LTL: Linear Temporal Logic

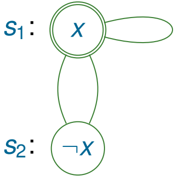
- Computation Tree
- Linear Temporal Logic
- Using Temporal Formulas
- Equivalences of Temporal Formulas

Computation Tree

Let $\mathbb{S} = (S, In, T, \mathcal{X}, dom)$ be a transition system and $s \in S$ be a state. The computation tree for $\mathbb{S}$ starting at s is the following (possibly infinite) tree.

1. The nodes of the tree are labeled by states in S.
2. The root of the tree is labeled by s.
3. For every node s' in the tree, its children are exactly such nodes $s'' \in S$ that $(s', s'') \in T$.

Computation Trees and Paths



A computation path for $\mathbb{S}$: any branch $s_0, s_1, \ldots$ in the tree.

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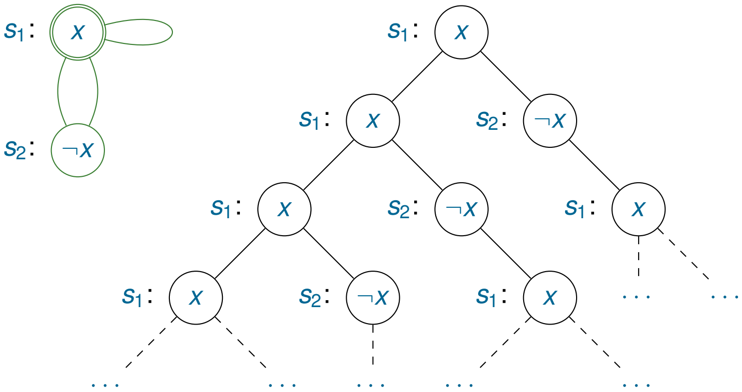

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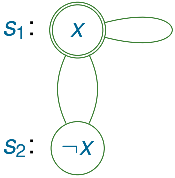

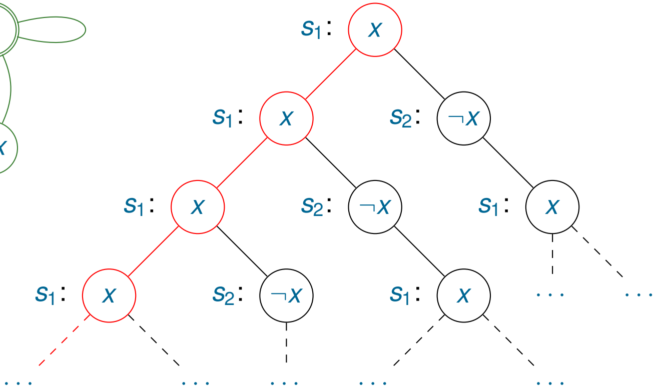

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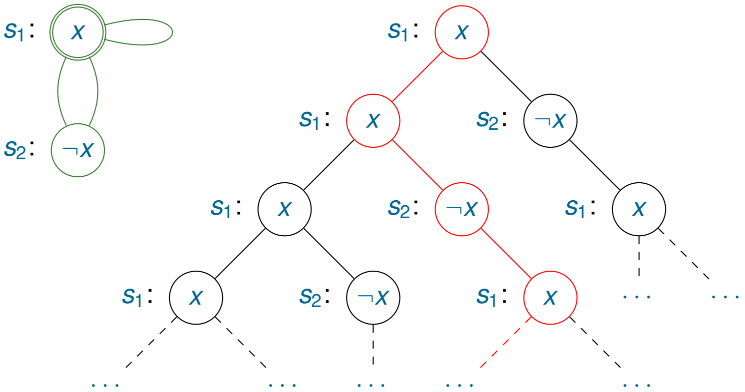

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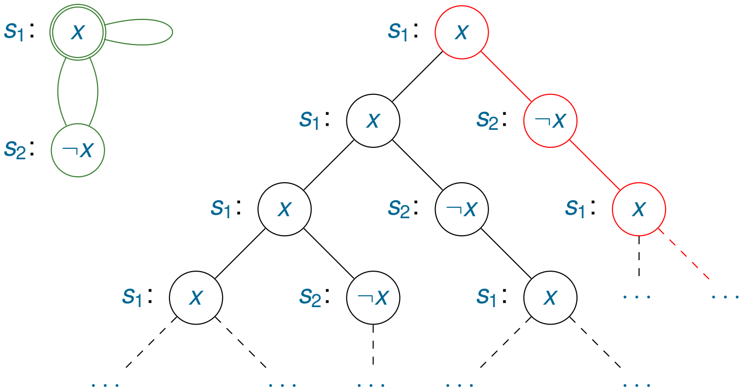

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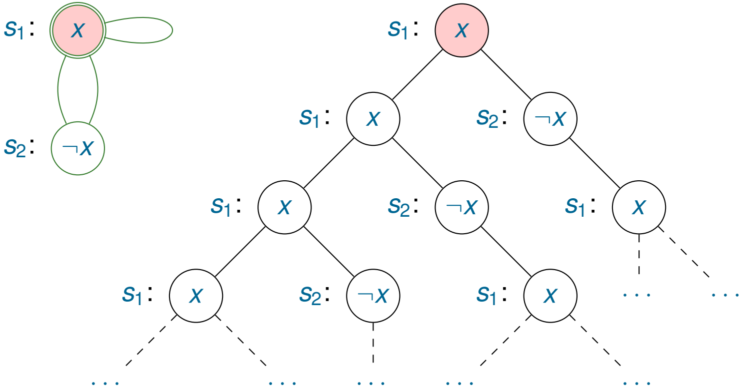

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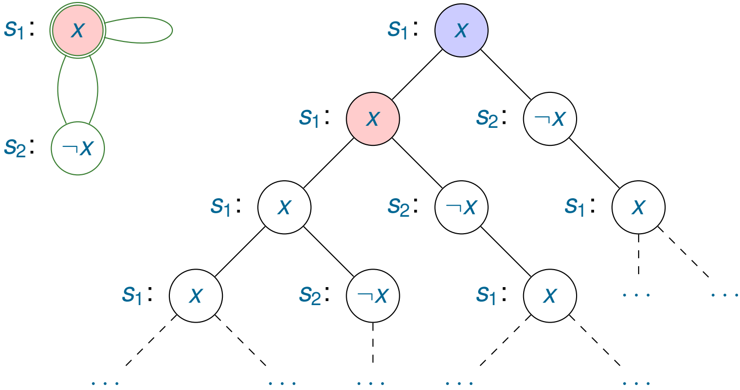

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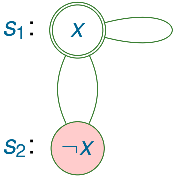

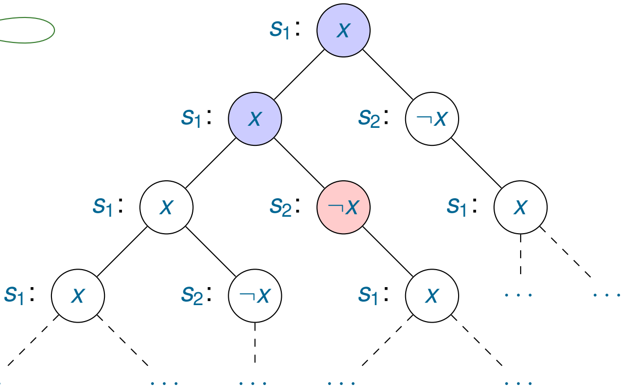

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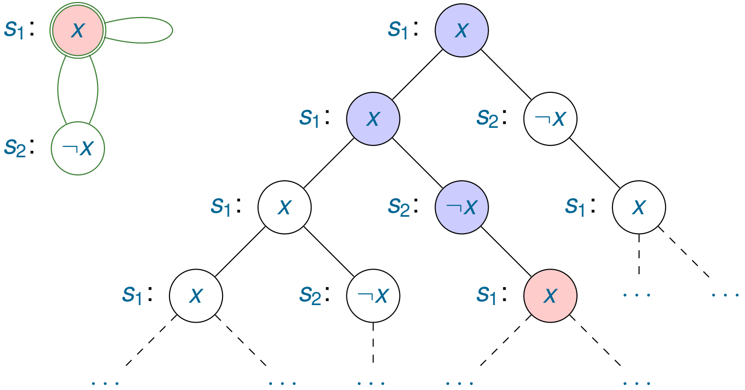

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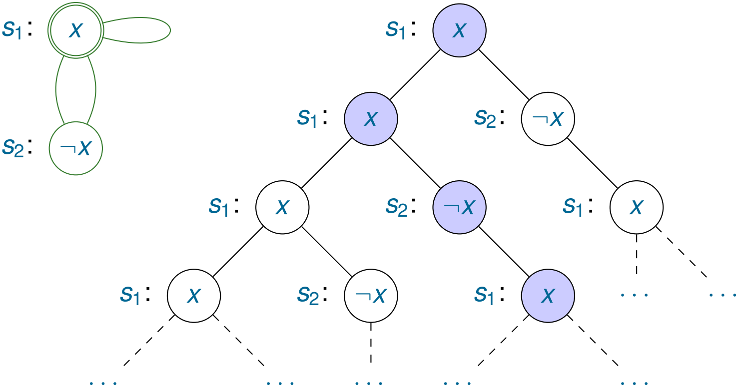

Properties

- Computation paths for a transition system are exactly all branches in the computation trees for this transition system.
- Let n be a node in a computation tree C for $\mathbb{S}$ labeled by s'. Then the subtree of C rooted at s' is the computation tree for $\mathbb{S}$ starting at s'. In other words, every subtree of a computation tree rooted at some node is itself a computation tree.
- For every transition system $\mathbb{S}$ and state s there exists a unique computation tree for $\mathbb{S}$ starting at s, up to the order of children.

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LTL

Linear Temporal Logic is a logic for reasoning about properties of computation paths.

Formulas are built in the same way as in propositional logic, with the following additions:

1. If F is a formula, then $\bigcirc F$, $\Box F$, and $\Diamond F$ are formulas;
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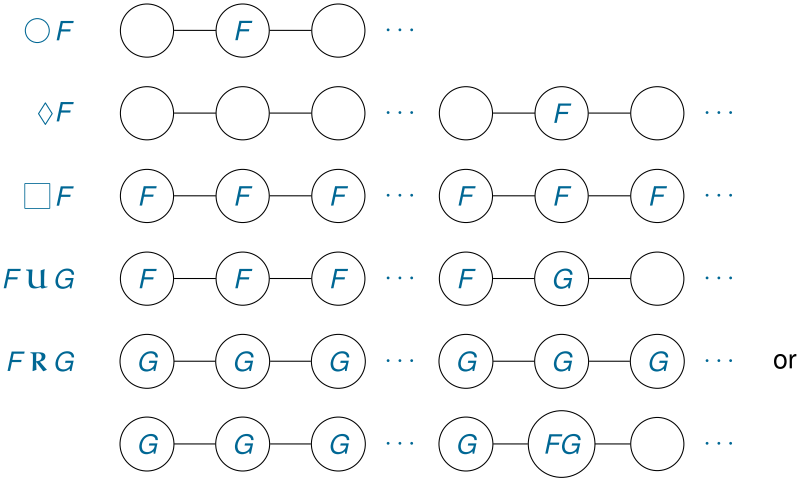
$\bigcirc$	next
$\Box$	always (in future)
$\Diamond$	sometimes (in future)
$\mathcal{U}$	until
$\mathcal{R}$	release

Semantics (intuitive)



Semantics

Unlike propositonal formulas, LTL formulas express properties of computations or computation paths.

Let $\pi = s_0, s_1, s_2 \ldots$ be a sequence of states and F be an LTL formula.

We define the notion F is true on π (or F holds on π), denoted by $\pi \models F$, by induction on F as follows.

For all $i = 0, 1, \ldots$ denote by π_i the sequence of states $s_i, s_{i+1}, s_{i+2} \ldots$ (note that $\pi_0 = \pi$).

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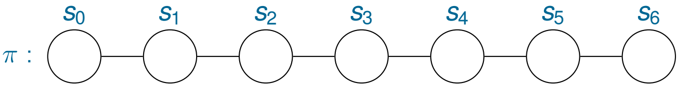

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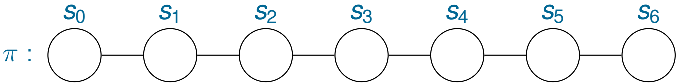

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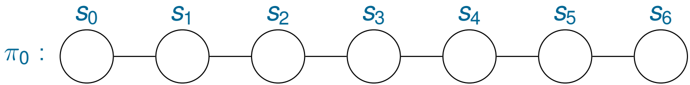

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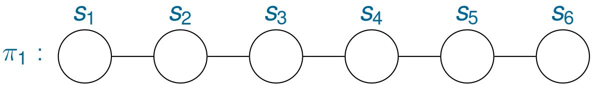

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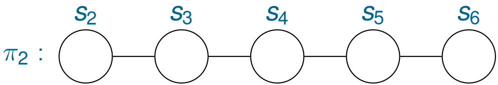

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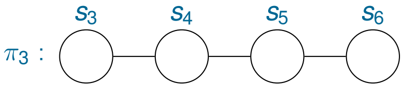

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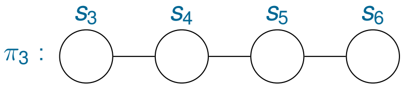

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Semantics, formally

The semantics of propositional connectives is standard.

Atomic formulas are true iff they are true in s_0.

The semantics of formulas built using propositional connectives on π is the same as in propositional logic where all subformulas are also evaluated on π.

1. $\pi \models \top$ and $\pi \not\models \bot$.
2. $\pi \models x = v$ if $s_0 \models x = v$.
3. $\pi \models F_1 \land \ldots \land F_n$ if for all $j = 1, \ldots, n$ we have $\pi \models F_j$;
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4. $\pi \models \neg F$ if $\pi \not\models F$.
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Semantics of temporal operators

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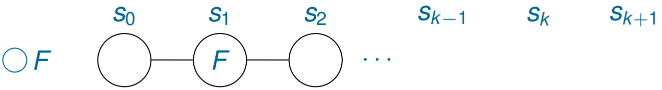

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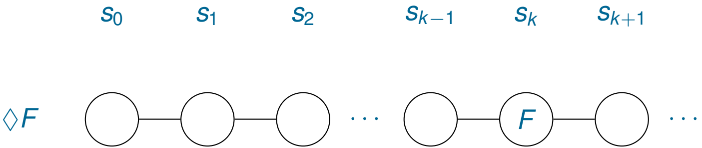

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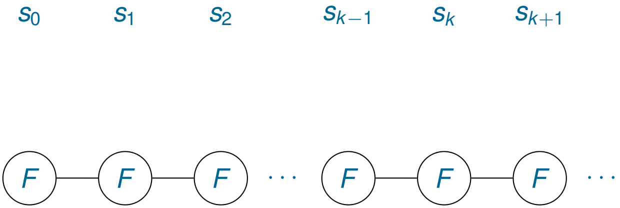

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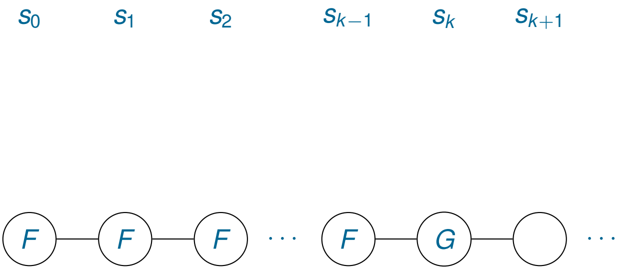

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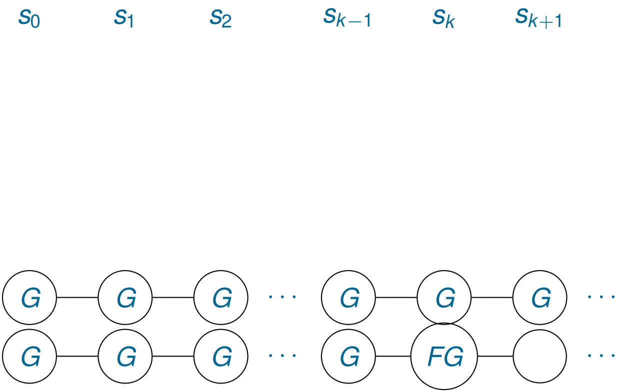
   such that $\pi_j \models F$.

$F \,\mathcal{R}\, G$



or

Semantics of temporal operators

6. $\pi \models \bigcirc F$ if $\pi_1 \models F$;
   $\pi \models \Diamond F$ if for some $k \geq 0$ we have $\pi_k \models F$;
   $\pi \models \Box F$ if for all $i \geq 0$ we have $\pi_i \models F$.
7. $\pi \models F \mathbf{U} G$ if for some $k \geq 0$ we have $\pi_k \models G$ and
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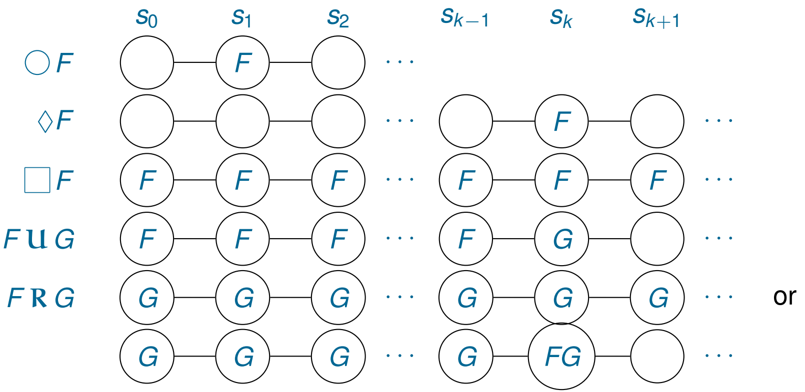

Standard properties

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Consider a transition system $\mathbb{S}$. We are interested in checking the following properties of LTL formulas

For an LTL formula F we can consider two kinds of properties of $\mathbb{S}$:

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Precedences of Connectives and Temporal Operators

Connective	Precedence
$\neg, \bigcirc, \Diamond, \Box$	6
$\mathbf{U}, \mathbf{R}$	5
$\wedge$	4
$\vee$	3
$\rightarrow$	2
$\leftrightarrow$	1

Expressing Some Properties

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Some useful properties

- Reachability and safety properties.
  Let unsafe describe states which are unsafe.
  Then $\Box\neg$unsafe express a safety requirement.
  Ex: $\Box\neg(\text{disp} = \text{beer} \wedge \text{customer} = \text{prof})$
- Mutual exclusion. Two processes are not in the critical section.
  Ex: $\Box\neg(\text{critical}_1 \wedge \text{critical}_2)$
- Fairness. Ex: $\Box(\text{customer} = \text{student} \rightarrow \Diamond\text{customer} = \text{prof})$
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Some Equivalences

Two LTL formulas F and G are called equivalent, denoted $F \equiv G$, if for every path π we have $\pi \models F$ if and only if $\pi \models G$.

Negation:

$$
\begin{aligned}
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\neg (A \,\mathcal{U}\, B) &\equiv
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Equivalences: Unwinding Properties

$$
\begin{array}{rcl}
\Diamond F & \equiv & F \vee \bigcirc \Diamond F \\
\Box F & \equiv & F \wedge \bigcirc \Box F \\
F \,\mathbf{U}\, G & \equiv & G \vee (F \wedge \bigcirc (F \,\mathbf{U}\, G)) \\
F \,\mathbf{R}\, G & \equiv & G \wedge (F \vee \bigcirc (F \,\mathbf{R}\, G))
\end{array}
$$

Other Equivalences

$$\begin{aligned}\Diamond(F \vee G) &\equiv \Diamond F \vee \Diamond G \\ \Box(F \wedge G) &\equiv \Box F \wedge \Box G\end{aligned}$$

Other Equivalences

$$\begin{aligned} \Diamond(F \lor G) &\equiv \Diamond F \lor \Diamond G \\ \Box(F \land G) &\equiv \Box F \land \Box G \end{aligned}$$

But

$$\begin{aligned} \Box(F \lor G) &\not\equiv \Box F \lor \Box G \\ \Diamond(F \land G) &\not\equiv \Diamond F \land \Diamond G \end{aligned}$$

How to Show that Two Formulas are not Equivalent?

Find a path that satisfies one of the formulas but not the other. For example for $\Box(F \vee G)$ and $\Box F \vee \Box G$.

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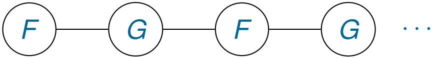

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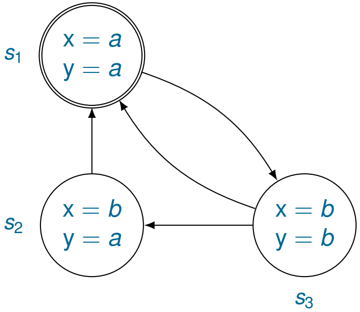
Example

Consider a transition system with the following state transition graph.



Are the following formulas true on all/some paths (from the initial state)?

$\Box(x = a \leftrightarrow y = a)$

$\Box(x = b \rightarrow \Diamond y = a)$

$\Box\Diamond y = b$

$\Box\Diamond y \neq b$

Summary LTL

Linear temporal logic (LTL) – expressing properties of computations.

- Computation tree, path
- LTL Syntax $\bigcirc$, $\Box$, $\Diamond$, $\mathcal{U}$, $\mathcal{R}$
- LTL Semantics
- Equivalences of temporal formulas
- expressing properties of state-changing systems