Deep Equilibrium Nets for Solving Dynamic Stochastic Models

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Lecture 4: Quadrature

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This course is inspired by Simon Scheidegger and based on his various teaching materials that I was relying on as a student and those which came as a result of our scientific collaboration

Outline of the presentation

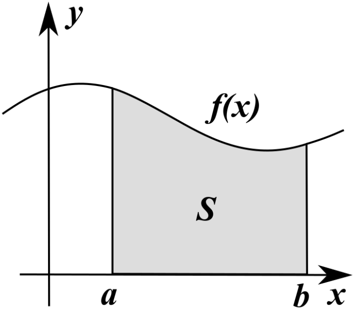
1. Numerical integration
2. Practical session

What is the problem?

- We have an AR(1) shock: $\log(z_{t+1}) = \rho \log(z_t) + \sigma\epsilon_{t+1}$, where $\epsilon_{t+1} \sim \mathcal{N}(0,1)$
- The shock is in essence a function $h(\omega)$, where $\omega \sim \mathcal{N}(0,1)$
- We are interested in the expectation of $h(\omega)$, which is: $\mathbb{E}[h(\omega)] = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\omega-\mu)^2}{2\sigma^2}\right) h(\omega) d\omega$
- We need to compute an integral above

How to numerically compute an integral?

- To compute an integral of a function: $\int_a^b f(x)dx$ means to approximate numerically the area S under the function $f(x)$
- How to do that? Numerical integration (quadrature)!



From Wikipedia

Numerical integration

- There is a bunch of quadrature rules out there
- For the reference you can turn to Judd (1998)
- We will discuss only one quadrature rule that is needed for us today: Gauss-Hermite quadrature

Gauss-Hermite quadrature

- GH is used to approximate the function of the type: $\int_{-\infty}^{\infty} \exp(-x^2) f(x) dx \approx \sum_{i=1}^{n} w_i f(x_i)$ where x_i are GH nodes and w_i are GH weights computed based on the Hermite polynomial (we won't go into details now)
- Thanks to Judd (1998) (to be more precise to Stroud and Secrest (1966)) we know which weights and nodes to use:

N	x_i	w_i
2	0.7071067811	0.8862269254
	- 1.224744871	0.2954089751
3	0.000000000	1.181635900
	1.224744871	0.2954089751

Table: GH wieghts and nodes

Gauss-Hermite quadrature for our case

- GH is used to approximate the function of the type: $\int_{-\infty}^{\infty} \exp(-x^2) f(x) dx \approx \sum_{i=1}^{n} w_i f(x_i)$
- We are interested in the expectation of $h(\omega)$, which is: $\mathbb{E}[h(\omega)] = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\omega-\mu)^2}{2\sigma^2}\right) h(\omega) d\omega$
- The functions do not look exactly the same...

Gauss-Hermite: change of variable

- We introduce a new variable $\vartheta = \frac{\omega - \mu}{\sqrt{2}\sigma}$
- We can infer $\omega = \sqrt{2}\sigma\vartheta + \mu$
- Then we get: $\mathbb{E}[h(\omega)] = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(-\vartheta^2\right) h(\sqrt{2}\sigma\vartheta + \mu) d\vartheta$
- This function we can approximate with GH weights and nodes as: $\frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} w_i h(\sqrt{2}\sigma\vartheta_i + \mu)$

Outline of the presentation

Brock-Mirman stochastic

- Go to the file 02_Brock_Mirman_Uncertainty_DEQN.ipynb

Questions?