

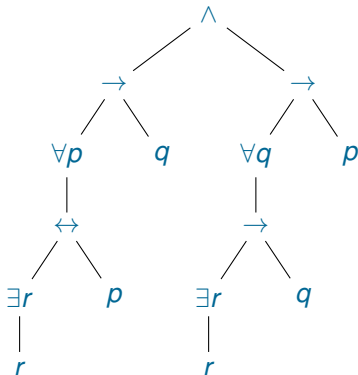
## Exercise 7 (Problem 1)

Draw the parse tree for the following formula:

$$(\forall p(\exists r r \leftrightarrow p) \rightarrow q) \wedge (\forall q(\exists r r \rightarrow q) \rightarrow p).$$

Mark all bound occurrences of variables in this formula.

**Solution**



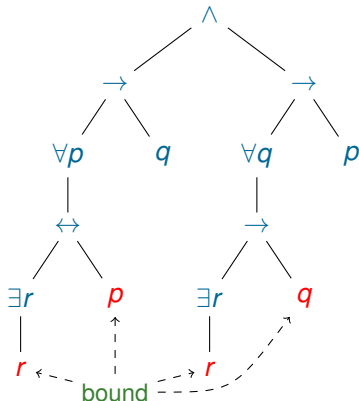
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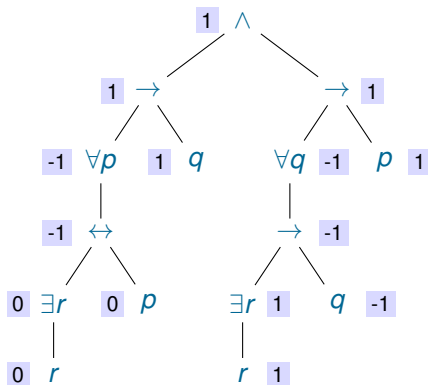
**Solution**



## Exercise 7 (Problem 2)

For the parse tree of Problem 1, mark each node with the polarity of this node.

Solution



## Exercise 7 (Problem 3)

Transform the following formulas into prenex form:

$$\forall p \neg p \vee \forall p p \rightarrow \neg p$$

### Solution

One possible sequence of transformations is shown below. The first two steps rectify the formula.

$$\begin{aligned}\forall p \neg p \vee \forall p p \rightarrow \neg p &\Rightarrow \\ \forall q \neg q \vee \forall p p \rightarrow \neg p &\Rightarrow \\ \forall q \neg q \vee \forall r r \rightarrow \neg p &\end{aligned}$$

## Exercise 7 (Problem 3)

Transform the following formulas into prenex form:

$$\forall p \neg p \vee \forall p p \rightarrow \neg p$$

### Solution

One possible sequence of transformations is shown below. The first two steps rectify the formula. The remaining steps push all quantifiers out.

$$\begin{aligned} \forall p \neg p \vee \forall p p \rightarrow \neg p &\Rightarrow \\ \forall q \neg q \vee \forall p p \rightarrow \neg p &\Rightarrow \\ \forall q \neg q \vee \forall r r \rightarrow \neg p &\Rightarrow \\ \forall q (\neg q \vee \forall r r) \rightarrow \neg p &\Rightarrow \\ \forall q \forall r (\neg q \vee r) \rightarrow \neg p &\Rightarrow \\ \exists q (\forall r (\neg q \vee r) \rightarrow \neg p) &\Rightarrow \\ \exists q \exists r (\neg q \vee r \rightarrow \neg p) \end{aligned}$$