Worked examples

Daniel Kelly

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Abstract

Time division multiple access

1 Example

Question 3: Consider the following idealised LTE scenario. The downstream channel is slotted in time, across F frequencies. There are four nodes, A, B, C, and D, reachable from the base station at rates of 10 Mbps, 5 Mbps, 2.5 Mbps, and 1 Mbps, respectively, on the downstream channel. These rates assume that the base station utilises all time slots available on all F frequencies to send to just one station. The base station has an infinite amount of data to send to each of the nodes, and can send to any one of these four nodes using any of the F frequencies during any time slot in the downstream sub-frame.

b: For the fairness requirement such that each node receives an equal amount of data during each downstream sub-frame, let n1, n2, n3, and n4 respectively represent the number of slots that A, B, C and D get.

Nodes	Downlink(Mbps) *
A	10
B	5
C	2.5
D	1

Table 1: *Downlink speed if all frequencies and time slots are used to communicate with the one node

$$10 \cdot t \cdot n1 = 5 \cdot t \cdot n2 = 2.5 \cdot t \cdot n3 = 1 \cdot t \cdot n4$$

Make **n2** the subject

$$\begin{aligned} 5 \cdot t \cdot n2 &= 10 \cdot t \cdot n1 \\ 5 \cdot n2 &= 10 \cdot n1 \\ n2 &= \frac{10 \cdot n1}{5} \\ n2 &= 2 \cdot n1 \end{aligned}$$

Make **n3** the subject

$$\begin{aligned} 2.5 \cdot t \cdot n3 &= 10 \cdot t \cdot n1 \\ 2.5 \cdot n3 &= 10 \cdot n1 \\ n3 &= \frac{10 \cdot n1}{2.5} \\ n3 &= 4 \cdot n1 \end{aligned}$$

Make **n4** the subject

$$\begin{aligned} 1 \cdot t \cdot n4 &= 10 \cdot t \cdot n1 \\ 1 \cdot n4 &= 10 \cdot n1 \\ n4 &= \frac{10 \cdot n1}{1} \\ n4 &= 10 \cdot n1 \end{aligned}$$

Define the total number of slots **N**

$$\begin{aligned} &N = n1 + n2 + n3 + n4 \\ &N = n1 + 2 \cdot n1 + 4 \cdot n1 + 10 \cdot n1 \\ &N = 17 \cdot n1 \end{aligned}$$

Make **n1**,**n2**,**n3**,**n4** the subject

$$\begin{aligned} n1 = n1 &= \frac{N}{17} \\ n2 = 2 \cdot n1 = 2 \cdot \frac{N}{17} &= \frac{2N}{17} \\ n3 = 4 \cdot n1 = 4 \cdot \frac{N}{17} &= \frac{4N}{17} \\ n4 = 10 \cdot n1 = 10 \cdot \frac{N}{17} &= \frac{10N}{17} \end{aligned}$$

The average transmission rate is

$$\overline{tr} = \frac{10 \cdot t \cdot n1 + 5 \cdot t \cdot n2 + 2.5 \cdot t \cdot n3 + 1 \cdot t \cdot n4}{t \cdot N}$$
$$\overline{tr} = \frac{10 \cdot n1 + 5 \cdot n2 + 2.5 \cdot n3 + 1 \cdot n4}{N}$$
$$\overline{tr} = \frac{10 \cdot n1 + 5 \cdot 2 \cdot n1 + 2.5 \cdot 4 \cdot n1 + 1 \cdot 10 \cdot n1}{N}$$
$$\overline{tr} = \frac{10 \cdot n1 + 10 \cdot n1 + 10 \cdot n1 + 10 \cdot n1}{N}$$
$$\overline{tr} = \frac{40 \cdot n1}{N}$$
$$\overline{tr} = \frac{40 \cdot \frac{N}{17}}{N}$$
$$\overline{tr} = \frac{\frac{40N}{17}}{\frac{N}{1}}$$
$$\overline{tr} = \frac{40N}{17} \cdot \frac{1}{N}$$
$$\overline{tr} = \frac{40}{17}$$
$$\overline{tr} = 2.353$$

2 Example

In this example only 1 of the nodes has a difference in the max downlink speed.

Nodes	Downlink(Mbps) *
A	10
B	10
C	10
D	5

Table 2: *Downlink speed if all frequencies and time slots are used to communicate with the one node

$$10 \cdot t \cdot n1 = 10 \cdot t \cdot n2 = 10 \cdot t \cdot n3 = 5 \cdot t \cdot n4$$

Make **n2** the subject

$$\begin{aligned} 10 \cdot t \cdot n2 &= 10 \cdot t \cdot n1 \\ 10 \cdot n2 &= 10 \cdot n1 \\ n2 &= \frac{10 \cdot n1}{10} \\ n2 &= n1 \end{aligned}$$

Make **n3** the subject

$$\begin{aligned} 10 \cdot t \cdot n3 &= 10 \cdot t \cdot n1 \\ 10 \cdot n3 &= 10 \cdot n1 \\ n3 &= \frac{10 \cdot n1}{10} \\ n3 &= n1 \end{aligned}$$

Make **n4** the subject

$$\begin{aligned} 5 \cdot t \cdot n4 &= 10 \cdot t \cdot n1 \\ 5 \cdot n4 &= 10 \cdot n1 \\ n4 &= \frac{10 \cdot n1}{5} \\ n4 &= 2 \cdot n1 \end{aligned}$$

Define the total number of slots **N**

$$N = n1 + n2 + n3 + n4$$
$$N = n1 + n1 + n1 + 2 \cdot n1$$
$$N = 5 \cdot n1$$

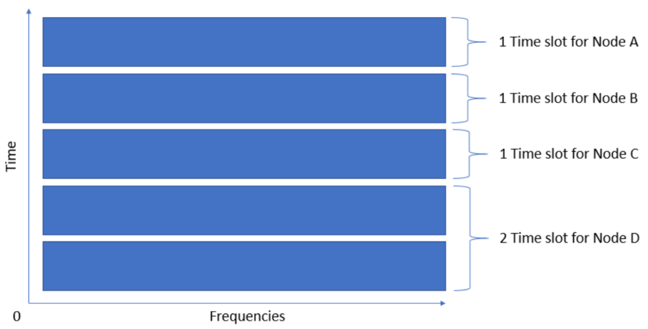
Make **n1**,**n2**,**n3**,**n4** the subject

$$\begin{aligned} n1 = n1 &= \frac{N}{5} \\ n2 = n1 &= \frac{N}{5} \\ n3 = n1 &= \frac{N}{5} \\ n4 = 2 \cdot n1 = 2 \cdot \frac{N}{5} &= \frac{2N}{5} \end{aligned}$$

The average transmission rate is

$$\overline{tr} = \frac{10 \cdot t \cdot n1 + 10 \cdot t \cdot n2 + 10 \cdot t \cdot n3 + 5 \cdot t \cdot n4}{t \cdot N}$$
$$\overline{tr} = \frac{10 \cdot n1 + 10 \cdot n2 + 10 \cdot n3 + 5 \cdot n4}{N}$$
$$\overline{tr} = \frac{10 \cdot n1 + 10 \cdot n1 + 10 \cdot n1 + 5 \cdot 2 \cdot n1}{N}$$
$$\overline{tr} = \frac{10 \cdot n1 + 10 \cdot n1 + 10 \cdot n1 + 10 \cdot n1}{N}$$
$$\overline{tr} = \frac{40 \cdot n1}{N}$$
$$\overline{tr} = \frac{40 \cdot \frac{N}{5}}{N}$$
$$\overline{tr} = \frac{\frac{40N}{5}}{\frac{N}{1}}$$
$$\overline{tr} = \frac{40N}{5} \cdot \frac{1}{N}$$
$$\overline{tr} = \frac{40}{5}$$
$$\overline{tr} = 8$$

Figure 1: Interpreting the results



Considering the assumptions of the question hold

- Have we assumed there is only 1 Frequency to simplify the analysis?
- Do we have a total of 5 time slots available?
- Specifically for this example, if each time slot was 1 second long. Would that mean each downlink frame would be 2 Mb for nodes A, B, and C communicating with the base station. With D have 2 downlink frames of 1 Mb?

3 Example

In this example we make all the max download speeds 10 Mbps. That is the downlink across all frequencies using all time slots for the base station to communicate with a single node.

Nodes	Downlink(Mbps) *
A	10
B	10
C	10
D	10

Table 3: *Downlink speed if all frequencies and time slots are used to communicate with the one node

$$10 \cdot t \cdot n1 = 10 \cdot t \cdot n2 = 10 \cdot t \cdot n3 = 10 \cdot t \cdot n4$$

Make **n2** the subject

$$n2 \ = n1$$

Make **n3** the subject

$$n3 \ = n1$$

Make **n4** the subject

$$n4 \ = n1$$

Define the total number of slots **N**

$$N = n1 + n2 + n3 + n4$$
$$N = n1 + n1 + n1 + n1$$
$$N = 4 \cdot n1$$

Make **n1**,**n2**,**n3**,**n4** the subject

$$n1 = n1 = \frac{N}{4}$$
$$n2 = n1 = \frac{N}{4}$$
$$n3 = n1 = \frac{N}{4}$$
$$n4 = n1 = \frac{N}{4}$$

The average transmission rate is

$$
\begin{aligned}
\overline{tr} &= \frac{10 \cdot t \cdot n1 + 10 \cdot t \cdot n2 + 10 \cdot t \cdot n3 + 10 \cdot t \cdot n4}{t \cdot N} \\
\overline{tr} &= \frac{10 \cdot n1 + 10 \cdot n2 + 10 \cdot n3 + 10 \cdot n4}{N} \\
\overline{tr} &= \frac{10 \cdot n1 + 10 \cdot n1 + 10 \cdot n1 + 10 \cdot n1}{N} \\
\overline{tr} &= \frac{40 \cdot n1}{N} \\
\overline{tr} &= \frac{40 \cdot \frac{N}{4}}{N} \\
\overline{tr} &= \frac{\frac{40N}{4}}{\frac{N}{1}} \\
\overline{tr} &= \frac{40N}{4} \cdot \frac{1}{N} \\
\overline{tr} &= \frac{40}{4} \\
\overline{tr} &= 10
\end{aligned}
$$

4 Example

Downlink max rate drops by a factor of 4 per node

Nodes	Downlink(Mbps) *
A	80
B	20
C	5
D	1.25

Table 4: *Downlink speed if all frequencies and time slots are used to communicate with the one node

$$80 \cdot t \cdot n1 = 20 \cdot t \cdot n2 = 5 \cdot t \cdot n3 = 1.25 \cdot t \cdot n4$$

Make **n2** the subject

$$
\begin{aligned}
20 \cdot t \cdot n2 &= 80 \cdot t \cdot n1 \\
20 \cdot n2 &= 80 \cdot n1 \\
n2 &= \frac{80 \cdot n1}{20} \\
n2 &= 4 \cdot n1
\end{aligned}
$$

Make **n3** the subject

$$
\begin{aligned}
5 \cdot t \cdot n3 &= 80 \cdot t \cdot n1 \\
5 \cdot n3 &= 80 \cdot n1 \\
n3 &= \frac{80 \cdot n1}{5} \\
n3 &= 16 \cdot n1
\end{aligned}
$$

Make **n4** the subject

$$
\begin{aligned}
1.25 \cdot t \cdot n4 &= 80 \cdot t \cdot n1 \\
1.25 \cdot n4 &= 80 \cdot n1 \\
n4 &= \frac{80 \cdot n1}{1.25} \\
n4 &= 64 \cdot n1
\end{aligned}
$$

Define the total number of slots **N**

$$
\begin{aligned}
N &= n1 + n2 + n3 + n4 \\
N &= n1 + 4 \cdot n1 + 16 \cdot n1 + 64 \cdot n1 \\
N &= 85 \cdot n1
\end{aligned}
$$

Make **n1**,**n2**,**n3**,**n4** the subject

$$
\begin{aligned}
n1 = n1 &= \frac{N}{85} \\
n2 = 4 \cdot n1 = 4 \cdot \frac{N}{85} &= \frac{4N}{85} \\
n3 = 16 \cdot n1 = 16 \cdot \frac{N}{85} &= \frac{16N}{85} \\
n4 = 64 \cdot n1 = 64 \cdot \frac{N}{85} &= \frac{64N}{85}
\end{aligned}
$$

The average transmission rate is

$$
\begin{aligned}
\overline{tr} &= \frac{80 \cdot t \cdot n1 + 20 \cdot t \cdot n2 + 5 \cdot t \cdot n3 + 1.25 \cdot t \cdot n4}{t \cdot N} \\
\overline{tr} &= \frac{80 \cdot n1 + 20 \cdot n2 + 5 \cdot n3 + 1.25 \cdot n4}{N} \\
\overline{tr} &= \frac{80 \cdot n1 + 20 \cdot 4 \cdot n1 + 5 \cdot 16 \cdot n1 + 1.25 \cdot 64 \cdot n1}{N} \\
\overline{tr} &= \frac{80 \cdot n1 + 80 \cdot n1 + 80 \cdot n1 + 80 \cdot n1}{N} \\
\overline{tr} &= \frac{320 \cdot n1}{N} \\
\overline{tr} &= \frac{320 \cdot \frac{N}{85}}{N} \\
\overline{tr} &= \frac{\frac{320 \cdot N}{85}}{\frac{N}{1}} \\
\overline{tr} &= \frac{320 \cdot N}{85} \cdot \frac{1}{N} \\
\overline{tr} &= \frac{320}{85} \\
\overline{tr} &= 3.765
\end{aligned}
$$

5 Example

Downlink max rate drops by a factor of 4 per node

Nodes	Downlink(Mbps) *
A	20
B	15
C	10
D	5

Table 5: *Downlink speed if all frequencies and time slots are used to communicate with the one node

$$20 \cdot t \cdot n1 = 15 \cdot t \cdot n2 = 10 \cdot t \cdot n3 = 5 \cdot t \cdot n4$$

Make **n2** the subject

$$\begin{aligned} 15 \cdot t \cdot n2 &= 20 \cdot t \cdot n1 \\ 15 \cdot n2 &= 20 \cdot n1 \\ n2 &= \frac{20 \cdot n1}{15} \\ n2 &= 1.\overline{3} \cdot n1 \end{aligned}$$

Make **n3** the subject

$$\begin{aligned} 10 \cdot t \cdot n3 &= 20 \cdot t \cdot n1 \\ 10 \cdot n3 &= 20 \cdot n1 \\ n3 &= \frac{20 \cdot n1}{10} \\ n3 &= 2 \cdot n1 \end{aligned}$$

Make **n4** the subject

$$\begin{aligned} 5 \cdot t \cdot n4 &= 20 \cdot t \cdot n1 \\ 5 \cdot n4 &= 20 \cdot n1 \\ n4 &= \frac{20 \cdot n1}{5} \\ n4 &= 4 \cdot n1 \end{aligned}$$

Define the total number of slots **N**

$$\begin{aligned} &N = n1 + n2 + n3 + n4 \\ &N = n1 + 1.\overline{3} \cdot n1 + 2 \cdot n1 + 4 \cdot n1 \\ &N = 8.\overline{3} \cdot n1 \end{aligned}$$

Make **n1**,**n2**,**n3**,**n4** the subject

$$\begin{aligned} n1 = n1 &= \frac{N}{8.\overline{3}} \\ n2 = 1.\overline{3} \cdot n1 = 1.\overline{3} \cdot \frac{N}{8.\overline{3}} &= \frac{1.\overline{3}N}{8.\overline{3}} \\ n3 = 2 \cdot n1 = 2 \cdot \frac{N}{8.\overline{3}} &= \frac{2N}{8.\overline{3}} \\ n4 = 4 \cdot n1 = 4 \cdot \frac{N}{8.\overline{3}} &= \frac{4N}{8.\overline{3}} \end{aligned}$$

The average transmission rate is

$$\begin{aligned} &\overline{tr} = \frac{20 \cdot t \cdot n1 + 15 \cdot t \cdot n2 + 10 \cdot t \cdot n3 + 5 \cdot t \cdot n4}{t \cdot N} \\ &\overline{tr} = \frac{20 \cdot n1 + 15 \cdot n2 + 10 \cdot n3 + 5 \cdot n4}{N} \\ &\overline{tr} = \frac{20 \cdot n1 + 15 \cdot 1.\overline{3} \cdot n1 + 10 \cdot 2 \cdot n1 + 5 \cdot 4 \cdot n1}{N} \\ &\overline{tr} = \frac{20 \cdot n1 + 20 \cdot n1 + 20 \cdot n1 + 20 \cdot n1}{N} \\ &\overline{tr} = \frac{80 \cdot n1}{N} \\ &\overline{tr} = \frac{80 \cdot \frac{N}{8.\overline{3}}}{N} \\ &\overline{tr} = \frac{\frac{80 \cdot N}{8.\overline{3}}}{\frac{N}{1}} \\ &\overline{tr} = \frac{80 \cdot N}{8.\overline{3}} \cdot \frac{1}{N} \\ &\overline{tr} = \frac{80}{8.\overline{3}} = \frac{240}{25} = \frac{48}{5} \\ &\overline{tr} = 9.6 \end{aligned}$$