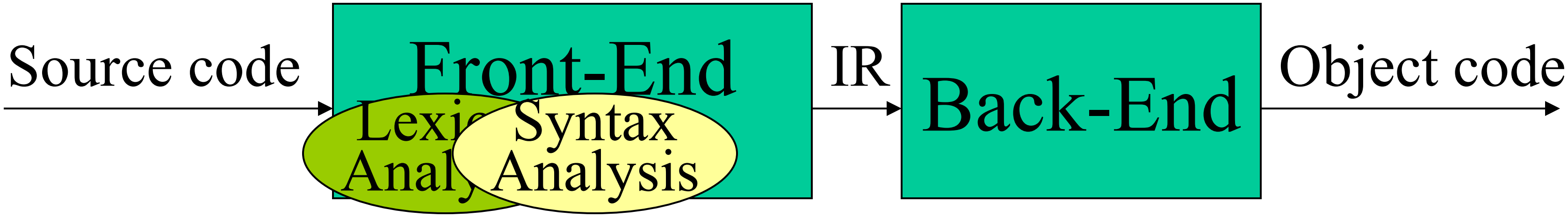
Lecture 9: Bottom-Up Parsing



(from last lecture) Top-Down Parsing:

- Start at the root of the tree and grow towards leaves.
- Pick a production and try to match the input.
- We may need to backtrack if a bad choice is made.
- Some grammars are backtrack-free (predictive parsing).

Today's lecture:

Bottom-Up parsing

Bottom-Up Parsing: What is it all about?

Goal: Given a grammar, G, construct a parse tree for a string (i.e., sentence) by starting at the leaves and working to the root (i.e., by working from the input sentence back toward the start symbol S).

Recall: the point of parsing is to construct a derivation:

$$S \Rightarrow \delta_0 \Rightarrow \delta_1 \Rightarrow \delta_2 \Rightarrow \ldots \Rightarrow \delta_{n-1} \Rightarrow sentence$$

To derive δ_{i-1} from δ_i, we match some *rhs b* in δ_i, then replace *b* with its corresponding *lhs*, *A*. This is called a **reduction** (it assumes $A \rightarrow b$).

The **parse tree** is the result of the **tokens** and the **reductions**.

Example: Consider the grammar below and the input string **abbcde**.

```
1.  Goal→aABe
2.     A→Abc
3.       |b
4.     B→d
```

Sentential Form	Production	Position
abbcde	3	2
a A bcde	2	4
a A de	4	3
a A B e	1	4
Goal	-	-

Finding Reductions

- What are we trying to find?
  - A substring b that matches the right-side of a production that occurs as one step in the rightmost derivation. Informally, this substring is called a **handle**.
- Formally, a handle of a right-sentential form δ is a pair $<A \rightarrow b,k>$ where $A \rightarrow b \in P$ and k is the position in δ of b's rightmost symbol.

(right-sentential form: a sentential form that occurs in some rightmost derivation).

  - Because δ is a right-sentential form, the substring to the right of a handle contains only terminal symbols. Therefore, the parser doesn't need to scan past the handle.
  - If a grammar is unambiguous, then every right-sentential form has a unique handle (sketch of proof by definition: if unambiguous then rightmost derivation is unique; then there is unique production at each step to produce a sentential form; then there is a unique position at which the rule is applied; hence, unique handle).

  If we can find those handles, we can build a derivation!

Motivating Example

Given the grammar of the left-hand side below, find a rightmost derivation for $x - 2*y$ (starting from Goal there is only one, the grammar is not ambiguous!). In each step, identify the handle.

1. $Goal \rightarrow Expr$
2. $Expr \rightarrow Expr + Term$
3. $\quad\quad | \; Expr - Term$
4. $\quad\quad | \; Term$
5. $Term \rightarrow Term * Factor$
6. $\quad\quad | \; Term \,/\, Factor$
7. $\quad\quad | \; Factor$
8. $Factor \rightarrow number$
9. $\quad\quad | \; id$

Production	Sentential Form	Handle
-	*Goal*	-
1	*Expr*	1,1
3	*Expr – Term*	3,3

<u>Problem</u>: given the sentence $x - 2*y$, find the handles!

A basic bottom-up parser

- The process of discovering a handle is called handle pruning.
- To construct a rightmost derivation, apply the simple algorithm:

  for i=n to 1, step -1

  find the handle $<A \rightarrow b,k>_i$ in δ_i

  replace b with A to generate δ_{i-1}

  (needs 2n steps, where n is the length of the derivation)
- One implementation is based on using a stack to hold grammar symbols and an input buffer to hold the string to be parsed. Four operations apply:
  - **shift**: next input is shifted (pushed) onto the top of the stack
  - **reduce**: right-end of the handle is on the top of the stack; locate left-end of the handle within the stack; pop handle off stack and push appropriate non-terminal left-hand-side symbol.
  - **accept**: terminate parsing and signal success.
  - **error**: call an error recovery routine.

Implementing a shift-reduce parser

```
push $ onto the stack
token = next_token()
repeat
   if the top of the stack is a handle A→b
        then /* reduce b to A */
                pop the symbols of b off the stack
                push A onto the stack
        elseif (token != eof) /* eof: end-of-file = end-of-input */
                then /* shift */
                        push token
                        token=next_token()
                else /* error */
                        call error_handling()
until (top_of_stack == Goal && token==eof)
```

Errors show up: a) when we fail to find a handle, or b) when we hit EOF and we need to shift. The parser needs to recognise syntax errors.

Example: x–2*y

Stack	Input	Handle	Action	
$	id – num * id	None	Shift	
$ id	– num * id	9,1	Reduce 9	
$ Factor	– num * id	7,1	Reduce 7	
$ Term	– num * id	4,1	Reduce 4	
$ Expr	– num * id	None	Shift	!!
$ Expr –	num * id	None	Shift	
$ Expr – num	* id	8,3	Reduce 8	
$ Expr – Factor	* id	7,3	Reduce 7	
$ Expr – Term	* id	None	Shift	!!
$ Expr – Term *	id	None	Shift	
$ Expr – Term * id		9,5	Reduce 9	
$ Expr – Term * Factor		5,5	Reduce 5	
$ Expr – Term		3,3	Reduce 3	
$ Expr		1,1	Reduce 1	
$ Goal		none	Accept	

- 1. Shift until top of stack is the right end of the handle
- 2. Find the left end of the handle and reduce

(5 shifts, 9 reduces, 1 accept)

What can go wrong?

(think about the steps with an exclamation mark in the previous slide)

- **Shift/reduce conflicts**: the parser cannot decide whether to shift or to reduce.

  Example: the dangling-else grammar; usually due to ambiguous grammars.

  Solution: a) modify the grammar; b) resolve in favour of a shift.

- **Reduce/reduce conflicts**: the parser cannot decide which of several reductions to make.

  Example: `id(id,id)`; reduction is dependent on whether the first `id` refers to array or function.

  May be difficult to tackle.

Key to efficient bottom-up parsing: the handle-finding mechanism.

LR(1) grammars

(a beautiful example of applying theory to solve a complex problem in practice)

A grammar is LR(1) if, given a rightmost derivation, we can (I) isolate the handle of each right-sentential form, and (II) determine the production by which to reduce, by scanning the sentential form from left-to-right, going at most 1 symbol beyond the right-end of the handle.

- LR(1) grammars are widely used to construct (automatically) efficient and flexible parsers:
  - Virtually all context-free programming language constructs can be expressed in an LR(1) form.
  - LR grammars are the most general grammars parsable by a non-backtracking, shift-reduce parser (deterministic CFGs).
  - Parsers can be implemented in time proportional to tokens+reductions.
  - LR parsers detect an error as soon as possible in a left-to-right scan of the input.

L stands for left-to-right scanning of the input; R for constructing a rightmost derivation in reverse; 1 for the number of input symbols for lookahead.

LR Parsing: Background

- Read tokens from an input buffer (same as with shift-reduce parsers)
- Add an extra state information after each symbol in the stack. The state summarises the information contained in the stack below it. The stack would look like:

  $\$ S_0\ Expr\ S_1 - S_2\ num\ S_3$

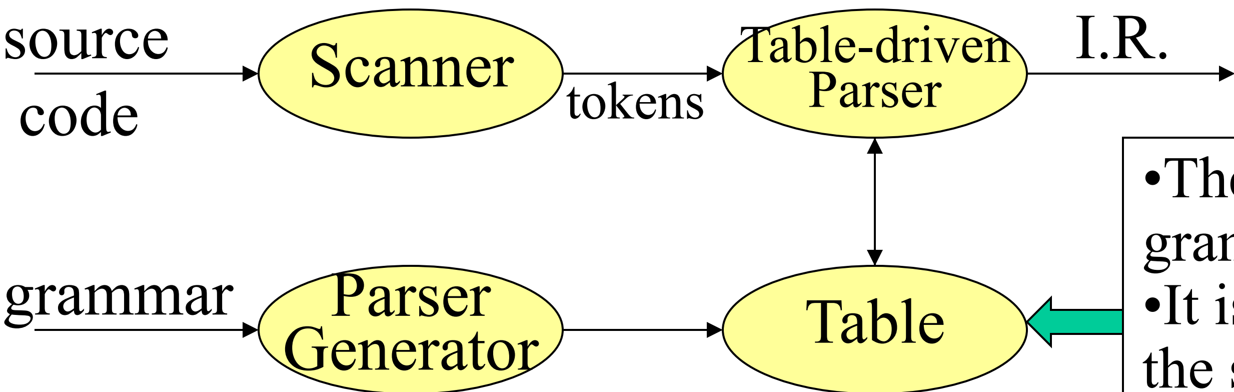
- Use a table that consists of two parts:
  - **action**[state_on_top_of_stack, input_symbol]: returns one of: shift s (push a symbol and a state); reduce by a rule; accept; error.
  - **goto**[state_on_top_of_stack,non_terminal_symbol]: returns a new state to push onto the stack after a reduction.

Skeleton code for an LR Parser

```
Push $ onto the stack
push s0
token=next_token()
repeat
  s=top_of_the_stack  /* not pop! */
  if ACTION[s,token]=='reduce A→b'
     then  pop 2*(symbols_of_b) off the stack
          s=top_of_the_stack  /* not pop! */
          push A; push GOTO[s,A]
     elseif ACTION[s,token]=='shift sx'
               then push token; push sx
                    token=next_token()
               elseif ACTION[s,token]=='accept'
                    then break
                    else report_error
end repeat
report_success
```

The Big Picture: Prelude to what follows

- LR(1) parsers are table-driven, shift-reduce parsers that use a limited right context for handle recognition.
- They can be built by hand; perfect to automate too!
- <u>Summary</u>: Bottom-up parsing is more powerful!



- The table encodes grammatical knowledge
- It is used to determine the shift-reduce parsing decision.

<u>Next</u>: we will automate table construction!

<u>Reading</u>: Aho2 Section 4.5; Aho1 pp.195-202; Hunter pp.100-103; Grune pp.150-152

Example

Consider the following grammar and tables:

1. $Goal \rightarrow CatNoise$
2. $CatNoise \rightarrow CatNoise\ miau$
3. $\qquad\qquad\qquad\quad |\ miau$

STATE	ACTION		GOTO
	eof	miau	CatNoise
0	-	Shift 2	1
1	accept	Shift 3	
2	Reduce 3	Reduce 3	
3	Reduce 2	Reduce 2	

<u>Example 1</u>: (input string miau)

Stack	**Input**	**Action**
$ s0	miau eof	Shift 2
$ s0 miau s2	eof	Reduce 3
$ s0 CatNoise s1	eof	Accept

<u>Example 2</u>: (input string miau miau)

Stack	**Input**	**Action**
$ s0	miau miau eof	Shift 2
$ s0 miau s2	miau eof	Reduce 3
$ s0 CatNoise s1	miau eof	Shift 3
$ s0 CatNoise s1 miau s3	eof	Reduce 2
$ s0 CatNoise s1	eof	accept

Note that there cannot be a syntax error with CatNoise, because it has only 1 terminal symbol. "miau woof" is a lexical problem, not a syntax error!

eof is a convention for end-of-file (=end of input)

Example: the expression grammar (slide 4)

1. $Goal \rightarrow Expr$
2. $Expr \rightarrow Expr + Term$
3. $\quad | \; Expr - Term$
4. $\quad | \; Term$
5. $Term \rightarrow Term * Factor$
6. $\quad | \; Term \, / \, Factor$
7. $\quad | \; Factor$
8. $Factor \rightarrow number$
9. $\quad | \; id$

STATE	ACTION							GOTO		
	eof	+	–	*	/	num	id	Expr	Term	Factor
0						S 4	S 5	1	2	3
1	Acc	S 6	S 7							
2	R 4	R 4	R 4	S 8	S 9					
3	R 7	R 7	R 7	R 7	R 7					
4	R 8	R 8	R 8	R 8	R 8					
5	R 9	R 9	R 9	R 9	R 9					
6						S 4	S 5		10	3
7						S 4	S 5		11	3
8						S 4	S 5			12
9						S 4	S 5			13
10	R 2	R 2	R 2	S 8	S 9					
11	R 3	R 3	R 3	S 8	S 9					
12	R 5	R 5	R 5	R 5	R 5					
13	R 6	R 6	R 6	R 6	R 6					

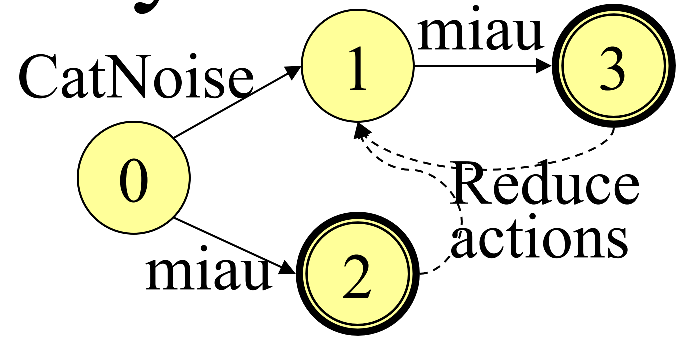
*Apply the algorithm in slide 3 to the expression x-2*y*
The result is the rightmost derivation (as in Lect.8, slide 7), but …
…no conflicts now: state information makes it fully deterministic!

Summary

- **Top-Down Recursive Descent**: Pros: Fast, Good locality, Simple, good error-handling. Cons: Hand-coded, high-maintenance.
- **LR(1):** Pros: Fast, deterministic languages, automatable. Cons: large working sets, poor error messages.
- **What is left to study?**
  - Checking for context-sensitive properties
  - Laying out the abstractions for programs & procedures.
  - Generating code for the target machine.
  - Generating good code for the target machine.
- **Reading**: Aho2 Sections 4.7, 4.10; Aho1 pp.215-220 & 230-236; Cooper 3.4, 3.5; Grune pp.165-170; Hunter 5.1-5.5 (too general).

LR(1) – Table Generation

LR Parsers: How do they work?



- Key: language of handles is regular
  - build a handle-recognising DFA
  - Action and Goto tables encode the DFA
- How do we generate the Action and Goto tables?
  - Use the grammar to build a model of the DFA
  - Use the model to build Action and Goto tables
  - If construction succeeds, the grammar is LR(1).
- Three commonly used algorithms to build tables:
  - LR(1): full set of LR(1) grammars; large tables; slow, large construction.
  - SLR(1): smallest class of grammars; smallest tables; simple, fast construction.
  - LALR(1): intermediate sized set of grammars; smallest tables; very common. (Space used to be an obsession; now it is only a concern)

LR(1) Items

- An LR(1) item is a pair [A,B], where:
  - A is a production $\alpha\rightarrow\beta\gamma\delta$ with a • at some position in the *rhs*.
  - B is a lookahead symbol.
- The • indicates the position of the top of the stack:
  - $[\alpha\rightarrow\beta\gamma\bullet\delta,a]$: the input seen so far (ie, what is in the stack) is consistent with the use of $\alpha\rightarrow\beta\gamma\delta$, and the parser has recognised $\beta\gamma$.
  - $[\alpha\rightarrow\beta\gamma\delta\bullet,a]$: the parser has seen $\beta\gamma\delta$, and a lookahead symbol of a is consistent with reducing to α.
- The production $\alpha\rightarrow\beta\gamma\delta$ with lookahead a, generates:
  - $[\alpha\rightarrow\bullet\beta\gamma\delta,a]$, $[\alpha\rightarrow\beta\bullet\gamma\delta,a]$, $[\alpha\rightarrow\beta\gamma\bullet\delta,a]$, $[\alpha\rightarrow\beta\gamma\delta\bullet,a]$
- ***The set of LR(1) items is finite.***
  - *Sets of LR(1) items represent LR(1) parser states.*

The Table Construction Algorithm

- Table construction:
  - **1.** Build the canonical collection of sets of LR(1) items, S:
    - I) Begin in S_0 with [Goal→•α, eof] and find all equivalent items as **closure**(S_0).
    - II) Repeatedly compute, for each S_k and each symbol α (both terminal and non-terminal), **goto**(S_k,α). If the set is not in the collection add it. This eventually reaches a fixed point.
  - **2.** Fill in the table from the collection of sets of LR(1) items.
- The canonical collection completely encodes the transition diagram for the handle-finding DFA.
- The lookahead is the key in choosing an action:

Remember Expr-Term from Lecture 8 slide 7, when we chose to shift rather than reduce to Expr?

Closure(state)

Closure(s)   // s is the state
  while (s is still changing)
    for each item [$\alpha \rightarrow \beta \bullet \gamma\delta$,a] in s
      for each production $\gamma \rightarrow \tau$
        for each terminal b in FIRST(δa)
          if [$\gamma \rightarrow \bullet \tau$,b] is not in s, then add it.

Recall (Lecture 7, Slide 7): *FIRST(A) is defined as the set of terminal symbols that appear as the first symbol in strings derived from A.*

E.g.: FIRST(Goal) = FIRST(CatNoise) = FIRST(miau) = miau

Example: (using the CatNoise Grammar) S0: {[Goal→•CatNoise,eof], [CatNoise→•CatNoise miau, eof], [CatNoise→•miau, eof], [CatNoise→•CatNoise miau, miau], [CatNoise→•miau, miau]}
(the 1st item by definition; 2nd,3rd are derived from the 1st; 4th,5th are derived from the 2nd)

Goto(s,x)

Goto(s,x)

new=$\emptyset$

for each item $[\alpha\to\beta\bullet x\delta,a]$ in s

add $[\alpha\to\beta x\bullet\delta,a]$ to new

return closure(new)

Computes the state that the parser would reach if it recognised an x while in state s.

Example:

S1 (x=CatNoise): [Goal→CatNoise•,eof], [CatNoise→CatNoise• miau, eof], [CatNoise→CatNoise• miau, miau]

S2 (x=miau): [CatNoise→miau•, eof], [CatNoise→miau•, miau]

S3 (from S1): [CatNoise→CatNoise miau•, eof], [CatNoise→CatNoise miau•, miau]

Example (slide 1 of 4)

Simplified expression grammar:

$Goal \rightarrow Expr$

$Expr \rightarrow Term\text{-}Expr$

$Expr \rightarrow Term$

$Term \rightarrow Factor * Term$

$Term \rightarrow Factor$

$Factor \rightarrow id$

$FIRST(Goal)=FIRST(Expr)=FIRST(Term)=FIRST(Factor)=FIRST(id)=id$

$FIRST(-)=-$

$FIRST(*)=*$

Example: first step (slide 2 of 4)

- S0: closure({[Goal→•Expr,eof]})

  {[Goal→•Expr,eof], [Expr→•Term-Expr,eof], [Expr→•Term,eof], [Term→•Factor*Term,eof], [Term→•Factor*Term,-], [Term→•Factor,eof], [Term→•Factor,-], [Factor→•id, eof], [Factor→•id,-], [Factor→•id,*]}
- Next states:
  – Iteration 1:
    - S1: goto(S0,Expr), S2: goto(S0,Term), S3: goto(S0, Factor), S4: goto(S0, id)
  – Iteration 2:
    - S5: goto(S2,-), S6: goto(S3,*)
  – Iteration 3:
    - S7: goto(S5, Expr), S8: goto(S6, Term)

Example: the states (slide 3 of 4)

S1: {[Goal→Expr•,eof]}
S2: {[Goal→Term•-Expr,eof], [Expr→Term•,eof]}
S3: {[Term→Factor•*Term,eof],[Term→Factor•*Term,-],
[Term→Factor•,eof], [Term→Factor•,-]}
S4: {[Factor→id•,eof], [Factor→id•,-], [Factor→id•,*]}
S5: {[Expr→Term-•Expr,eof], [Expr→•Term,eof],
[Term→•Factor*Term,eof], [Term→•Factor*Term,-],
[Term→•Factor,eof], [Term→•Factor,-], [Factor→•id,eof],
[Factor→•id,-], [Factor→•id,-]}
S6: {[Term→Factor*•Term,eof],[Term→Factor*•Term,-],
[Term→•Factor*Term,eof], [Term→•Factor*Term,-],
[Term→•Factor,eof], [Term→•Factor,-], [Factor→•id,eof],
[Factor→•id,-], [Factor→•id,-]}
S7: {[Expr→Term-Expr•,eof]}
S8: {[Term→Factor*Term•,eof], Term→Factor*Term•,-]}

Table Construction

- 1. Construct the collection of sets of LR(1) items.
- 2. State i of the parser is constructed from state j.
  - If $[A \rightarrow \alpha \bullet a\beta, b]$ in state i, and goto(i,a)=j, then set action[i,a] to "shift j".
  - If $[A \rightarrow \alpha \bullet, a]$ in state i, then set action[i,a] to "reduce $A \rightarrow \alpha$".
  - If $[Goal \rightarrow A \bullet, eof]$ in state i, then set action[i,eof] to "accept".
  - If goto[i,A]=j then set goto[i,A] to j.
- 3. All other entries in action and goto are set to "error".

Example: The Table (slide 4 of 4)

Goal→*Expr*
Expr→*Term-Expr*
Expr→*Term*
Term→*Factor***Term*
Term→*Factor*
Factor→*id*

STATE	ACTION				GOTO		
	id	-	*	eof	Expr	Term	Factor
0	S 4				1	2	3
1				Accept			
2		S 5		R 3			
3		R 5	S 6	R 5			
4		R 6	R 6	R 6			
5	S 4				7	2	3
6	S 4					8	3
7				R 2			
8		R 4		R 4			

Further remarks

- If the algorithm defines an entry more than once in the ACTION table, then the grammar is not LR(1).
- Other table construction algorithms, such as LALR(1) or SLR(1), produce smaller tables, but at the cost of larger space requirements.
- **`yacc`** can be used to convert a context-free grammar into a set of tables using LALR(1) (see **`% man yacc`**)
- **<u>In practice</u>**: "…the compiler-writer does not really want to concern himself with how parsing is done. So long as the parse is done correctly, …, he can live with almost any reliable technique…" [J.J.Horning from "Compiler Construction: An Advanced Course", Springer-Verlag, 1976]