Convolution

- Convolution is a technique for manipulating images by shifting convolution matrices across it.
- When a convolution matrix is placed on an image pixel, the following operation takes place:

Image Pixel

i_a	i_b	i_c
i_d	i_e	i_f
i_g	i_h	i_i

Convolution Matrix

C_a	C_b	C_c
C_d	C_e	C_f
C_g	C_h	C_i

$$i_e = \sum_{k=a}^{i} i_k \cdot C_k$$

Edge Detection with Sobel Operator

G: Edge response pixel | i: pixel index

A: Image pixel | horiz: horizontal; vert: vertical

$$G_{horiz}(i) = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \cdot A(i) \quad \| \quad G_{vert}(i) = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \cdot A(i)$$

$$G = \sqrt{G_{horiz}^2 + G_{vert}^2} \quad \text{or} \quad G(i) = \sqrt{G_{horiz}(i)^2 + G_{vert}(i)^2}$$

Θ: direction of gradient | D: direction pixel

$$\Theta = \arctan(G_{vert}, G_{horiz}) \quad \text{or} \quad \Theta(i) = \arctan(G_{vert}(i), G_{horiz}(i))$$

$$D(i) = \frac{\Theta(i) + \pi}{2\pi} \cdot 255$$