n = 1000; % generated

pdf_unif = 1;
pdf_beta = @(x) 6 * x .* (1-x);
```

Sample Generation

```
tic;
uz = rand(n/c, 2);
u = uz(:,1);
z = uz(:,2);
x = z;

% treat as Not a Number (NaN) rejected samples
x(u > c * pdf_beta(z) / pdf_unif) = NaN;
toc;

muhat = mean(x,'omitnan');
sigmahat = var(x,'omitnan');
```

```
Elapsed time is 0.000899 seconds.
```

Output resutls

```
% expectation and variance of the beta distribution
mu = a/(a + b);
sigma = a * b / ((a + b)^2 * (a + b + 1));

disp(['The mean of our samples is ' num2str(muhat)])
disp(['while its expected value is ' num2str(mu)])
disp(' ')
disp(['The variance of our samples is ' num2str(sigmahat)])
disp(['while its expected value is ' num2str(sigma)])
```

```
The mean of our samples is 0.49679
while its expected value is 0.5

The variance of our samples is 0.0483
while its expected value is 0.05
```

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Published with MATLAB® R2021a