

# Outline

## Exercise 1

Problem 1

Problem 2

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The following formula has its parentheses removed. Restore the parentheses.

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Take the interpretation  $I_1 = \{p \mapsto 0, q \mapsto 0, r \mapsto 0\}$ . We have

$$\begin{aligned} I_1(p \rightarrow (q \rightarrow r)) &= 1; \\ I_1((p \rightarrow q) \rightarrow r) &= 0. \end{aligned}$$



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Show that the formulas  $p \rightarrow (q \rightarrow r)$  and  $(p \rightarrow q) \rightarrow r$  are **not equivalent** by finding an interpretation in which they have different truth values.

Another solution is the interpretation  $I_2 = \{p \mapsto 0, q \mapsto 1, r \mapsto 0\}$ . We have

$$\begin{aligned} I_2(p \rightarrow (q \rightarrow r)) &= 1; \\ I_2((p \rightarrow q) \rightarrow r) &= 0. \end{aligned}$$