Exercise 10 (Problem 1)

Let F be a formula. Represent in LTL the following property of a path $s_0, s_1 \ldots$: a formula F holds in all states of the form s_{4k} and s_{4k+1}, where $k = 0, 1, \ldots$ and does not hold in all other states.

Solution

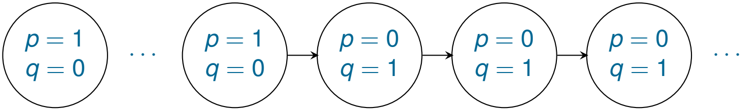
$$F \land \bigcirc F \land \square(F \leftrightarrow \bigcirc\bigcirc\neg F).$$

Exercise 10 (Problem 2)

Consider the formula $(p \wedge \neg q)\, \mathbf{U}\, \Box(q \wedge \neg p)$. Describe the set of all paths that make this formula true.

Solution

Each path making this formula true has the following form



with zero or more states having $p = 1$.
More precisely, a path $\pi = s_0, s_1, \dots$ satisfies this formula if there exists $k \geq 0$ such that

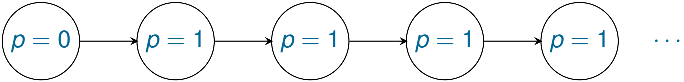
1. for all $i < k$ we have $s_i \models p \wedge \neg q$;
2. for all $i \geq k$ we have $s_i \models q \wedge \neg p$.

Exercise 10 (Problem 3)

Show that the following formulas are not equivalent: $\Box \Diamond p$ and $\neg(p \, \mathbf{U} \, \neg p)$.

Solution

Consider the following path π:



We have $\pi \models \Box \Diamond p$ but $\pi \not\models \neg(p \, \mathbf{U} \, \neg p)$.