Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Symbolic AI

Date: Friday 15th May 2015

Time: 14:00 - 16:00

Please answer any THREE Questions from the FOUR Questions provided.

This is a CLOSED book examination

The use of electronic calculators is permitted provided they are not programmable and do not store text

1. a) Define a Prolog predicate `otherFromThree(I,J,K)`, which, when `I` and `J` are instantiated to distinct elements of $\{1,2,3\}$ and K is uninstantiated, will return with K instantiated to the remaining element, thus:

```
?- otherFromThree(3,1,K).
K = 2.
```

The predicate should not re-satisfy. You may use the pre-defined Prolog predicate `member/3`. (4 marks)

b) The Towers of Hanoi problem features n-discs $D_1,\ldots,D_n$ of increasing diameters, stacked on three poles in such a way that no disc is placed above any other disc of lesser diameter. The top-most disc on any pole may be moved and placed on any other pole as long as doing so would not place it on a disc of lesser diameter.

A permitted arrangement of the n discs may be represented as a list of integers $[p_1,\ldots,p_n]$, where, for all i ($1 \leq i \leq n$), p_i is the number of the pole (1, 2 or 3) on which disc D_i is placed. Let G_n denote the *state-graph* of this problem: i.e. the graph whose vertices are the permitted arrangements of the n discs, and whose edges join arrangements that can be reached from each other by moving a single disc.

Draw G_n for the values $n = 1$, $n = 2$ and $n = 3$. (6 marks)

c) Describe how G_{n+1} is related to G_n. (You may use a diagram if that helps.) (2 marks)

d) What is the length of the shortest path from $[1,\ldots,1]$ (all discs on pole 1) to $[2,\ldots,2]$ (all discs on pole 2) in G_n? Express your answer as a function of n. (2 marks)

e) Suppose we represent the action of moving the top disc on pole I to pole J by a Prolog term `move`(I,J). Define a Prolog predicate `plan(N,I,J,P)` which, when called with `N` instantiated by a positive integer, `I` and `J` instantiated by elements of the set $\{1,2,3\}$, and `P` uninstantiated, returns with `P` instantiated to a path in G_n from `[I, ..., I]` to `[J, ..., J]`—i.e. a a list of legal moves that, when executed in order, will transfer the discs from pole `I` to pole `J`, thus:

```
?- plan(2,1,2,P).
P = [move(1,3), move(1,2), move(3,2)].
```

You may use the predefined predicate `append/3` and the predicate `oherFromThree/3`, which you defined above. (6 marks)

2. a) Write first-order formulas, ϕ_1, ϕ_2 and ψ, representing the following English sentences, using a natural signature of unary and binary predicates.

   i) ϕ_1: Every artist hates some beekeeper
   ii) ϕ_2: Some beekeeper hates no artist
   iii) ψ: Some beekeeper is not an artist.

   (6 marks)

   b) Write ϕ_1, ϕ_2 and $\neg\psi$ (note the negation) in clause form. (Hint: you should have five clauses, with one Skolem constant and one Skolem function.) (6 marks)

   c) Call the set of clauses obtained in Part b) Γ. Use resolution theorem proving to derive the empty clause from Γ. (8 marks)

3. a) Explain the operation of the predefined Prolog predicate `name/2`. (2 marks)

b) Using the predefined Prolog predicates `name/2` and `append/2`, define a predicate `unglom/3` which removes one atom from the end of another (failing if this is impossible):

```
?- unglom(mousetrap,trap,Word).
Word = mouse.

?- unglom(mousetrap,ap,Word).
Word = mousetr.

?- unglom(mousetrap,cheese,Word).
no.
```

(2 marks)

c) The inhabitants of the planet Htrae speak a language one fragment of which is given by the following grammar.

IP $\rightarrow$ I′ NP	N $\rightarrow$ nam
I′ $\rightarrow$ VP I	N $\rightarrow$ god
VP $\rightarrow$ NP V	V $\rightarrow$ nettib
NP $\rightarrow$ N Det	I $\rightarrow$ sah
	Det $\rightarrow$ eht.

Draw the phrase-structure of the Htraean sentence (IP)

nam eht nettib sah god eht.

(6 marks)

d) Write a Prolog definite clause grammar (dcg) to parse this fragment of Htraean. Your dcg should output the phrase-structure of the input sentence as a Prolog term. (4 marks)

e) Members of the Htraean priesthood speak a more complex language. The noun in the subject of the sentence (IP specifier) must carry the suffix ‘-bus’, and the noun in the object (VP complement) must carry the suffix ‘-jbo’. Thus, in their dialect,

namjbo eht nettib sah godbus eht

is grammatical, while

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nambus eht nettib sah godbus eht

would not be. Modify your dcg so as to recognize the priestly dialect. The rules should work when new Htraean nouns are added: *tac*, *esuom*, *elidocorc* ....

(6 marks)

4. a) Consider the following expressions in the simply-typed lambda calculus:

$$\begin{aligned} L_1 &= \lambda P \lambda Q.(\forall x((P\ x) \rightarrow (Q\ x))) \\ L_2 &= \lambda x.(\text{boy}\ x) \\ L_3 &= \lambda x.(\text{coughed}\ x). \end{aligned}$$

Compute the fully β-reduced forms of:

i) $(L_1\ L_2)$

ii) $((L_1\ L_2)\ L_3)$,

clearly showing any intermediate steps. (6 marks)

b) Write a (linguistically natural) semantically annotated context free grammar able to reproduce the correct semantics for the sentence

Every boy coughed,

and illustrate the derivation of the semantics of this sentence using a phrase-structure diagram. (10 marks)

c) What happens if you attempt fully to β-reduce the expression

$$(\lambda x(x\ x)\ \lambda x(x\ x)) \quad ?$$

(4 marks)