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Chair of Systems Design
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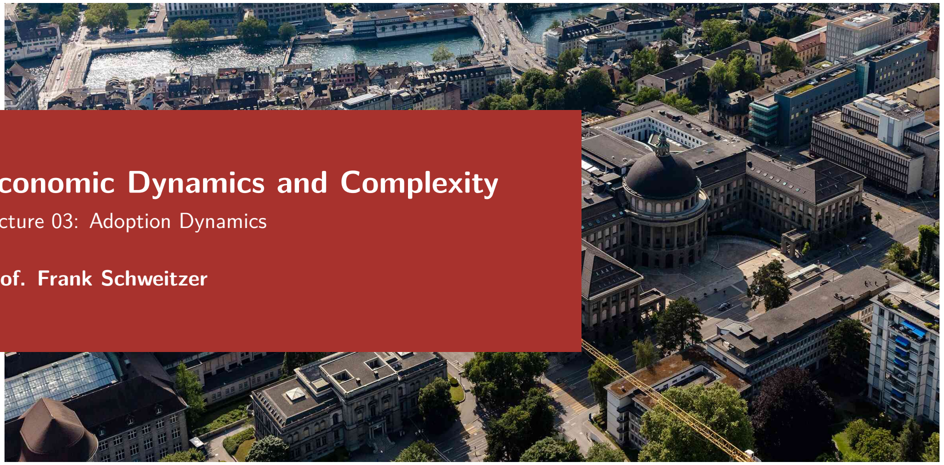
Economic Dynamics and Complexity

Lecture 03: Adoption Dynamics

Prof. Frank Schweitzer



Outline

Adoption of products

"Diffusion" models

Epidemic models

Adoption of behavior

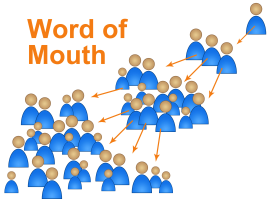
Resource-dependent growth

Notes:

Notes:

Adoption through "word of mouth" persuasion

- **Problem:** Get customers to adopt innovations
  - Applies to technologies, new "products"
  - Assumption: innovation is superior ⇒ advantage
  - **NO** market model: no demand, supply, prices
- **Solution:** Spread information
  - (a) Broadcasting ⇒ common source model
  - (b) **"Word of mouth" persuasion**
- **Modeling approaches:**
  - **"Micro" level:** Interactions between **individuals**
    ⇒ Agent-based modeling
  - **"Macro" level:** Interactions between **subpopulations**
    ⇒ System dynamics modeling ⇒ **aggregated level**
- **"Diffusion" model:** Spread within population
  - depends only on *contact rate*, adoption *probability*, interaction *frequency*
  - **NO** restricted access/supply, no preferences



Source: stock.adobe.com

Notes:

Bass innovation model

- **System dynamics model**: populations (aggregated)
  - two subpopulations: A (adopters), P (potential adopters)
  - population size: N=const., ⇒ **Maximum number of adopters**
  - actual number of adopters $N_A(t)$, potential adopters $N_P(t)$
  - Boundary condition: $N = N_A(t) + N_P(t)$
  - Systemic variable: **frequency** or **fraction** of adopters

$$x(t) = \frac{N_A(t)}{N}\ ; 1 - x(t) = \frac{N - N_A(t)}{N} = \frac{N_P(t)}{N}$$

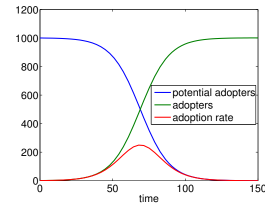
- **System dynamics**:

$$\frac{dN_A}{dt} = c\, i\, \frac{N_A}{N} N_P \quad \Rightarrow \quad \frac{dx(t)}{dt} = \beta\, x(t)\, [1 - x(t)]$$

  - Perfect mixing → **"Chemical reaction:"** $P \xrightarrow{\beta} A\ ; \quad \beta = c\, i$
  - β: Number of reactions per time interval $\Delta t \to [1/s]$
    c: contact rate, $0 \leq i \leq 1$: probability of adoption
  - Time-dependent prob. of encountering adopters: $x(t) = N_A(t)/N$
  - **Adoption "rate":** $dN_A/dt \to dx(t)/dt$

solution: S-curve/*logistic curve*

$$\boxed{x(t) = \frac{1}{1 + e^{-\beta(t-\mu)}}}$$



- *"S-curve"* ⇒ **Total adoption**
- *"bell-shaped curve"* ⇒ **Adpotion rate**

Notes:

- Bass, Frank (1969). "A new product growth model for consumer durables". Management Science 15 (5): p215-227.

- β is not the number of new adopters per unit time; that is dN_A/dt. β should be seen as a measure of the probability of one potential adopter to become an adopter at time t.
- The model distinguishes only two types of customers: those who adopted the product already (*A: adopters*), and those who will potentially adopt the product (*P: potential adopters*)
- To check the solution, just plug the solution into the differential equation. μ is the time at which $\frac{dx(t)}{dt}$ is maximal.

Remarks on the Bass innovation model

1. **Simple dynamic model** of spread of a new product
   - Analogy to **epidemics**: SI $\rightarrow$ Susceptible – Infected
   - Simple positive feedback (reinforcement) with limited resources
   - First like exponential growth, then saturation effects kick in
2. **Strong assumptions:**
   - Direct persuasion $\Rightarrow$ Depends only on A and P
   - Homogeneous populations (no spatial effects)
   - No threshold for the dynamics (i.e. no "critical mass")
   - No economic ingredients: No competition of products, preferences, lockin effects
3. **Limitations:**
   - Requires initialization: $N_A(t=0) \neq 0$
   - Final state: Only **adopters** $\Rightarrow$ $N_A = N$, $N_P = 0$
   - No reversal of adoption $\Rightarrow$ would require SIR (R: recovered)
   - Empirical evidence? $\Rightarrow$ Adoption of donation behavior

Notes:

- Processes which behave like in the Word of Mouth model are also called "logistic".

SS03: Adoption of innovations

- Develop a `python` program of the Bass innovation model
- Show plots with the time dependent number of adopters, potential adopters and the adoption rate.
- Run the model with different sets of parameters
  - What is the role of the contact rate?
  - What happens if the number of early adopters $N_A(t=0)$ is increased?
- Calculate the maximum of the adoption rate. Why is the curve for the adoption rate symmetric in time?

Notes:

Closed-form solution: The $\mathcal{S}$-curve

❶ Separation of variables:

$$\int \frac{dx}{x(a-x)} = \frac{1}{a}\left[\int \frac{dx}{x} + \int \frac{dx}{a-x}\right] = \int \beta dt + c_0$$

❷ Integration

$$\ln\left|\frac{x}{a-x}\right| = a\beta t + ac_0 \Rightarrow \frac{x}{a-x} = e^{ac_0}e^{a\beta t} = ce^{a\beta t}$$

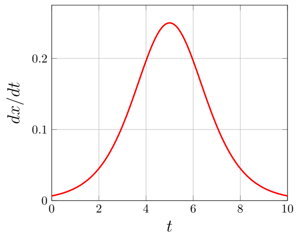
❸ Initial condition $x(t=0) = x_0(t_0)$

$$\frac{x_0}{a-x_0} = ce^{a\beta t_0} \Rightarrow c = \left[\frac{x_0}{a-x_0}\right]e^{-a\beta t_0}$$

❹ Solving for x

$$x(t) = \frac{a\left[\frac{x_0}{a-x_0}\right]e^{a\beta(t-t_0)}}{1+\left[\frac{x_0}{a-x_0}\right]e^{a\beta(t-t_0)}} = \frac{ax_0}{(a-x_0)e^{-a\beta(t-t_0)}+x_0}$$

$$\frac{dx(t)}{dt} = \beta\, x(t)\,[a - x(t)]$$



The $\mathcal{S}$-curve: Parameters

$$\frac{dx(t)}{dt} = \beta\, x(t)\,[a - x(t)]$$

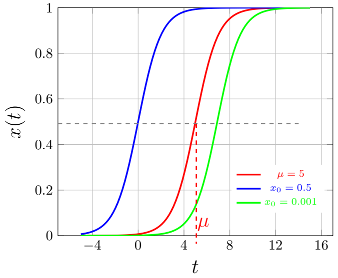
- **Closed-form solution:**

$$x(t) = \frac{1}{1+e^{-\beta(t-\mu)}} \quad \Leftrightarrow \quad x(t) = \frac{ax_0}{(a-x_0)e^{-a\beta(t-t_0)}+x_0}$$

  - **Ceiling** $a \to 1$
  - **Maximum growth**: $dx/dt = 0 \to \hat{x} = a/2$
  - **Time to reach maximum growth:** $x(t) = \hat{x}$

$$\beta\mu = \beta t_0 + \ln\frac{1-x_0}{x_0}$$

  - **Initial condition:** $x_0(t_0)$ shifts curve
    - $x_0(t_0) = 0$ not working $\Rightarrow$ "cold-start problem"
    - $x_0(t_0) = 0.5 \to \mu = 0$



Notes:

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Outline

Adoption of products

“Diffusion” models

Epidemic models

Adoption of behavior

Resource-dependent growth

Generalized diffusion models

- **Typical diffusion model**

$$\frac{dN_A(t)}{dt} = g(t)\big[M - N_A(t)\big]$$

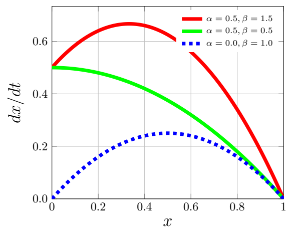
  - **Generalized growth factor**: $g(t) = \alpha + \beta N_A(t)$
  - **Importance of** a: solves problems for $N_A(0) = 0$
  - M: Maximum number of adopters (resource)
  - Divide by M and define $x(t) = N_A(t)/M$

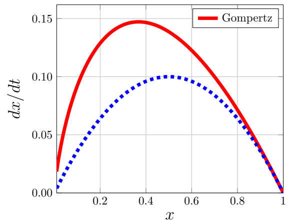
$$\frac{dx(t)}{dt} = \big[\alpha + \beta x(t)\big]\big[1 - x(t)\big]$$

$$x(t) = \frac{1 - e^{-(\alpha+\beta)t}}{1 + (\beta/\alpha)e^{-(\alpha+\beta)t}} \quad \text{if } x_0(0) \approx 0$$

- **Alternative: Gompertz equation**

$$\frac{dx(t)}{dt} = \beta x(t)\big[-\ln x(t)\big]$$





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Growth dynamics

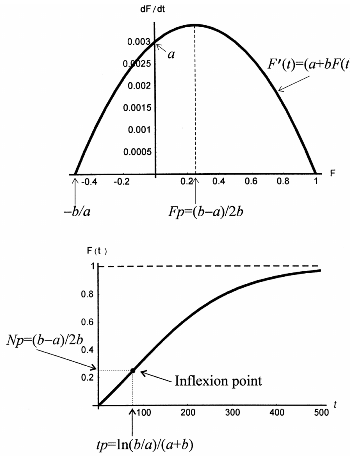
- **Plot of $dx(t)/dt$ vs. $x(t)$**

$$\frac{dx(t)}{dt} = \left[\alpha + \beta x(t)\right]\left[1 - x(t)\right]$$

  - Equilibria: $\frac{dx(t)}{dt} = 0 \Rightarrow x_1^0 = -\frac{\beta}{\alpha},\ x_2^0 = 1$
  - Maximum: $\frac{d^2x(t)}{dt^2} = 0 \Rightarrow \hat{x} = \frac{(\beta - \alpha)}{2\beta}$
- Time to reach maximum growth (inflexion point):
  - solve $x(t) = \hat{x}$ for t

$$\hat{t} = \frac{1}{\alpha + \beta} \ln\left(\frac{\alpha}{\beta}\right)$$

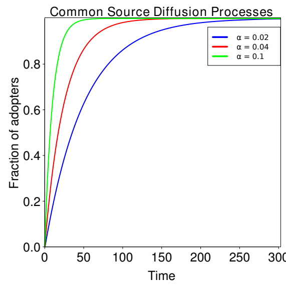
  - $t < \hat{t}$: exponential growth dominates



Source: Shone, p.55

Common source model

- **Problem:** How to solve the "cold start" problem?
- **Solution:** Broadcasted information about new technology
  - **Management science:** "Common Source Model"
  - Use of advertisement, marketing $\Rightarrow$ convinces ($\alpha \cdot N_P$) users during each time step to adopt the product
  - α: (constant) rate of *innovation adoption*
  - does **not** result in $\mathcal{S}$-curve



$$\frac{dN_A(t)}{dt} = \alpha N_P(t)$$

$$N_P(t) = N - N_A(t)\ ;\ x(t) = \frac{N_A(t)}{N}$$

$$\frac{dx(t)}{dt} = \alpha[1 - x(t)] \quad \Rightarrow \quad x(t) = 1 - e^{-\alpha t}$$

Notes:

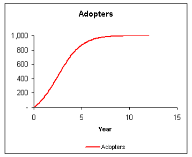
Notes:

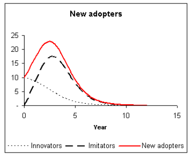
- These slides repeat the general growth model from the perspective of management science.
- "Cold Start" refers to the fact that there must be at least one initial adopter for the adoption to spread; as well as the extremely slow spreading at the beginning
- x is the fraction of adopters $\frac{N_A}{N_A+N_P}$.
- The dynamics of the number of adopters is

$$\frac{dN_A}{dt} = \alpha N_P$$

rewriting this equation for the fraction of adopters x gives the equation in the slide.

Mixed source model



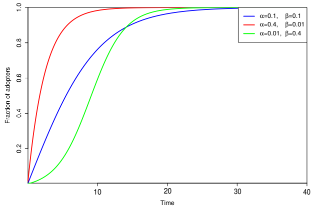


- **Combine** common source model and "word of mouth" effect
- **Generalized** or "Mixed Source Model": $\mathcal{S}$-curve

$$\frac{dx(t)}{dt} = \alpha[1 - x(t)] + \beta x(t)\,[1 - x(t)]$$

$$x(t) = \frac{\left[\alpha + \beta x_0\right] - \alpha\left[1 - x_0\right] e^{-[\alpha+\beta]t}}{\left[\alpha + \beta x_0\right] + \beta\left[1 - x_0\right] e^{-[\alpha+\beta]t}}$$

  - Initial condition: $x_0 = x(0) = N_A(t = 0)/N$
  - α: 0.02 (sometimes < 0.01), β: 0.38 (between 0.3–0.5)



Limitations of density dependent diffusion models

1. Arrival of new technologies is *exogenous* (not explained)
   - *Competition* between technologies?
   - Subsequent introduction of new technologies – effect on S-curves?
2. Leaves out firm's strategic behavior $\Rightarrow$ why do firms adopt?
   - Early adoption increases if firms anticipate later consequences
3. Most new technologies fail (never diffuse)
   - Combination of path dependence and diffusion models
4. Empirical evidence shows asymmetric S-curves
   - Extensions of epidemic models (various populations with different parameters)

Notes:

Average α, β values empirically found by:

- Mahajan, Vijay; Muller, Eitan and Bass, Frank (1995). Diffusion of new products: Empirical generalizations and managerial uses. Marketing Science 14 (3): G79-G88.
- Pictures above taken from Wikipedia. The second graph shows on the y-axis the new adopters per time unit (growth rate)

Notes:

- **Path dependence** refers to the amplification of early decisions. If your company early on decides to choose technology A, it is unlikely that it later reverts to B. That means, if B is the new technology, it is unlikely that it will be adopted if everyone chose A early on.

Outline

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Intermezzo: Basics of Epidemic Models

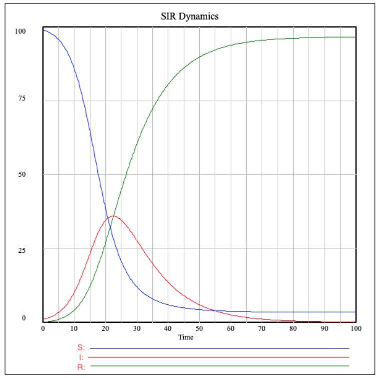
- **SI model:** Susceptible $\mathcal{S} = N_P$, Infected $\mathcal{I} = N_A$
  - **multiplier:** $\beta = ci$: infectious contacts per time unit

$$\frac{d\mathcal{S}}{dt} = -ci\frac{\mathcal{I}}{N}\mathcal{S} = -\beta\frac{\mathcal{I}\mathcal{S}}{N}; \quad \frac{d\mathcal{I}}{dt} = \beta\frac{\mathcal{I}\mathcal{S}}{N}$$

  - **final state:** only **Infected** $\Rightarrow \mathcal{I} = N, \mathcal{S} = 0$
- **SIR model:** 3rd population: Removed $\mathcal{R}$
  - fraction of the infected population $\mathcal{I}$ is removed because of immunity or death $\Rightarrow$ removal rate γ

$$\frac{d\mathcal{S}}{dt} = -\beta\frac{\mathcal{I}\mathcal{S}}{N}; \quad \frac{d\mathcal{I}}{dt} = \beta\frac{\mathcal{I}\mathcal{S}}{N} - \gamma\mathcal{I}; \quad \frac{d\mathcal{R}}{dt} = \gamma\mathcal{I}$$

  - **final state:** $\mathcal{I} = 0$, disease cannot spread
  - **Infected** becomes an **intermediate** state



Notes:

Notes:

- There exists other variations of this model with the same underlying concepts:
  - **SI** model: All the population becomes infected at $t \to \infty$.
  - **SIR** model: part of the population (R) is removed from the susceptible pool. If β is large enough: the whole population is infected at some point and then recovers. $R = N$, $S = 0$ for large t. If the infected population recovers faster than the disease can spread: $I = 0$ and $S > 0$ for large t similarly to the plot above ($\beta = .3$, $\gamma = .1$).
  - **SIS** model: part of the population can recover and becomes *immediately* susceptible again.
  - **SIRS** model: part of the population can recover (R), and a fraction of this can become susceptible again.

Reproduction number and herd immunity

- dynamics of infections $\mathcal{I}$ depends on R_0 (R-nought)

$$R_0 = \frac{\beta}{\gamma}$$

  - known as the **basic reproduction number** of an infection
  - expected number of new infections from a single infection
  - assumption: all subjects in a population are susceptible
- **herd immunity if** immunized fraction is

$$\frac{\mathcal{R}}{N} > 1 - \frac{1}{R_0} = 1 - \frac{\gamma}{\beta}$$

- **effective reproduction number**: $R = R_0 \left(\mathcal{S}/N\right)$
  - $R > 1$: R new cases from each infection case
  - $R < 1$: disease effectively eliminated

Disease	R_0
Measles	12-18
Chickenpox	10-12
COVID-19 (Delta)	≥ 7 (2021)
COVID-19	2-6 (2020)
Common Cold	2-3
Influenza	0.9 - 2.8

Modifying R_0

- R_0 depends on many parameters: $R_0 = \beta/\gamma$, $\beta = ci$
  - $\tau = 1/\gamma$: **mean infectious period** of a individual.

$$R_0 = \beta\tau = c \cdot i \cdot \tau$$

- Which parameters can be changed to control an epidemic?
  - **infectivity** i: intrinsic property of the disease $\Rightarrow$ cannot be modified.
  - **contact rate** c and **infectious period** τ: can be modified to reduce R_0
    - reducing c means **reducing the number of contacts** per unit time.
    - reducing τ means eliminating, treating or isolating infectious individuals.
    - $c(t)$ and $\tau(t)$ change over time (e.g., due to public health policies)
    - from data we measure the **effective reproduction number** $R_t = R(t) = i \cdot c(t) \cdot \tau(t)$

Notes:

- Note that $\mathcal{S}/N$ can be hardly estimated under practical circumstances.
- The new cases resulting from infected individuals are also called "secondary" cases.

Notes:

Limitations of SI/SIR/SIS/SIRS Models

Simple epidemics models are **unrealistic** in most real-world scenarios

- assumptions about **homogeneous populations** are wrong
  - all individuals are characterised by the same parameters c, i, and γ.
  - homogenous interactions: c does not depend on specific individuals interacting
  - spatial homogeneity $\Leftrightarrow$ hot spots, mobility
- R_0 for **past** outbreaks $\neq R_0$ of current outbreaks (same disease)
- inhomogeneous systems: **threshold dynamics** (starts beyond a critical value)

Possible model improvements:

- spatial models (to account for heterogenous populations)
- network models (to account for heterogenous interactions)
- combinations of spatial models and network models

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Notes:

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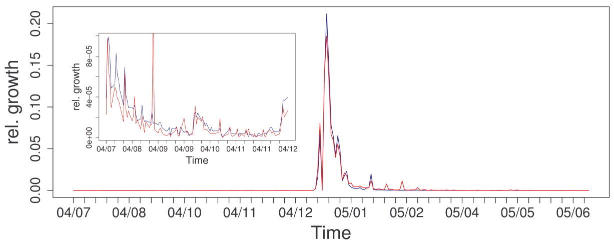
Case Study: Adoption of behavior

Wave of donations after tsunami disaster (inset: before tsunami)

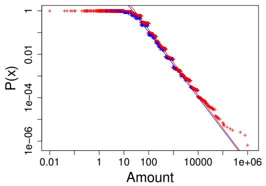


01-06-2005: $N_{\rm tot}=1'556'626$, $A_{\rm tot}=126'879'803$

F. Schweitzer, R. Mach: *The epidemics of donations: logistic growth and power-laws*, PLoS ONE, January (2008)

Scale-free distribution

Cumulative probability distribution: $P(x) \sim x^{-\alpha}$



- clear power law over several orders of magnitude
  - scale free nature of donations
- exponent α similar before ($\alpha = 1.501 \pm 0.023$) and after ($\alpha = 1.515 \pm 0.002$) the desaster
  - similarities to other German and Swiss donor organizations

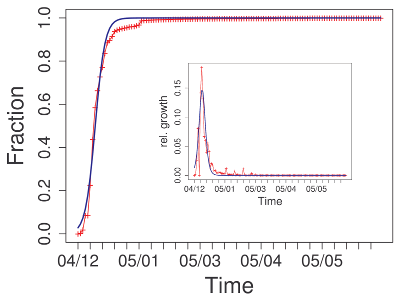
F.S., R. Mach, PLoS ONE, Jan (2008)

Notes:

- $N_{\rm tot}$ is total number of donations, $A_{\rm tot}$ the total amount of all donations.
- The figure shows daily number (blue) and amount (red) of donations shown as a fraction of the total number/amount over a period of one year (mid of 2004 until mid of 2005 for time series of "Deutschland hilft". The inset magnifies the relative growth of number and amount of donations for the half-year period preceeding the earthquake.
- Source: Schweitzer, Frank; Mach, Robert: The Epidemics of Donations: Logistic Growth and Power Laws, *PLoS ONE* vol. 3, no.1 (2008) e1458
- You can download this paper from: `http://www.plosone.org/doi/pone.0001458`

Notes:

Saturated growth dynamics



- fraction of the total number of donations
  (inset: relative growth of amount of donations)
  - fit: $\mu = 8.05 \pm 0.07$, $\beta = 0.5 \pm 0.01$

Notes:

Remember:

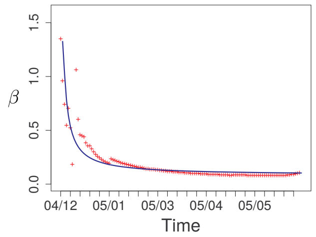
$$\frac{df(t)}{dt} = \beta f(t)\left[1 - f(t)\right] \quad \Rightarrow \quad f(t) = \frac{1}{1 + e^{-\beta(t-\mu)}}$$

- μ: time when $df(t)/dt$ has reached maximum

In this and the next slide the blue curves in the figure result from fits, while the red points represent the empirical distribution.

- Source: Schweitzer, Frank; Mach, Robert: The Epidemics of Donations: Logistic Growth and Power Laws, *PLoS ONE* vol. 3, no.1 (2008) e1458

Influence of mass media



- slowing down of mean field interaction

$$\beta = \left[a_0 + (a_1/t) + (a_2/t)^2\right]$$

- $\beta = c\,i$: number of successful interactions per time interval
  - early stage: people were more enthusiastic to donate money
  - later stage: became more indifferent
- decrease of β in time $\Rightarrow$ lack of public attention

Notes:

F.S., R. Mach, PLoS ONE, Jan (2008)

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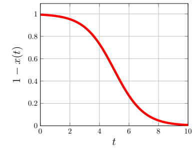
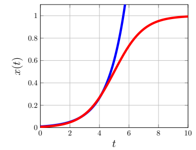
Resource-dependent growth

Growth depends on resources

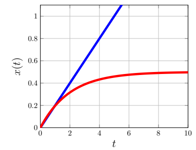
1. **Non-renewable resource:** time dependent depletion
   - **Example:** Adoption of products: $N_a + N_p = N$
   - Declining number of **potential adopters** $dN_p = -dN_a$
   - **1-dim problem, stationary solution**: $N_p^\star = 0$



2. **Renewable Resource:** time dependent re-growth
   1. **Exponential re-growth:** $\dot{x} = \alpha x$
   2. **Saturated re-growth:** $\dot{x} = \beta x(1-x)$
   - **Example:** Fisheries, maximum defined by **Carrying Capacity**
   - **1-dim problem, stationary solution**: $N^\star = K = (b-d)/(k_b + k_d)$



3. **Renewable Resource with depletion:** exponential decay
   1. **Constant re-growth:** $\dot{x} = q_0$
   2. **Saturated re-growth:** $\dot{x} = q_0 - \gamma x$
   - **Example:** Capital stock (γ : depreciation rate)
   - **1-dim problem, stationary solution**: $x^\star = q_0/\gamma$



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Questions

1. What are the main assumptions for the "word-of-mouth" effect? How realistic are they?
2. Why is the Bass innovation model described as a 1-dimensional dynamic problem if there are adopters and potential adopters?
3. Discuss the limitations of the Bass innovation model (and note the difference to Question 01).
4. Explain the different parameters of the $\mathcal{S}-$curve. Why shifts the initial condition the curve?
5. How is the $\mathcal{S}-$curve related to the growth dynamics? What is the role of μ? What happens at the inflexion point?
6. What are the differences between the common source and the mixed source model?
7. Discuss the limitations of "diffusion" models. What would you improve first? How?
8. What is an SIR model? What would be the impact of "recovered" in an adoption dynamics?
9. Discuss the differences for saturated re-growth dependent on the growth factor.

Notes: