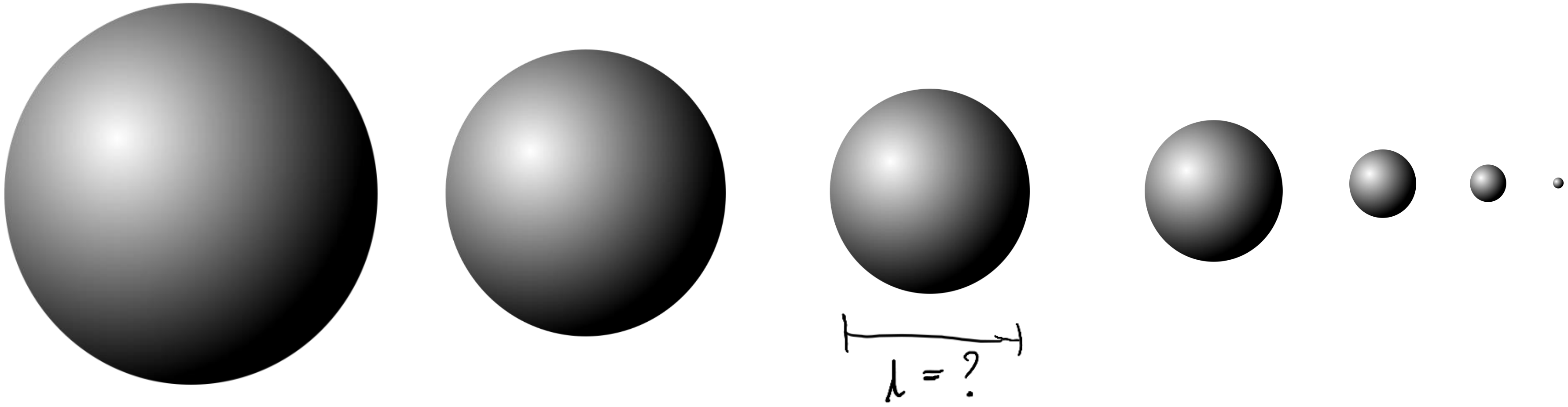
The Planck length: two electrons and how to make a black hole

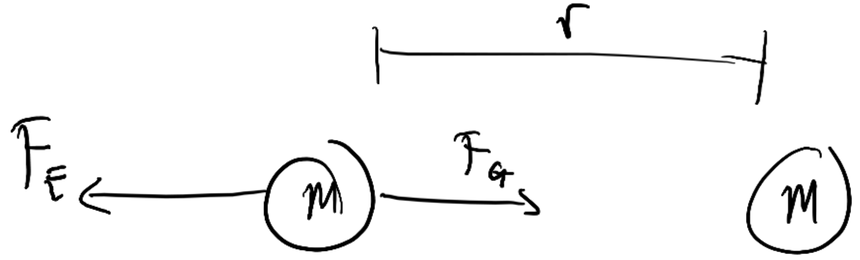
Physics

The question I'll be answering:

- Is there a fundamental limit to the size an object or a region of space can be? If so, how do we calculate it?



A pair of electrons.

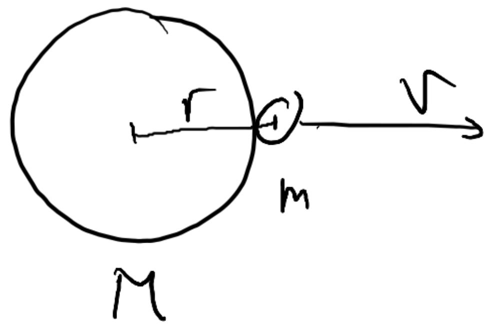


- Gravitational force of attraction
  - $F_G = \frac{Gm_1m_2}{r^2} = \frac{Gm^2}{r^2}$
- Electrostatic force of repulsion
  - $F_E = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2} = \frac{q^2}{4\pi\varepsilon_0 r^2}$
- Which is larger?
  - $\frac{F_E}{F_G} = \frac{\frac{q^2}{4\pi\varepsilon_0 r^2}}{\frac{Gm^2}{r^2}} = \frac{q^2}{4\pi\varepsilon_0 Gm^2} = 4.16 \times 10^{42}$. Independent of separation?
  - $= 4{,}160{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000$ times stronger…

Not that simple – Quantum mechanics.

- Uncertainty Principle
  - $\Delta p \geq \frac{\hbar}{2\Delta x} \Rightarrow \Delta p \geq \frac{\hbar}{2r} \Rightarrow \Delta E \geq \frac{\hbar c}{2r}$ and $\Delta E = mc^2$
  - $r \leq \frac{\hbar}{mc}$ therefore $r \approx 3.86 \times 10^{-13}$
- Generally, $r \downarrow,\ \Delta E \uparrow,\ therefore\ m \uparrow and\ F_G \uparrow$
- When does $F_G = F_E$? At $r = 1.86 \times 10^{-34}$ (can't be bothered to do maths no time)

BLACK HOLES



- Work done to move mass m from the surface of M to infinity
  - $F_G = \frac{GMm}{r^2}$, note F changes with distance [calculus…].
  - To find energy required, use $W = Fs$
  - $dW = Fdr = \frac{GMm}{r^2}dr$
  - $W = \int_r^\infty \frac{GMm}{r^2}dr = GMm\left[-\frac{1}{r}\right]\Big|_r^\infty = \frac{GMm}{r}$
  - Work done is equivalent to the gain in GPE – what velocity is required for m to escape to infinity?
  - $\frac{GMm}{r} = \frac{1}{2}mv^2$
  - $v = \sqrt{\frac{2GM}{r}}$
- What if escape velocity equals the speed of light
  - $c = \sqrt{\frac{2GM}{r}} \Rightarrow r_s = \frac{2GM}{c^2}$ - the Schwarzschild radius

Things we know and a small black hole

- From QM and SR:
  - $r \sim \frac{\hbar}{mc}$
- From.. Gravity:
  - $r \sim \frac{Gm}{c^2}$
- Eliminate m and equate:
  - $r = \ell_P \sim \sqrt{\frac{G\hbar}{c^3}} \sim 1.616 \times 10^{-35} m$ (small) – or you could do this simply by unit consistency, but that would be no fun.
  - $m \sim \sqrt{\frac{\hbar c}{G}} \sim 2.21 \times 10^{-8} kg$