COMP21111

Two hours

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Logic and Modelling

Date: Wednesday 19th January 2011

Time: 09:45 - 11:45

Please answer any THREE questions from the FIVE questions provided

This is a CLOSED book examination

The use of electronic calculators is NOT permitted

[PTO]

1.

a) Make a table of all positions, the corresponding subformulas and polarities in the following formula (4 marks)

$$p \wedge \neg q \wedge ((q \leftrightarrow \neg q) \vee p \vee (q \rightarrow (r \vee p))).$$

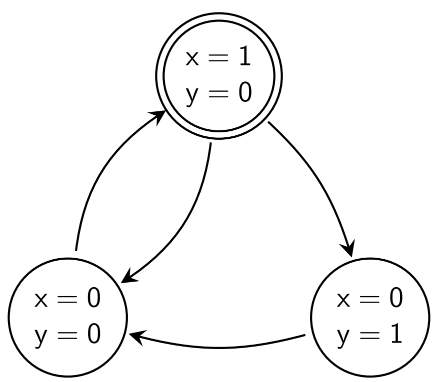
b) Apply the DPLL method to the following set of clauses, i.e., show the splitting tree and the results of all unit propagation steps. Is this set of clauses satisfiable? If yes, give an interpretation which satisfies it. (7 marks)

$$\begin{gathered} \neg q \vee r \\ \neg p \vee \neg q \vee \neg r \\ p \vee r \\ p \vee \neg q \vee \neg r \\ p \vee q \\ p \vee \neg q \vee r \end{gathered}$$

c) Let p be a variable with the domain $\{a, b, c\}$ and q be a boolean variable. Transform the following formula of PLFD into a propositional logic formula: (3 marks)

$$p = a \rightarrow \neg(p = b) \vee q = 0.$$

d) Consider a transition system with the following state transition graph.



Let S_1 be the set of states symbolically represented by the formula $\mathsf{x} = 1$ and S_2 be the set of states symbolically represented by the formula $\mathsf{x} = 1 \wedge \mathsf{y} = 0$.

1. State whether or not S_1 coincides with the set of initial states, justifying your answer with a brief explanation. (2 marks)
2. Find a symbolic representation of the set of states reachable from S_2 in exactly two steps. (2 marks)
3. Find a symbolic representation of the set of states backward reachable from S_2 in exactly two steps. (2 marks)

2.

a) Transform the formula $(p \leftrightarrow q) \rightarrow r$ into a set of clauses using the standard CNF transformation. (5 marks)

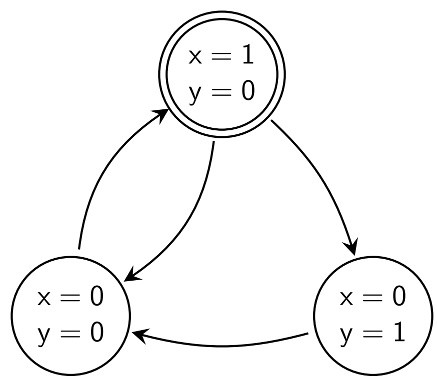
b) Consider the set consisting of the following clauses:

$p_0 \vee \neg p_1 \vee p_2$	$p_0 \vee \neg p_1 \vee p_2 \vee p_4$	$\neg p_0 \vee p_1 \vee \neg p_2$	$\neg p_0 \vee \neg p_1 \vee \neg p_2$
$p_0 \vee \neg p_1 \vee p_4$	$p_3 \vee p_2 \vee p_4 \vee \neg p_0$	$\neg p_2 \vee \neg p_2 \vee p_4 \vee p_3$	$\neg p_2 \vee \neg p_0 \vee p_4 \vee p_4$
$p_0 \vee p_3 \vee \neg p_4$	$p_0 \vee \neg p_1 \vee \neg p_2 \vee \neg p_3$	$\neg p_1 \vee \neg p_2 \vee \neg p_3$	$p_1 \vee \neg p_2 \vee \neg p_3 \vee \neg p_4$
$p_1 \vee p_2$	$p_2 \vee p_3 \vee \neg p_4 \vee p_3$	$\neg p_0 \vee \neg p_2 \vee \neg p_3 \vee \neg p_4$	$p_0 \vee p_2 \vee p_4$

For each of the variables p_0, p_1, p_2, p_3, p_4 find the probability that WSAT will choose this variable for flipping when the current interpretation is
$\{p_0 \mapsto 0, p_1 \mapsto 0, p_2 \mapsto 0, p_3 \mapsto 0, p_4 \mapsto 0\}$. (6 marks)

c) Consider an instance of propositional logic of finite domains where the variable x has the domain $\{b, c, d\}$. Write down the domain axiom for this variable. (3 marks)

d) Consider a transition system with the following state transition graph.



Which of the following formulas are true on all paths? For the formulas that are false, give the path on which they are false.

1. $\square(x = 0 \vee y = 0)$; (1 marks)
2. $\square\Diamond(y = 0)$; (1 marks)
3. $\square\Diamond(y = 1)$; (2 marks)
4. $\square(x = 1 \rightarrow \Diamond y = 1)$. (2 marks)

3.

a) Consider the set consisting of the following clauses:

$$p_0 \vee \neg p_1 \vee \neg p_2, \quad \neg p_0 \vee \neg p_2, \quad p_0 \vee p_1, \quad p_1 \vee p_2, \quad p_0 \vee \neg p_1 \vee p_2.$$

Show how the GSAT algorithm can find a model of this set starting with the initial random interpretation $\{p_0 \mapsto 0, p_1 \mapsto 0, p_2 \mapsto 1\}$. (9 marks)

b) Draw the OBDD for the formula $r \wedge (p \vee q)$ and the order $p > q > r$. (5 marks)

c) Let p and q be atoms. Show that the following two formulas are not equivalent by giving a path which satisfies one of them but not the other: (3 marks)

$$\Box(p \vee q);$$
$$\Box p \vee \Box q.$$

d) Let F and G be temporal formulas. Express in LTL the following property: whenever F holds at a state, G will hold at some state afterwards. (3 marks)

4.

a) Show, using semantic tableaux, that the formula

$$(p \to q \lor r) \to (p \to q) \lor (p \to r)$$

is valid. (7 marks)

b) A propositional formula A of variables $p_1, \ldots, p_n$ is true in an interpretation I if and only if at least one atom from $p_1, \ldots, p_n$ is false in I. Draw the OBDD for A and the order $p_1 > p_2 > \ldots$. (7 marks)

c) A variable x in propositional logic of finite domains has the domain $\{a, b, c\}$. Consider a transition system consisting of the following two transitions:

$$\begin{array}{lcl} t_1 & \rightleftharpoons & \mathsf{x} = a \land (\mathsf{x}' = b \lor \mathsf{x}' = c) \\ t_2 & \rightleftharpoons & \mathsf{x} = b \land (\mathsf{x}' = b \lor \mathsf{x}' = c) \end{array}$$

The set of initial states is described by the formula $\mathsf{x} \neq c$. Draw the state transition graph of the system. (6 marks)

5.

a) Consider the following formula in CNF

$$\exists p \forall q \exists r ((\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (p \vee q \vee r) \wedge (p \vee q \vee \neg r))$$

Evaluate this formula using the DPLL algorithm. Show all steps of the algorithm. Is this formula true or false? (11 marks)

b) Let F and G be temporal formulas. Express in LTL the following properties:

1. Whenever F holds at a state, G will hold at some state afterwards. (3 marks)
2. Either F holds infinitely often or G holds infinitely often. (2 marks)
3. F holds at all even states and does not hold at all odd states. (4 marks)

END OF EXAMINATION