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Article *in* Artificial Satellites · January 1995

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ARTIFICIAL SATELLITES, Planetary Geodesy No 24, Vol. 30 No 1 – 1995

THE FOURIER TRANSFORM BAND PASS FILTER AND ITS APPLICATION FOR POLAR MOTION ANALYSIS

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ABSTRACT. In the paper the Fourier Transform (FT) Band Pass Filter (BPF) with appropriate transfer functions is presented and applied for polar motion analysis. The trapezial, parabolic and cosine-bell transfer functions are introduced in the FT filter and tested on the model data similar to polar motion series. The FT BPF with the trapezial transfer function is also compared with the Ormsby BPF by computation of the differences between the model data and filtered oscillations. The optimum parameter values of the transfer function were found and used to filter out single oscillations from polar motion data as well as to compute the spectra with a good frequency resolution. The FT BPF applied to complex-valued time series enables determination of prograde and retrograde oscillations in polar motion data.

1. INTRODUCTION

Difficulties in polar motion analysis are caused by variability of amplitudes and phases of its oscillations with periods ranging from hours to decades. Two most energetic oscillations the Chandler and annual one have very close periods, and are difficult to separate. One and a half century long astrometric pole coordinate series with accuracy of 0.01-0.02 arcsecond enable their study. Many different methods of spectral analysis and filtering were applied to study these oscillations (Chandler 1982; Lambeck 1980; Brzeziński 1987; Gross 1992; King et al. 1991; Nastula et al. 1993; Vondrak et al. 1994; IERS 1988-1994) as well as the 30 year oscillation (Markowitz 1970; Dickman 1981). Variable Chandler oscillation was deeply investigated by Guinot (1972), Yatskiv et al. (1972), Dickman (1981), Okubo (1982) and Vondrak (1985), Nastula et al. (1993). Polar motion data obtained by the new techniques like: SLR, VLBI and GPS with the accuracy corresponding to 1-5 cm on the Earth's surface and resolution of 1 to a few days (Eanes et al. 1989; Carter 1986; Lichten et al. 1992) enable investigations of short periodic oscillations with amplitudes of several milliarcseconds (mas). These short periodic oscillations in polar motion have variable amplitudes and can be detected using stochastic methods of spectral analysis and filtering (Brzeziński 1992; Gross et al. 1992; Chao 1993;

Dickman 1989; Eubanks et al. 1988; Eubanks 1993; Kolaczek et al. 1991, 1993; Kolaczek 1992, 1993; Kosek 1987a,b, 1991; Kosek et al. 1994, 1995; Nastula 1988; Nastula et al. 1990; Salstein et al. 1989). These stochastic methods are still being improved and need more improvements for analysis of different geophysical processes. In this paper the Fourier Transform Band Pass Filter (FT BPF) (Koopmans 1974; Speed 1985) is presented, in which some transfer function (spectral window) method is introduced and examined on the basis of the model data and polar motion data. This method can be applied for determination of long and short periodic oscillations of polar motion and computation of the BPF spectra with a good frequency resolution.

2. THE FOURIER TRANSFORM BAND PASS FILTER

Theory of linear band pass filters of convolution type

$$y_t = \sum_{k=-\infty}^{\infty} a_k \, x_{t-k} \tag{1}$$

making use of the discrete FT is presented by Koopmans (1974) and Speed (1985). The idea of such approach is described by the following formula:

$$u_t = FT^{-1}\left[FT[x_t] \; A(\omega)\right], \quad t=1,2,\ldots,N, \tag{2}$$

where

$$A(\omega) = \sum_{k=-\infty}^{\infty} a_k \exp(-i2\pi k\omega), \quad -\frac{1}{2} \leq \omega \leq \frac{1}{2} \tag{3}$$

is the filter transfer function, ω denotes the normalized frequency, N is the number of points of x_t series, u_t denotes the filtered oscillation, a_k are the Fourier coefficients, FT denotes the discrete Fourier Transform operator, which in this paper is represented by the Fast Fourier Transform (FFT) operator (Singleton 1969).

When a_k decrease quickly then the filtered oscillation values u_t, $t=1,2,\ldots,N$ are close to:

$$v_t = \sum_{k=1}^{N} a_{t-k} x_k \qquad \text{(Brillinger 1975).} \tag{4}$$

If the time series x_t is weakly stationary, then according to Koopmans (1974) actual results u_t, $t=1,2,\ldots,N$, differ from the theoretical y_t in the mean-square sense as follows:

$$2\pi m S_{N,t} \leq E(u_t - y_t)^2 \leq 2\pi M S_{N,t}, \tag{5}$$

where m and M are the lower and upper bounds of the spectral density function $d(\omega)$ $(-1/2 \leq \omega \leq 1/2)$ of the x_t series,

$$S_{N,t} = \sum_{k=-\infty}^{t-N-1} |a_k|^2 + \sum_{k=t}^{\infty} |a_k|^2 + \sum_{k=t-N}^{t-1} |(a_k - \overline{a_k})|^2 , \tag{6}$$

$$\bar{a}_k = \sum_{p=-\infty}^{\infty} a_{k+pN} \quad . \tag{7}$$

From the above estimates it follows, that the transfer function with quickly decreasing weights a_k can give smaller mean-square errors than the one with slowly decreasing weights. On the other hand, according to (6), one can also omit the values of u_t near the ends (t=1 and t=N) in order to retain only the values with possibly small mean-square errors. It results from decreasing character of the Fourier coefficients (Koopmans 1974). Ideal filters having transfer functions of the boxcar type have slowly decreasing weights, what magnifies filtration errors caused by finite data length (Speed 1985). Application of the window function method of designing nonrecursive digital filters reduces the filtration errors caused by finite data length (Tretter 1976). Thus, such window function method can be also introduced in the FT BPF by designing different transfer functions. In this paper 3 types of transfer function are considered:

- trapezial

$$A(\omega,\omega_T)=\begin{cases} 0 & \text{if } |\omega-\omega_T|>\alpha+\lambda \\ 1 - \dfrac{(|\omega-\omega_T|-\alpha)}{\lambda} & \text{if } \alpha<|\omega-\omega_T|\leq\lambda+\alpha \\ 1 & \text{if } |\omega-\omega_T|\leq\alpha \end{cases} \tag{8}$$

- parabolic

$$A(\omega,\omega_T)=\begin{cases} 0 & \text{if } |\omega-\omega_T|>\lambda \\ 1 - \left(\dfrac{\omega-\omega_T}{\lambda}\right)^2 & \text{if } |\omega-\omega_T|\leq\lambda \end{cases} \tag{9}$$

- cosine-bell

$$A(\omega,\omega_T)=\begin{cases} 0 & \text{if } |\omega-\omega_T|>\lambda \\ \dfrac{1}{2}+\dfrac{1}{2}\cos\left(\dfrac{\pi(\omega-\omega_T)}{\lambda}\right) & \text{if } |\omega-\omega_T|\leq\lambda \end{cases} \tag{10}$$

where $\omega_T = \Delta t/T$ is the normalized frequency, T is a period of an oscillation to be filtered, Δt is the sampling interval, α is the transfer function pass band and λ is the transition band.

For $\lambda=0$, $\alpha>0$ the trapezial function becomes the boxcar and for $\alpha=0$, $\lambda>0$ it becomes the triangular one. The functions of all three types are initially defined for $0\leq\omega\leq 1/2$ and extended on the interval $-1/2\leq\omega\leq 0$ to make the resulting transfer function defined for $-1/2\leq\omega\leq 1/2$ even. It assures, that the filtered oscillation is real-valued. These three types of transfer function were chosen, because they represent functions of different smoothness, namely, noncontinuous (boxcar), continuous (parabolic) and differentiable (cosine-bell). The FT BPF can be also applied for two-dimensional time series represented as complex-valued series $x_t + iy_t$. In the case of complex-valued time series the filter transfer function does not have to be even as in the case of real-valued series, what is connected with the well-known properties of the discrete FT and allows to compute prograde and retrograde oscillations (Brillinger 1975).

3. THE ORMSBY BAND PASS FILTER

The output of the Ormsby BPF is given by the following formula (Ormsby 1961; Kosek 1991):

$$u_t(\omega_T) = \sum_{k=-L}^{L} x_{t+k} h_k(\omega_T) \,, \qquad t=1,2,\dots,N, \tag{11}$$

where

$$h_k(\omega_T) = \frac{2\sin(\pi k\lambda/2)\sin(2\pi k\beta)\cos(2\pi k\omega_T)}{(\pi k)^2}, \tag{12}$$

$\beta = \alpha + \lambda/2$, L is the filter length.

The accuracy of an oscillation computed by this filter depends on the two parameters λ and L and is constant when λL=const (Ormsby 1961). The filter length L should be at least greater than $2T_{max}/\Delta t$, where T_{max} is the longest period of oscillations being filtered.

4. COMPARISON OF THE ORMSBY AND THE FT BPF USING MODEL DATA.

The comparison of the Ormsby and the FT BPF can be carried out when the transfer function of both filters is trapezial and the considered data series is real-valued. The shape of such function depends on the parameters α and λ. The oscillation computed by the BPF is a function of these two parameters. To check the filter accuracy the standard deviations of the differences between the assumed model data and the sums of corresponding oscillations determined by these two filters were computed.

Two model data series were generated. First one was the sum of 7 oscillations (MODEL 1, Tab. 1) and the second one was the sum of 4 oscillations (MODEL 2, Tab. 1). In both model data series constant amplitudes and phases of oscillations were adopted. The sampling intervals of the MODEL 1 and the MODEL 2 series are Δt=1 day and Δt=0.01 year, respectively. These two model data series represent the short periodic (MODEL 1) and the long periodic (MODEL 2) part of polar motion data spectrum.

Table 1. Periods and amplitudes of the model data series (all phases are equal to zero).

	MODEL 1, Δt=1 day							
Periods	-13.7	-27.	38.	49.	63.	95.	122.	(days)
Amplitudes	0.5	1.0	1.0	1.5	2.0	2.5	3.0	(mas)
	MODEL 2, Δt=0.01 year							
Periods	0.3	0.5	1.0	1.2	(years)			
Amplitudes	0.006	0.010	0.100	0.200	(arcsec)			

The standard deviations of the differences between the MODEL 1 data with N=6000 points and the sum of its 7 oscillations computed by these filters are shown in Figure 1 as a function of α and λ parameters. To compute these standard deviations in the case of the FT BPF, 300 data points at the ends of data time interval were omitted. The region of optimum parameter values of the trapezial transfer function is larger for the FT BPF than for the Ormsby BPF. The parameter values of the trapezial transfer function for the Ormsby and FT filters, for which the minimum standard deviations were obtained are similar. In the case of the MODEL 2 data with N=2000 points the region of optimum parameter values of the trapezial transfer function is also wider for the FT BPF than for the Ormsby BPF (Fig. 2).

5. COMPARISON OF DIFFERENT TRANSFER FUNCTIONS OF THE FT BPF USING MODEL DATA.

Three types of transfer function: the boxcar, cosine-bell and parabolic, defined in the paragraph 2, were tested using two model data series (Tab. 1). Additionally, the MODEL 1 data were disturbed by white noise with standard deviation of 2 mas. The standard deviation of the differences between the real-valued MODEL 1 data and the sum of its oscillations computed by the FT BPF was calculated as a function of α or λ parameters or their sum $\alpha + \lambda$ (Fig. 3a). The region of the optimum filter parameter values is the widest for the cosine-bell transfer function and it is the smallest for the boxcar one. Addition of white noise to the model data increases the standard deviation of residuals and does not seem to have influence on the optimum parameter values of the transfer function . In the case of the real-valued MODEL 2 data the region of parameter values corresponding to small standard deviations of residuals is bigger for the parabolic and cosine-bell transfer functions than for the boxcar one (Fig. 3b).

The influence of white noise on the FT BPF oscillation determination can be examined by taking into account the theoretical results concerning stochastic properties of the discrete FT (Brillinger 1975). According to these results for a series of real independent random variables n_t, $t=1,2,\ldots,N$ with the normal distribution $N(0,\sigma^2)$, the discrete FT real and imaginary parts $Re[FT(s/N)]$, $Im[FT(s/N)]$, $s=1,2,\ldots,(N-1)/2$, are approximately independent random variables with the normal distribution $N(0,N\sigma^2/2)$. Multiplying by a transfer function (vanishing outside intervals $|\omega_T-\omega| < \lambda+\alpha$ and $|-\omega_T-\omega| < \lambda+\alpha$, where $0 < \omega_T-\lambda-\alpha$) and performing the inverse FT, we finally obtain random variables μ_t, $t=1,2,\ldots,N$ distributed approximately as $N(0,8(\alpha+\lambda)\sigma^2)$. Thus, the variance of the filtered oscillation random component, corresponding to white noise added to the model series, is reduced by the factor $8(\lambda+\alpha)$ in comparison with the white noise variance σ^2.

The standard deviations of residuals corresponding to chosen FT BPF transfer function parameter values were computed for particular time interval length (Fig. 1,2). To check the influence of time interval length on the minimum standard deviation of residuals such standard deviations were computed for both model data as a function of time interval length using the optimum transfer function parameter values. These optimum transfer function parameter values were determined for each time interval length. The minimum standard deviations in the case of the real-valued MODEL 1 data are presented in Figure 4a as a function of time interval length for different transfer functions. The biggest standard deviation values correspond to the boxcar transfer function and the smallest to the cosine-bell one. The increase of the time interval length

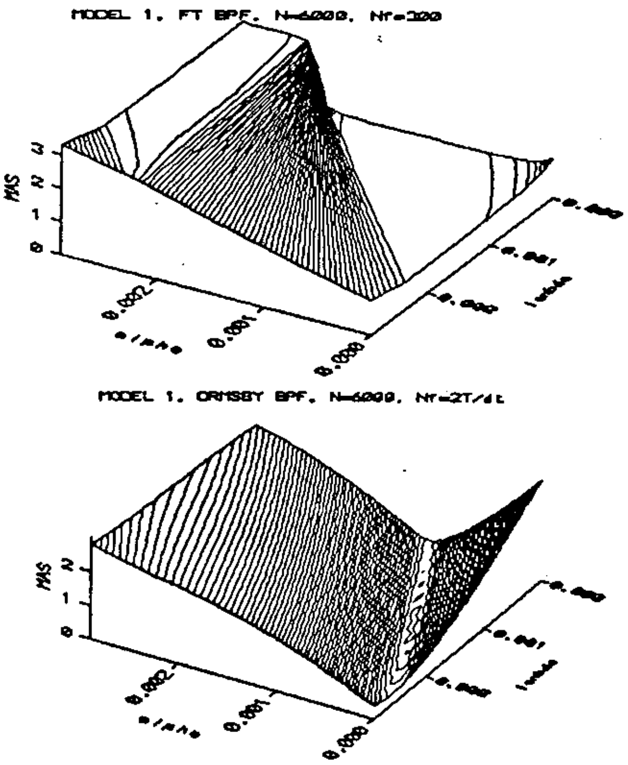

Fig. 1. The standard deviation of the differences between the MODEL 1 data and the sum of its 7 oscillations computed by the FT and Ormsby band pass filters with trapezial transfer function for different α and λ parameter values.

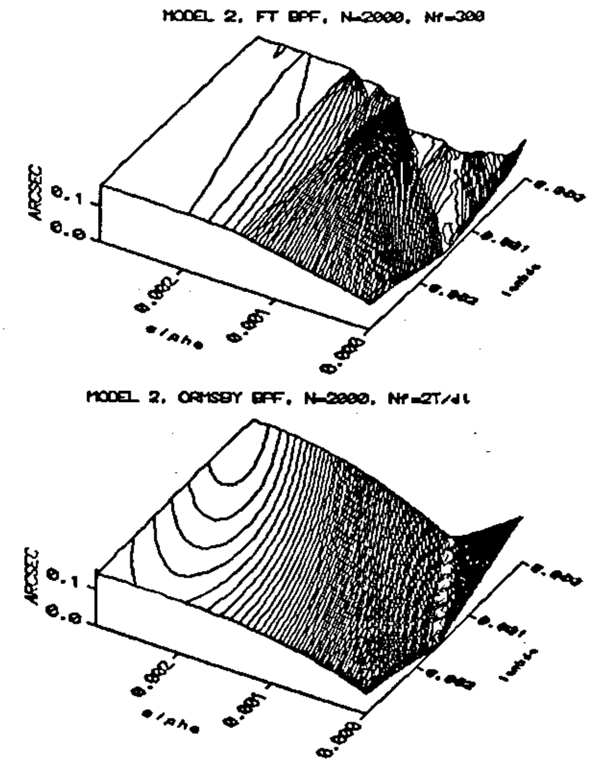

Fig. 2. The standard deviation of the differences between the MODEL 2 data and the sum of its 4 oscillations computed by the FT and Ormsby band pass filters with trapezial transfer function for different α and λ parameter values.

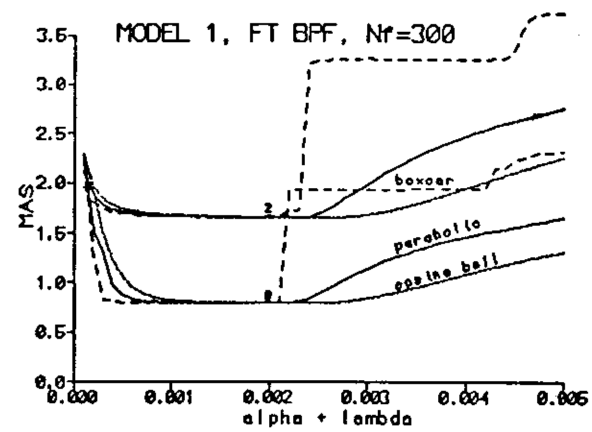

Fig. 3a. The standard deviation of the differences between the MODEL 1 data (lower lines denoted by 0) or MODEL 1 data + white noise (upper lines denoted by 2) and the sum of its 7 oscillations computed by the FT BPF with different transfer functions and their parameter values.

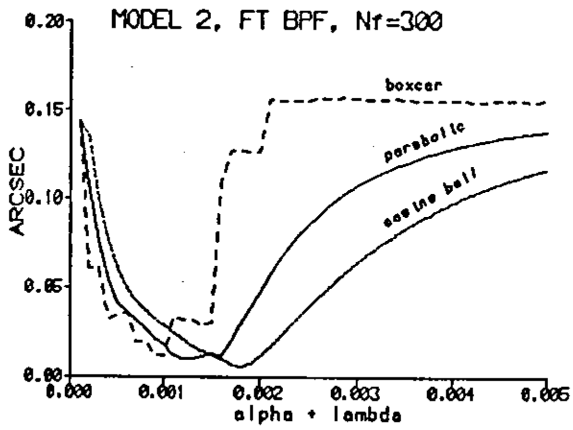

Fig. 3b. The standard deviation of the differences between the MODEL 2 data and the sum of its 4 oscillations computed by the FT BPF with different transfer functions and their parameter values.

does not diminish significantly the minimum standard deviations especially for the parabolic and cosine-bell transfer functions. The results of the similar analysis for the real-valued MODEL 2 data are presented in Figure 4b. The average value of the computed standard deviations is the biggest for the boxcar transfer function and it is the smallest for the cosine-bell one. The standard deviation values change periodically for each transfer function and they reach the smallest values for data time interval lengths of 6, 12, 18,... years, which are the common multiples of the 0.3, 0.5, 1.0 and 1.2 year periods. According to Koopmans (1974) the biggest oscillation errors occur symmetrically at both ends of data time interval. However, it is worth remarking, that according to the results shown in Figure 4b such big errors at the ends do not occur when $N\Delta t$ is a common multiple of oscillation periods of the MODEL 2 data. Indeed, in the MODEL 2 data each oscillation can be filtered exactly, if $N\Delta t = s_k T_k$, $k=1,2,3,4$, where $0 < s_k < (N-1)/2$ are integers, what results from the orthogonality properties of trigonometric functions on sets of equidistant time points (Priestley 1981).

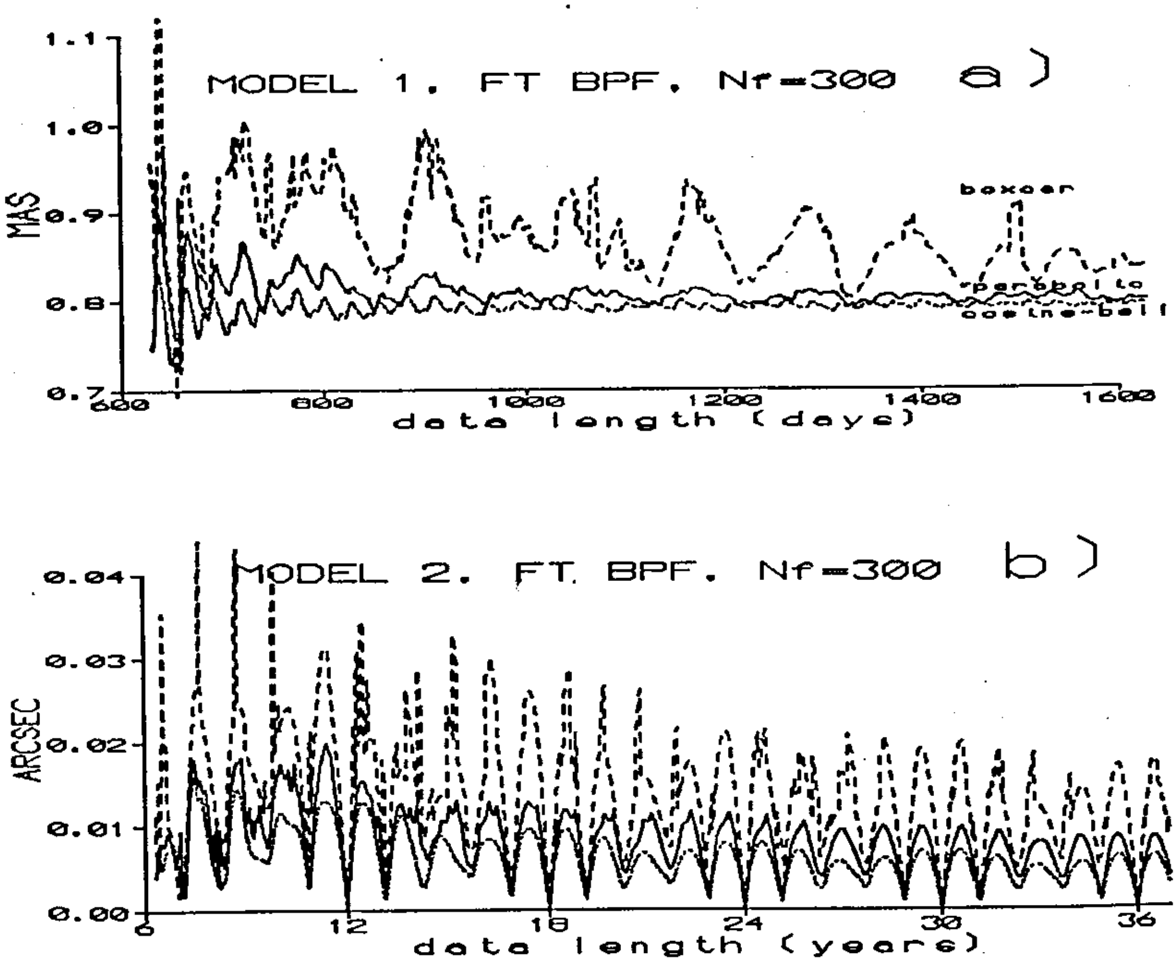

Fig. 4. Minimum standard deviation of the differences between **a)** MODEL 1 data and **b)** MODEL 2 data and the sum of corresponding oscillations computed by the FT BPF with application of the boxcar (dashed line), parabolic (solid line) and cosine-bell (dotted line) transfer functions with the optimum parameter values.

6. APPLICATION OF THE FT BPF TO SPECTRAL ANALYSIS OF THE COMPLEX-VALUED MODEL DATA.

An oscillation with particular period T filtered by the FT BPF can be used to compute the BPF spectrum at this point (Otnes and Enochson 1972; Kosek 1991). The BPF spectrum is defined by the following formula:

$$\hat{S}(\omega_T)=\frac{1}{N-2N_f}\sum_{t=N_f+1}^{N-N_f}|u_t(\omega_T)|^2 \tag{13}$$

where $u_t(\omega_T)$ is the complex-valued oscillation with period T computed by the FT BPF (eq. 2), N_f=300 is the number of oscillation points to be cut off at the ends.

The FT BPF spectra with application of the cosine-bell transfer function and its optimum parameter values (Fig. 3a,b) were computed for 6000 days long complex-valued MODEL 1 data, the same model data with added white noise with standard deviation of σ=8 mas and 90 years long complex-valued MODEL 2 data with added white noise with standard deviation of σ=0.02 arcsec. The sampling intervals of MODEL 1 and 2 data were equal to Δt=1 day and Δt=0.05 years, respectively. The square roots of the spectra presented in Figure 5a,b. show the mean radius of prograde (T > 2Δt) and retrograde (T < -2Δt) oscillations.

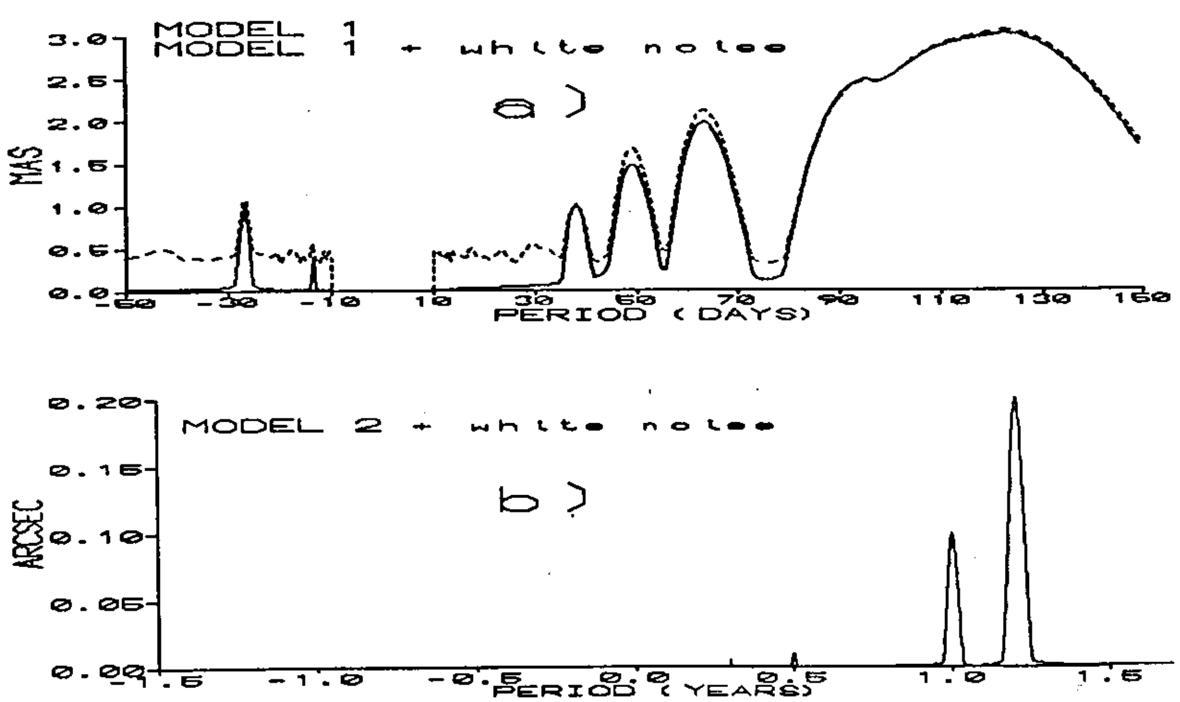

Fig. 5. The square root of the FT BPF spectrum computed with application of the cosine-bell transfer function for: **a)** the complex-valued MODEL 1 data (solid line) and the complex-valued MODEL 1 data with added white noise with the standard deviation σ=8 mas (dotted line) (λ=0.0022), **b)** the complex-valued MODEL 2 data with added white noise with the standard deviation σ=0.02 arcsec (λ=0.018).

In the case of the MODEL 1 and 2 data the increase of the white noise standard deviation does not change the heights of the spectrum peaks, which are equal to the amplitudes of the model oscillations. In the case of the MODEL 1 data the white noise influences only the less energetic peak corresponding to -13.7 day oscillation. Addition of white noise with standard deviation of 0.02 arcsec to the MODEL 2 data has practically no influence on the spectrum, in which peaks corresponding to the four model oscillations were detected.

7. APPLICATION OF THE FT BPF TO POLAR MOTION DATA ANALYSIS

The IERS93C01 pole coordinates spanning from 1846 to 1993.45 and IERS90C04 pole coordinates spanning from 1962 to 1994.4 (IERS, 1988-1993) were analyzed by the FT BPF in order to demonstrate a possibility of its application. The BPF spectra of complex-valued pole coordinates were computed and the most energetic oscillations were filtered. The sampling interval of the IERS93C01 pole coordinates data is $\Delta t=0.1$ and $\Delta t=0.05$ years before and after 1890, respectively. The IERS90C04 pole coordinate data have the sampling interval $\Delta t=1$ day. To remove long periodic terms the data have been filtered by the Butterworth High Pass Filter (HPF) with the cutoff period of 4 years and 250 days for the IERS93C01 and IERS90C04 pole coordinates data, respectively.

Next, the FT BPF spectral analyses with the cosine-bell transfer function and different λ parameter values were performed to compute the spectra of such filtered x, y pole coordinate data (Fig. 6). The filtered IERS93C01 pole coordinate data before 1903.45 and the filtered IERS90C04 pole coordinates data before 1984.0 were omitted for spectrum computation, due to very small signal to noise ratio. For geophysical time series it is difficult to find the appropriate λ parameter value, because their oscillations have variable amplitudes and frequencies. Each spectrum was computed for λ parameter values equal to 0.001, 0.002 and 0.003. The number of peaks in the spectrum increases with decrease of the λ parameter value, but at the same time the spectrum peak heights decrease. In the case of the spectra of the filtered IERS90C04 polar motion data the most energetic peaks correspond to 180, 120, 100, 62, and 49 day oscillations (Fig. 6a). In the case of the spectra of the IERS93C01 polar motion data there are the energetic peaks corresponding to the Chandler and annual oscillations, however much less energetic peaks can also be noticed (Fig. 6b). The Chandler and annual peaks are well separated. The presence of the weak retrograde annual peak with the amplitude of ~ 5 mas (Fig. 6b) shows that the annual oscillation is elliptic. There is no a meaningful retrograde Chandler peak in these spectra.

The FT BPF was applied also to filter out the Chandler (1.18), annual (1.00), semiannual (0.50) and 0.33 year oscillations from the x, y IERS93C01 pole coordinates data filtered by the Butterworth HPF with the 4-year cutoff period. The differences between the described pole coordinate data and the sum of the mentioned above filtered oscillations were computed for different parameter values of the cosine-bell transfer function. In order to find the appropriate parameter values of such transfer function the standard deviations of the considered differences were computed as a function of λ parameter (Fig. 7). The minimum standard deviations of 0.0167 arcsec for x and 0.0169 arcsec for y coordinate residuals were reached for $\lambda=0.0080$ and $\lambda=0.0081$, respectively.

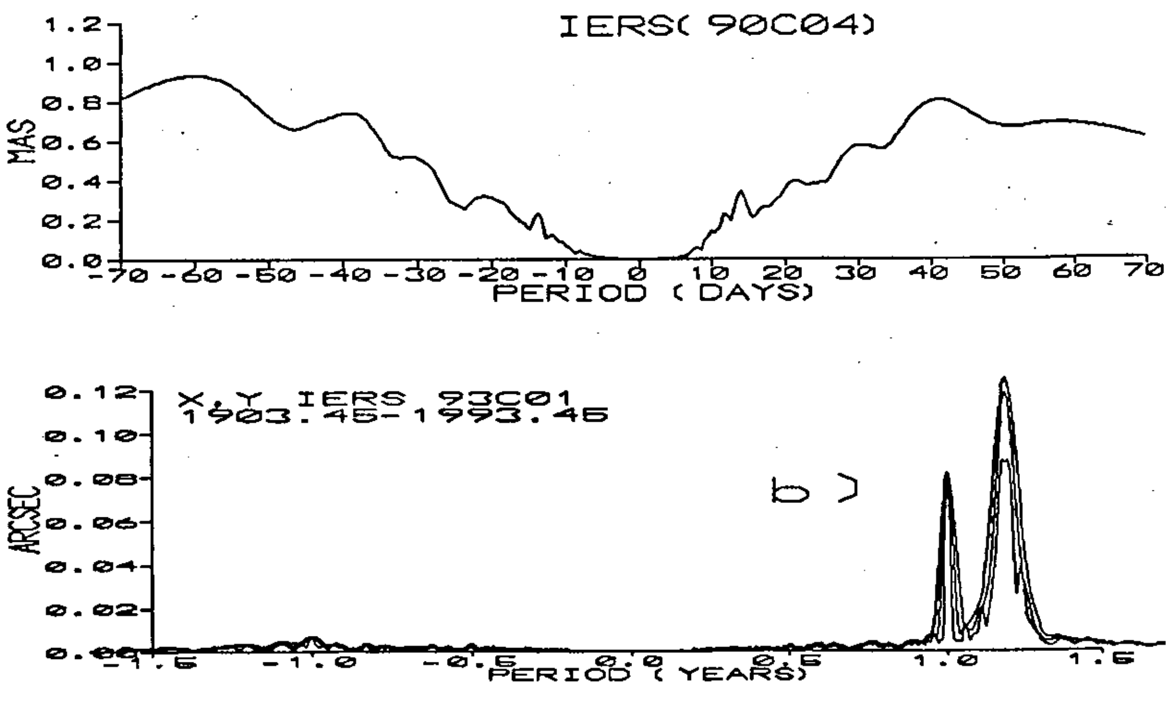

Fig. 6. The square root of the FT BPF spectrum computed with application of the cosine-bell transfer function (λ=0.001, λ=0.002 and λ=0.003) for **a)** the filtered IERS90C04 polar motion data spanning from 1984.0 to 1994.3 and **b)** the filtered IERS93C01 polar motion data spanning from 1903.45 to 1993.45 (greater peak heights correspond to the larger λ values).

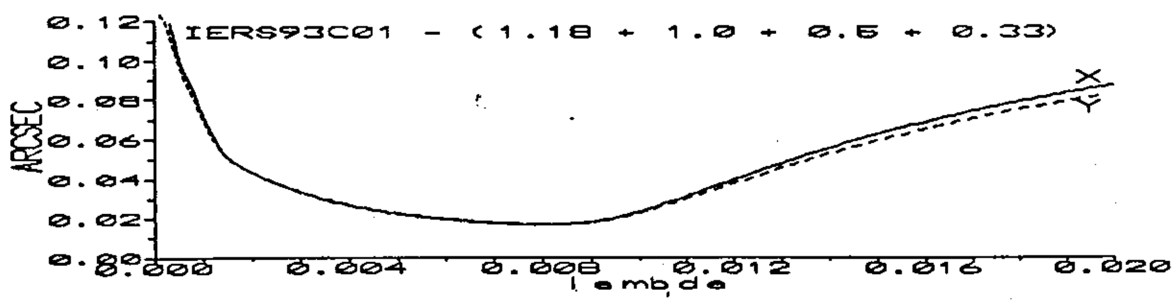

Fig. 7. The standard deviation of the differences between the IERS93C01 x, y pole coordinates filtered by the Butterworth HPF with the cutoff period of 4 years and the sum of the 4 most energetic oscillations in polar motion with periods of 1.18, 1.00, 0.50 and 0.33 years, computed by the FT BPF with the cosine-bell transfer function.

For the estimated optimum cosine-bell transfer function parameter value λ=0.008 (Fig. 7), the most energetic Chandler and annual oscillations as well as the less energetic half a year and 0.33-year oscillations were computed by the FT BPF (Fig. 8, 9).

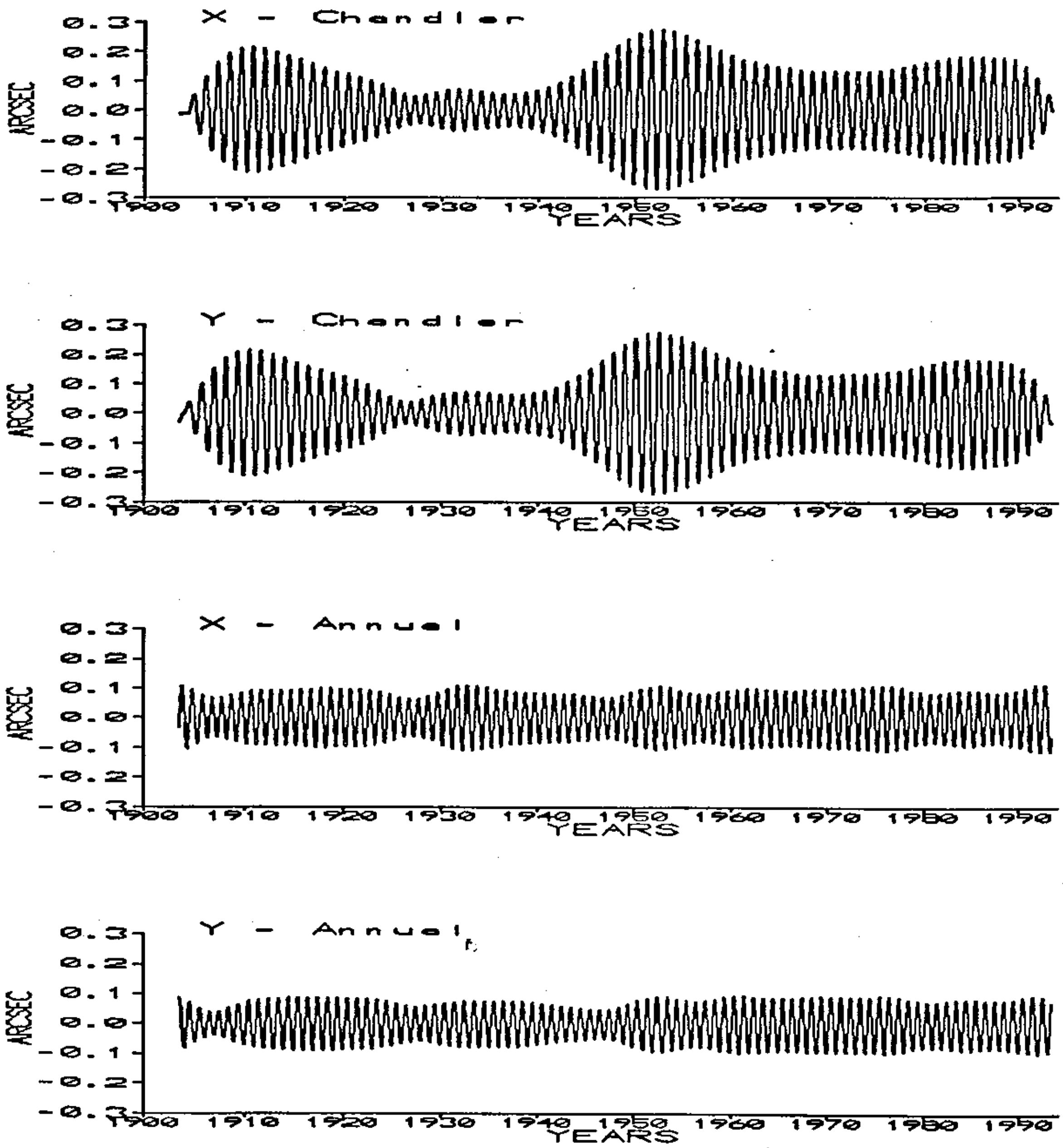

Fig. 8. The Chandler and annual oscillations computed from the x, y IERS93C01 pole coordinates data by the FT BPF with the cosine-bell transfer function (λ=0.008).

The obtained results for the Chandler and annual oscillations are in a good agreement with the previous results derived from different methods (IERS 1994; Nastula et al. 1993). The Chandler and annual oscillations computed by the FT BPF are similar to those computed by the CENSUS X-11 filter (IERS, 1994). The Chandler oscillation has variable amplitude, which has the minima in 1927-1940 and in 1970 years as well as maxima in 1911, 1952 and 1985 years for x and y pole coordinates. The amplitudes of this oscillation for x and y coordinates have the same value, which means that the Chandler oscillation is approximately circular. The amplitude of the annual oscillation is approximately constant and it is smaller for y coordinate than for the x one, which means that this oscillation is elliptic.

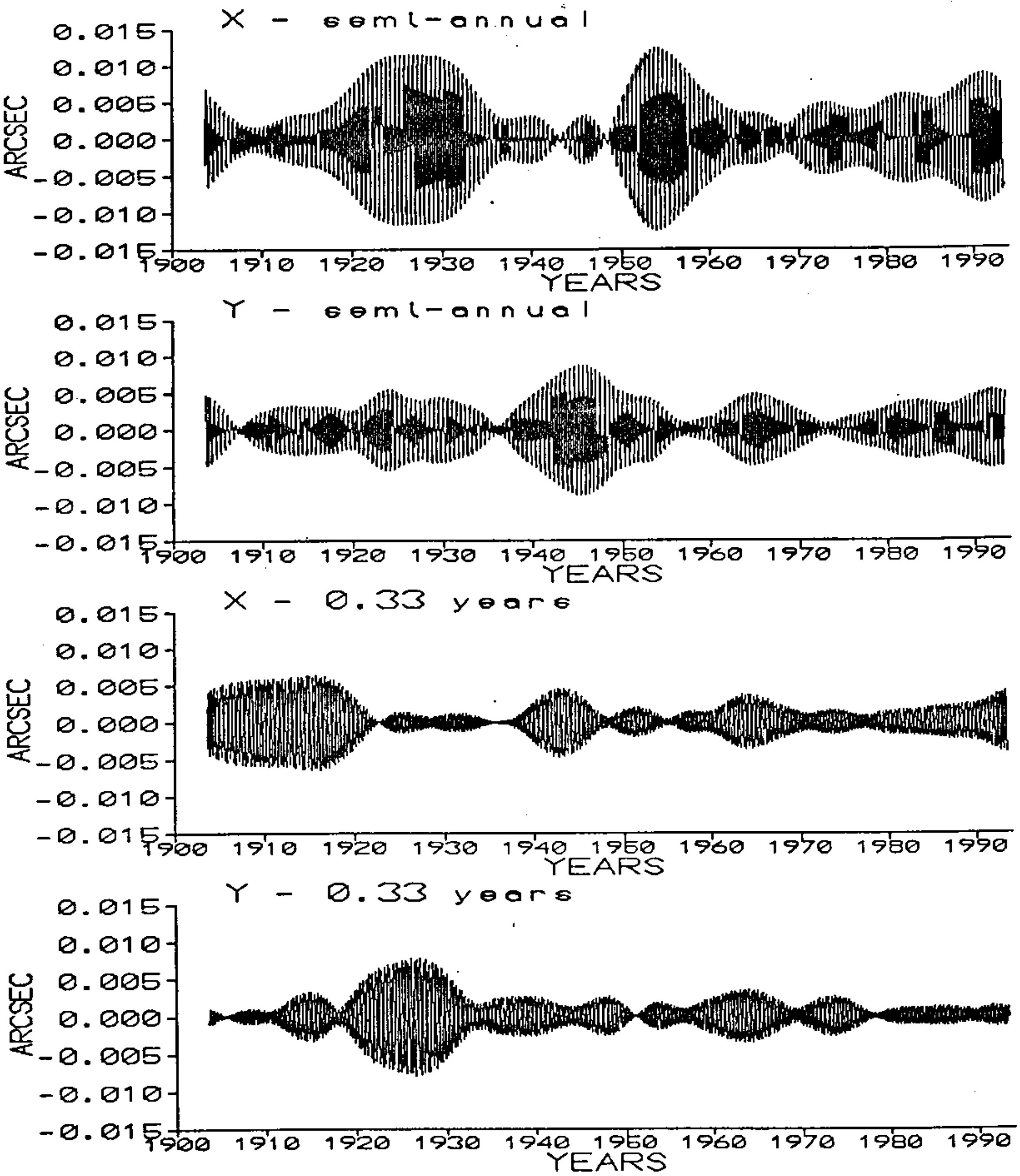

Fig. 9. The half a year and 0.33 year oscillations computed from the IERS93C01 pole coordinates data by the FT BPF with the cosine-bell transfer function (λ=0.008).

The semi-annual oscillation has variable amplitude and it is more than 10 times smaller than the annual oscillation amplitude (Fig. 9). Its amplitude is smaller for y coordinate than for x one, thus it is elliptic (Kołaczek et al. 1991). The amplitude maxima for these two coordinates occur in different years and their behaviour is not so regular before 1960 year especially for x pole coordinate. The amplitude modulation period after 1960 year is of the order of about 7 years for x coordinate and it seems to be greater for y pole coordinate.

It can be noticed that the amplitude of the 0.33-year oscillation is variable and it is smaller than the amplitude of the semi-annual oscillation (Fig. 9). Before 1960 year the amplitude maxima occur in different years for both coordinates. Beginning 1960 year the 0.33-year oscillation has similar amplitude modulation in both coordinates. Amplitude variations of this oscillations with periods of the order of about 6 years were found by the previous analyzes (Kołaczek et al. 1993; Kosek et al. 1995). Other oscillations in polar motion with periods ranging from 10 to 150 days have been already investigated by Kolaczek (1992, 1993), Kolaczek et al. (1993), Kosek (1987a,b, 1991), Kosek et al. (1994, 1995) and all of them have variable amplitudes too.

The differences between the IERS93C01 pole coordinates and the sum of the filtered Chandler, annual, semi-annual and 0.33-year oscillations are shown in Figure 10. There are also long period variations in pole coordinate data. These differences filtered by the Butterworth Low Pass Filter (LPF) with the 2-year cutoff period show these long period oscillations, from which the about 30-year Markowitz oscillation (Markowitz 1970) is the best known. The differences filtered by the Butterworth HPF with the 2-year cutoff period (Fig. 11) are very similar to those shown in the IERS Annual Report (IERS 1994).

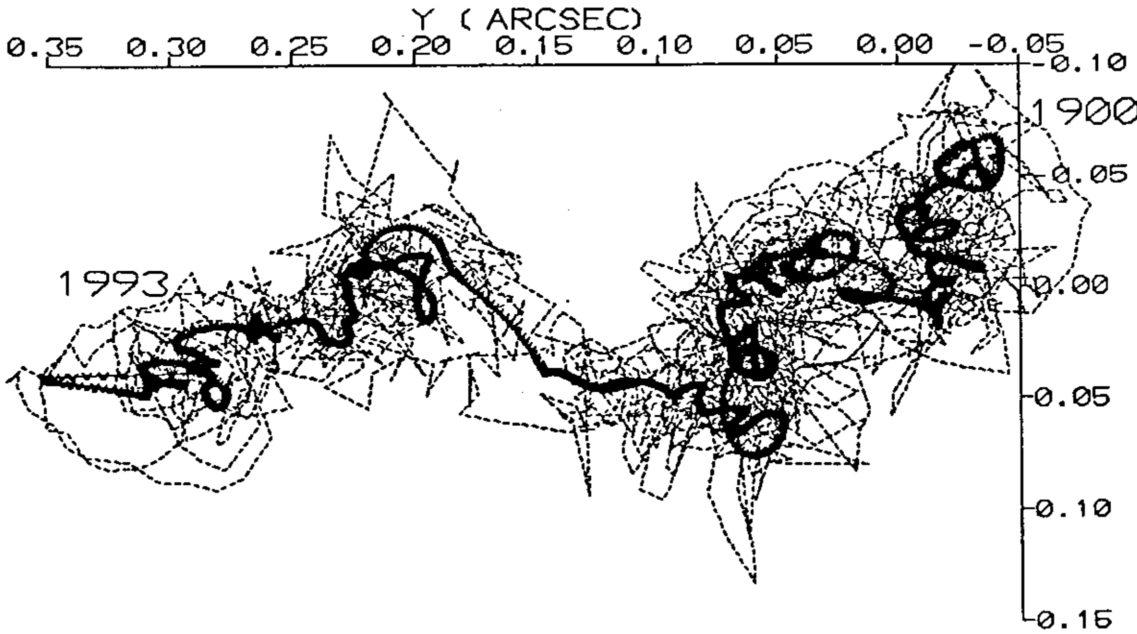

Fig. 10. The difference between the IERS93C01 pole coordinates data and the sum of the Chandler, annual, semi-annual and 0.33 year oscillations (dotted line) and filtered by the Butterworth LPF with the 2-year cutoff period (heavy line).

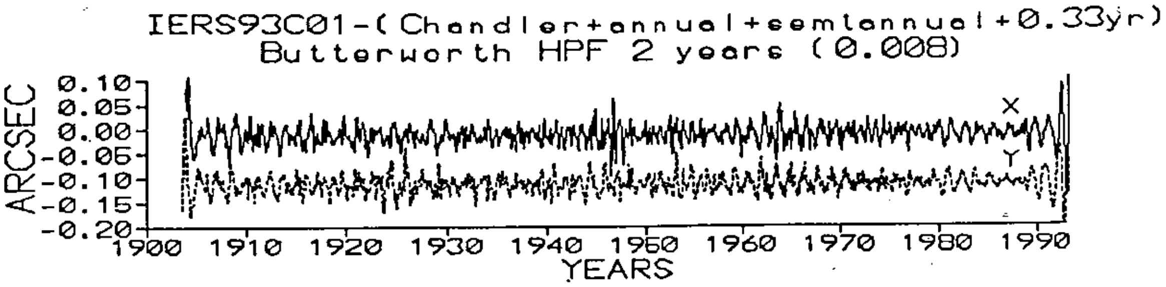

Fig. 11. The differences between the filtered by the Butterworth HPF (cutoff period = 2 years) IERS93C01 x (solid line) and y (dotted line) pole coordinates and the sum of the Chandler, annual, semi-annual and 0.33 year oscillations of x and y pole coordinates, respectively (Figs 8 and 9). The y coordinate graph has been shifted for clearer presentation.

CONCLUSIONS

Presented in this paper FT BPF applied for the complex-valued time series enables computation of the prograde and retrograde oscillations and the BPF spectra.

To reduce the computation time of the FT BPF the following improvements to the computation program were introduced:
- the FT of the analyzed series is computed only once and is used many times, to filter out oscillations with different periods or transfer function parameter values,
- the inverse FT is applied to the data, which are zero except for the frequency interval, where the transfer function is not zero,
- the FT was replaced by the FFT.

To reduce the filtration errors caused by the finite data length the trapezial, parabolic and cosine-bell transfer functions instead of the boxcar one were introduced.

Comparison of the Ormsby BPF and the FT BPF showed that the region of the optimum transfer function parameter values for the Ormsby filter is much narrower than for the FT one.

The FT BPF spectrum peak heights approximate the squared amplitudes of oscillations with accuracy depending on the frequency resolution of the filter. The increase of the transfer function transition band or pass band increases peak heights in the spectrum, but at the same time decreases their number.

In the case of different geophysical phenomena it is difficult to estimate the optimum parameter values of the BPF transfer function, because at the beginning we do not know the frequencies precisely. The FT BPF with application of the cosine-bell or parabolic transfer functions is recommended to analyze such geophysical time series.

The Chandler and annual oscillations can be filtered from polar motion data with a good frequency resolution by the FT BPF with the appropriate λ parameter value of the cosine-bell or parabolic transfer functions. Application of the parameter value $\lambda=0.008$ for filtering the Chandler, annual, semi-annual and 0.33-year oscillations from the considered pole coordinates data gave the standard deviation of 0.017 arcsec of the residuals obtained after removing these oscillations from the used data. The filtered Chandler and annual oscillations as well as the computed residuals are similar to those shown in the IERS (1994) Annual Report. The semi-annual and 0.33-year oscillations filtered by the FT BPF have variable amplitudes as it has been shown previously for the shorter time span of data.

Acknowledgements. This paper was supported by the grant of U.S.-Poland Maria Sklodowska Joint Found II Project No PAN/SMI-92-97. The authors thank Prof. B. Kołaczek for her valuable comments that helped to improve this paper.

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Received: March 29, 1995

Revised: June 5, 1995, by J. Vondrák

Accepted: June 6, 1995