COMP21111

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Logic and Modelling

Date: Monday 21st January 2013

Time: 09:45 - 11:45

Please answer any THREE Questions from the FOUR Questions provided

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]

1.

a) Convert the following formula $\neg(p \rightarrow q \wedge r) \vee q$ into clausal form using definitional normal form transformation. Is the obtained set of clauses equivalent to the initial formula? (6 marks)

b) Apply the DPLL method to the following set of clauses, i.e., show the DPLL tree and the results of all unit propagation steps. Is this set of clauses satisfiable? If yes, give an interpretation which satisfies it. (9 marks)

$$\boxed{\begin{array}{r} \neg p \vee q \vee r \\ p \vee \neg q \vee \neg r \\ \neg p \vee \neg q \vee \neg r \\ p \vee q \\ p \vee r \\ \neg p \vee \neg q \vee r \end{array}}$$

c) Formalise the following in propositional logic: exactly two variables are false among $p_1, \ldots, p_n$. If you are using the T notation you should explicitly define it. (5 marks)

2.

a) Briefly explain what is a pure atom. Describe the pure atom rule for checking satisfiability of propositional formulas. (3 marks)

b)

i Draw the OBDD for the formula $(q \rightarrow \neg p) \wedge (p \vee q)$ with ordering $p > q$. (4 marks)

ii Apply the $\forall$-quantification algorithm to the OBDD constructed in i) to obtain an OBDD node representing $\forall q((q \rightarrow \neg p) \wedge (p \vee q))$. (4 marks)

c) Consider the following QBF formula in CNF (9 marks)

$$\boxed{\begin{array}{l} \exists p \forall q \exists r \\ \neg q \vee r \\ \neg p \vee q \vee r \\ \neg p \vee \neg q \vee \neg r \\ p \vee q \vee \neg r \end{array}}$$

Evaluate this formula using the DPLL algorithm. Show all steps of the algorithm. Is the given QBF formula true or false?

3.

a) Briefly describe two main differences between the splitting and the DPLL algorithms for propositional satisfiability. (2 marks)

b) Consider the set consisting of the following clauses:

$$p_0 \lor \neg p_1 \lor \neg p_2, \quad \neg p_0 \lor \neg p_2, \quad p_0 \lor p_1, \quad p_1 \lor p_2, \quad p_0 \lor \neg p_1 \lor p_2.$$

Show how the GSAT algorithm can find a model of this set starting with the initial random interpretation $\{p_0 \mapsto 0, p_1 \mapsto 0, p_2 \mapsto 1\}$. Show the candidates for flipping at each stage. (9 marks)

c) Based on an OBDD representation of a formula F, in each of the following cases explain how to check that F is:

i valid;

ii satisfiable;

iii equivalent to a formula G represented in the same OBDD structure.

(3 marks)

d) Consider formulas $I(\bar{x})$ and $T(\bar{x}, \bar{x}')$ symbolically representing a set of initial states and a transition relation.

i Define the formula $R_{=1}(\bar{x})$ representing the set of states reachable from the initial states in exactly one step; (2 marks)

ii Define formulas $R_{\leq 0}(\bar{x}), \ldots, R_{\leq n}(\bar{x})$ representing sets of states reachable from the initial states in $\leq 0, \ldots, \leq n$ steps, respectively. (4 marks)

4.

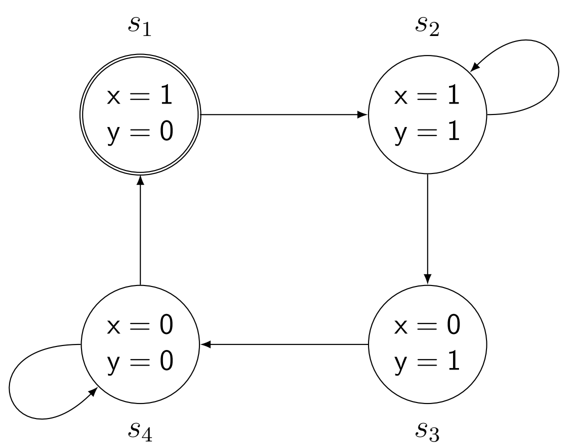
a) Let A be a propositional formula over variables $p_1, \ldots, p_{n-1}$. What is the number of models of $p_n \leftrightarrow A$? Explain your answer. (3 marks)

b) Apply the splitting algorithm to evaluate the following QBF formula:

$$\exists q \forall p(p \leftrightarrow (q \rightarrow p)).$$

Is this formula true or false? (8 marks)

c) Consider a transition system with the following state transition graph.



Which of the following formulas are true on at least one of the paths starting from the initial state? If a formula is true on a path, draw one such path.

i $\Diamond(x \rightarrow \Box y)$ (1 marks)

ii $\Box\Diamond(x \leftrightarrow \neg y)$ (1 marks)

iii $\Diamond\Box(x \leftrightarrow \neg y)$ (2 marks)

iv $(\Diamond y)\,\mathbf{U}\,(\Box\neg y)$ (2 marks)

d) Let a variable x in propositional logic of finite domains have the domain $\{u, v, f\}$. Write down the domain axiom for the variable x. (3 marks)

END OF EXAMINATION