COMP21111

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Logic and Modelling

Date: Friday 20th January 2012

Time: 14:00 - 16:00

Please answer any THREE questions from the FIVE questions provided

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This is a CLOSED book examination

The use of electronic calculators is NOT permitted

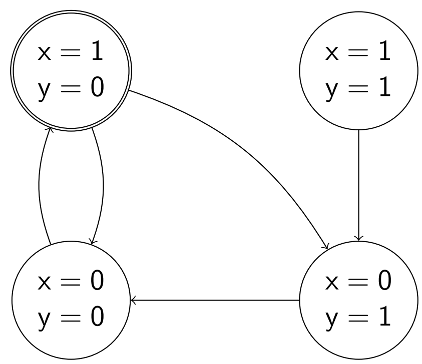
[PTO]

1.

a) Apply the DLL method to the following set of clauses, i.e., show the splitting tree and the results of all unit propagation steps. Use the pure literal rule, whenever applicable. Is this set of clauses satisfiable? If yes, give an interpretation which satisfies it. (8 marks)

$$
\begin{gathered}
p \lor q \lor \neg r, \\
p \lor \neg q \lor r, \\
p \lor \neg q \lor \neg r, \\
\neg p \lor q \lor r, \\
\neg p \lor q \lor \neg r, \\
\neg p \lor \neg q \lor r, \\
\neg p \lor \neg q \lor \neg r.
\end{gathered}
$$

b) Consider a transition system with the following state transition graph.



For each of the following formulas determine whether it is true or false on all paths.

1. $\Box\Diamond(\mathsf{x} = 0 \land \mathsf{y} = 0)$; (1 marks)
2. $\Box\Diamond(\mathsf{y} = 1)$; (2 marks)
3. $\Box(\mathsf{x} = 0 \lor \mathsf{y} = 0)$; (2 marks)
4. $\Box(\mathsf{x} = 1 \land \mathsf{y} = 1 \rightarrow \bigcirc \mathsf{x} = 1)$. (2 marks)

For each formula that is false, give a path on which it is false.

c) Consider the set consisting of the following clauses:

$p_0 \lor \neg p_1 \lor p_2$	$p_0 \lor \neg p_1 \lor p_2 \lor p_4$	$\neg p_0 \lor p_1 \lor \neg p_2$	$\neg p_0 \lor \neg p_1 \lor \neg p_2$
$p_0 \lor \neg p_1 \lor p_4$	$p_3 \lor p_2 \lor p_4 \lor \neg p_0$	$\neg p_2 \lor \neg p_2 \lor p_4 \lor p_3$	$\neg p_2 \lor \neg p_0 \lor p_4 \lor p_4$
$p_0 \lor p_3 \lor \neg p_4$	$p_0 \lor \neg p_1 \lor \neg p_2 \lor \neg p_3$	$\neg p_1 \lor \neg p_2 \lor \neg p_3$	$p_1 \lor \neg p_2 \lor \neg p_3 \lor \neg p_4$
$p_1 \lor p_2$	$p_2 \lor p_3 \lor \neg p_4 \lor p_3$	$\neg p_0 \lor \neg p_2 \lor \neg p_3 \lor \neg p_4$	$p_0 \lor p_2 \lor p_4$

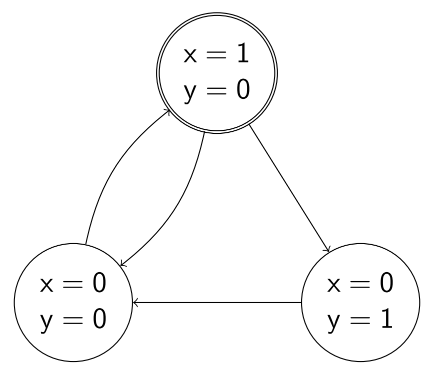
For each of the variables p_0, p_1, p_2, p_3, p_4 find the probability that WSAT will choose this variable for flipping when the current interpretation is
$\{p_0 \mapsto 0, p_1 \mapsto 0, p_2 \mapsto 0, p_3 \mapsto 0, p_4 \mapsto 0\}$. (5 marks)

2.

a) What is the total number of occurrences of variables in the domain axiom for a variable whose domain contains 1000 values? (4 marks)

b) Show, using the tableau method, that the formula $(p \land (q \to r)) \to (p \land \neg q) \lor (p \land r)$ is a tautology. (10 marks)

c) Consider a transition system with the following state transition graph.



Let S_1 be the set of states symbolically represented by the formula $\mathsf{x} = 1$ and S_2 be the set of states symbolically represented by the formula $\mathsf{x} = 1 \land \mathsf{y} = 0$.

1. Does S_1 coincide with the set of initial states? (2 marks)

2. Find a symbolic representation of formulas reachable from S_2 in exactly two steps. (2 marks)

3. Find a symbolic representation of formulas backward reachable from S_2 in exactly two steps. (2 marks)

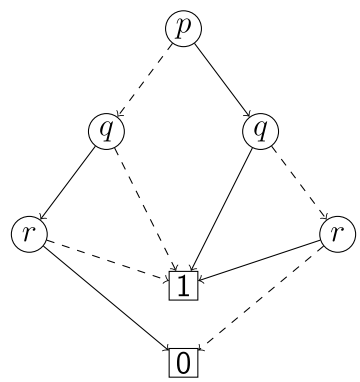
3.

a) Consider the following formula A:

$$\forall p(p \to q) \land p \leftrightarrow \exists q(\neg q \lor r).$$

1. Draw the parse tree for A. (1 marks)
2. Underline the free occurrences of variables in A. (2 marks)
3. Explain why A is not rectified and make it into a rectified formula by renaming bound variables. (3 marks)

b) A formula A has the following OBDD.



Find all models of A. (4 marks)

c) Write down LTL formulas expressing the following properties:

1. Formula A never holds in two consecutive states; (3 marks)
2. If A holds in a state s, it also holds in all states after s; (3 marks)
3. A holds in at most one state. (4 marks)

4.

a) Consider the following formula in CNF (11 marks)

$$\exists p \forall q \exists r ((\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (p \vee q \vee r) \wedge (p \vee q \vee \neg r))$$

Evaluate this formula using the DLL algorithm. Show all steps of the algorithm. Is this formula true or false?

b) Consider the set consisting of the following clauses:

$$p_0 \vee \neg p_1 \vee \neg p_2, \quad \neg p_0 \vee \neg p_2, \quad p_0 \vee p_1, \quad p_1 \vee p_2, \quad p_0 \vee \neg p_1 \vee p_2.$$

Show how the WSAT algorithm can find a model of this set starting with the initial random interpretation $\{p_0 \mapsto 0, p_1 \mapsto 0, p_2 \mapsto 1\}$. (9 marks)

5.

a) Transform the formula $(p \leftrightarrow q) \rightarrow r$ into a set of clauses using the standard CNF transformation. (5 marks)

b) Let p and q be atoms. Show that the following two formulas are not equivalent by giving a path which satisfies one of them but not the other: (7 marks)

$$\Box(p \rightarrow q);$$
$$\neg\Diamond p \vee \Box q.$$

c) Evaluate the formula $\exists q \forall p(q \leftrightarrow p)$ using the following algorithm. First, build an OBDD which represents the propositional part of the formula $q \leftrightarrow p$. Then apply the quantification algorithm to the OBDD until all quantifiers are eliminated. Use the order $q > p$. (Give the initial OBDD and the OBDDs resulting from each quantification step). (8 marks)

END OF EXAMINATION