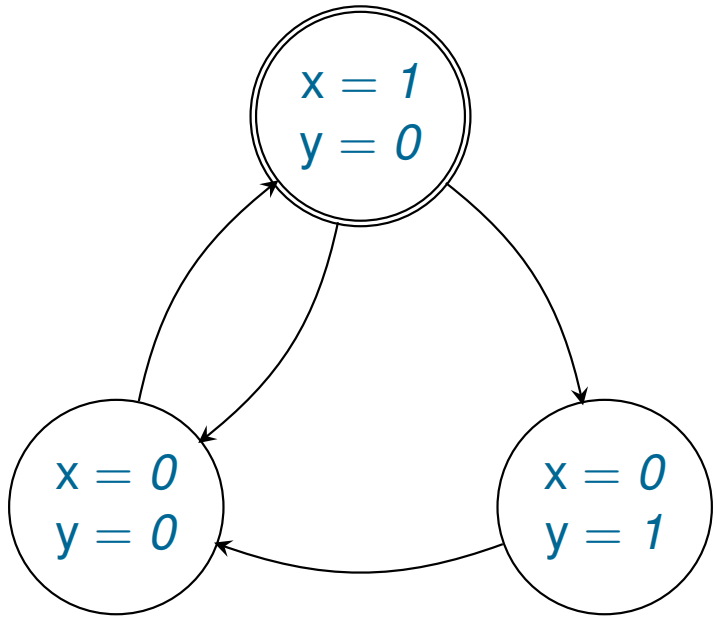
Exercise 11

Consider a transition system with the following state transition graph.



Let S_1 be the set of states symbolically represented by the formula $x = 1$ and S_2 be the set of states symbolically represented by the formula $x = 0 \wedge y = 1$.

1. State whether or not S_1 coincides with the set of initial states.
2. Find a symbolic representation of the set of states reachable from S_2 in exactly two steps.
3. Find a symbolic representation of the set of states backward reachable from S_2 in exactly three steps.

Solution

1. S_1 does not coincide with the set of initial states. The set of initial states is represented by the formula $x = 1 \wedge y = 0$, which is not equivalent to $x = 1$. The problem here is that, when we deal with the symbolic representation of this transition system, a new state $x = 1 \wedge y = 1$ appears, which is not an initial state, but satisfies S_1.
2. $x = 1 \wedge y = 0$ (exactly one state can be reached from S_2 in two steps.
3. $x = 1 \leftrightarrow y = 0$ (exactly two state can be backward reached from S_2 in three steps, namely the two states shown in the system apart from the leftmost one). Of course, any equivalent formula can be used here, for example, $(x = 1 \wedge y = 0) \vee (x = 0 \wedge y = 1)$.