Exercise 9 (Problem 1)

Take the domain axiom for a variable whose domain contains 1000 values and transform this axiom into CNF using the standard CNF transformation. What is the number of clauses in the resulting CNF?

Solution

The domain axiom is the following formula:

$$(x_{v_1} \vee \ldots \vee x_{v_{1000}}) \wedge \bigwedge_{i<j\leq 1000} (\neg x_{v_i} \vee \neg x_{v_j}),$$

It is already in CNF, so the set of clauses obtained by the CNF transformation will contain the clause

$$x_{v_1} \vee \ldots \vee x_{v_{1000}})$$

and all binary clauses

$$\neg x_{v_i} \vee \neg x_{v_j}$$

such that $i < j \leq 1000$. It is not hard to argue that the number of such binary clauses is $\frac{999 \cdot 1000}{2} = 499500$. Thus, the CNF contains 499501 clause.

Exercise 9 (Problem 2)

Let x be a variable with the domain $\{a, b, c, d\}$ and p be a boolean variable. Consider the following formula:

$$\neg((p \rightarrow \neg x = a) \rightarrow x = b \vee x = c \vee \neg p).$$

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- Find a model of this formula;

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- Find a model of this formula;

Solution

This formula has a unique model $\{p \mapsto 1, x \mapsto d\}$.

Exercise 9 (Problem 2), continued

- Transform this formula to propositional logic;

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To transform this formula to propositional logic, we introduce propositional variables x_a, x_b, x_c, x_d, rewrite the formula using these variables and add domain axioms for x. This gives us the following formula:

$$
\begin{array}{l}
\neg((p \rightarrow \neg x_a) \rightarrow x_b \vee x_c \vee \neg p) \wedge \\
(x_a \vee x_b \vee x_c \vee x_d) \wedge \\
(\neg x_a \vee \neg x_b) \wedge \\
(\neg x_a \vee \neg x_c) \wedge \\
(\neg x_a \vee \neg x_d) \wedge \\
(\neg x_b \vee \neg x_b) \wedge \\
(\neg x_b \vee \neg x_d) \wedge \\
(\neg x_c \vee \neg x_d).
\end{array}
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Exercise 9 (Problem 2), continued

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(\neg x_a \vee \neg x_b) \wedge \\
(\neg x_a \vee \neg x_c) \wedge \\
(\neg x_a \vee \neg x_d) \wedge \\
(\neg x_b \vee \neg x_b) \wedge \\
(\neg x_b \vee \neg x_d) \wedge \\
(\neg x_c \vee \neg x_d).
\end{array}
$$

- Find a model of the resulting propositional formula

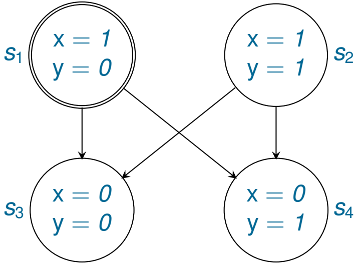
This formula has a unique model

$$\{p \mapsto 1, x_a \mapsto 0, x_b \mapsto 0, x_c \mapsto 0, x_d \mapsto 1\}.$$

Exercise 9 (Problem 3)

Consider the transition system with the state transition graph shown on the right.

1. Find a symbolic representation of the set of states $\{s_1, s_3\}$.
2. Find a symbolic representation of the transition $\{(s_1, s_3)\}$.
3. Find a symbolic representation of the transition relation of this system.



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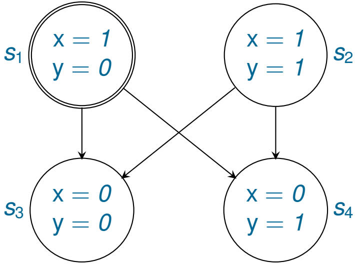

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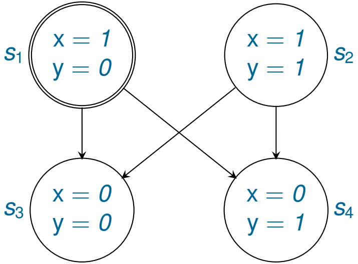

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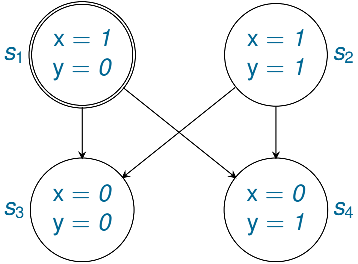

Solution

1. $y = 0$

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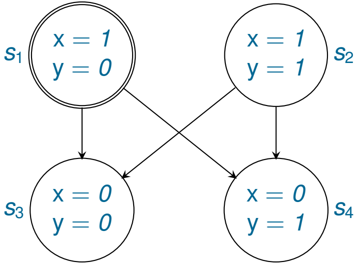

2. Find a symbolic representation of the transition $\{(s_1, s_3)\}$.

Solution

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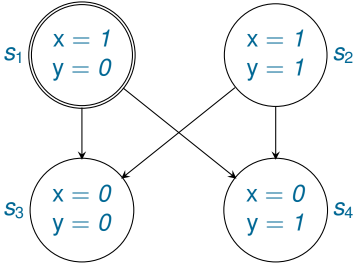

2. Find a symbolic representation of the transition $\{(s_1, s_3)\}$.

Solution

2. $x = 1 \wedge y = 0 \wedge x' = 0 \wedge y' = 0$

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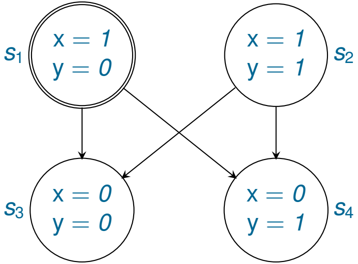

3. Find a symbolic representation of the transition relation of this system.

Solution

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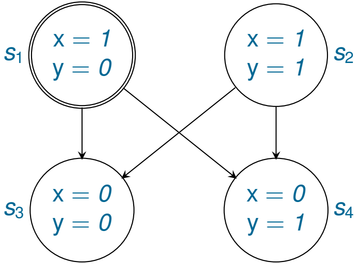

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Solution

3. $x = 1 \wedge x' = 0$

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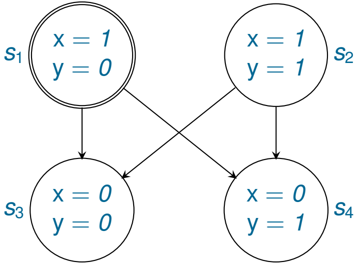

Solution

1. $y = 0$
2. $x = 1 \wedge y = 0 \wedge x' = 0 \wedge y' = 0$
3. $x = 1 \wedge x' = 0$