Wave-Particle Duality and Wave Mechanics

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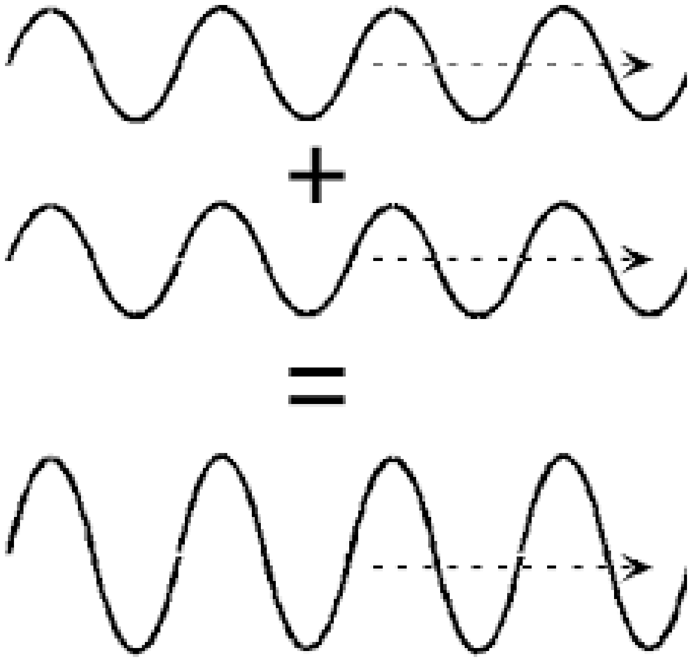
The Wave Particle Duality of Matter

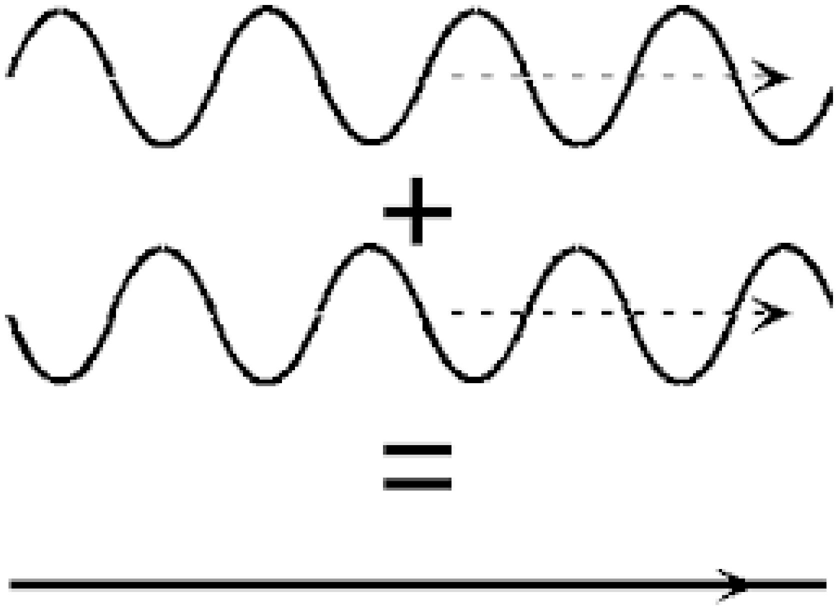
- Wave-particle duality refers to the fundamental property of matter where, at one moment it appears like a wave, and yet at another moment it acts like a particle.

- The unique properties of (light) waves
  - Are oscillatory in nature i.e have frequency and wavelength
  - Transport energy from one point to the next
  - **They exhibit interference**
  - **They exhibit diffraction**

Wave interference



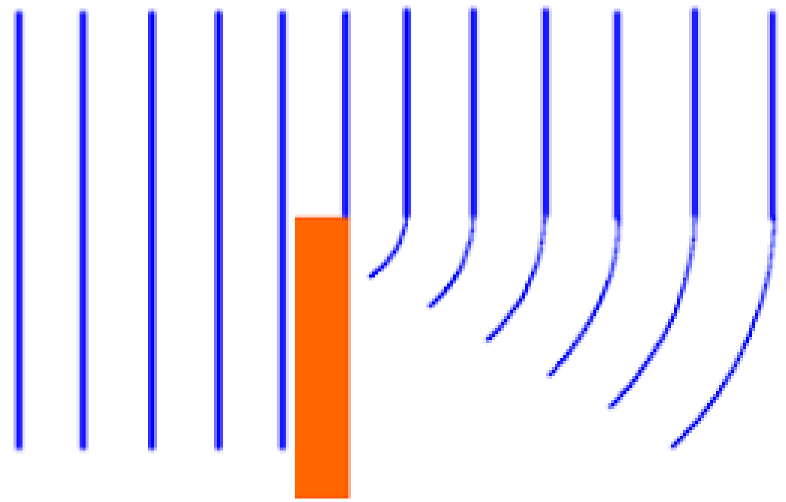
Constructive Wave interference

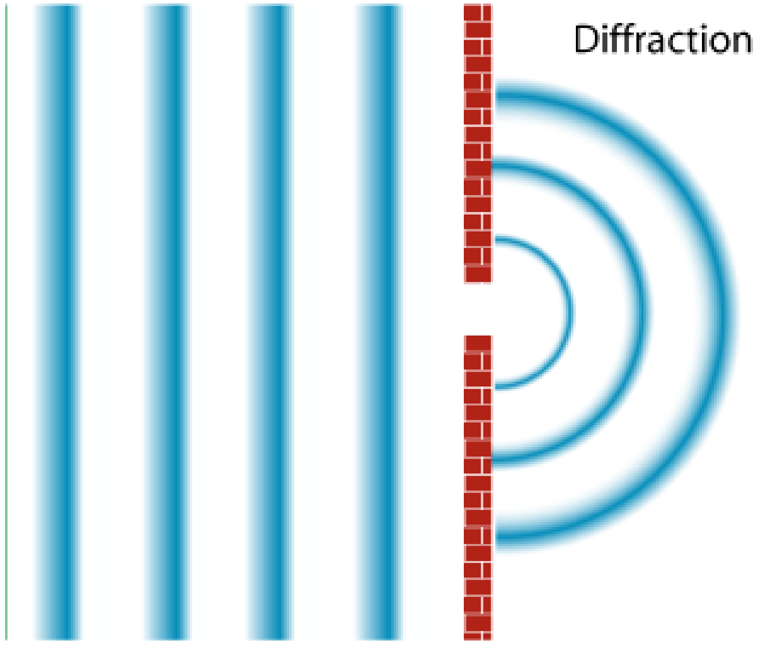


Destructive Wave interference

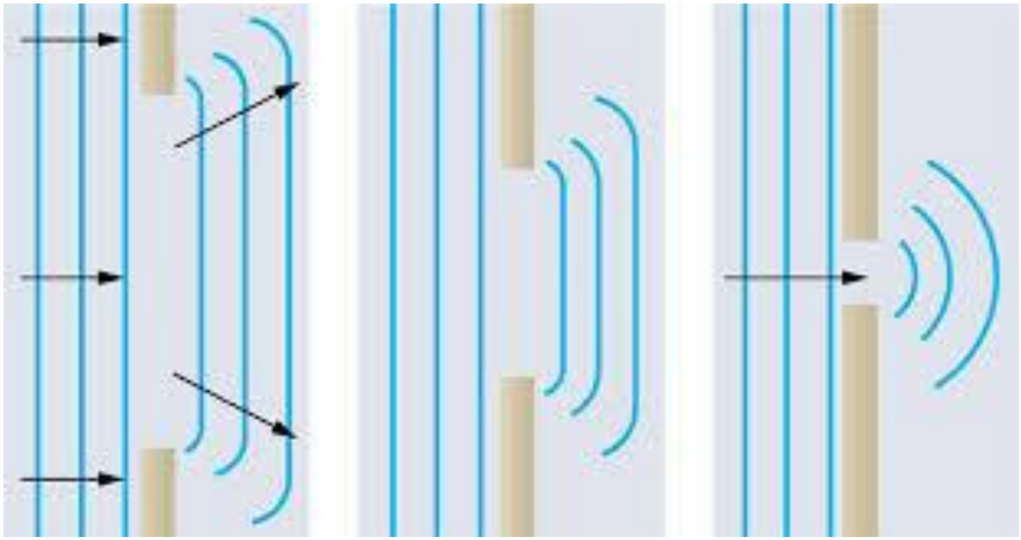
Wave Diffraction

➢ The <u>bending or spreading of waves around the corners of an obstacle or through an aperture</u> into the region of geometrical shadow of the obstacle/aperture.

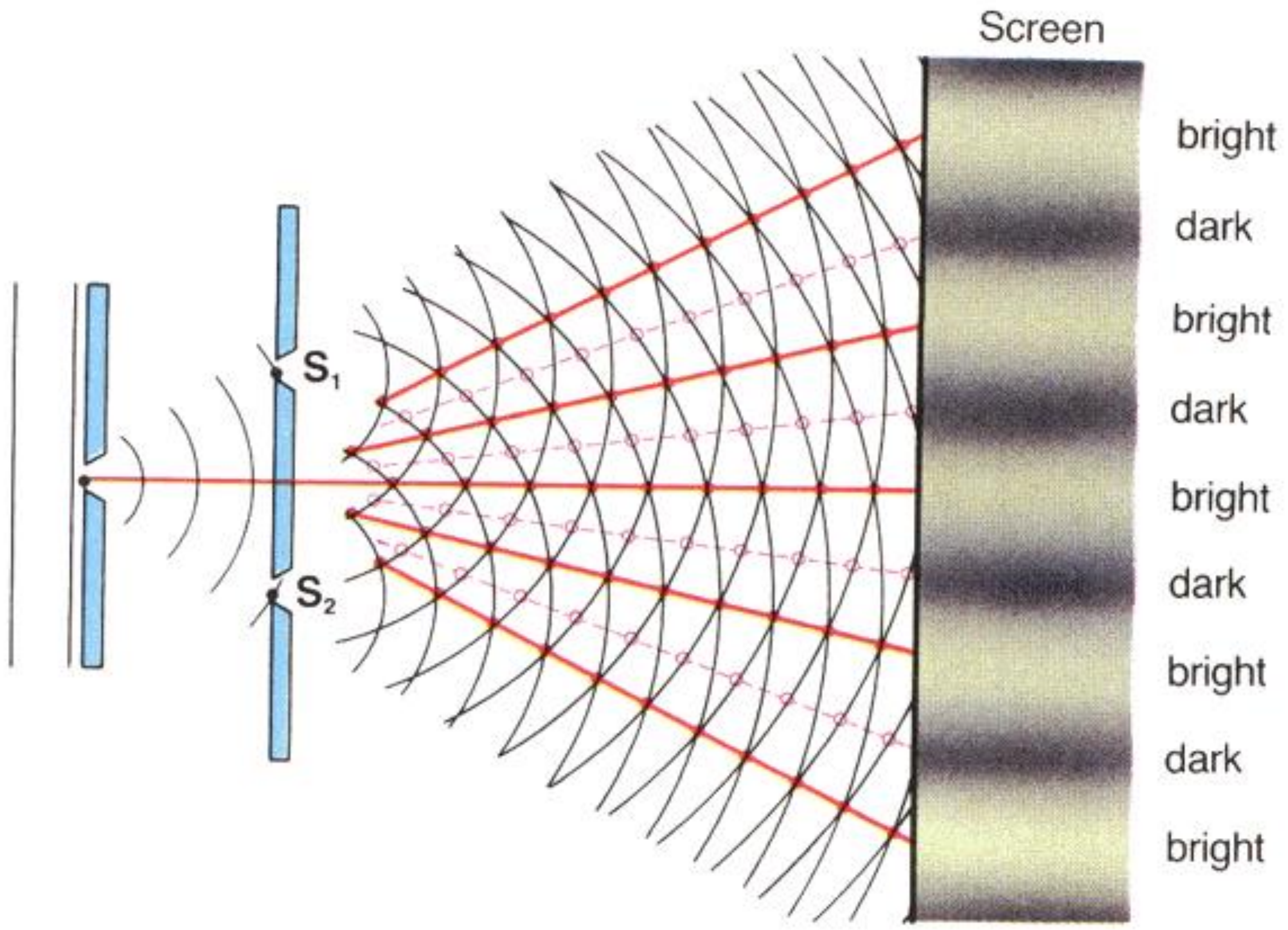




❑ It occurs significantly when the size of the aperture or obstacle is of similar linear dimensions to the wavelength of the incident **wave**.



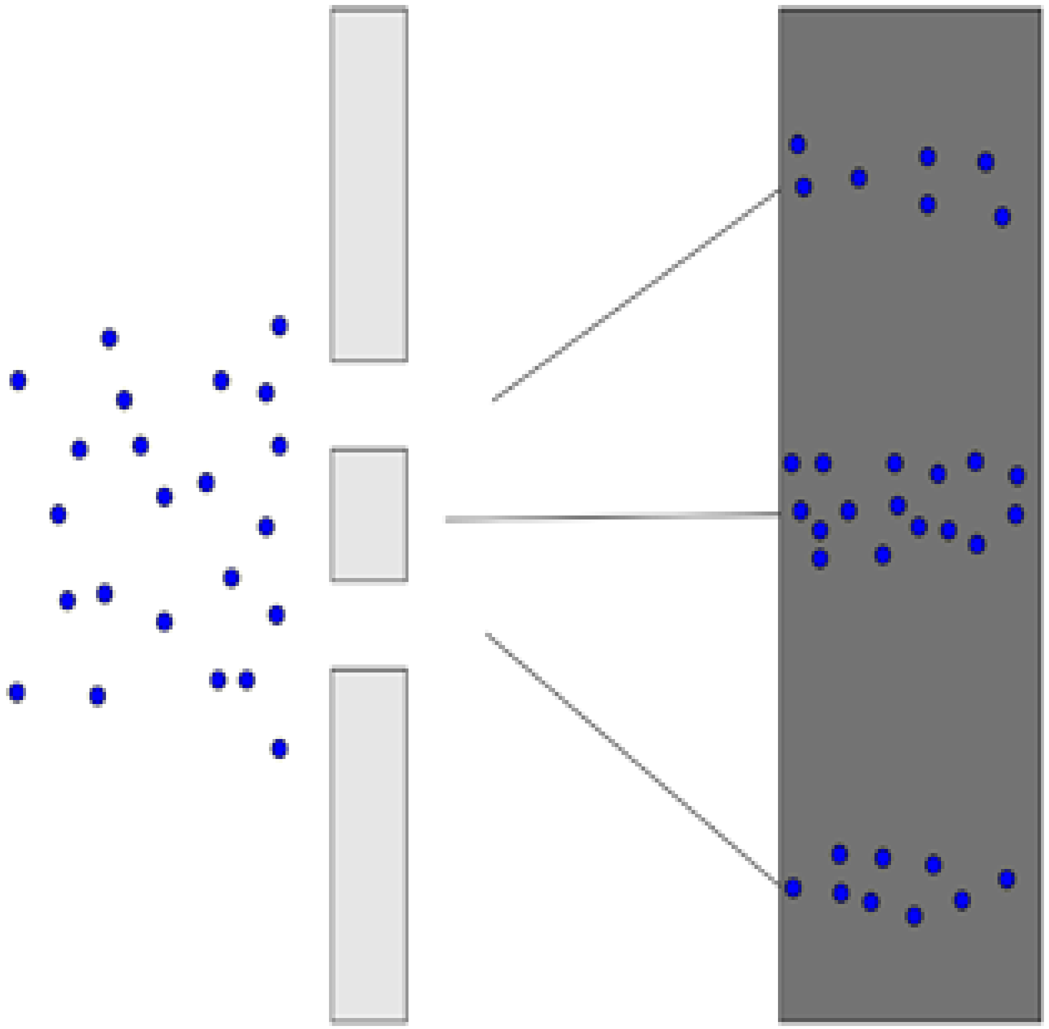
Light Wave interference pattern caused by diffraction



Electron Diffraction experiment

- Electrons undergo diffraction just like light waves do.

- <u>Particles also have wave properties!!!</u>

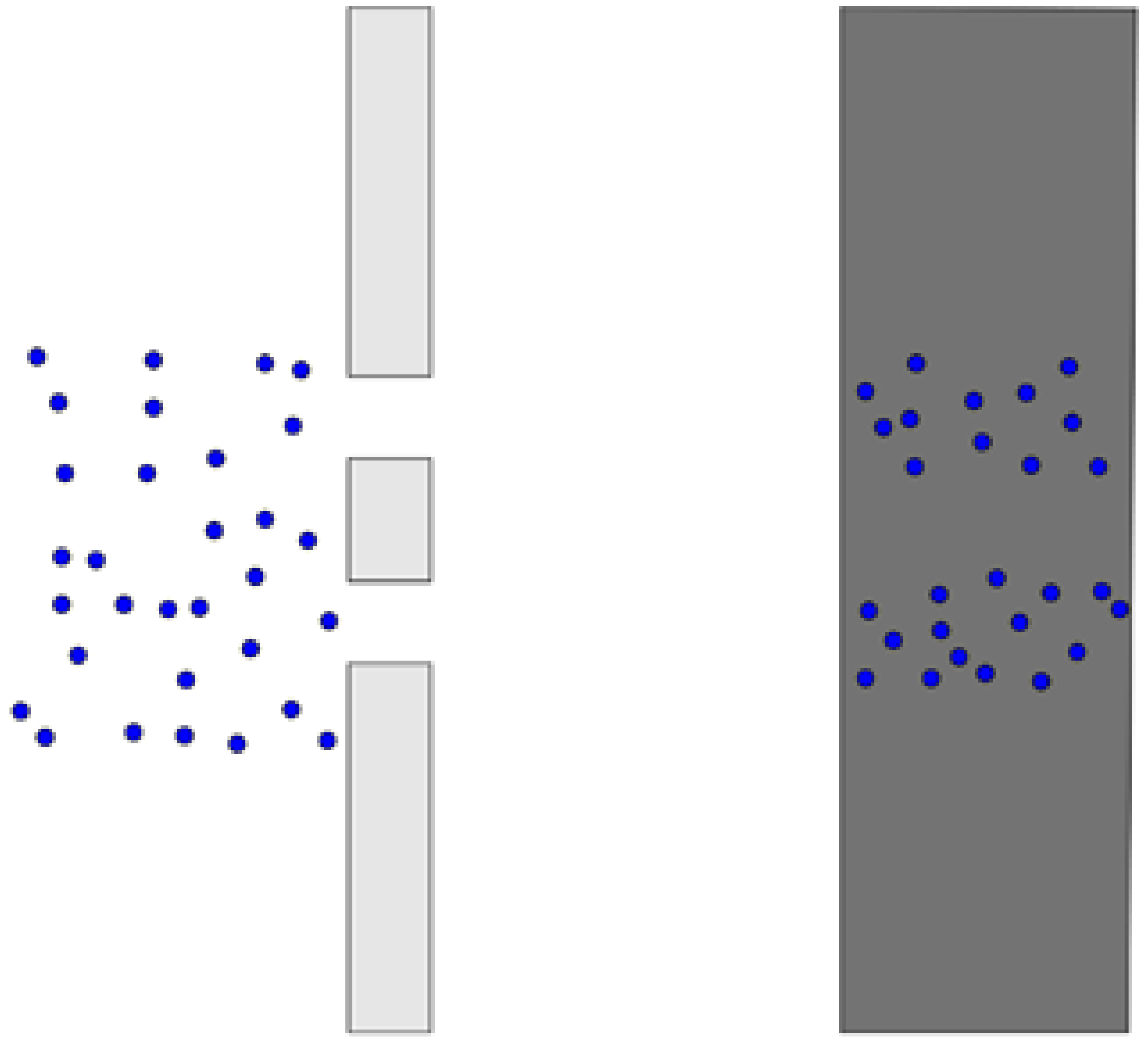


Does Big Objects exhibit wave particle duality?

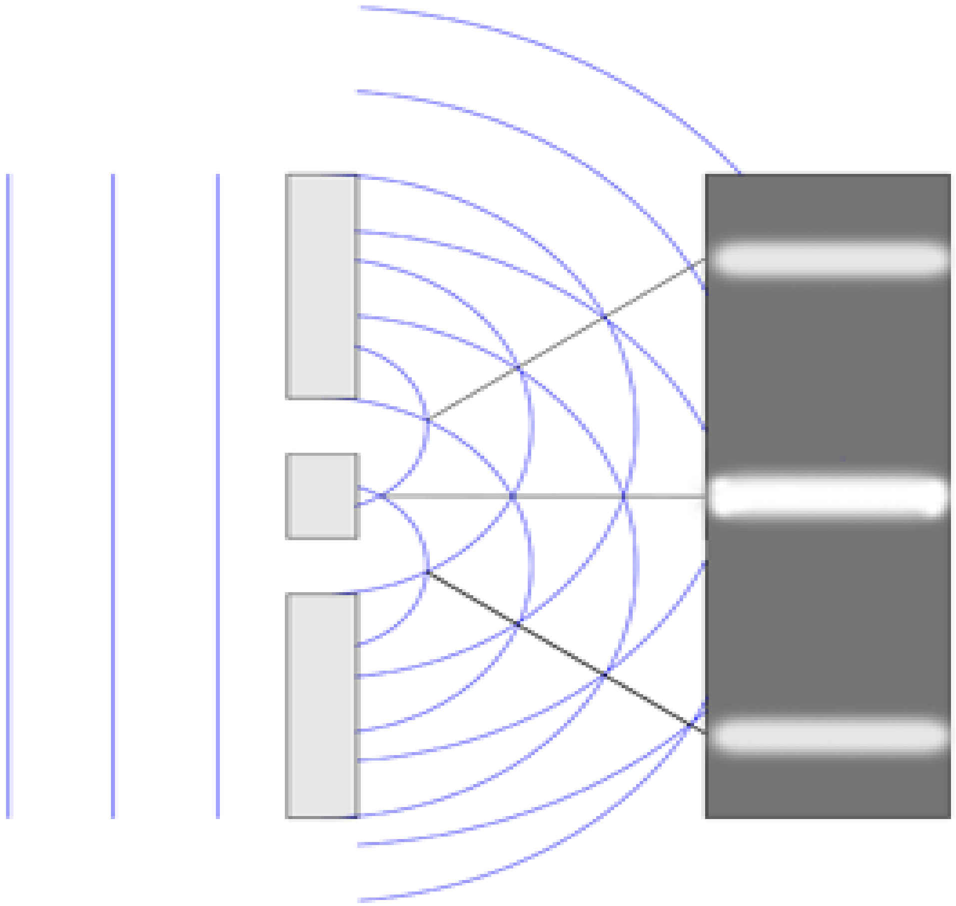
<u>Tennis ball through two slits</u>

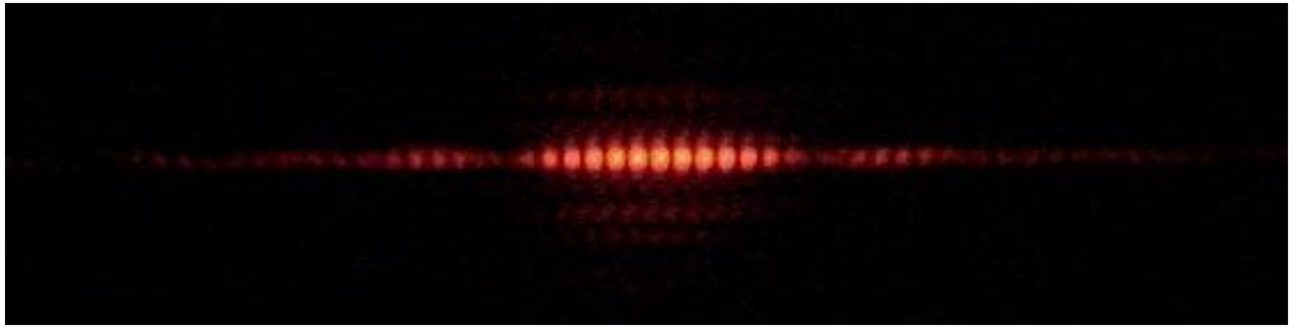
Tennis ball does not undergo diffraction.

<u>No wave properties for big objects!!</u>



Pattern made by electrons similar to those of Blue Light after passing through two slits





The De Broglie Wavelength

Louis de Broglie (1924) proposed *that* ***if EM radiation exhibits wave-particle duality, then perhaps material objects such as electrons may at times act like waves***.

For a photon $E = h\upsilon$ and $E=Pc$; $P=E/c$. Recall that $\upsilon=c/\lambda$ and $P=mv$ are the frequency and momentum of a photon

$$v = \frac{E}{h} \qquad \text{and} \qquad \lambda = \frac{h}{P}$$

thus the corresponding **de Broglie wavelength** (*a moving body is associated with de Broglie waves)* is given by

$$P = m\upsilon = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p} = \frac{h}{m\upsilon}$$

The de Broglie's relation, predicts the wavelength λ of matter associated with the motion of a material particle possessing momentum *P*.

$\Lambda = h/P = h/mv$

Examples:

- What is the de Broglie wavelength of **a ball of mass *m* = 1 kg**, moving at a speed $\upsilon = 10\ ms^{-1}$?

$$\lambda = \frac{h}{P} = \frac{h}{m\upsilon} = \frac{6.6\ x\,10^{-34}\ J-s}{1.0kg\ \ x10ms^{-1}} = 6.6\,x10^{-25}\ \overset{o}{A}.$$

- What is the de Broglie wavelength of **an electron** whose K.E. is 100eV?

$$\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mK}} = \frac{6.6\ x10^{-34}\ J-s}{\sqrt{\left(2x9.11x10^{-31}\,kgx100evx1.6x10^{-19}\ J\right)}} = \frac{6.6\,x10^{-34}\ J-s}{5.4\,x10^{-24}\ kg.ms^{-1}} = 1.2\,x10^{-10}\ m.$$

- *Thus for macroscopic (big) objects the wavelength associated with them is so small that the wave-like aspects of matter are negligible, hence the unobserved matter waves in daily life*

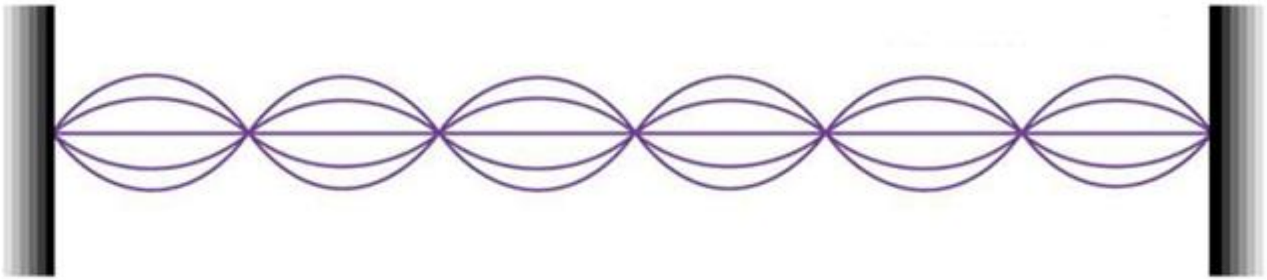
In terms of the de Broglie wavelength (λ), the Bohr's equation becomes

$$\frac{hr}{\lambda} = \frac{nh}{2\pi} \qquad\qquad 2\pi r = n\lambda$$

- Thus the allowed orbits are those in which the circumference ($2\pi r$) of the orbit can contain exactly an integral number of de Broglie wavelengths.

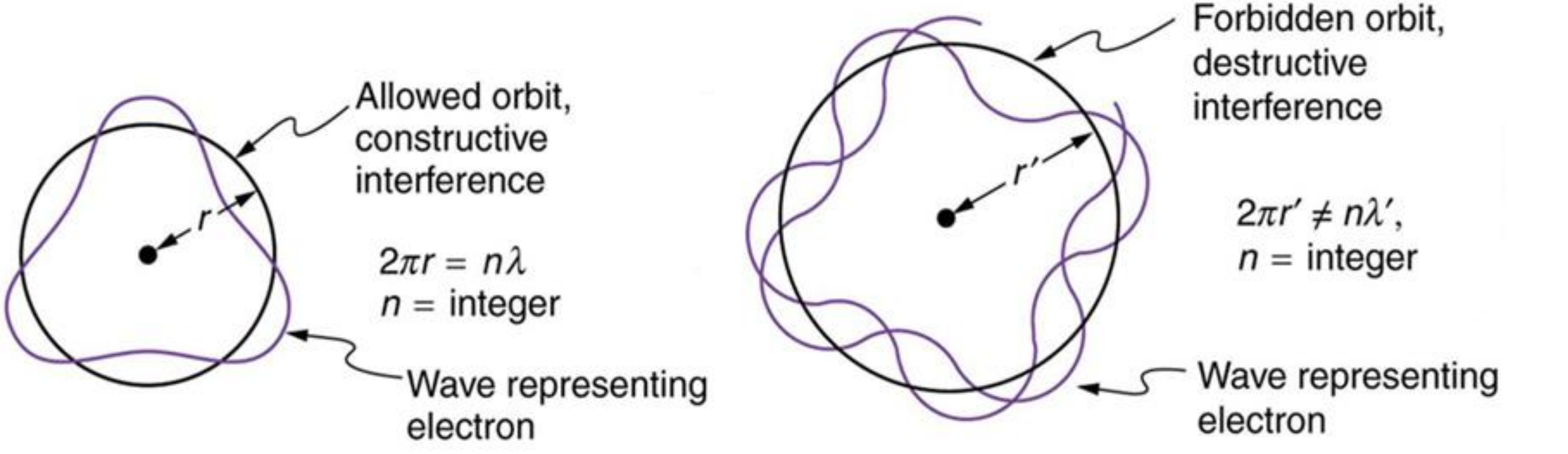
- Orbits where the circumference is not an exact multiple of the wavelength is not allowed.

Particle-wave nature of matter: **De Broglie** wavelength



(a)

The electron in Bohr's circular orbits could be described as a **standing wave**, one that does not move through space.



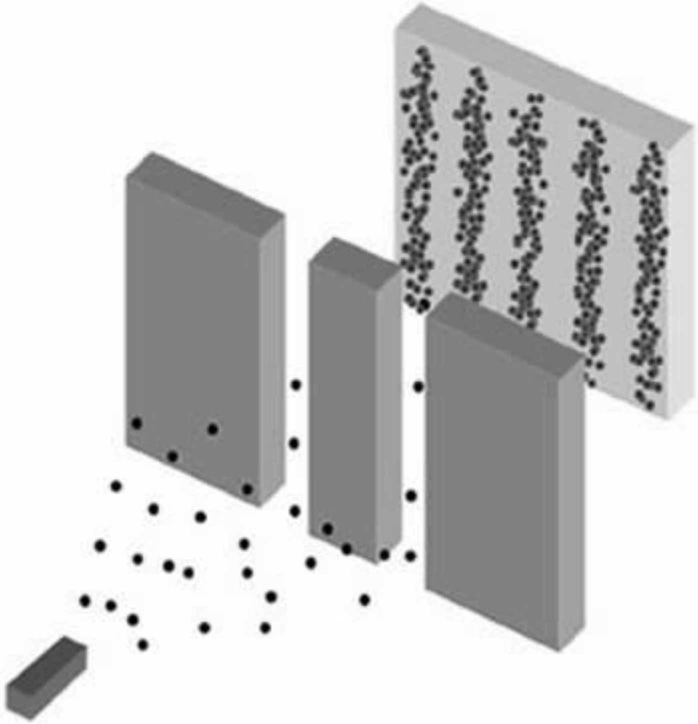
(b) (c)

$\lambda = h/p = h/mv$

Allowed orbit which corresponds to a complete de Broglie wave joined on itself

Prohibited orbit since the fractional number of wavelengths result in destructive interference

Example: DeBroglie Wavelength of an Electron



If an electron has a mass of $9.11 \times 10^{-31}\ kg$ and a velocity of $8000\ m/s$ what is the DeBroglie wavelength of that electron?

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{mv}$$

$$\lambda = \frac{6.626 \times 10^{-34}\ m^2 kg/s}{(9.11 \times 10^{-31}\ kg)(8000\ m/s)}$$

The Wave-Particle duality and the Complementarity's Principle

The particle and wave models are complementary; if a measurement confirms the wave nature of radiation of matter, then it is impossible to prove the particle nature in the same experiment, and vice-versa.

The probability Interpretation of de Broglie Waves

According to the **wave model**, the **intensity *I*** (energy per unit area per unit time) at a point on the screen is given by

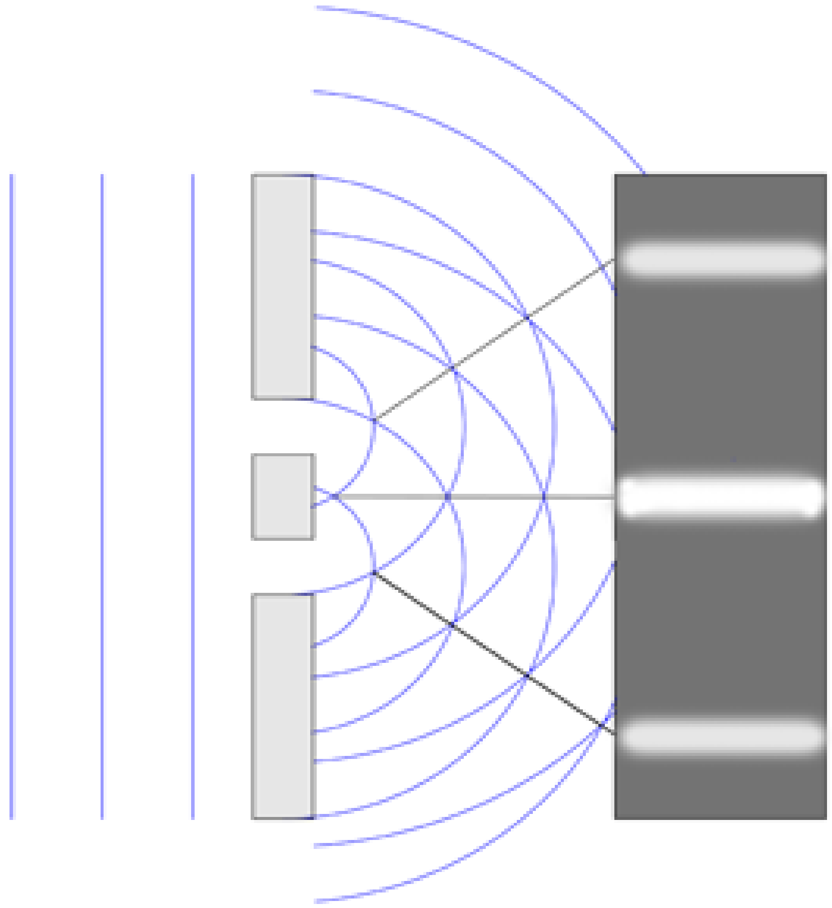
$$\mathrm{I} = c\varepsilon_o E^2$$

According to the photon picture, on the other hand, the intensity at a point is given by

$$I = h\nu N$$

N = the photon flux (number of photons per unit area per unit time) arriving at a particular point of the screen.

We **cannot predict** in advance **where any individual photon will strike** the screen, producing a flash. However, since the final pattern comprises alternating bright and dark bands, any photon has a *high probability* of striking a bright band and a *zero probability* of arriving at a dark band. ***N*** **is thus a measure of the probability of finding a photon near a point.**



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