

Two hours

**UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE**

Logic and Modelling

Date: Friday 23rd January 2015

Time: 14:00 - 16:00

Please answer any THREE Questions from the FOUR Questions provided

This is a CLOSED book examination

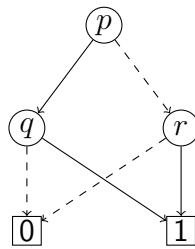
The use of electronic calculators is NOT permitted

[PTO]

1.

a) Let F and G be temporal formulas. Express in LTL the following properties:

1. Whenever F holds at a state, G will hold at some state afterwards. (2 marks)
2. Either F holds infinitely often or both F and G hold finitely often. (3 marks)
3. F holds at all even states and does not hold at all odd states (note that states are numbered from 0) (3 marks)

b) Consider this OBDD for some formula F :The order of variables is $p > q > r$. Draw the OBDD for the formula $(\forall p)F$. (7 marks)c) Consider a transition system with two boolean variables x, y . Draw the state transition graph corresponding to the transition with the symbolic representation

$$\neg x \wedge (y \leftrightarrow \neg y') \wedge (x' \leftrightarrow x').$$

(5 marks)

2.

- a) Consider the following formula in CNF (8 marks)

$$\exists p \forall q \exists r ((p \vee q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r))$$

Evaluate this formula using the DPLL algorithm. Show all steps of the algorithm. Is this formula true or false?

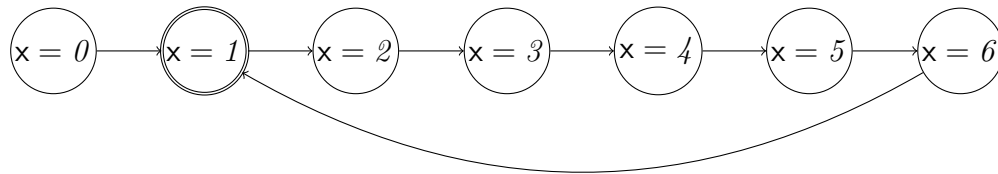
- b) Consider the set consisting of the following two clauses

$$p \quad \neg p \vee q$$

Suppose that the initial random interpretation is $\{p \mapsto 0, q \mapsto 0\}$ and k is a positive integer.

1. What is the probability that GSAT will find a model of this set after $2k$ flips? (2 marks)
2. What is the probability that WSAT will find a model of this set after $2k$ flips? (4 marks)

- c) Consider a transition system with the following state transition graph:



One should check, using reachability algorithms whether the set of state with the symbolic representation $x = 0$ is reachable from the initial states.

Answer the following questions:

1. Is this set of states reachable or not? (1 mark)
2. What is the number of satisfiability checks made by the forward reachability algorithm? (1 mark)
3. What is the number of satisfiability checks made by the backward reachability algorithm? (1 mark)
4. What is the number of equivalence checks made by the forward reachability algorithm? (1 mark)
5. What is the number of equivalence checks made by the backward reachability algorithm? (1 mark)
6. Explain how you calculated the number of steps for the forward reachability algorithm (1 mark)

3.

- a) Let p_1, \dots, p_5 be variables. Draw the OBDD for the formula expressing that an even number of variables among p_1, \dots, p_5 is true and the order $p_5 > p_4 > p_3 > p_2 > p_1$. (Hint: think of the dual property: an odd number of variables is true). (7 marks)

- b) Consider the set consisting of the following clauses:

$$p_1 \vee p_2, \quad p_1 \vee p_3, \quad \neg p_1 \vee \neg p_2 \vee p_3, \quad \neg p_1 \vee p_2, \quad p_1 \vee \neg p_2 \vee \neg p_3, \quad \neg p_1 \vee p_2 \vee p_3.$$

Show how the GSAT algorithm can find a model of this set starting with the initial random interpretation $\{p_1 \mapsto 0, p_2 \mapsto 0, p_3 \mapsto 0\}$. (6 marks)

- c) Convert the formula $\neg(s \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)))$ to CNF using the standard CNF transformation algorithm. (3 marks)

- d) Let p be a propositional variable and consider the LTL formulas $\Box(p \leftrightarrow \bigcirc p)$ and $\Box(\bigcirc p \leftrightarrow \bigcirc \bigcirc p)$.

1. Show that these formulas are not equivalent by giving a path in which one of them is true, while the other one is false. (2 marks)
2. Show all paths with this property. (2 marks)

4.

- a) Prove validity of the formula $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ using the tableau method. (7 marks)
- b) Write down an if-then-else normal form of the formula $(p \leftrightarrow q) \leftrightarrow \perp$. (4 marks)
- c) Consider an instance of propositional logic of finite domains with two variables x, y , both ranging over the domain $\{1, 2, 3\}$. Write down a symbolic representation of the transition, which strictly increases both the values of x and y and makes y greater than x . In other words,
1. The value of x after the transition is greater than its value before the transition.
 2. The value of y after the transition is greater than its value before the transition.
 3. After the transition $y > x$.
- (4 marks)
- d) Find all models of the formula $p \leftrightarrow (q \leftrightarrow r)$. (5 marks)

END OF EXAMINATION