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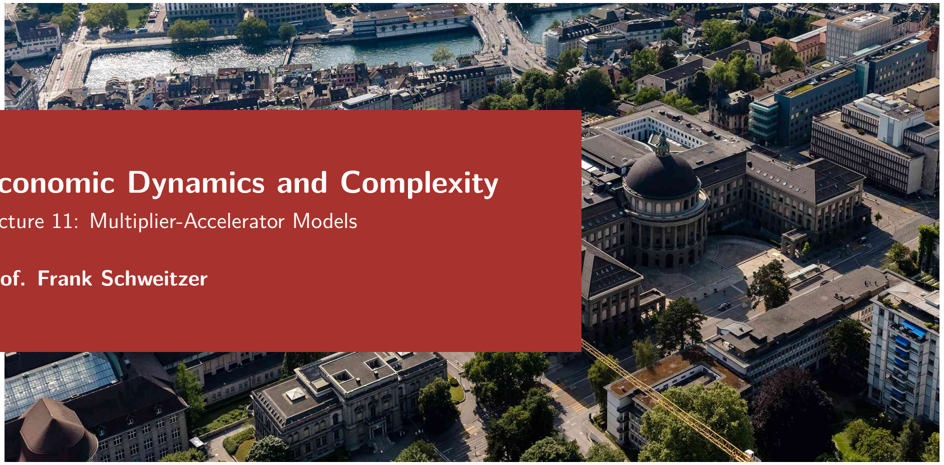


Economic Dynamics and Complexity

Lecture 11: Multiplier-Accelerator Models

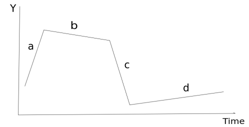
Prof. Frank Schweitzer

Notes:

Standard Definition of Business Cycle

Business cycles are a type of *fluctuation* found in the *aggregate economic activity of nations* that organize their work mainly in business enterprises:

A cycle consists of *expansions* occurring at about the same time in many economic activities, followed by similarly general *recessions*, *contractions*, and *revivals* which merge into the expansion phase of the next cycle.



In duration, business cycles *vary* from more than one year to ten or twelve years; they are *not divisible* into shorter cycles of similar characteristics with amplitudes approximating their own.

Arthur F. Burns and Wesley C. Mitchell (1946)

Notes:

Source: A. F. Burns and W. C. Mitchell, Measuring business cycles, New York, National Bureau of Economic Research, 1946.

- The four stages in this definition often have slightly differing names: *boom (expansion), recession, depression (contraction), recovery (revival)*

Reasons for oscillations

- **Shocks**
  - Most often: **external**, but also internal shocks (e.g. demand, inflation rate)
- **Prediction errors**
  - Most often for expected price, but also for expected consumption ($\rightarrow$ inventory)
- **Delays**
  - (i) Different time scales (fast, slow), (ii) Lagged response (supply, price, inventory)
- **Nonlinear functions**
  - Supply and demand curves, delay equations
- **Couplings**
  - Between markets, between capital stock and production

Notes:

Outline

Nonlinear supply and demand

Cobwebs revisited

Multiplier-accelerator models

Notes:

Nonlinear demand curve

- **Remember: Elasticities** $\to$ relative changes

$$E_d = \frac{\Delta Q_d/Q_d}{\Delta p/p} < 0\,;\; E_s = \frac{\Delta Q_s/Q_s}{\Delta p/p} > 0$$

- **Linear supply curve:** $Q_s = \gamma + \delta p$
  - Constant unitary elasticity: $E_s = 1$, $\gamma = 0$ (no basic supply)

$$\Delta Q_s = \frac{Q_s}{p}\Delta p = \frac{\gamma + \delta p}{p}\Delta p = \delta \Delta p$$

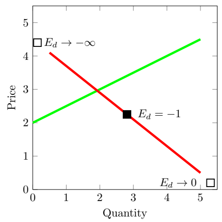
- **Linear demand curve**: $Q_d = \alpha - \beta p$

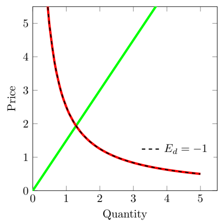
$$\Delta Q_d = -\frac{Q_d}{p}\Delta p = \frac{\beta p - \alpha}{p}\Delta p = \left[\beta - \frac{\alpha}{p}\right]\Delta p$$

  - $\alpha \neq 0$: always basic demand $\to$ $E_d = -1$ only at $\alpha = 2\beta\, p$
- **Nonlinear demand curve**: $Q_d = \beta p^{-\varepsilon}$

$$dQ_d = -\beta\varepsilon p^{-\varepsilon-1}dp\,;\; \frac{dQ_d}{Q_d} = -\varepsilon\frac{dp}{p}$$

  - **Always** constant unitary elasticity $E_d = -1$ if $\varepsilon = 1$





Notes:

Let's face it: The linear demand curve in the above graph is: $Q = 4.5 - 0.8p$. Then $E_d = -1$ implies

$$\frac{\Delta Q/Q}{\Delta p/p} = -1 \to \frac{\Delta Q}{\Delta p} = -\frac{Q}{p}$$

$$\frac{\Delta Q}{\Delta p} = -0.8 = -\frac{4.5 - 0.8p}{p} \to p = \frac{4.5}{1.6} = 2.81$$

$$Q = 4.5 - 0.8 \times 2.81 = 2.25$$

Example: Productivity growth

- **Modeling assumptions**
  1. **Demand** for patents: $Y_t \to$ Exponential growth: $Y_{t+1} = a + bY_t$
  2. Industry growth rate q_t (productivity) depends on patents: $q_t \propto Y_t$
  3. Higher growth rates lead to relative price increase:
  4. Nonlinear demand curve with constant elasticity ε

$$\frac{p_{t+1} - p_t}{p_t} = \frac{\Delta p_t}{p_t} = \nu q_{t+1}\,;\quad \frac{\Delta Y_t/Y_t}{\Delta p_t/p_t} = -\varepsilon$$

- **Dynamics:** *Logistic map*

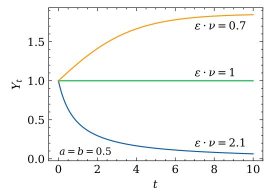
$$\frac{\Delta Y_t}{Y_t} = \frac{Y_{t+1} - Y_t}{Y_t} = -\varepsilon\nu q_t = -\varepsilon\nu(a + bY_t)$$

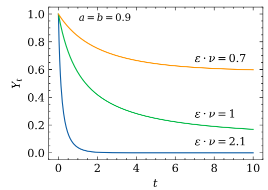
$$Y_{t+1} = (1 - a\varepsilon\nu)Y_t - b\varepsilon\nu Y_t^2$$

- **Solution:**

$$Y_t = \frac{(1 - a\varepsilon\nu)Y_0}{b\varepsilon\nu Y_0 + (2 - a\varepsilon\nu)^{-t}(1 - a\varepsilon\nu - b\varepsilon\nu Y_0)}$$

Solutions depend on ν, ε





Notes:

- The example follows Shone (2002) p. 121, who attributes it to Baumol and Wolff (1992)

Nonlinear supply curve

- Supply depends on **expected** price p_t^e

$$Q_t^s = \arctan(\mu p_t^e)\ ; \quad Q_t^d = a - bp_t$$
$$p_t^e = p_{t-1}^e + \lambda\left(p_{t-1} - p_{t-1}^e\right)$$

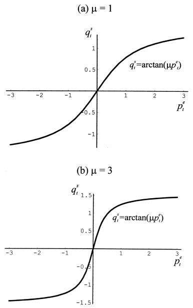
- Price dynamics from market clearing: $Q_t^s = Q_t^d$

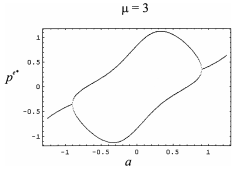
$$p_t = \frac{a - Q_t^d}{b} = \frac{a - Q_t^s}{b} = \frac{a - \arctan(\mu p_t^e)}{b}$$
$$p_t = \frac{p_{t+1}^e - (1-\lambda)p_t^e}{\lambda}$$

- Dynamics of **expected price**

$$p_{t+1}^e = (1-\lambda)p_t^e + \frac{\lambda a}{b} - \frac{\lambda \arctan(\mu p_t^e)}{b}$$

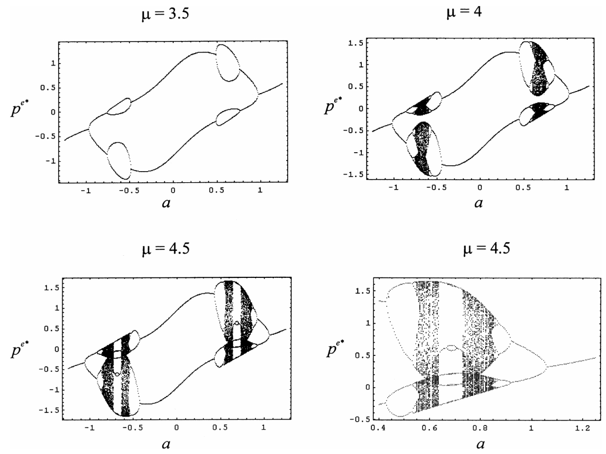
- **Bifurcation diagram** $(p_e^\star, a)$ depends on μ





Period doubling scenario and chaos

- Strong oscillations: Limited predictability of expected price
- Critical values of μ depend on a, b, c, d
- Sensitivity to initial conditions



Shone, p. 366

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The example is taken from Shone (2002), p. 363, who attributes it to Hommes (1991)

Outline

Nonlinear supply and demand

Cobwebs revisited

Multiplier-accelerator models

Cobwebs in interdependent markets

- Market 1 supplies market 2 (but no switch of suppliers!)
  - Market 1: computer chips → "c"
  - Market 2: computer hardware → "h"

$$d_t^c = a_1 - b_1 p_t^c \qquad d_t^h = a_2 - b_2 p_t^h$$
$$s_t^c = c_1 + d_1 p_{t-1}^c \qquad s_t^h = c_2 + d_2 p_{t-1}^c + e p_{t-1}^c$$

  - Market 2 depends on expected price of chips $\hat{p}_t^c = p_{t-1}^c$, $e < 0$!
  - Separate market clearing: $d_t^c = s_t^c$, $d_t^h = s_t^h$
- **Equilibria** $$p_c^\star = \frac{a_1 - c_1}{b_1 + d_1}\ ;\ p_h^\star = \frac{a_2 - c_2}{b_2 + d_2} - \frac{e}{b_2 + d_2}\left(\frac{a_1 - c_1}{b_1 + d_1}\right)$$
- **Dynamics** close to equilibrium

$$p_t^c - p_c^\star = -\left(\frac{d_1}{b_1}\right)\left(p_{t-1}^c - p_c^\star\right)$$
$$p_t^h - p_h^\star = -\left(\frac{d_2}{b_2}\right)\left(p_{t-1}^h - p^\star\right) - \left(\frac{e}{b_2}\right)\left(p_{t-1}^c - p_c^\star\right)$$

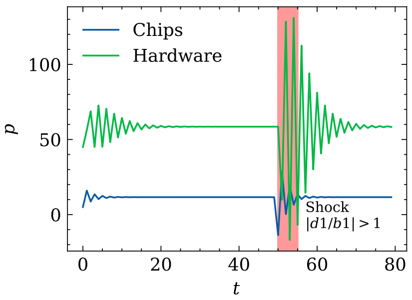
Notes:

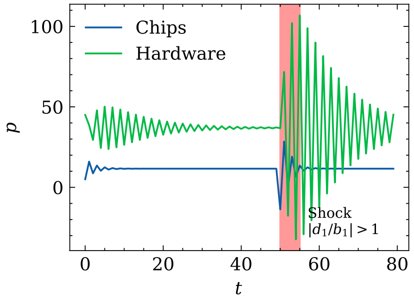
Notes:

The equations should all look familiar to you. If not, go back to the previous lecture and recapitulate the cobweb dynamics. The example is taken from Shone (2002), p. 346, who attributes it to Ezekiel (1938) and Waugh (1964).

Can supply market 1 destabilize market 2?

- Stability condition for market 1: $|-d_1/b_1| < 1$
- Market 2: $|-d_2/b_2| < 1$ **only for fixed prices** $p_c^\star$
- Assume shocks $|-d_1/b_1| > 1$ at random periods of time $t_e^i - t_a^i$





Solving 2nd order difference equations

- Non-homogeneous equation:

$$y_{t+2} + a y_{t+1} + b y_t = c$$

- General solution $y_t = \hat{y} + y^c$
- Particular solution $\hat{y} \to y^\star$ (equilibrium for y_t)

$$y^\star + a y^\star + b y^\star = c \to y^\star = \frac{c}{1 + a + b}$$

- Complimentary solution $y^c \to y_t = r_1 x_1^t + r_2 x_2^t$
  - Constants $r_{1,2}$ from

$$y_0 = r_1 x_1^0 + r_2 x_2^0 = r_1 + r_2 \; ; \; y_1 = r_1 x_1 + r_2 x_2$$
$$r_1 = \frac{y_1 - r_2 y_0}{r_1 - r_2} \; ; \; r_2 = \frac{y_1 - r_1 y_0}{r_2 - r_1}$$

- $x_{1,2}$: solutions of $x^2 - ax - b = 0$ (characteristic equation)

$$x_{1,2} = \frac{a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2}$$

- Distinct real roots: $a^2 > 4b$ $\to$ *damped* oscillations
- Equal real roots: $a^2 = 4b$ $\to$ *stable* oscillations
- Complex roots: $a^2 < 4b$ $\to$ *explosive* oscillations

Notes:

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Cobweb with inventory

- Inventory S_t: Built up from excess supply
- Market clearing does not hold, price not set from $Q^s_t = Q^d_t$
- Price p_t decreases if inventory grows

$$\Delta S_t = S_t - S_{t-1} = Q^s_t - Q^d_t \;;\; p_t - p_{t-1} = -\gamma \Delta S_{t-1}$$

- Linear curves: $Q^d_t = a - bp_t$, $Q^s_t = c + dp_t$ (not: $t-1$)

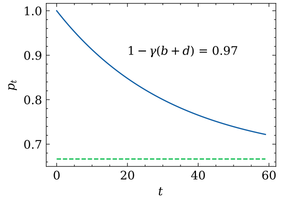
$$p_t = \gamma(a-c) + [1 - \gamma(b+d)]p_{t-1} \to p^\star = \frac{(a-c)}{(b+d)}$$

- Solution for 1st order difference equation (near equilibrium)

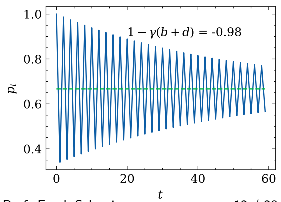
$$(general:)\; y_{t+1} = ay_t + c \;\to\; y_t = y^\star + a^t[y_0 - y^\star]$$

$$p_t = \frac{a-c}{b+d} + [1-\gamma(b+d)]^t \left(p_0 - \frac{a-c}{b+d}\right)$$

$0 < [1 - \gamma(b+d)] < 1$



$-1 < [1 - \gamma(b+d)] < 0$



Cobweb with target inventory

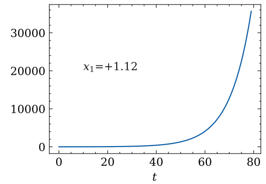
- Price changes if inventory deviates from target inventory $\hat{S}$

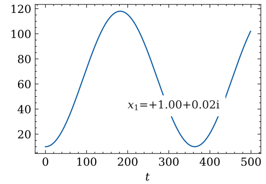
$$p_t - p_{t-1} = -\gamma\left[S_{t-1} - \hat{S}\right]$$
$$p_{t-1} - p_{t-2} = -\gamma\left[S_{t-2} - \hat{S}\right]$$
$$p_t - p_{t-1} = p_{t-1} - p_{t-2} - \gamma\,[S_{t-1} - S_{t-2}]$$
$$p_t = \gamma(a-c) + [2 - \gamma(b+d)]\, p_{t-1} - p_{t-2}$$

- General dynamics: $p_t = p^\star + r_1 x_1^t + r_2 x_2^t$
- $x_{1,2} \to$ characteristic equation: $x^2 - [2 - \gamma(b+d)]x + 1 = 0$

$$x_{1,2} = \frac{[2-\gamma(b+d)]}{2} \pm \frac{\sqrt{[2-\gamma(b+d)]^2 - 4}}{2} \qquad |x_1| > |x_2|$$

- If $[2-\gamma(b+d)]^2 \geq 4$ then both $x_{1,2}$ are real and
  - if $|x_1| > 1$ then $p_t \to \pm\infty$ (unstable)
  - if $-1 < x_1 < 1$ then $p_t \to 0$ (stable)
- If $[2-\gamma(b+d)]^2 < 4$ then both $x_{1,2}$ are complex leading to oscillations





Notes:

The example is taken from Shone (2002), p. 349
Remember how 1st order differential equations are solved. This was covered in Lecture 10: "Dynamics close to equilibrium"

Notes:

The example is taken from Shone (2002), p. 350

Outline

Nonlinear supply and demand

Cobwebs revisited

Multiplier-accelerator models

Lagged consumption

- Shift of perspective: Output $Y_t \to$ **Income** Y_t
  - Intermediate variable: Total expenditure $E_t = I_t + C_t \to Y_t = E_t$
  - $I_t = I_0 =$ const., Requires to model (only) consumption: C_t
  - Consider time lags $\to$ 1st step: separate dynamics
- **General assumption**: $C = c_0 + cY_t$
  - c: "propensity to *consume*", c_0: basic consumption
  - Equilibrium for income: $Y^\star = (c_0 + I_0)/(1-c)$
- **Lagged consumption:** $C_t = c_0 + cY_{t-1}$ OR **lagged expenditure:** $Y_t = E_{t-1}$
  - Same recursive equation: $Y_t = (c_0 + I_0) + cY_{t-1}$
  - same equilibrium: $Y^\star = (c_0 + I_0)/(1-c)$
- **Dynamics:** (Remember: $y_{t+1} = ay_t + c \to y_t = y^\star + a^t[y_0 - y^\star]$)

$$Y_t = \frac{c_0 + I_0}{1-c} + c^t\left(Y_0 - \frac{c_0 + I_0}{1-c}\right) \quad \Rightarrow \quad C_t = \frac{c_0 + cI_0}{1-c} + c^{t+1}\left(Y_0 - \frac{c_0 + I_0}{1-c}\right)$$

Notes:

Notes:

The example is taken from Shone (2002), p. 425
You should not confuse this model with the neoclassical growth model. There we introduced a savings rate s with $C = (1-s)Y$ and $I = sY$. Here, in contrast, we assume a constant investment I_0 and a consumption that only partly increases with the available income, cY, but has a baseline c_0.

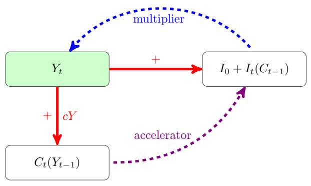
Multiplier-accelerator models

- **Aim:** Explain *business cycles* from lagged response
  - Assume a shock of demand at time t: Decrease consumption by increasing investment $I_0 + \Delta I$
  - How will the available income Y_{t+1} change?
  - New equilibrium after some time: $\hat{Y}^{\star} = (c_0 + I_0 + \Delta I)/(1-c)$
- *1. Feedback cycle:* Investment increases income
  - **Income multiplier** $\Delta Y_t = \alpha_t \Delta I$
  - $\alpha_t \to \alpha = (\hat{Y}^{\star} - Y^{\star})/\Delta I$: Change between equilibria
- *2. Feedback cycle:* Increase in income induces increase in investment
  - **Lagged investment** $\to$ **Accelerator** $\Delta Y_t = \beta \Delta Y_{t-1}$

$$I_0 + I_t = I_0 + \beta(Y_{t-1} - Y_{t-2}) = I_0 + \beta(C_t - C_{t-1})$$

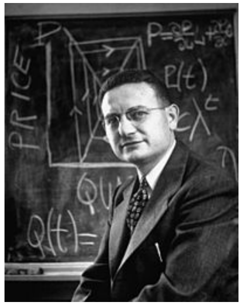
- **Different forms of accelerators**
  - Example: Duesenberry (1951): $I_t = \beta Y_{t-1} - \kappa K_{t-1}$

$$I_t = K_t - K_{t-1} \to Y_{t-1} = (c+\beta) Y_{t-2} - \kappa K_{t-2}$$



Samuelson

Samuelson's Multiplier-Accelerator Model



- Studied at University of Chicago (age 16)
- PhD from Harvard, teaching at MIT
- First American to win a Nobel prize in 1970
- Best-selling textbook, *Economics: An Introductory Analysis*
- Seminal figure in the development of neoclassical economics
- Mathematical analysis provides foundation for modern economics

- Multiplier α: *consumers spend* fraction of income on investment $\to$ increase of income
- Accelerator β: Increase of income induces later *increase of investment* $\to$ Further increase of income

Notes:

Notes:

Samuelson's Multiplier-Accelerator Model

- For a *closed* economy: Net income fully spend: $\boxed{Y_t = C_t + I_t}$
- *Consumption* C_t in period t depends on Y_{t-1} in $(t-1)$: $\boxed{C_t = cY_{t-1}}$
  - $0 < c < 1$: Propensity to consume
- *Investment* I: $\boxed{I = I_0 + I_t^{ind}}$
  - *autonomous* component: $I_0 = \mathrm{const},$
  - *induced* component $I_t^{ind} = \beta(C_t - C_{t-1})$, Investment *accelerator* $\beta > 0$
- *Net national income* in each period:

$$Y_t = \underbrace{cY_{t-1}}_{C_t} + \underbrace{I_0 + \beta(C_t - C_{t-1})}_{I_t}$$

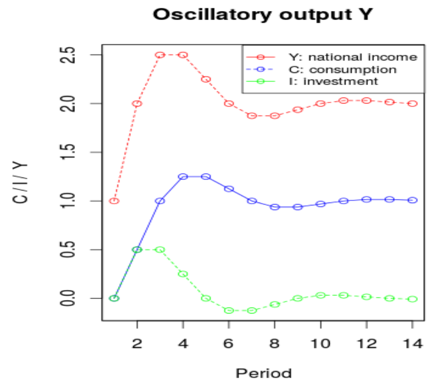
  - Second-order, linear, non-homogenous *difference equation* using $C_t = cY_{t-1}$:

$$Y_t = I_0 + c(1+\beta)Y_{t-1} - c\beta Y_{t-2}$$

(*Unit: one dollar*)

Period	Current governmental expenditure	Current consumption induced by previous expenditure	Current private investment proportional to time increase in consumption	Total national income
1	1.00	0.00	0.00	1.00
2	1.00	0.50	0.50	2.00
3	1.00	1.00	0.50	2.50
4	1.00	1.25	0.25	2.50
5	1.00	1.25	0.00	2.25
6	1.00	1.125	−0.125 *	2.00
7	1.00	1.00	−0.125	1.875
8	1.00	0.9375	−0.0625	1.875
9	1.00	0.9375	0.00	1.9375
10	1.00	0.96875	0.03125	2.00
11	1.00	1.00	0.03125	2.03125
12	1.00	1.015625	0.015625	2.03125
13	1.00	1.015625	0.00	2.015625
14	1.00	1.0078125	−0.0078125	2.00
.. ..	...	....	....	....

* Negative induced private investment is interpreted to mean that for the system as a whole there is *less* investment in this period than there otherwise would have been. Since this is a marginal analysis, superimposed implicitly upon a going state of affairs, this concept causes no difficulty.



- *Feedback* causes (intermediate) *oscillatory* behavior of Y.
- *Equilibrium* can still be reached

Notes:

Literature:

- Paul A. Samuelson "Interactions between the Multiplier Analysis and the Principle of Acceleration" (1939)
- Guenter Gabisch and Hans-Walter Lorenz "Business Cycle Theory. A Survey of Methods and Concepts"(1989)
- M. Lines and F. Westerhoff "Expectations and the Multiplier-Accelerator Model" (2006)
- Victor Zarnowitz "Recent Work on Business Cycles in Historical Perspective: A Review of Theories and Evidence" (1985)

Notes:

- I_0: current government expenditures, constant
- In the table $c = 0.5$, $\beta = 1$

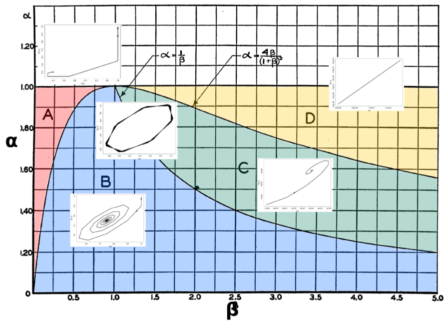
Source:
Paul A. Samuelson "Interactions between the Multiplier Analysis and the Principle of Acceleration" (1939)

Equilibrium Solutions

- Equilibrium condition: $Y_t = Y_{t-1} = Y_{t-2}$
- Possible *solutions*:
  - unstable
  - stable (fixed point, oscillations)
- Particular equilibrium solution:

$$Y^\star = \frac{I_0}{1-c}$$

  - This implies: $C^\star = cY^\star$ and $I^\star = I_0$
- *Stability* of the *fixed point* requires $c < \frac{1}{\beta}$



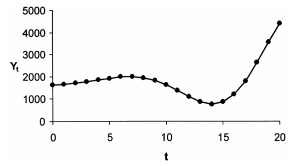
Top layer plots: *phase plane* plots with Y_t on the x-axis and Y_{t+1} on y-axis.

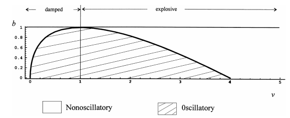
Dynamics

- Lagged income:
  $Y_t = C_t + I_t = c_0 + cY_{t-1} + I_0 + \beta(Y_{t-1} - Y_{t-2})$
  - 2nd order non-homogeneous recursive equation
  - Particular solution $Y_t = Y^\star \to Y^\star = (c_0 + I_0)/(1-c)$ equals the simple multiplier result
- Homogeneous equation: $Y_t - (c+\beta)Y_{t-1} + \beta Y_{t-2} = 0$
  - Characteristic equation: $x^2 - (c+\beta)x + \beta = 0$
  - Solutions:

$$x_{1,2} = \frac{(c+\beta)}{2} \pm \frac{\sqrt{(c+\beta)^2 - 4\beta}}{2}$$

- General form: $Y_t = Y^\star + r_1 x_1^t + r_2 x_2^t$
- The long term dynamics are determined by x_1 and
  - $\beta > 0$ divergence
  - $(\alpha+\beta)^2 < 4\beta$ oscillations





Notes:

I_0 := government expenditures Y := national income
What is plotted here?

- Region A: "If there is a constant level of governmental expenditure [I_0] through time, the national income [Y] will approach asymptotically a value $\frac{1}{1-c}$ times the constant level of governmental expenditure . Perfectly periodic net governmental expenditure will result eventually in perfectly periodic fluctuations in national income."
- Region B: "A constant continuing level of governmental expenditure [I_0] will result in damped oscillatory movements of national income [Y], gradually approaching the asymptote $\frac{1}{1-c}$ times the constant level of governmental expenditure. Perfectly regular periodic fluctuations in government expenditure will result eventually in fluctuations of income of the same period for $c = \frac{1}{\beta}$."
- Region C: "A constant level of governmental expenditure [I_0] will result in explosive, ever increasing oscillations around an asymptote."
- Region D: "A constant level of governmental expenditure [I_0] will result in an ever increasing national income [Y]."

Source: Guenter Gabisch and Hans-Walter Lorenz "Business Cycle Theory. A Survey of Methods and Concepts"(1989)

Notes:

SS11: 2nd Order Difference Equations and Multiplier-Accelerator Models

- Learn how to solve 1st and 2nd order difference equations
  - Find equilibria of a discrete system
  - Characterize behavior using the characteristic equation
  - Derive an analytical solution for 2nd order difference equations
- Explore the stability and evolution of multiplier-accelerator models

Hicks' Model

- **John Hicks (1904-1989)**
  - Knighted in 1964
  - Nobel prize Economics: work on general equilibrium, welfare theory (1972)
  - Prolific in areas as diverse as money and international trade, growth and fluctuations, industrial relations and comparative statics
- **Nonlinear trade cycle model** (1950)
  - C_t, I_t and Y_t similar to *Samuelson's* model: $I_t = \beta(Y_t - Y_{t-1})$ instead of $I_t = \beta(Y_{t-1} - Y_{t-2})$
  - proposes *upper* bound for *output* and *lower* bound for *investment*
  - *prevents explosion* for production

Notes:

Notes:

Source: `https://www.nobelprize.org/prizes/economic-sciences/1972/hicks/facts/`
Sources:

- Duesenberry James, *Hicks on the Trade Cycle*, 1950
- Knox A., *On a Theory of the Trade Cycle*, 1950
- Arndt H., *Mr. Hicks's Trade Cycle Theory*, 1951
- Hommes C., *Periodic, almost periodic and chaotic behaviour in Hicks' non-linear trade cycle model*, 1993

Hicks' Model

- **Problem 1** with Samuelson's model: I_t^{ind} can become **negative**
  - **Solution:** make sure that $I = I_0 + I_t^{\text{ind}} \geq 0$
  - **lower bound** I^f, such that *induced* investment: $\tilde{I}_t^{\text{ind}} = \max[\, I_t^{\text{ind}}, -I^f]$
  - if: $I^f = I_0 \Rightarrow$ *Total* investment: $I_t = I_0 + \tilde{I}_t^{\text{ind}} \geq 0$
- **Problem 2** with Samuelson's model: if $I_t^{\text{ind}} > 0$, Y_t can be come **very large**
  - **Solution:** make sure that $Y_t \leq Y^c \Rightarrow$ **upper bound**
    - when output rises, cost of raw materials do and labour force becomes limiting (*diseconomy of scale*)
    - *bounding output* implies *bounding investment*
- **Income dynamics:** $\boxed{Y_t = \min[\, C_t + I_t, Y^c]}$
  - *three different types of dynamics*
    - *fixed* point: $Y_t = Y_{t-1}$ gives $Y^\star = I_0/(1-c)$
    - *periodic* oscillations
    - *aperiodic* oscillations

Hicks' Model

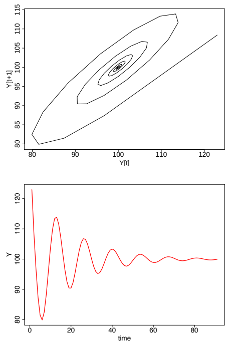


Figure: Damped Oscillations
$\beta = 0.9$, $c = 0.8$

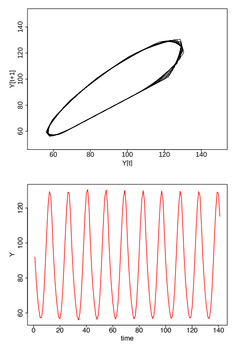


Figure: Aperiodic cycles
$\beta = 1.25$, $c = 0.75$

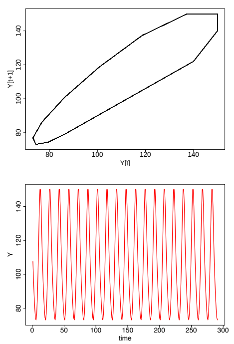


Figure: Periodic cycles
$\beta = 1.25$, $c = 0.8$

Notes:

Sources:

- Gallegati et. el., *Hicks' trade cycle revisited: cycles and bifurcations*, 2003
- Hommes Cars, *Periodic, almost periodic and chaotic behaviour in Hicks' non-linear trade cycle model*, 1993

Notation:

- $Y^c :=$ ceiling for the output Y_t
- $-I^f :=$ floor for the investment I_t^{ind}
- negative net investment, $-I^f$, corresponds to zero gross investment
- $Y^\star$: Income at the fixed point

Notes:

Other parameters:

- $Y^c = 150$, $I^f = 10$, $I_t^a = 20$, $Y_1 = Y_2 = 1$

Note that the aperiodicity in the middle column can be seen only on the phase plot (top).

Exogenous growth of autonomous investment

- $I_t = I_t^a + I_t^{\mathrm{ind}}$, $I_t^a = I_0(1+g)^t$
- Dynamics of income (using Samuelson's lag)

$$Y_t = C_t + I_t = (c+\beta)Y_{t-1} - \beta Y_{t-2} + I_0(1+g)^t$$

- Moving equilibrium at time t: $Y_t \to \hat{Y}(1+g)^t$

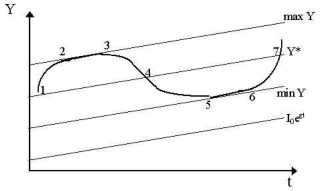
$$\hat{Y}(1-g)^t - (c+\beta)\hat{Y}(1-g)^{t-1} + \beta\hat{Y}(1-g)^{t-2} = I_0(1-g)^t$$

- Solution with $g=0 \to \hat{Y} = Y^\star = I_0/(1+c)$

$$\hat{Y} = \frac{I_0(1+g)^2}{(1+g)^2 - (c+\beta)(1+g) + \beta}$$

- Complete dynamics (Discussion $\to$ Samuelson model ....)

$$Y_t = \hat{Y}(1+g)^t + r_1 x_1^t + r_2 x_2^t$$
$$x_{1,2} \to x^2 + (b+\nu)x + \nu = 0$$



Discussion: Pros and Cons

- Different *scenarios* dependent on *control parameters* c, β
  - Most solutions *explode* or *die out* $\to$ *Non-realistic* outcome for most c, β
  - *Permanent* business *cycles* only for a *boundary* case of $c = 1/\beta$
- *Oscillations* can be either *periodic* or *aperiodic*
  - Cycles with fixed period and amplitude - *at odds* with *empirical* evidence
  - Each periodic time path has a *unique period* of oscillation
- Varying β gives business cycles of any *desirable length*
  - (c) and (β) are constant, *but* in reality change with the level of income
- Nonlinear Hicks model cures **some** shortcomings
  - Model's *behavior* does *not* depend on *upper* bound Y^c
- **General criticism:**
  - *Anticipation* on investment decisions *not* considered
  - *Cyclic* behavior is still *exogenously* generated

Notes:

The example is taken from Shone (2002), p. 126

The figure is from: `http://www.hetwebsite.net/het/essays/multacc/image/hicksac2.jpg`

Notes:

Questions

1. Explain the meaning of “business cycles”. Why is not every oscillation a business cycle?
2. Derive the conditions to observe a demand elasticity $E_d = -1$ for the linear and the nonlinear demand curve.
3. Explain the role of the expected price in supply-demand models? See also previous lecture.
4. Why behaves a cobweb with inventory so differently from a cobweb with *target* inventory?
5. What is a multiplier-accelerator model? Why are the amplifiers called “multiplier” and “accelerator”?
6. Compare the ideas of Samuelson and Hicks regarding “investment”. What are their drawbacks?*

*This website may provide further arguments: `http://www.hetwebsite.net/het/essays/multacc/multacc.htm`

Notes: