Exercise 8 (Problem 1)

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)).$$

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Solution

$$\exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q))$$

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Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}[column sep=small]
 & \exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)) \arrow[dl, "r = 1"'] \arrow[d, phantom, "\vee"] \\
\forall q \exists p (p \leftrightarrow q) &
\end{tikzcd}$$

Exercise 8 (Problem 1)

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}
 & & \exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)) \arrow[dl, "r = 1"'] \arrow[d, phantom, "\lor"] \\
 & \forall q \exists p (p \leftrightarrow q) \arrow[dl, "q = 0"'] \arrow[d, phantom, "\land"] & \\
\exists p (\neg p) & &
\end{tikzcd}$$

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Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}[column sep=small]
 & & & \exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)) \;\; \bigvee \arrow[dl, "r=1"'] \\
 & & \forall q \exists p (p \leftrightarrow q) \;\; \bigwedge \arrow[dl, "q=0"'] & \\
 & \exists p (\neg p) \;\; \bigvee \arrow[dl, "q=0"'] & & \\
\boxed{1} \;\; \top & & &
\end{tikzcd}$$

Exercise 8 (Problem 1)

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}
 & & & \exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)) \quad \vee \arrow[dl, "r = 1"'] \\
 & & \forall q \exists p (p \leftrightarrow q) \quad \wedge \arrow[dl, "q = 0"'] & \\
 & \boxed{1} \; \exists p (\neg p) \quad \vee \arrow[dl, "q = 0"'] & & \\
\boxed{1} \; \top & & &
\end{tikzcd}$$

Exercise 8 (Problem 1)

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)).$$

Solution

$$
\begin{tikzcd}
& & & \exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)) \quad \vee \arrow[dl, "r = 1"'] \\
& & \forall q \exists p (p \leftrightarrow q) \arrow[dl, "q = 0"'] \arrow[dr, "q = 1"] \arrow[d, phantom, "\wedge"] & \\
& 1 \quad \exists p (\neg p) \quad \vee \arrow[dl, "q = 0"'] & & \exists p (p) \\
1 \quad \top & & &
\end{tikzcd}
$$

Exercise 8 (Problem 1)

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}
 & & \exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)) \quad \vee \arrow[d, "r = 1"'] & \\
 & & \forall q \exists p (p \leftrightarrow q) \quad \wedge \arrow[dl, "q = 0"'] \arrow[dr, "q = 1"] & \\
 & \boxed{1} \ \exists p (\neg p) \quad \vee \arrow[dl, "q = 0"'] & & \exists p (p) \quad \vee \arrow[dl, "p = 1"'] \\
\boxed{1} \ \top & & \boxed{1} \ \top &
\end{tikzcd}$$

Exercise 8 (Problem 1)

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}[column sep=small]
 & & \exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)) \quad \lor \arrow[d, "r = 1"'] & \\
 & & \forall q \exists p (p \leftrightarrow q) \quad \land \arrow[dl, "q = 0"'] \arrow[dr, "q = 1"] & \\
 & \boxed{1}\ \exists p (\neg p) \quad \lor \arrow[dl, "q = 0"'] & & \exists p (p) \ \boxed{1} \quad \lor \arrow[dl, "p = 1"'] \\
\boxed{1}\ \top & & \boxed{1}\ \top &
\end{tikzcd}$$

Exercise 8 (Problem 1)

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \to r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}
 & & & \exists r\forall q\exists p(p \leftrightarrow ((p \to r) \leftrightarrow q)) \quad \lor \arrow[dl, "r = 1"'] \\
 & & \forall q\exists p(p \leftrightarrow q)\ \boxed{1} \quad \land \arrow[dl, "q = 0"'] \arrow[dr, "q = 1"] & \\
 & \boxed{1}\ \exists p(\neg p) \quad \lor \arrow[dl, "q = 0"'] & & \exists p(p)\ \boxed{1} \quad \lor \arrow[dl, "p = 1"'] \\
 \boxed{1}\ \top & & \boxed{1}\ \top &
\end{tikzcd}$$

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Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}
 & & & \boxed{1}\ \exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q))\ \ \vee \arrow[dl, "r = 1"'] \\
 & & \forall q \exists p (p \leftrightarrow q)\ \boxed{1}\ \ \wedge \arrow[dl, "q = 0"'] \arrow[dr, "q = 1"] & \\
 & \boxed{1}\ \exists p (\neg p)\ \ \vee \arrow[dl, "q = 0"'] & & \exists p (p)\ \boxed{1}\ \ \vee \arrow[dl, "p = 1"'] \\
\boxed{1}\ \top & & \boxed{1}\ \top &
\end{tikzcd}$$

Exercise 8 (Problem 1)

Evaluate the following formula using the Splitting Algorithm:

$$\exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)).$$

Solution

$$\begin{tikzcd}
 & & \boxed{1}\ \exists r \forall q \exists p (p \leftrightarrow ((p \rightarrow r) \leftrightarrow q)) \quad \vee \arrow[d, "r = 1"'] & & \\
 & & \forall q \exists p (p \leftrightarrow q)\ \boxed{1} \quad \wedge \arrow[dl, "q = 0"'] \arrow[dr, "q = 1"] & & \\
 & \boxed{1}\ \exists p (\neg p) \quad \vee \arrow[dl, "q = 0"'] & & \exists p (p)\ \boxed{1} \quad \vee \arrow[dl, "p = 1"'] & \\
\boxed{1}\ \top & & \boxed{1}\ \top & &
\end{tikzcd}$$

The formula is true.

Exercise 8 (Problem 2)

Evaluate the following formula using only the pure literal rule, universal literal deletion and unit propagation.

$$\exists p \forall q \exists r \forall s ((p \vee q \vee s) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg q \vee \neg r \vee s)).$$

Solution

$$\exists p \forall q \exists r \forall s ((p \vee q \vee s) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg q \vee \neg r \vee s)).$$

Solution

$$\exists p \forall q \exists r \forall s ((p \vee q \vee s) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg q \vee \neg r \vee s)).$$

The literal s is pure. Since s it is universally quantified, we make it s false and the formula is simplified into

$$\exists p \forall q \exists r ((p \vee q) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg q \vee \neg r)).$$

Solution

$$\exists p \forall q \exists r \forall s ((p \lor q \lor s) \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r \lor s)).$$

The literal s is pure. Since s it is universally quantified, we make it s false and the formula is simplified into

$$\exists p \forall q \exists r ((p \lor q) \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r)).$$

Now in the clause $p \lor q$ the literal q is universally quantified and it is quantified after p, so we can delete it from this clause obtaining

$$\exists p \forall q \exists r (p \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r)).$$

Solution

$$\exists p \forall q \exists r \forall s ((p \lor q \lor s) \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r \lor s)).$$

The literal s is pure. Since s it is universally quantified, we make it s false and the formula is simplified into

$$\exists p \forall q \exists r ((p \lor q) \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r)).$$

Now in the clause $p \lor q$ the literal q is universally quantified and it is quantified after p, so we can delete it from this clause obtaining

$$\exists p \forall q \exists r (p \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r)).$$

We can now apply unit propagation to p obtaining

$$\forall q \exists r ((\neg q \lor r) \land (\neg q \lor \neg r)).$$

Solution

$$\exists p \forall q \exists r \forall s ((p \lor q \lor s) \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r \lor s)).$$

The literal s is pure. Since s it is universally quantified, we make it s false and the formula is simplified into

$$\exists p \forall q \exists r ((p \lor q) \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r)).$$

Now in the clause $p \lor q$ the literal q is universally quantified and it is quantified after p, so we can delete it from this clause obtaining

$$\exists p \forall q \exists r (p \land (\neg p \lor \neg q \lor r) \land (\neg q \lor \neg r)).$$

We can now apply unit propagation to p obtaining

$$\forall q \exists r ((\neg q \lor r) \land (\neg q \lor \neg r)).$$

The literal $\neg q$ is pure. Since q is universally quantified, we make q true and the formula is simplified into

$$\exists r (r \land \neg r)$$

Solution

$$\exists p \forall q \exists r \forall s((p \vee q \vee s) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg q \vee \neg r \vee s)).$$

The literal s is pure. Since s it is universally quantified, we make it s false and the formula is simplified into

$$\exists p \forall q \exists r((p \vee q) \wedge (\neg p \vee \neg q \vee r) \wedge (\neg q \vee \neg r)).$$

Now in the clause $p \vee q$ the literal q is universally quantified and it is quantified after p, so we can delete it from this clause obtaining

$$\exists p \forall q \exists r(p \wedge (\neg p \vee \neg q \vee r) \wedge (\neg q \vee \neg r)).$$

We can now apply unit propagation to p obtaining

$$\forall q \exists r((\neg q \vee r) \wedge (\neg q \vee \neg r)).$$

The literal $\neg q$ is pure. Since q is universally quantified, we make q true and the formula is simplified into

$$\exists r(r \wedge \neg r)$$

Finally, unit propagation applied to r gives $\perp$, so the formula is false.

Exercise 8 (Problem 3)

Evaluate the following formula using DPLL:

$$\forall p \exists q \forall s \exists r ((p \vee q \vee s) \wedge (p \vee \neg q \vee \neg r \vee \neg s) \wedge (p \vee \neg q \vee r \vee \neg s))$$

Solution

$$\forall p \exists q \forall s \exists r$$
$$p \vee q \vee s$$
$$p \vee \neg q \vee \neg r \vee \neg s$$
$$p \vee \neg q \vee r \vee \neg s$$

Solution

$$\forall p \exists q \forall s \exists r$$
$$p \vee q \vee s$$
$$p \vee \neg q \vee \neg r \vee \neg s$$
$$p \vee \neg q \vee r \vee \neg s$$

$$\xrightarrow{\neg p}$$

$$\exists q \forall s \exists r$$
$$q \vee s$$
$$\neg q \vee \neg r \vee \neg s$$
$$\neg q \vee r \vee \neg s$$

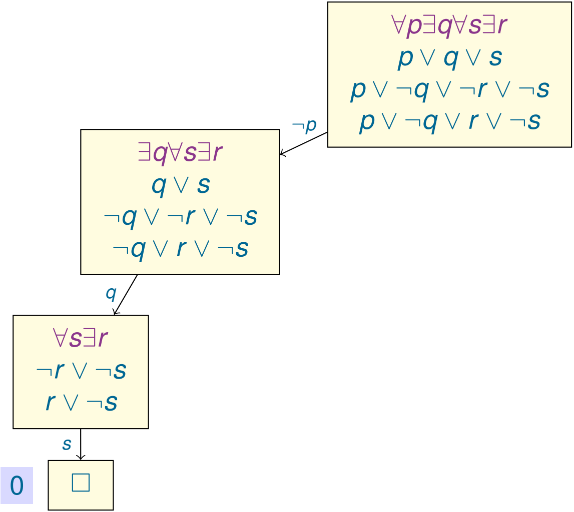
Solution

$$\begin{tikzcd}
& \boxed{\begin{array}{c} \forall p \exists q \forall s \exists r \\ p \lor q \lor s \\ p \lor \neg q \lor \neg r \lor \neg s \\ p \lor \neg q \lor r \lor \neg s \end{array}} \arrow[dl, "\neg p"'] \\
\boxed{\begin{array}{c} \exists q \forall s \exists r \\ q \lor s \\ \neg q \lor \neg r \lor \neg s \\ \neg q \lor r \lor \neg s \end{array}} \arrow[d, "q"'] & \\
\boxed{\begin{array}{c} \forall s \exists r \\ \neg r \lor \neg s \\ r \lor \neg s \end{array}} &
\end{tikzcd}$$

Solution

$$
\begin{tikzcd}
 & & \boxed{\begin{array}{c} \forall p \exists q \forall s \exists r \\ p \lor q \lor s \\ p \lor \neg q \lor \neg r \lor \neg s \\ p \lor \neg q \lor r \lor \neg s \end{array}} \arrow[dl, "\neg p"'] \\
 & \boxed{\begin{array}{c} \exists q \forall s \exists r \\ q \lor s \\ \neg q \lor \neg r \lor \neg s \\ \neg q \lor r \lor \neg s \end{array}} \arrow[dl, "q"'] & \\
\boxed{\begin{array}{c} \forall s \exists r \\ \neg r \lor \neg s \\ r \lor \neg s \end{array}} \arrow[d, "s"'] & & \\
\boxed{\square} & &
\end{tikzcd}
$$

Solution

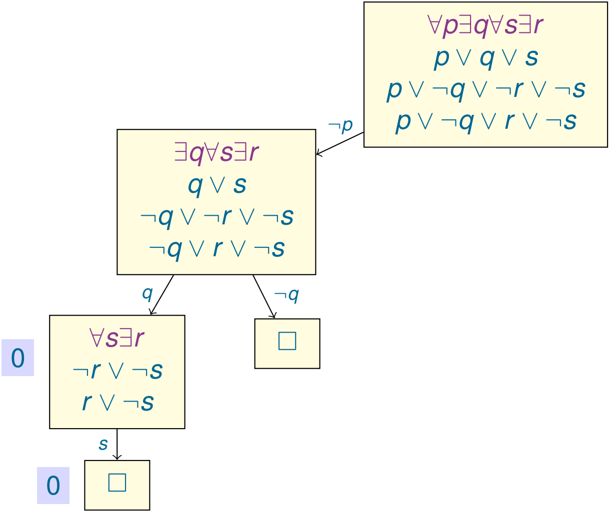


$\forall p \exists q \forall s \exists r$
$p \vee q \vee s$
$p \vee \neg q \vee \neg r \vee \neg s$
$p \vee \neg q \vee r \vee \neg s$
$\neg p$
$\exists q \forall s \exists r$
$q \vee s$
$\neg q \vee \neg r \vee \neg s$
$\neg q \vee r \vee \neg s$
q
$\forall s \exists r$
$\neg r \vee \neg s$
$r \vee \neg s$
s
0
$\Box$

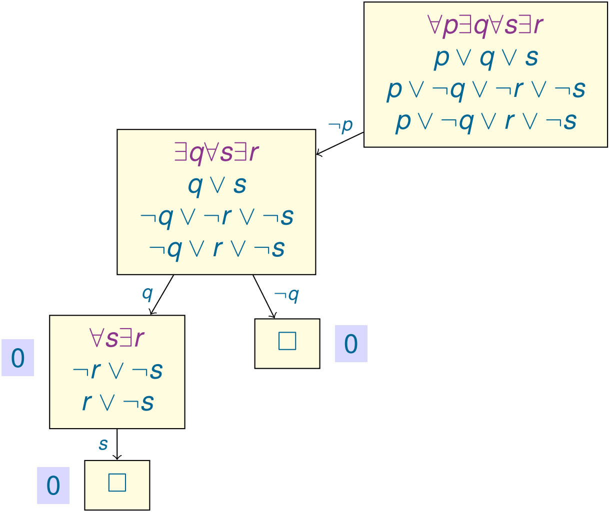
Solution

$$\begin{tikzcd}
& & \boxed{\begin{array}{c} \forall p \exists q \forall s \exists r \\ p \vee q \vee s \\ p \vee \neg q \vee \neg r \vee \neg s \\ p \vee \neg q \vee r \vee \neg s \end{array}} \arrow[dl, "\neg p"'] \\
& \boxed{\begin{array}{c} \exists q \forall s \exists r \\ q \vee s \\ \neg q \vee \neg r \vee \neg s \\ \neg q \vee r \vee \neg s \end{array}} \arrow[d, "q"'] & \\
0 & \boxed{\begin{array}{c} \forall s \exists r \\ \neg r \vee \neg s \\ r \vee \neg s \end{array}} \arrow[d, "s"'] & \\
0 & \boxed{\square} &
\end{tikzcd}$$

Solution

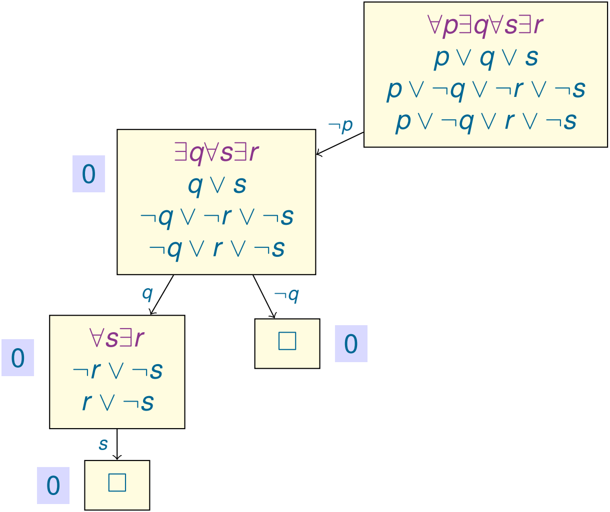


Solution

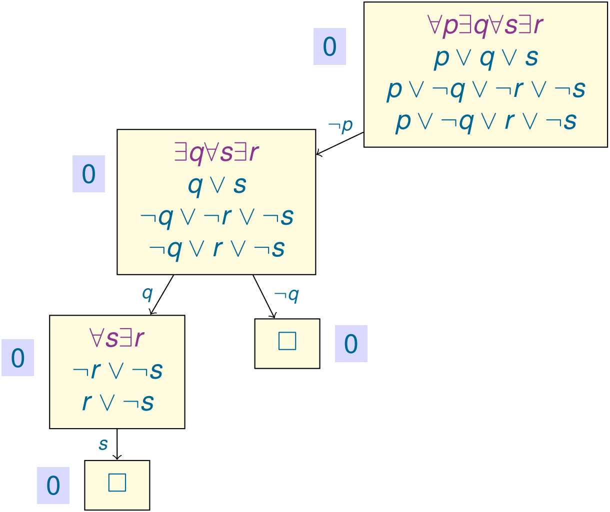


$\forall p \exists q \forall s \exists r$
$p \vee q \vee s$
$p \vee \neg q \vee \neg r \vee \neg s$
$p \vee \neg q \vee r \vee \neg s$
$\neg p$
$\exists q \forall s \exists r$
$q \vee s$
$\neg q \vee \neg r \vee \neg s$
$\neg q \vee r \vee \neg s$
q
$\neg q$
$\forall s \exists r$
$\neg r \vee \neg s$
$r \vee \neg s$
0
$\square$
0
s
$\square$
0

Solution

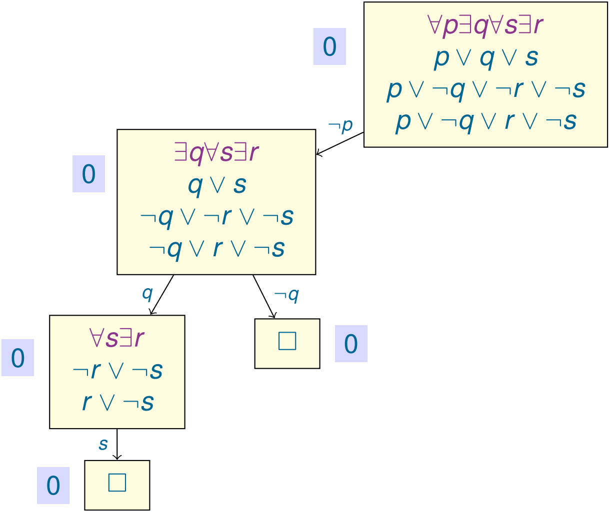

$\forall p \exists q \forall s \exists r$
$p \vee q \vee s$
$p \vee \neg q \vee \neg r \vee \neg s$
$p \vee \neg q \vee r \vee \neg s$
$\neg p$
0
$\exists q \forall s \exists r$
$q \vee s$
$\neg q \vee \neg r \vee \neg s$
$\neg q \vee r \vee \neg s$
q
$\neg q$
0
$\forall s \exists r$
$\neg r \vee \neg s$
$r \vee \neg s$
$\square$
0
s
0
$\square$

Solution



0
$\forall p \exists q \forall s \exists r$
$p \vee q \vee s$
$p \vee \neg q \vee \neg r \vee \neg s$
$p \vee \neg q \vee r \vee \neg s$
$\neg p$
0
$\exists q \forall s \exists r$
$q \vee s$
$\neg q \vee \neg r \vee \neg s$
$\neg q \vee r \vee \neg s$
q
$\neg q$
0
$\forall s \exists r$
$\neg r \vee \neg s$
$r \vee \neg s$
$\square$
0
s
0
$\square$

Solution



The formula is false.