Exercise 5. Problem 1

Consider the set consisting of the following clauses:

$$\neg p_0 \lor \neg p_1 \lor \neg p_2, \quad p_0 \lor \neg p_2, \quad \neg p_0 \lor p_1, \quad p_1 \lor p_2, \quad \neg p_0 \lor \neg p_1 \lor p_2.$$

Show how GSAT can find a model of this set starting with the initial random interpretation $\{p_0 \mapsto 1, p_1 \mapsto 0, p_2 \mapsto 1\}$.

Solution

$$
\begin{array}{ccccc}
1 & & 0 & & 1 \\
\hline
\neg p_0 & \vee & \neg p_1 & \vee & \neg p_2 \\
p_0 & & & \vee & \neg p_2 \\
\neg p_0 & \vee & p_1 & & \\
 & & p_1 & \vee & p_2 \\
\neg p_0 & \vee & \neg p_1 & \vee & p_2
\end{array}
$$

Solution

1		0		1
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
p_0			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	p_1		
		p_1	$\vee$	p_2
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	p_2

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	p_0	p_1	p_2		p_0	p_1	p_2		
1	1	0	1	4					

Solution

1		0		1
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
p_0			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	p_1		
		p_1	$\vee$	p_2
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	p_2

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	p_0	p_1	p_2		p_0	p_1	p_2		
1	1	0	1	4	4	4	3		

Solution

1		1		1
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
p_0			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	p_1		
		p_1	$\vee$	p_2
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	p_2

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	p_0	p_1	p_2		p_0	p_1	p_2		
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1						

Solution

$$\begin{array}{ccccc}
1 & & 1 & & 1 \\
\hline
\neg p_0 & \vee & \neg p_1 & \vee & \neg p_2 \\
p_0 & & & \vee & \neg p_2 \\
\neg p_0 & \vee & p_1 & & \\
 & & p_1 & \vee & p_2 \\
\neg p_0 & \vee & \neg p_1 & \vee & p_2
\end{array}$$

flip no.	interpretation p_0	p_1	p_2	satisfied clauses	p_0	p_1	p_2	candidates for flipping	flipped variable
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1	4					

Solution

1		1		1
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
p_0			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	p_1		
		p_1	$\vee$	p_2
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	p_2

flip no.	interpretation p_0	p_1	p_2	satisfied clauses	p_0	p_1	p_2	candidates for flipping	flipped variable
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1	4	4	4	4		

Solution

1		1		0
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
p_0			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	p_1		
		p_1	$\vee$	p_2
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	p_2

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	p_0	p_1	p_2		p_0	p_1	p_2		
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1	4	4	4	4	p_0, p_1, p_2	p_2
3	1	1	0						

Solution

$$
\begin{array}{ccccc}
1 & & 1 & & 0 \\
\hline
\neg p_0 & \vee & \neg p_1 & \vee & \neg p_2 \\
p_0 & & & \vee & \neg p_2 \\
\neg p_0 & \vee & p_1 & & \\
 & & p_1 & \vee & p_2 \\
\neg p_0 & \vee & \neg p_1 & \vee & p_2
\end{array}
$$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	p_0	p_1	p_2		p_0	p_1	p_2		
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1	4	4	4	4	p_0, p_1, p_2	p_2
3	1	1	0	4					

Solution

$$
\begin{array}{ccccc}
1 & & 1 & & 0 \\
\hline
\neg p_0 & \vee & \neg p_1 & \vee & \neg p_2 \\
p_0 & & & \vee & \neg p_2 \\
\neg p_0 & \vee & p_1 & & \\
 & & p_1 & \vee & p_2 \\
\neg p_0 & \vee & \neg p_1 & \vee & p_2
\end{array}
$$

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	p_0	p_1	p_2		p_0	p_1	p_2		
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1	4	4	4	4	p_0, p_1, p_2	p_2
3	1	1	0	4	5	3	4		

Solution

0		1		0
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
p_0			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	p_1		
		p_1	$\vee$	p_2
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	p_2

flip no.	interpretation			satisfied clauses				candidates for flipping	flipped variable
	p_0	p_1	p_2		p_0	p_1	p_2		
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1	4	4	4	4	p_0, p_1, p_2	p_2
3	1	1	0	4	5	3	4	p_0	p_0
	0	1	0						

Solution

0		1		0
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	$\neg p_2$
p_0			$\vee$	$\neg p_2$
$\neg p_0$	$\vee$	p_1		
		p_1	$\vee$	p_2
$\neg p_0$	$\vee$	$\neg p_1$	$\vee$	p_2

flip no.	interpretation p_0	p_1	p_2	satisfied clauses	p_0	p_1	p_2	candidates for flipping	flipped variable
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1	4	4	4	4	p_0, p_1, p_2	p_2
3	1	1	0	4	5	3	4	p_0	p_0
	0	1	0	5					

Solution

$$
\begin{array}{ccccc}
0 & & 1 & & 0 \\
\hline
\neg p_0 & \vee & \neg p_1 & \vee & \neg p_2 \\
p_0 & & & \vee & \neg p_2 \\
\neg p_0 & \vee & p_1 & & \\
 & & p_1 & \vee & p_2 \\
\neg p_0 & \vee & \neg p_1 & \vee & p_2
\end{array}
$$

flip	interpretation			satisfied clauses				candidates	flipped
no.	p_0	p_1	p_2		p_0	p_1	p_2	for flipping	variable
1	1	0	1	4	4	4	3	p_0, p_1	p_1
2	1	1	1	4	4	4	4	p_0, p_1, p_2	p_2
3	1	1	0	4	5	3	4	p_0	p_0
	0	1	0	5					

The model found after 3 flips is $\{p_0 \mapsto 0, p_1 \mapsto 1, p_2 \mapsto 0\}$.

Exercise 5. Problem 2

Consider the set consisting of the following clauses:

$p_0 \lor \neg p_1 \lor p_2$	$p_0 \lor \neg p_1 \lor p_2 \lor p_4$	$\neg p_0 \lor p_1 \lor \neg p_2$	$\neg p_0 \lor \neg p_1 \lor \neg p_2 \lor \neg p_4$
$p_0 \lor \neg p_1 \lor p_4$	$p_3 \lor p_2 \lor p_4 \lor \neg p_0$	$\neg p_2 \lor \neg p_2 \lor p_4 \lor p_3$	$\neg p_2 \lor \neg p_0 \lor p_4 \lor p_4$
$p_0 \lor p_3 \lor \neg p_4$	$p_0 \lor \neg p_1 \lor \neg p_2 \lor \neg p_3$	$\neg p_1 \lor \neg p_2 \lor \neg p_3$	$p_1 \lor \neg p_2 \lor \neg p_3 \lor \neg p_4$
$p_1 \lor p_2$	$p_2 \lor p_3 \lor \neg p_4 \lor p_3$	$\neg p_0 \lor \neg p_2 \lor \neg p_3 \lor \neg p_4$	$p_0 \lor p_2 \lor p_4$

For each of the variables p_0, p_1, p_2, p_3, p_4 find the probability that WSAT will choose this variable for flipping when the current interpretation is $\{p_0 \mapsto 0, p_1 \mapsto 0, p_2 \mapsto 0, p_3 \mapsto 0, p_4 \mapsto 0\}$.

Solution

WSAT will first select clauses false in the current interpretation. These are

$$p_1 \lor p_2$$
$$p_0 \lor p_2 \lor p_4$$

Solution

WSAT will first select clauses false in the current interpretation. These are

$$p_1 \lor p_2$$
$$p_0 \lor p_2 \lor p_4$$

Each of these clauses will be selected with the equal probability, that is, $\frac{1}{2}$.

Solution

WSAT will first select clauses false in the current interpretation. These are

$$p_1 \vee p_2$$
$$p_0 \vee p_2 \vee p_4$$

Each of these clauses will be selected with the equal probability, that is, $\frac{1}{2}$.
The following table shows the probabilities that a given clause and a variable in it will be selected for flipping:

clause	p_0	p_1	p_2	p_3	p_4
$p_1 \vee p_2$	0	$\frac{1}{4}$	$\frac{1}{4}$	0	0
$p_0 \vee p_2 \vee p_4$	$\frac{1}{6}$	0	$\frac{1}{6}$	0	$\frac{1}{6}$
total	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{5}{12}$	0	$\frac{1}{6}$

The bottom row of this table contains the answer.

Exercise 5. Problem 3

Show validity of the following formula using semantic tableaux:

$$(p \rightarrow r) \rightarrow (p \vee q \rightarrow r \vee q).$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \vee q \to r \vee q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \vee q \to r \vee q)) = 0$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

$$\begin{array}{c} | \\ (p \to r) = 1 \\ (p \lor q \to r \lor q) = 0 \end{array}$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$
$$|$$
$$(p \to r) = 1$$
$$(p \lor q \to r \lor q) = 0$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

$$\Big|$$

$$\begin{array}{c} (p \to r) = 1 \\ (p \lor q \to r \lor q) = 0 \end{array}$$

$$\Big|$$

$$\begin{array}{c} (p \lor q) = 1 \\ (r \lor q) = 0 \end{array}$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

$$\Big|$$

$$(p \to r) = 1$$
$$(p \lor q \to r \lor q) = 0$$

$$\Big|$$

$$(p \lor q) = 1$$
$$(r \lor q) = 0$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} (p \to r) = 1 \\ (p \lor q \to r \lor q) = 0 \end{array}$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} (p \lor q) = 1 \\ (r \lor q) = 0 \end{array}$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} r = 0 \\ q = 0 \end{array}$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} (p \to r) = 1 \\ (p \lor q \to r \lor q) = 0 \end{array}$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} (p \lor q) = 1 \\ (r \lor q) = 0 \end{array}$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} r = 0 \\ q = 0 \end{array}$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \vee q \to r \vee q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \vee q \to r \vee q)) = 0$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} (p \to r) = 1 \\ (p \vee q \to r \vee q) = 0 \end{array}$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} (p \vee q) = 1 \\ (r \vee q) = 0 \end{array}$$

$$\begin{array}{c} | \end{array}$$

$$\begin{array}{c} r = 0 \\ q = 0 \end{array}$$

$$\begin{array}{ccc} & / \quad \backslash & \\ p = 1 & & q = 1 \end{array}$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

$$\begin{array}{c} (p \to r) = 1 \\ (p \lor q \to r \lor q) = 0 \end{array}$$

$$\begin{array}{c} (p \lor q) = 1 \\ (r \lor q) = 0 \end{array}$$

$$\begin{array}{c} r = 0 \\ q = 0 \end{array}$$

$$\begin{array}{cc} p = 1 & q = 1 \\ & \text{closed} \end{array}$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

$$\begin{array}{c} (p \to r) = 1 \\ (p \lor q \to r \lor q) = 0 \end{array}$$

$$\begin{array}{c} (p \lor q) = 1 \\ (r \lor q) = 0 \end{array}$$

$$\begin{array}{c} r = 0 \\ q = 0 \end{array}$$

$$\begin{array}{ccc} p = 1 & \qquad & q = 1 \\ & & \text{closed} \end{array}$$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \lor q \to r \lor q)) = 0$$

$$(p \to r) = 1$$
$$(p \lor q \to r \lor q) = 0$$

$$(p \lor q) = 1$$
$$(r \lor q) = 0$$

$$r = 0$$
$$q = 0$$

$p = 1$ | $q = 1$ closed

$p = 0$ | $r = 1$

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \vee q \to r \vee q)) = 0$. Below we show how to build a closed tableau for this formula.

$$((p \to r) \to (p \vee q \to r \vee q)) = 0$$

$$(p \to r) = 1$$
$$(p \vee q \to r \vee q) = 0$$

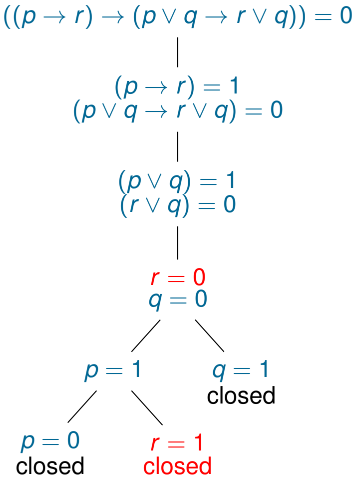
$$(p \vee q) = 1$$
$$(r \vee q) = 0$$

$$r = 0$$
$$q = 0$$

- $p = 1$
  - $p = 0$
    closed
  - $r = 1$
- $q = 1$
  closed

Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \vee q \to r \vee q)) = 0$. Below we show how to build a closed tableau for this formula.



Solution

A formula is valid if it is true in all interpretations. Therefore, to show validity of this formula, it is sufficient to show unsatisfiability of the signed formula $((p \to r) \to (p \lor q \to r \lor q)) = 0$. Below we show how to build a closed tableau for this formula.

- $((p \to r) \to (p \lor q \to r \lor q)) = 0$
  - $(p \to r) = 1$
$(p \lor q \to r \lor q) = 0$
    - $(p \lor q) = 1$
$(r \lor q) = 0$
      - $r = 0$
$q = 0$
        - $p = 1$
          - $p = 0$
closed
          - $r = 1$
closed
        - $q = 1$
closed