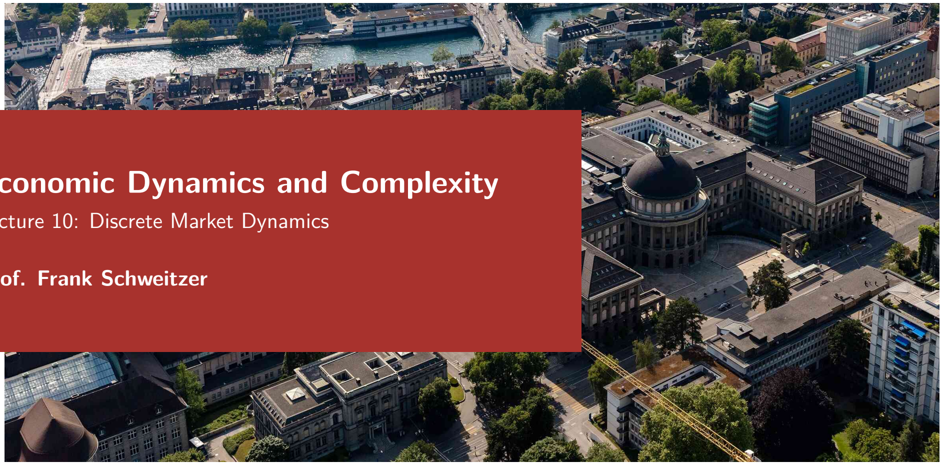
ETH zürich

Chair of Systems Design
www.sg.ethz.ch



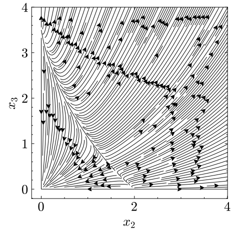
Economic Dynamics and Complexity

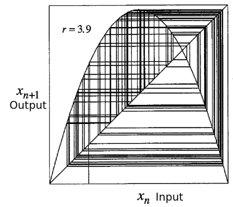
Lecture 10: Discrete Market Dynamics

Prof. Frank Schweitzer

Simple dynamics can lead to complex behavior

1. **Nonlinearity**
   - Functions with nonlinear terms, e.g. $ax - bx^2$
   - Multiple equilibria dependent on parameters
   - Instability of solutions
   - **Example:** Saturated growth
2. **Coupling**
   - Dynamics: Set of coupled equations $\rightarrow xy$
   - "Space": Set of similar systems i, j, k
   - Indirect coupling via resources ρ
   - **Example:** 2-Box system
3. **Discrete time**
   - Large $\Delta t \rightarrow$ No convergence
   - Time lag $t + \tau \rightarrow$ Delay
   - **Example:** Deterministic chaos





Notes:

Notes:

Outline

Supply and demand

Cobweb dynamics

Coupled cobwebs

Supply and Demand

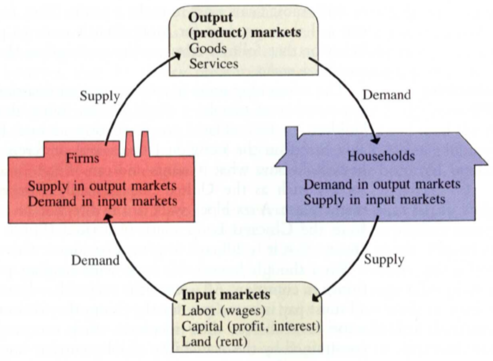
Economic systems

- two fundamental players: *decision-making units*
  - **firms**: producing units ⇒ supply
  - **households**: consuming units ⇒ demand
- market: match supply and demand by adjusting the **price**
  - holds for **competitive markets** with many sellers and buyers
  - both sellers and buyers have little market power individually → cannot set market price
- **market clearing mechanism**:
  - set price such that *demand* = *supply* → (quasi-stationary) equilibrium
  - no excess demand/supply ⇒ no pressure to change price
  - note: markets are not always in equilibrium, react with delay

Notes:

Notes:

The Market Allocation Mechanism

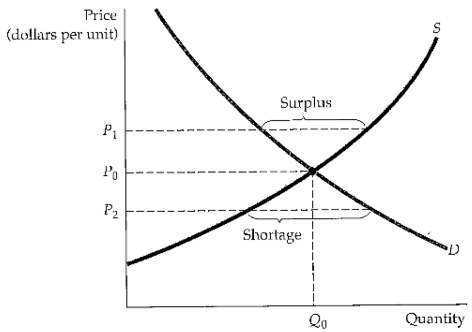


- circular flow of **economic activities** (clockwise, shown) and **money** (counterclockwise, not shown)

Notes:

- In this lecture, we focus on the *output* or *product market* (of goods, services).
- Note that households can also be suppliers (of labor) and firms can also have a demand (labor) – which is exchanged in the *input market* (labor, capital, land markets).
- Source: K.E. Case, R.C. Fair, *Principles of Economics*, Prentice Hall, 1997

The Market Mechanism



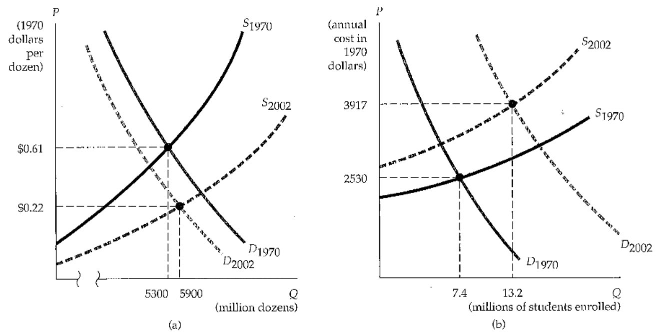
- Supply (S) and Demand (D) are two different quantities
  - At higher price P_1 a surplus develops and price falls
  - At lower price P_2 there is a shortage and price is bid up
  - The *market clears* at price P_0 and quantity Q_0 ($\Rightarrow$ equilibrium)

Notes:

Source: Pindyck/Rubinfeld, *Microeconomics*, Pearson Education, 2005 (1995), Chap. 2

Changes in Market Equilibrium

market for eggs — market for college education



- from 1970 to 2002 the real price of eggs fell by 74% while the real price of a college education rose by 55%

Notes:

- (a) market for eggs
  - supply of eggs shifted downward as production costs fell
  - demand shifted to the left as consumer preferences changed
  - result: the real price of eggs fell sharply and egg consumption rose slightly
- (b) market for college education
  - supply of college education shifted up as costs of equipment and staffing rose
  - demand shifted to the right as a growing number of highschool graduates desired a college education
  - result: both price and enrollments rose sharply

Source: Pindyck/Rubinfeld, *Microeconomics*, Pearson Education, 2005 (1995), Chap. 2

Elasticities of Supply and Demand

How much does the demand or supply change if price is changed?

- **price elasticities** of demand and supply
  - measure the *sensitivity* to price changes
  - refer to **relative** values (percentage), i.e. dependent on P and Q

$$E_d = \frac{\Delta Q_d/Q_d}{\Delta P/P} = \frac{P}{Q_d}\frac{\Delta Q_d}{\Delta P}\ ;\quad E_s = \frac{\Delta Q_s/Q_s}{\Delta P/P} = \frac{P}{Q_s}\frac{\Delta Q_s}{\Delta P}$$

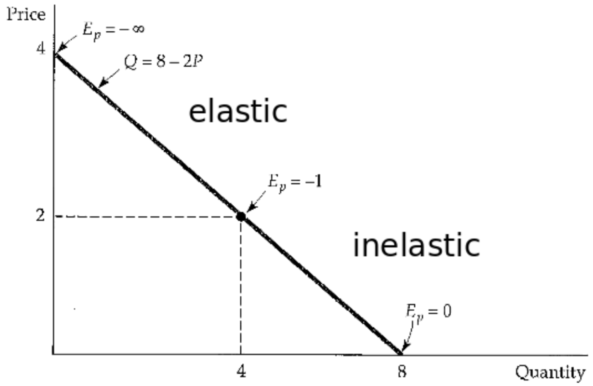
  - ΔQ_d is the *decrease* of Q_d with a price *increase* of ΔP
  - example: if the price increases by 1%, and $E_d = 5$, then the demand decreases by 5%.
  - $|E_d| > 1$: demand is *price elastic*
  - $|E_d| < 1$: demand is *price inelastic*
  - the elasticities change over the demand/supply curve
  - high elasticity: consumer can choose substitutes

Notes:

- Originally ΔQ_d referred to *increase* of demand with a price *increase* of ΔP, which means E_d is then (almost) always negative (since there are almost no goods for which demand increases with a higher price). It became convention to work with a positive E_d, which means ΔQ_d is the *decrease* of Q_d with a price *increase* of ΔP.
- Example: set $P = 100$ and $Q = 500$. Suppose we increase P by 1, i.e. $\Delta P = 1$, and then Q goes down to 490. Then $\Delta Q = 10$ (instead of -10), so:

$$E_d = \frac{100}{500}\frac{10}{1} = 2$$

So the demand is here price elastic.



- **Linear demand curve**
  - Price elasticity depends on: slope of demand, price, quantity
  - Near the top elasticity is large, near the bottom it is small

Outline

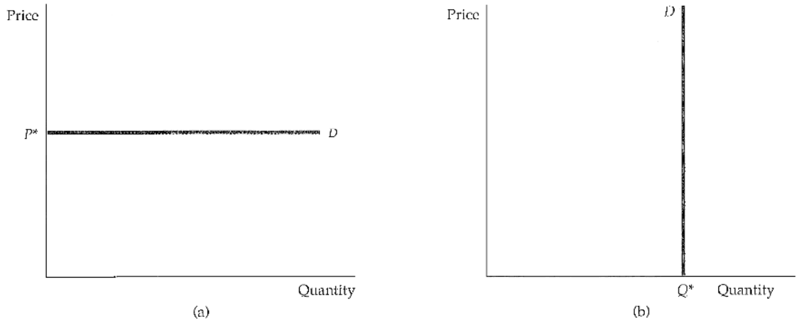
Supply and demand

Cobweb dynamics

Coupled cobwebs

Notes:

Source: Pindyck/Rubinfeld, *Microeconomics*, Pearson Education, 2005 (1995), Chap. 2



(a) infinitely elastic demand and (b) completely inelastic demand

- for a horizontal demand curve $\Delta Q/\Delta P$ is infinite
- for a vertical demand curve $\Delta Q/\Delta P$ is zero
- an example of inelastic good would be common food (bread, rice, eggs)
- an example of elastic good would be entertainment (restaurant, movie, travel)
- note: the elasticity changes along the the linear line. The linear line is an approximation often used in economics

Notes:

Example: Cobweb-Dynamics

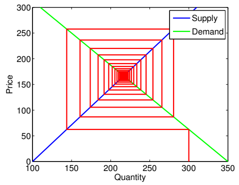
$$Q_t^D = D_t = \frac{a-P}{b} = \alpha - \beta P_t$$

$$Q_t^S = S_t = \frac{P-c}{d} = \gamma - \delta P_t$$

- Simple dynamic model of supply (S) and demand (D)
  - S, D coupled via the price (P)
  - *Linear dependence* of $S(P)$, $D(P)$
  - Feedback over time t, $t-1$
    - Demand D responds to price P immediately, i.e. $D_t = f_D(P_t)$
    - Supply S responds to price P with a lag, i.e. $S_t = f_S(P_{t-1})$
    - Usual illustration: Supply and demand of pork
      $\Rightarrow$ lag in supply adjustment since pig breeding takes time
  - $\alpha = a/b$, $\gamma = -c/d$: basic supply/ demand at $P = 0$
  - $\beta = 1/b$, $\delta = 1/d$: *price derivatives* of demand and supply

$$E_D = \beta \frac{P_t}{D_t}\ ; \quad E_S = \delta \frac{P_{t-1}}{S_t}$$

  - Larger β, δ: small variations of $P \rightarrow$ great variations in S, D



Source: pixabay.com

Market clearing and Equilibrium

- **Price dynamics:**

$$D_t = \alpha - \beta P_t \ \rightarrow \ P_t = \frac{\alpha}{\beta} - \frac{1}{\beta} D_t$$

$$S_t = \gamma + \delta P_{t-1} \ \rightarrow \ P_{t-1} = \frac{1}{\delta} S_t - \frac{\gamma}{\delta}$$

- **Market Clearing:** Supply = Demand

$$S_t = D_t \ \Rightarrow \ P_t = \frac{1}{\beta}(\alpha - \gamma) - \frac{\delta}{\beta} P_{t-1}$$

- **Equilibrium:** Supply and demand do **not change**

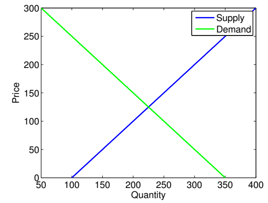
$$S_t = S_{t-1} \ \rightarrow \ S^\star = \gamma + \delta P^\star$$

$$D_t = D_{t-1} \ \rightarrow \ D^\star = \alpha - \beta P^\star$$

- Market clearing **and** equilibrium:

$$S^\star = D^\star \ \rightarrow \ P^\star = (\alpha - \gamma)/(\beta + \delta)$$

Control parameter: δ/β



Notes:

Kaldor, N. (1934): A Classificatory Note on the Determination of Equilibrium, in: Review of Economic Studies, Vol I (February), pp. 122-136.

- About the elasticity:

$$E_D = \frac{\Delta Q_D / Q_D}{\Delta P / P} = \frac{P}{Q_D} \frac{\Delta Q_D}{\delta P} = \frac{P_t}{D_t} \frac{\delta Q_d}{\delta P}$$

Is $\frac{\Delta Q_D}{\Delta P}$ now β or $-\beta$? Once again this is a matter of definition. If we choose ΔQ_D to measure *decrease* of demand with a price *increase*, we get:

$$E_D = \beta \frac{P_t}{D_t}$$

As long as you are consistent, the two possible definitions of ΔQ_D are perfectly equivalent.

Notes:

- Note: In the figures price over quantity is plotted. However, the price derivatives of supply $\delta = \frac{dS}{dP}$ and demand $\beta = \frac{dD}{dP}$ imply a change of quantity over price.

Dynamics close to equilibrium

- 1st order linear non-homogeneous dynamics with fixed point $y^\star$

$$y_{t+1} = ay_t + c \;\rightarrow\; y^\star = ay^\star + c$$
$$y_{t+1} - y^\star = a(y_t - y^\star)$$

- Deviation from equilibrium: $x_{t+1} = y_{t+1} - y^\star$: homogeneous eq

$$x_{t+1} = ax_t \;\rightarrow\; x_t = a^t x_0 \rightarrow y_t - y^\star = a^t[y_0 - y^\star]$$

- Solution $$y_t = \frac{c}{1-a} + a^t\left[y_0 - \frac{c}{1-a}\right]$$

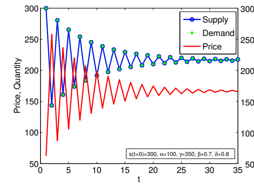
- Cobweb $$P_t - P^\star = \left(-\frac{\delta}{\beta}\right)^t [P_0 - P^\star]$$

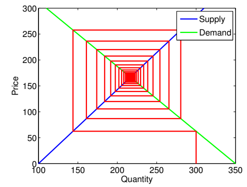
$$P_t = \frac{\alpha-\gamma}{\beta+\delta} + \left(-\frac{\delta}{\beta}\right)^t \left[P_0 - \frac{\alpha-\gamma}{\beta+\delta}\right]$$

- $\delta/\beta > 0 \rightarrow (-\delta/\beta)^t$ changes sign, convergence for $0 < |-\delta/\beta| < 1$

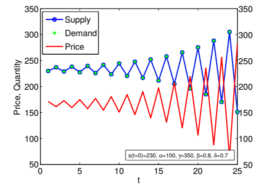
Cobweb dynamics

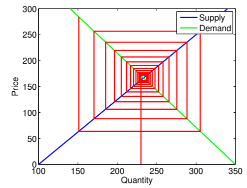
- $\delta/\beta < 1$: system reaches stable equilibrium ($s(t=0) = 300,\ \alpha = 100,\ \gamma = 350,\ \delta = 0.7,\ \beta = 0.8$)





- $\delta/\beta > 1$: dynamics 'explodes' ($\Rightarrow$ instable) ($s(t=0) = 230,\ \delta = 1.1,\ \beta = 0.8$)





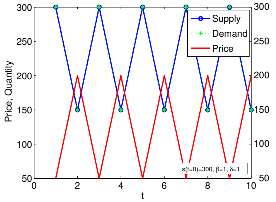
Notes:

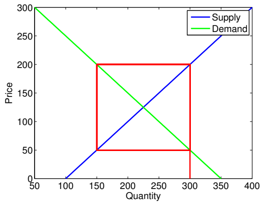
Notes:

- **Explanation of the Cobweb:** For a given quantity (300) the demand curve sets the price at which the whole quantity can indeed be sold (market clearing), e.g. 60 CHF. For this price, however, the production of the good is less attractive. I.e., the supply curve tells the quantity (135) that producers are willing to supply at 60 CHF, in the next time step. Because this quantity is small, demand responds immediately and the price rises to the high value of 250 CHF. For this price, it becomes very attractive to produce the good, i.e. in the next time step a quantity of 280 is supplied ....
- in the above example, demand always equal supply, the two curves overlap
- On the right graph, each horizontal line is an adjustment in quantity (happens between time steps). Each vertical line is adjustment in price, happens within a time step.
- As an intuitive, graphical explanation why $\delta/\beta < 1$ leads to equilibrium, think about the cobweb diagram, with the quantity on the horizontal axis and price on the vertical axis. Now, when you start drawing the evolution of the system from an arbitrary quantity, you do that in the clockwise direction. Then, in order for the oscillations to die out and converge to the intersection of the demand line and the supply line, you want the demand line to be less steep than the supply line. Now recall that the equations give supply and demand as functions of price, while we plot the price on vertical axis and supply and demand on horizontal axis – which is the opposite of the conventional function plotting. That is why the relation steepness (in absolute terms) on this "inverted" is the opposite of the relation between the corresponding coefficients δ and β.

Cobweb dynamics: Periodic oscillations

- $\delta/\beta = 1$: system changes periodically $\left(s(t=0)=300,\ \delta=0.8,\ \beta=0.8\right)$

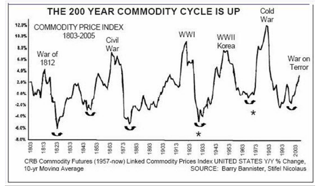


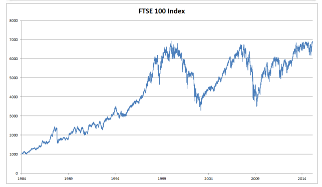


- **Remarks**
  1. Simple economic model: two coupled equations
     - Price is determined through supply and demand: $S_t = D_t$
     - No expectations about price $\Rightarrow P_t^{\mathrm{expect}}$
  2. Important role of **control parameters** (price derivatives)
     - Existence of **critical parameter**: $\delta/\beta = 1$
     - Small change at critical stage $\rightarrow$ transition **stability** $\Leftrightarrow$ **instability**

Limitations of Classical Cobweb Theory

- *standard assumptions*
  - single market, linear supply and demand
  - response of supply with a production lag
    - suppliers form naive expectations based on previous prices
  - *result:* cyclical price dynamics
    - confirmed in many commodity markets, for intermediate times
- *limitations of classical cobweb theory*
  - only simple outcomes: damped, exploding, periodic oscillations
  - real markets: irregular, "random looking" prices
    - example: stock markets
  - suppliers: no simple expectations, but anticipatory effects
  - no interaction between markets: consider inter-market influence





Notes:

Notes:

- The upper graph shows the price of commodity index (cyclical). (https://gannresources.blogspot.com)
- the lower graph shows the price of FTSE100 index (stock) (random). (https://en.wikipedia.org/wiki/FTSE_100_Index)

Cobweb dynamics with expectations

- **Adaptive expectations about price**

$$P^e_t = P^e_{t-1} - \lambda\left(P^e_{t-1} - P_{t-1}\right)$$
$$= (1-\lambda)P^e_{t-1} + \lambda P_{t-1}$$

  - aka *reinforcement learning* $\to$ adaptation
  - $\lambda = 1$: classic model, forecast error: $P^e_{t-1} - P_{t-1}$
  - Market clearing $Q^D_t = \alpha - \beta P_t = Q^S_t = \gamma + \delta P_{t-1}$
  - Correct expectation means: $P^e_t = P_{t-1}$

$$P^e_t = P_{t-1} = \frac{Q^S_t - \gamma}{\delta} = \frac{Q^D_t - \gamma}{\delta}$$
$$= \left(\frac{\alpha-\gamma}{\delta}\right) - \left(\frac{\beta}{\delta}\right) P_t$$
$$\alpha - \beta P_t = \gamma + \delta(1-\lambda)\left[\left(\frac{\alpha-\gamma}{\delta}\right) - \left(\frac{\beta}{\delta}\right) P_{t-1}\right] + \delta\lambda P_{t-1}$$
$$P_t = \lambda\left(\frac{\alpha-\gamma}{\delta}\right) + \left[1 - \lambda - \left(\frac{\delta\lambda}{\beta}\right)\right] P_{t-1}$$

- Equilibrium price
  $P^\star = (\alpha-\gamma)/(\beta+\delta)$
- Dynamics

$$P_t - P^\star = \left[1 - \lambda - \left(\frac{\delta\lambda}{\beta}\right)\right]^t (P_0 - P^\star)$$

- Convergence only if

$$\frac{1}{\lambda} - 1 < \frac{\delta}{\beta} < \frac{2}{\lambda} - 1$$

Outline

Supply and demand

Cobweb dynamics

Coupled cobwebs

Notes:

Notes:

Coupled Cobweb Dynamics

- N suppliers, two interacting markets: X, Z
  - fractions of suppliers in market X, Z at time t

$$W_{X,t}\ ;\quad W_{Z,t} = 1 - W_{X,t}$$

  - **individual** supplier: provides $S_{X,t}$ and $S_{Z,t}$ (supply from a particular supplier)
    total supply in both markets:

$$N \cdot W_{X,t} \cdot S_{X,t} \quad \text{or} \quad N \cdot W_{Z,t} \cdot S_{Z,t}$$

- *market clearing* at every time ⇒ **total demand** $D_{X,t}$ equals **total supply**

$$D_{X,t} = N \cdot W_{X,t} \cdot S_{X,t} \quad \text{and} \quad D_{Z,t} = N \cdot W_{Z,t} \cdot S_{Z,t}$$

What is the Price?

- prices: $P_{X,t}$, $P_{Z,t}$; differs in two markets, changes with time
  - *demand*: fast response on the price ⇒ $D_{X,t}(P_{X,t})$ (and $D_{Z,t}(P_{Z,t})$)
  - *supply*: response with a time lag ⇒ $S_{X,t}(P_{X,t-1})$ (and $S_{Z,t}(P_{Z,t-1})$)
  - assumption: **linear dependency** for *total demand* and *individual supply*

$$D_{X,t} = \frac{a_X - P_{X,t}}{b_X} \qquad D_{Z,t} = \frac{a_Z - P_{Z,t}}{b_Z}$$

$$S_{X,t} = \frac{P_{X,t-1} - c_X}{d_X} \qquad S_{Z,t} = \frac{P_{Z,t-1} - c_Z}{d_Z}$$

- **market clearing** ⇒ *recursive equations*

$$P_{X,t} = a_X - \frac{b_X}{d_X} N\, W_{X,t}\, (P_{X,t-1} - c_X)$$

$$P_{Z,t} = a_Z - \frac{b_Z}{d_Z} N\, W_{Z,t}\, (P_{Z,t-1} - c_Z)$$

Notes:

- The model and results are from
  Dieci, R. and Westerhoff, F. : Interacting cobweb markets, *Journal of Economic Behavior & Organization* Volume 75, Issue 3, September 2010, Pages 461-481
- A copy of the working paper can be found in our Literature section.

Notes:

- Note that $a_X,\ a_Z,\ b_X,\ b_Z > 0$, and $c_X, c_Z \geq 0$, $d_X,\ d_Z > 0$
- $P_{X,t} = a_x - b_X\, D_{X,t} = a_x - b_x\, N\, W_{X,t}\, S_{X,t} = a_X - \frac{b_X}{d_X} N\, W_{X,t}\, (P_{X,t-1} - c_X)$
- Note the parameters are defined slightly different from the orignal cobweb model:

$$s_t = \gamma + \delta P_{t-1} \;\rightarrow\; S_{X,t} = \frac{P_{X,t-1} - c_X}{d_X}$$

$$d_t = \alpha - \beta p_t \;\rightarrow\; D_{X,t} = \frac{a_X - P_{X,t}}{b_X}$$

There is unfortunately no consensus on the definition of parameters in the literature. For the following discussion, keep in mind what corresponds to what:

γ	$-c/d$
δ	$1/d$
α	a/b
β	$1/b$

So the control parameter δ/β is now d/b

Decoupled Markets

- $P_{X,t}$ and $P_{Z,t}$: decoupled linear first-order difference equations
- fixed point analysis (stationary solution): $P_{X,t} = P_{X,t-1}$, $P_{Z,t} = P_{Z,t-1}$
  - Example: suppliers evenly distributed between X and Z

$$W_{X,t} = W_{Z,t} = \bar{W} = 0.5$$

  - steady state of market X gives the price

$$\bar{P}_X = \frac{a_X d_X + N\bar{W} b_X c_X}{d_X + N\bar{W} b_X}$$

  - globally asymptotically stable if

$$\left| \frac{N\bar{W} b_X}{d_X} \right| < 1$$

    - Remember: $|b_X/d_X| < 1$, which holds for $N\bar{W} = 1$
    - Compare with the stability requirement for classical cobweb: $\beta/\delta < 1$

Market Interaction

- suppliers switch between X, Z, expressed in time dependent $W_{X,t}$, $W_{Z,t}$
- decision depends on profits $\pi_{X,t-1}$, $\pi_{Z,t-1}$ made in time $t-1$:

$$W_{X,t} = \frac{\exp(f\ \pi_{X,t-1})}{\exp(f\ \pi_{X,t-1}) + \exp(f\ \pi_{Z,t-1})}\ ; \quad W_{Z,t} = 1 - W_{X,t}$$

- $f \in (0, \infty)$: intensity of choice, shows how fast suppliers switch markets $\Rightarrow$ can be interpreted as *rationality* of agents

Notes:

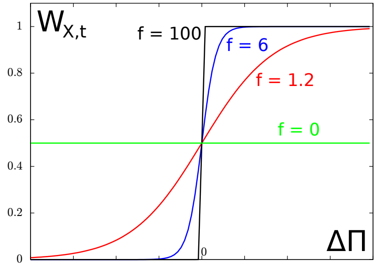
- $\bar{P}_X = a_X - \frac{b_X}{d_X} N\, \bar{W}\left(\bar{P}_X - c_X\right)$ (the condition for stability with the formula for $\bar{P}_X$ from the previous slide)
- $\left(1 + \frac{b_X}{d_X} N\, \bar{W}\right) \bar{P}_X = a_X - \frac{b_X}{d_X} N\, \bar{W}\, c_X$
- $\Rightarrow \bar{P}_X = \frac{a_X - \frac{c_X b_X}{d_X} N\, \bar{W}}{1 + \frac{b_X}{d_X} N\, \bar{W}} = \frac{a_X d_X - c_X b_X\, N\, \bar{W}}{d_X + b_X N\, \bar{W}}$

Notes:

- The function for $W_{X,t}$ is called the 'logit function'
- A discussion of the logit function can be found in
  "A rational route to randomness" by William A. Brock and Cars H. Hommes
  `http://www.ssc.wisc.edu/~wbrock/rp457a.pdf`

Logit Function

- $f = 0$: no response to market information, $W_{X,t} = W_{Z,t} \to 0.5$
- $f \to \infty$: suppliers respond extremely strong to past profit
  $W_{X,t} \to 1$ or $W_{X,t} \to 0$, depending on where profit was highest (in X or in Z)



Profits

- switch between markets depends crucially on *profits* in markets X and Z

$$\pi_{X,t} = P_{X,t}S_{X,t} - C_X(S_{X,t})\ ; \quad \pi_{Z,t} = P_{Z,t}S_{Z,t} - C_Z(S_{Z,t})$$

  - $P_{X,t}S_{X,t}$: revenue obtained from selling supply $S_{X,t}$ at price $P_{X,t}$ in market X
  - $C_X(S_{X,t})$: nonlinear cost *function* for producing $S_{X,t}$
  - assumption: $C_X(S_{X,t}) = c_X S_{X,t} + e_X S_{X,t}^2$

$$\boxed{\pi_{X,t} = P_{X,t}S_{X,t} - (c_X S_{X,t} + e_X S_{X,t}^2)}$$

$$\boxed{\pi_{Z,t} = P_{Z,t}S_{Z,t} - (c_Z S_{Z,t} + e_Z S_{Z,t}^2)}$$

Notes:

- A discussion of the logit function can be found in
  "A rational route to randomness" by William A. Brock and Cars H. Hommes
  `http://www.ssc.wisc.edu/~wbrock/rp457a.pdf`
- $\Delta\pi = \pi_{X,t-1} - \pi_{Z,t-1}$, so for a positive $\Delta\pi$ we expect there to be more suppliers in market X (since profit was higher in market X at $t-1$).

Notes:

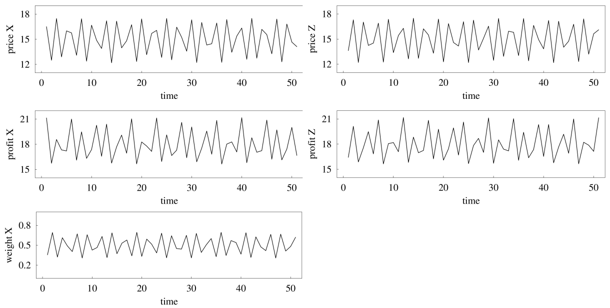
- $C_o(S_{o,t}) = c_o S_{o,t} + e_o S_{o,t}^2$ ($c_o > 0$ and $e_o > 0$) is a valid assumption for costs on the short term.

Computational Analysis

- assume symmetric markets:

  $a_X = a_Z = a\,;\quad b_X = b_Z = b\,;\quad c_X = c_Z = c\,;\quad d_X = d_Z = d$

  - setup: $a = 20$, $b = 6$, $c = 2$, $d = 8$, $e = 1$, $f = 0.17$
  - *quasiperiodic motion* along an orbit (equilibrium)

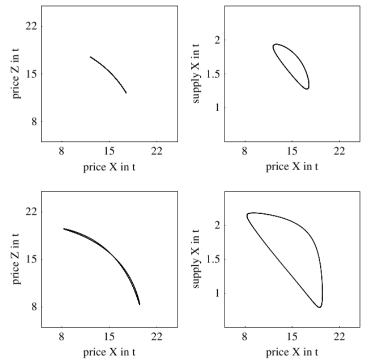


Notes:

- Note that in the simulation, the absolute value of the price derivative of the demand is larger than the price derivative of the supply. Otherwise the system is unstable.
- *Quasiperiodic* here means that, in addition to the periodic dynamics (which repeats after a larger number of time steps), there is some irregular dynamics which makes the value rather unpredictable *within* that longer period.
- **Important:** The figures above do *not* show the quasiperiodic motion. In fact, we do not see a sequence repeating. Does it mean, there is no quasiperiodic motion? No. Maybe, we have not waited long enough. This points to the problem of detecting chaos: Is the sequence periodic or aperiodic (as required for chaos). Remember that the positive Ljapunov exponent gives a first hint as a necessary, but not sufficient condition.
- Weight X represents the fraction of suppliers in market X. Remember that $W_{Z,t} = 1 - W_{X,t}$
- Note that while the price stays within bounds, the long term behavior of the system never settles to a fixed point. So, the equilibrium is described by a stable orbit that is repeated all over again. See last lecture for the explanation of "orbit".

Role of Sensitivity Parameter f

Phase space plots:



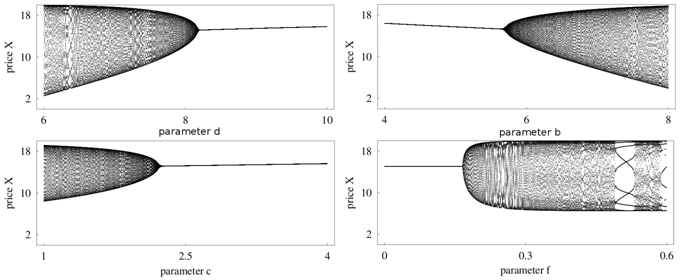
- top row: $f = 0.17$,
  bottom row: $f = 0.20$
- higher sensitivity increases the amplitude of fluctuations of prices, profits and distribution of suppliers across markets

Notes:

- The panels show the dynamics in phase space (after some time, i.e. the long-run behavior) for the case of symmetric markets
- $a = 20, b = 6, c = 2, d = 8, e = 1, f = 0.17$ (top panels) and $f = 0.20$ (bottom panels)
- The motion takes place on an invariant closed curve in the phase space. The left panels represent projections on the plane P_X , P_Z , the right panels show projections on the plane P_X , S_X. Notice how the phase space increases for a higher sensitivity to market information.

Bifurcation Diagrams

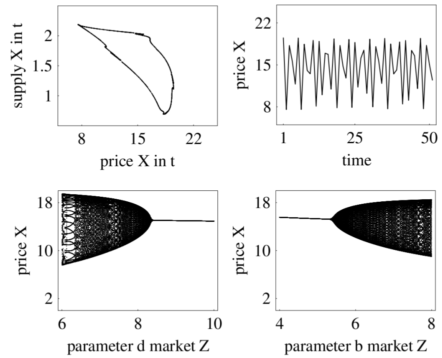
- complex behavior emerges from various control parameters
  - c/d: price-independent supply, $1/d$: price derivative of supply
  - $1/b$: price derivative of demand, f: sensitivity to profit differences



Notes:

- Note the period doubling scenarios. It is easier to see if you zoom in on the graphs
- Note the drastic change of the outcome as a result of slight changes in parameters at bifurcation points.
- Note it is not possible to determine from these graphs whether we are dealing with 'real chaos' or not - to determine that, we also need the Liapunov exponent which still is a necessary yet not sufficient condition.

Asymmetric Markets



- $d_X = 8$, $d_Z = 6$ $\Rightarrow$ more complex attractor
- bifurcations diagrams reveal destabilizing effects of one market on another

Notes:

Summary

- **realistic economic assumptions**
  - suppliers can change markets dependening on profitability
  - sensitivity to profits, cost functions are considered
- **limited predictability**
  - policies in interacting markets (such as influencing price elasticities) can have unforeseen consequences
  - *endogeneously* created fluctuations
- **result: realistic price dynamics**
  - coupled cobweb dynamics can explain the strong cyclical price motion observed in many commodity markets

SS 10: Coupled Cobweb Dynamics

- Using python explore the described Model
  - Create plots of the time dependent behaviour of market prices and profits, and the fraction of producers in each market.
- Investigate the role of the different control parameters on the dynamic behavior.
  - Change the intensity of choice and explain what happens.
  - Discuss the result with respect to predictability and profitability of markets.

Notes:

Notes:

Questions

1. What is the meaning of market clearing?
2. Explain the role of price elasticities for supply and demand. What is the meaning of a high elasticity? What is the difference to price derivatives?
3. What conditions have to be fulfilled for a stable equilibrium of supply and demand in the Cobweb model? What is their economic interpretation?
4. What are the limitations of the classical cobweb theory? How does coupled cobwebs overcome some of these limitations?
5. Explain the role of the profit and the sensitivity to it on the decision of the suppliers.

Notes: