Geometrical Transformations

Translation = move around

We want to shift by $\vec{t} = \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$

$$\Rightarrow \text{we go from } \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{\text{original}} \text{ to } \underbrace{\begin{pmatrix} x + t_x \\ y + t_y \\ z + t_z \end{pmatrix}}_{\text{3D-shifted}}$$

Scaling = make bigger or smaller

We want to scale by $\vec{S} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$, normally $S_x = S_y = S_z$

$$\Rightarrow \text{we go from } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ to } \begin{pmatrix} x \cdot S_x \\ y \cdot S_y \\ z \cdot S_z \end{pmatrix}$$

Rotation = turn around

Let's say we want to rotate by φ around the z-axis.

$$\Rightarrow \text{we go from } \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ to } \begin{pmatrix} x \cdot \cos(\varphi) - y \cdot \sin(\varphi) \\ x \cdot \sin(\varphi) + y \cdot \cos(\varphi) \\ z \end{pmatrix}$$

Matrix transformation = Translation + Scaling + Rotation with one matrix

Let's say we want to transform point P to P_{transf}.

Then $P_{transf} = T \cdot P$, so

$$\begin{pmatrix} x_{transf} \\ y_{transf} \\ z_{transf} \\ 1 \end{pmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

By setting the correct values for a, b, c, ..., p we can do all the magic of translation, scaling and rotation with just one matrix multiplication. Hurray!

We undo transformations by multiplying by the inverse of the applied matrix, remember!

$$\underbrace{M \times M^{-1}}_{M^{-1}\text{: inverse matrix}} = \underbrace{I}_{\text{identity matrix}}$$