Exercise 4. Problem 1

Apply the DPLL algorithm to the following sets of clauses:

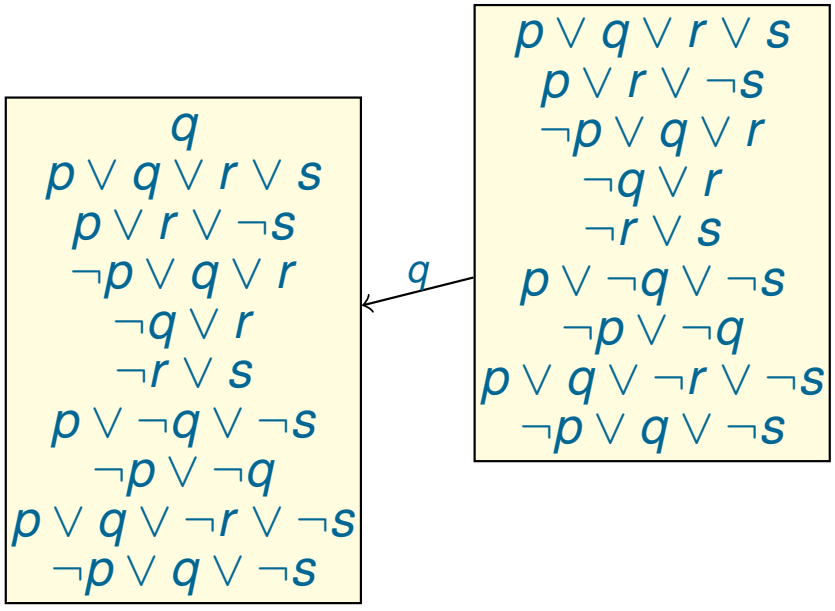
$$
\begin{aligned}
&p \vee q \vee r \vee s \\
&p \vee r \vee \neg s \\
&\neg p \vee q \vee r \\
&\neg q \vee r \\
&\neg r \vee s \\
&p \vee \neg q \vee \neg s \\
&\neg p \vee \neg q \\
&p \vee q \vee \neg r \vee \neg s \\
&\neg p \vee q \vee \neg s
\end{aligned}
$$

Is this set satisfiable? If yes, find a model of this set.

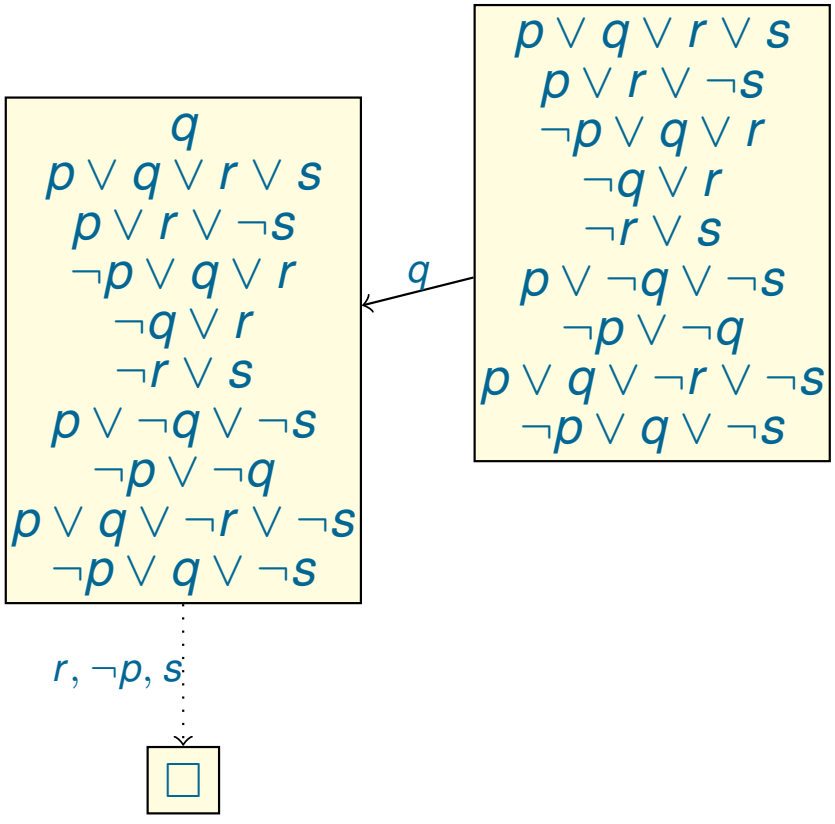
Solution

$$
\begin{array}{c}
p \vee q \vee r \vee s \\
p \vee r \vee \neg s \\
\neg p \vee q \vee r \\
\neg q \vee r \\
\neg r \vee s \\
p \vee \neg q \vee \neg s \\
\neg p \vee \neg q \\
p \vee q \vee \neg r \vee \neg s \\
\neg p \vee q \vee \neg s
\end{array}
$$

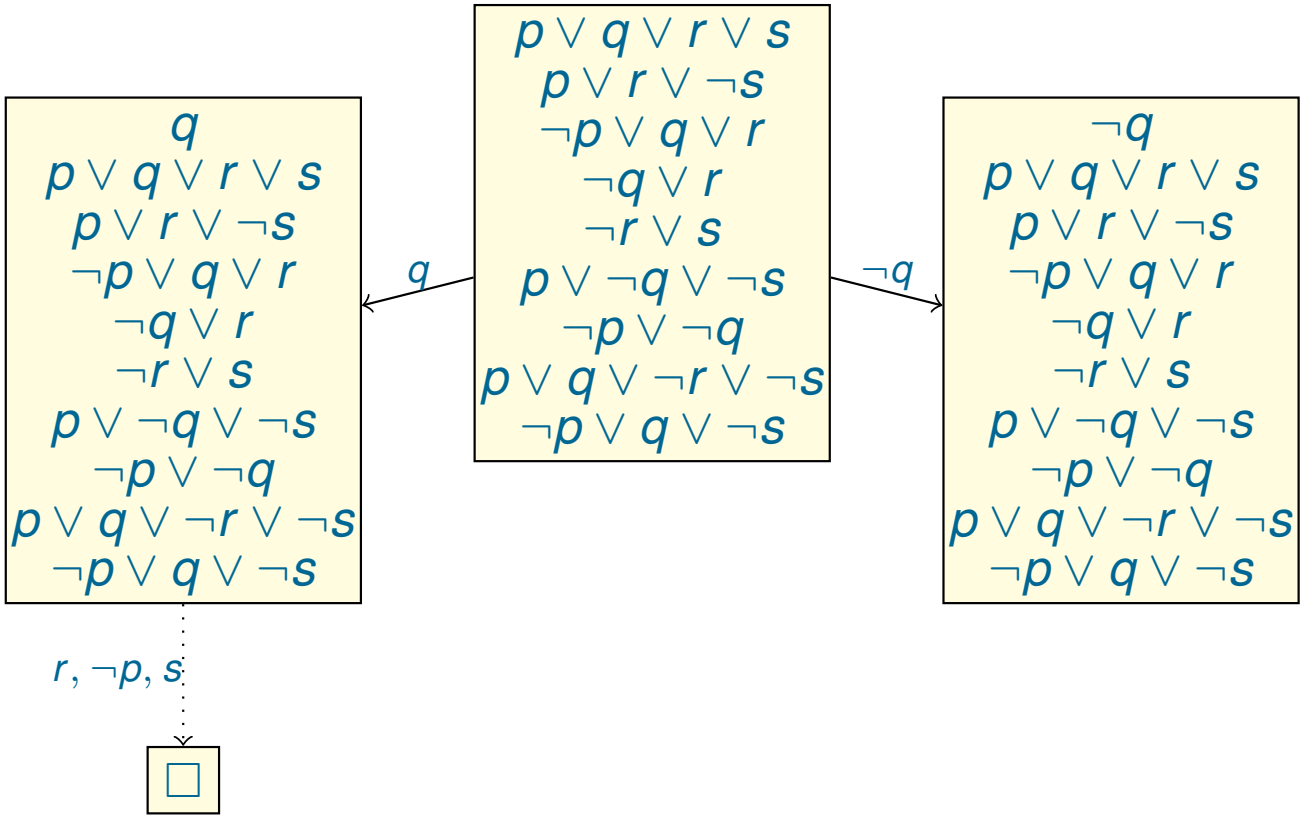
Solution


$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$
q
q
$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$

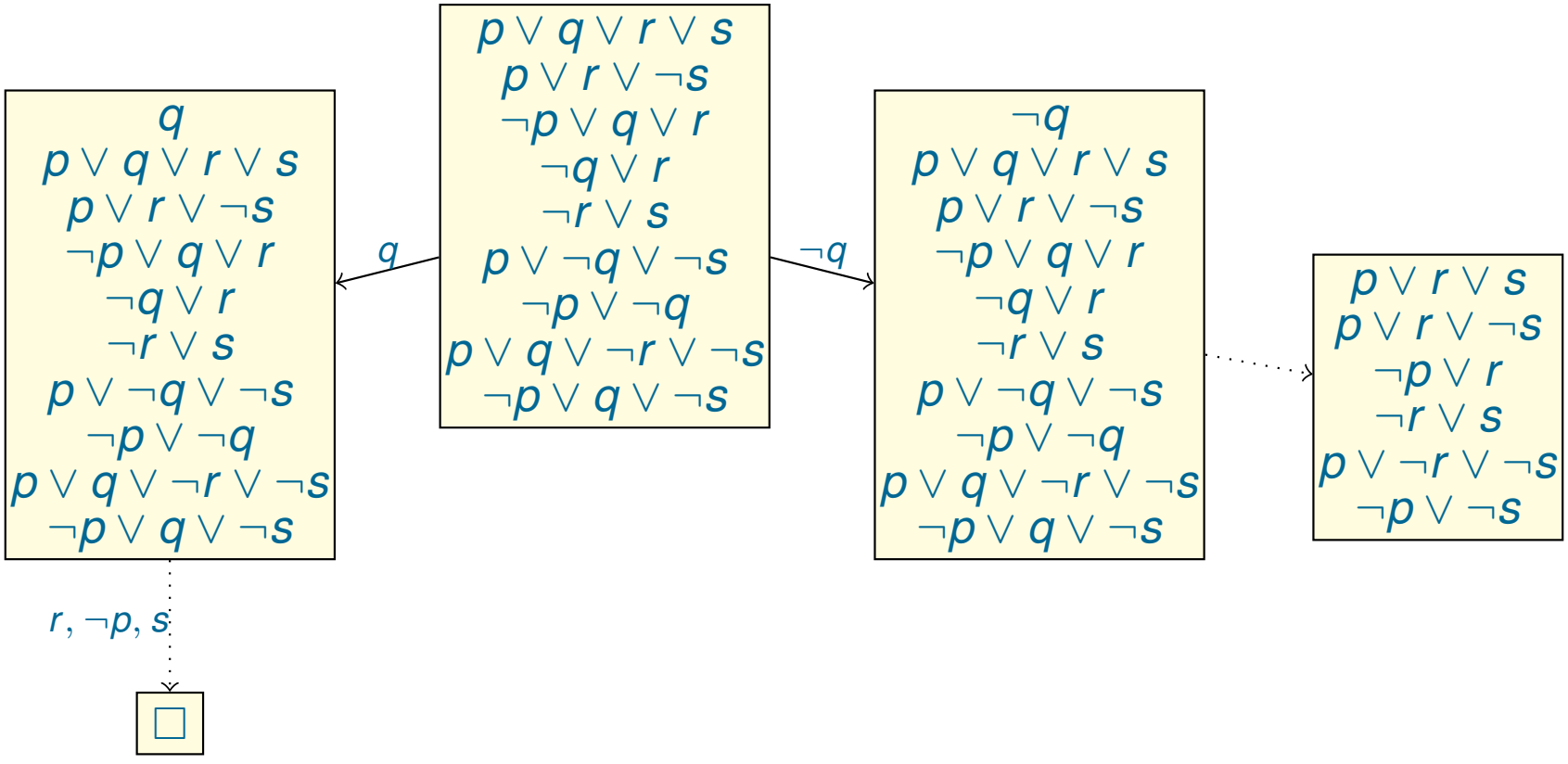
Solution


$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$
q
q
$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$
$r, \neg p, s$
$\square$

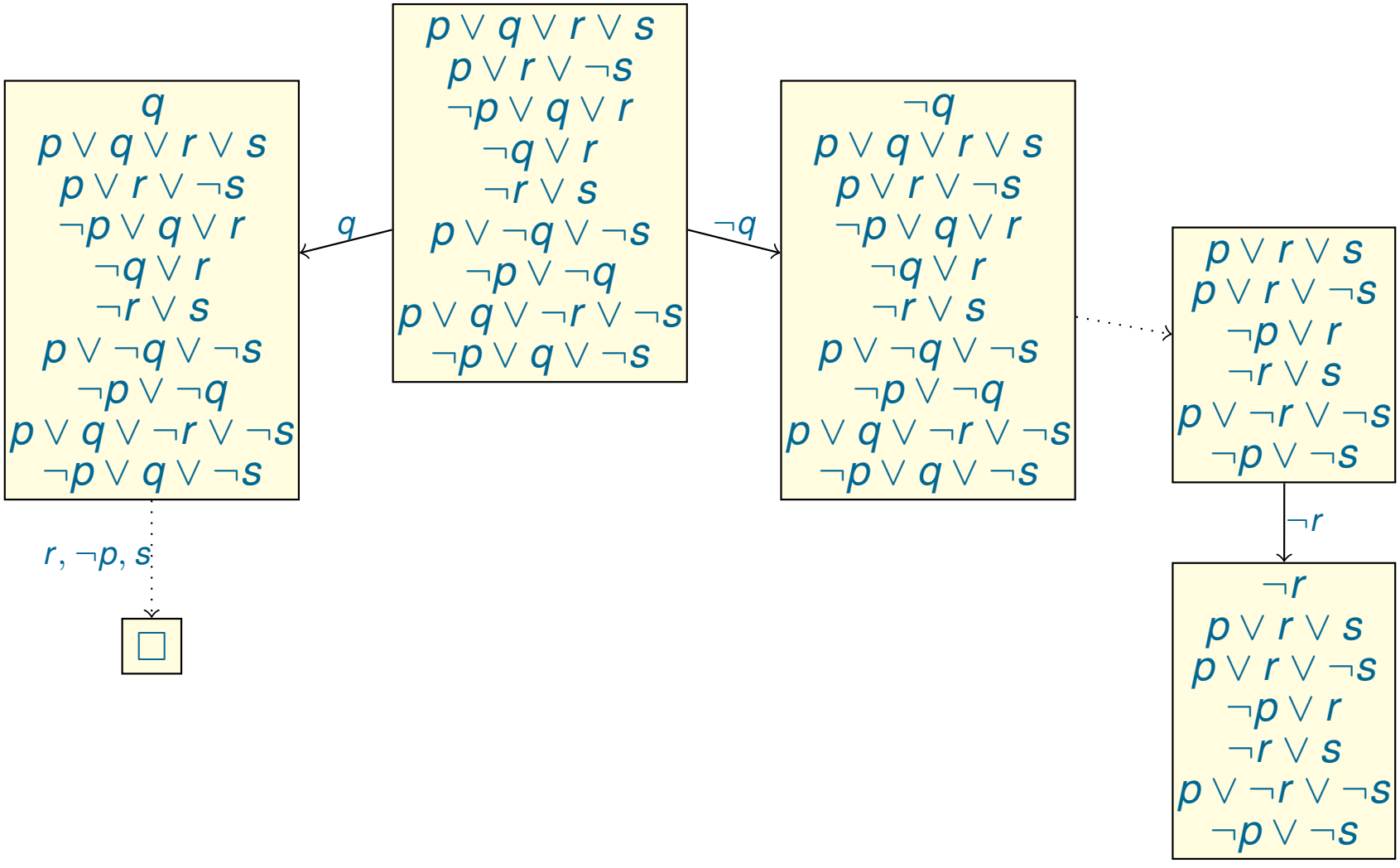
Solution


$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$
q
$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$
$\neg q$
$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$
q
$\neg q$
$r, \neg p, s$
$\square$

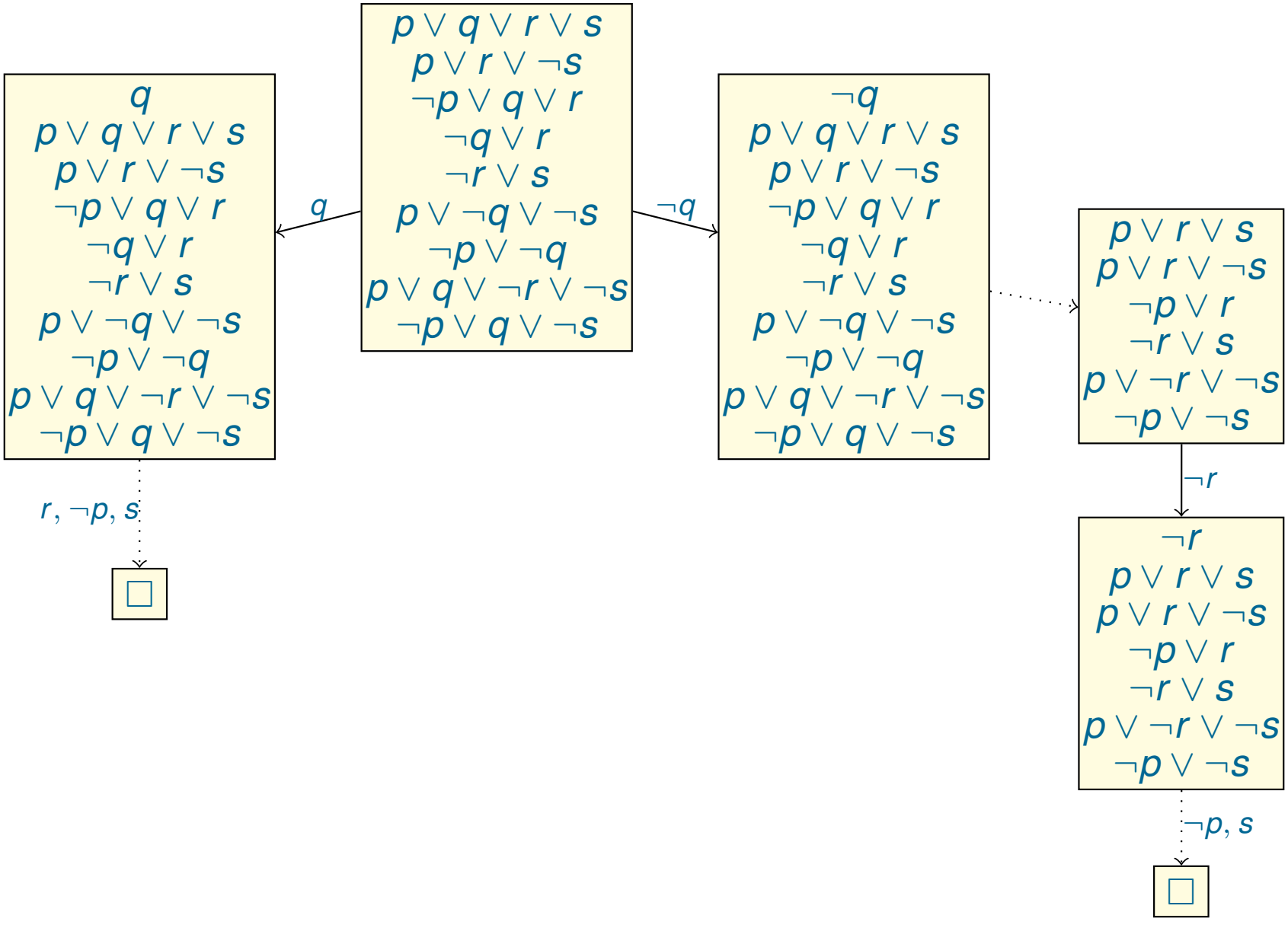
Solution


$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$
q
$\neg q$
$r, \neg p, s$
$\square$

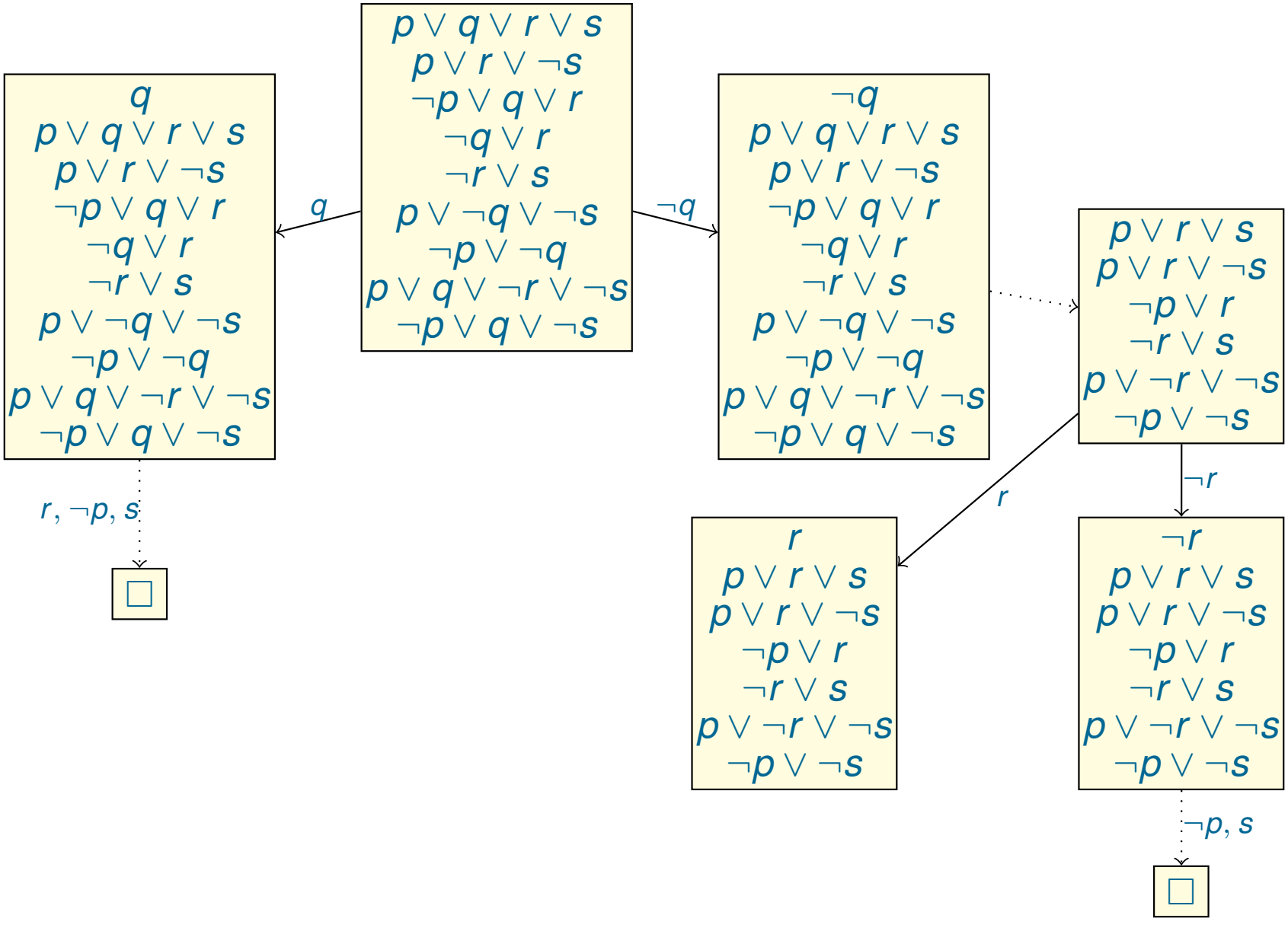
Solution



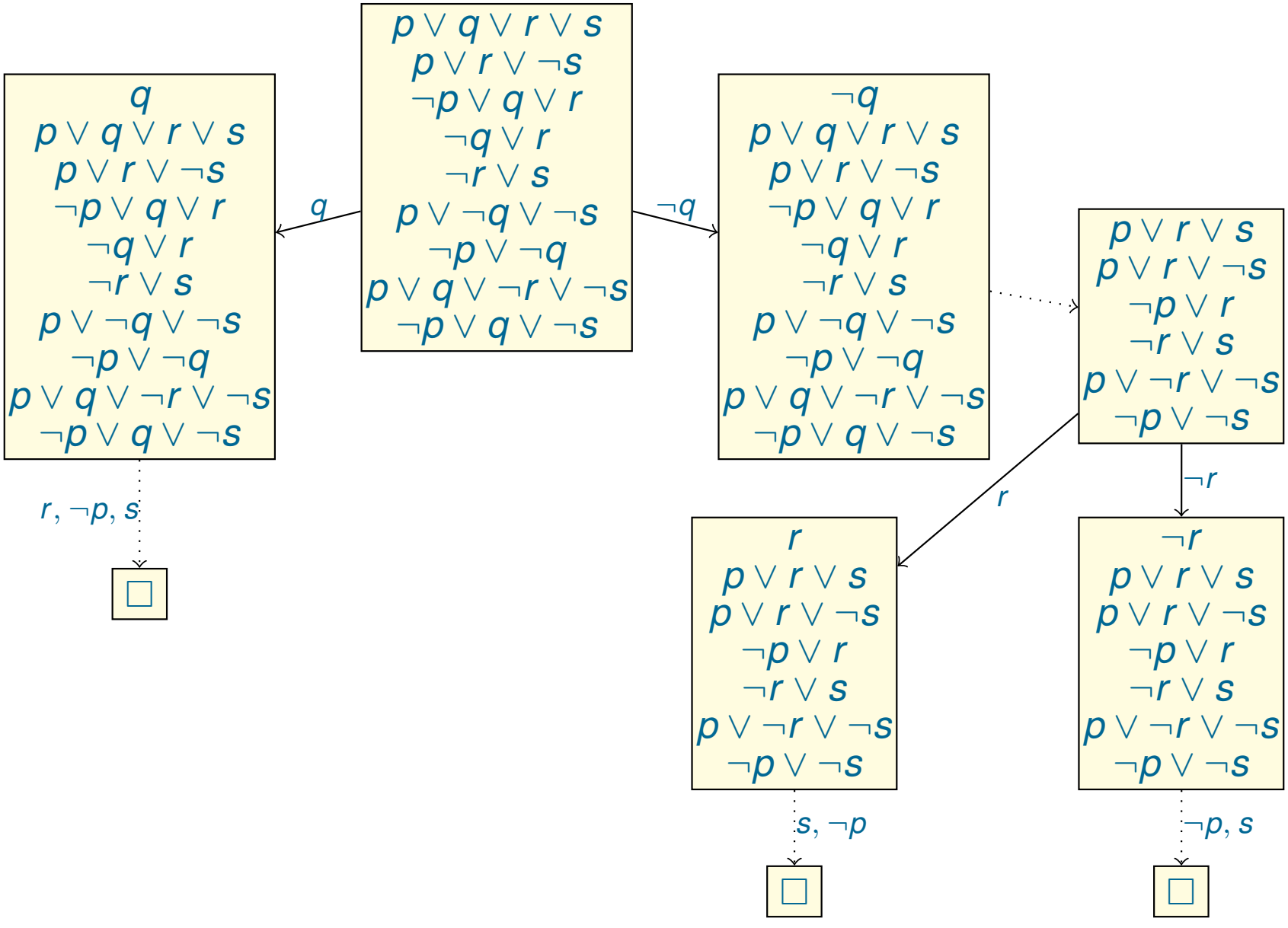
Solution


p ∨ q ∨ r ∨ s
p ∨ r ∨ ¬s
¬p ∨ q ∨ r
¬q ∨ r
¬r ∨ s
p ∨ ¬q ∨ ¬s
¬p ∨ ¬q
p ∨ q ∨ ¬r ∨ ¬s
¬p ∨ q ∨ ¬s
q
p ∨ q ∨ r ∨ s
p ∨ r ∨ ¬s
¬p ∨ q ∨ r
¬q ∨ r
¬r ∨ s
p ∨ ¬q ∨ ¬s
¬p ∨ ¬q
p ∨ q ∨ ¬r ∨ ¬s
¬p ∨ q ∨ ¬s
r, ¬p, s
□
¬q
p ∨ q ∨ r ∨ s
p ∨ r ∨ ¬s
¬p ∨ q ∨ r
¬q ∨ r
¬r ∨ s
p ∨ ¬q ∨ ¬s
¬p ∨ ¬q
p ∨ q ∨ ¬r ∨ ¬s
¬p ∨ q ∨ ¬s
p ∨ r ∨ s
p ∨ r ∨ ¬s
¬p ∨ r
¬r ∨ s
p ∨ ¬r ∨ ¬s
¬p ∨ ¬s
¬r
p ∨ r ∨ s
p ∨ r ∨ ¬s
¬p ∨ r
¬r ∨ s
p ∨ ¬r ∨ ¬s
¬p ∨ ¬s
¬p, s
□

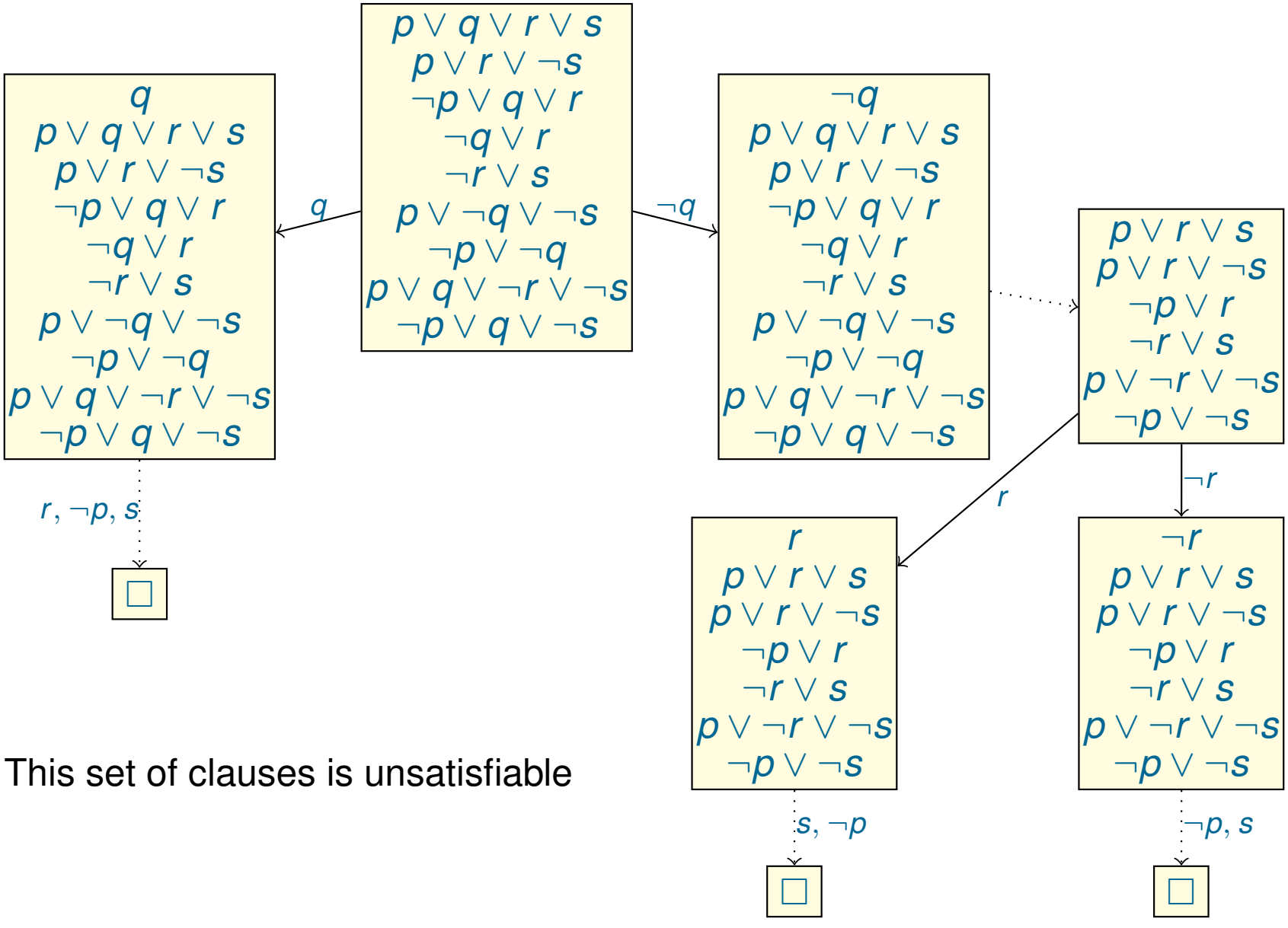
Solution


$p \vee q \vee r \vee s$
$p \vee r \vee \neg s$
$\neg p \vee q \vee r$
$\neg q \vee r$
$\neg r \vee s$
$p \vee \neg q \vee \neg s$
$\neg p \vee \neg q$
$p \vee q \vee \neg r \vee \neg s$
$\neg p \vee q \vee \neg s$
q
$\neg q$
r
$\neg r$
$r, \neg p, s$
$\neg p, s$
□

Solution



Solution



This set of clauses is unsatisfiable

Exercise 4. Problem 2

Convert the formula $p \wedge q \leftrightarrow \neg p \vee \neg q$ to CNF using the definitional clausal form transformation algorithm.

Exercise 4. Problem 2

Convert the formula $p \wedge q \leftrightarrow \neg p \vee \neg q$ to CNF using the definitional clausal form transformation algorithm.

Solution

The transformation is given in the following table. The clausal normal form is obtained by putting together all clauses in the rightmost column.

	subformula	definition	clauses
			n_1
n_1	$p \wedge q \leftrightarrow \neg p \vee \neg q$	$n_1 \leftrightarrow (n_2 \leftrightarrow n_3)$	$n_1 \vee n_2 \vee n_3$ $n_1 \vee \neg n_2 \vee \neg n_3$ $\neg n_1 \vee n_2 \vee \neg n_3$ $\neg n_1 \vee \neg n_2 \vee n_3$
n_2	$p \wedge q$	$n_2 \leftrightarrow p \wedge q$	$\neg n_2 \vee p$ $\neg n_2 \vee q$ $\neg p \vee \neg q \vee n_2$
n_3	$\neg p \vee \neg q$	$n_3 \leftrightarrow \neg p \vee \neg q$	$\neg n_3 \vee \neg p \vee \neg q$ $p \vee n_3$ $q \vee n_3$